Space-Time Cut-off Rate for the Flat Rayleigh Fading Channel

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Abstract — **The computational cut-off rate is studied for the complex Rayleigh flat fading spatio-temporal channel under a peak power constraint. Any optimal finite constellation of signals must admit an equalizer distribution which equalizes the conditional decoding error probabilities. For small constellations the set of equiprobable mutually-orthogonal unitary matrices attains cut-off rate. For low SNR these matrices are rank one and a single transmit antenna is as good as multiple antennas.**

I. Introduction

There are M transmitter antennas and N receiver antennas and the MN channel fading coefficients are i.i.d. constant complex Gaussian over the coherent fade interval of length T time periods [2]. While the SNR η is known the fading coefficients are unknown to both transmitter and receiver. In a frequency hop system each coherent fade interval corresponds to a different frequency band. A baseband transmitted signal S_i is a $T \times M$ matrix having complex valued entries. When the S_i's are drawn from a constellation of K possible signals the signalling rate is K/T symbols/sec/hz. The computational cut-off rate specifies the maximum practical rate that can be supported by the channel and is often simpler to calculate than channel capacity. For more details on the following results see [1].

II. Cutoff Rate Representations

For any K-dimensional constellation $\{S_1, \ldots, S_K\}$ define the $K \times K$ *dissimilarity matrix* $E_K = \left(\left(e^{-ND(S_i\|S_j)}\right)\right)_{i,j=1}^K$ where

$$D(S_i\|S_j) \quad \stackrel{\text{def}}{=} \frac{1}{2}\log \frac{\left|I_T+\frac{\eta}{2}(S_iS_i^H+S_jS_j^H)\right|^2}{\left|I_T+\eta S_iS_i^H\right|\left|I_T+\eta S_jS_j^H\right|}.$$

is a pairwise distance function between signal matrices in the constellation.

Proposition 1 *[1] Let $\tilde{R}_o(K)$ denote the peak power constrained cut-off rate restricted to K-dimensional constellations. Then*

$$\tilde{R}_o(K) \quad = \quad \log\left\{\max_{0<k\le K}\ \max_{\{S_i\}_{i=1}^k \in \tilde{\mathcal{S}}_{\text{peak}}^k} \underline{1}_k^T E_k^{-1} \underline{1}_k\right\}.$$

where $\tilde{\mathcal{S}}_{\text{peak}}^K$ is the set of K-dimensional peak constrained constellations for which there exists an "equalizer probability vector" $\underline{P}_K$ satisfying $E_K\underline{P}_K = c\underline{1}_k$ for some $c > 0$.

III. Optimality of Unitary Orthogonal Matrices

For given η, T and M define the integer M_o

$$M_o = \text{argmax}_{m\in\{1,\ldots,M\}}\left\{m\log\frac{(1+\eta TM/(2m))^2}{1+\eta TM/m}\right\}. \quad (1)$$

and

$$D_{\max} \stackrel{\text{def}}{=} \max_{S_1,S_2\in\mathcal{S}_{\text{peak}}^K} D(S_1\|S_2) = M_o\log\frac{(1+\eta TM/(2M_o))^2}{1+\eta TM/M_o}.$$

$D_{\max}$ is the maximum value of the minimum distance achievable by any constellation of dimension $K \le T/M_o$.

Proposition 2 *[1] Let $2M \le T$ and let M_o be as defined in (1). Suppose that $M_o \le \min\{M, T/K\}$. Then*

$$\tilde{R}_o(K) = \log\left(\frac{K}{1+(K-1)e^{-ND_{\max}}}\right)$$

and $D_{\max}$ is given by (2). Furthermore, the optimal constellation attaining $\tilde{R}_o(K)$ is the set of K equiprobable rank M_o mutually orthogonal unitary matrices: $S_i^H S_i = I_{M_o}$ and $S_i^H S_j = 0$, $i \ne j$.

The rank M_o of the matrices in the optimal constellation increases from 1 to M as the SNR parameter ηTM increases from 0 to ∞. If SNR is sufficiently large, e.g. for $M = 6$ and $T \ge 12$ if $\eta TM \ge 17$, then $M_o = M$ and the optimal signal matrices utilize all M transmit antennas. On the other hand for small SNR, e.g. $\eta TM < 4$, then $M_o = 1$ and the optimal signal matrices apply all available transmit power to a single antenna element over each coherent fade interval.

References

[1] A. O. Hero and T. L. Marzetta, "On computational cut-off rate for space time coding," Technical memo, Bell Laboratories, Lucent Technologies, Murray Hill, NJ, 2000.

[2] T. L. Marzetta and B. M. Hochwald, "Capacity of a mobile multiple-antenna communication link in Rayleigh fading," *IEEE Trans. on Inform. Theory*, vol. IT-45, pp. 139–158, Jan. 1999.

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