UNICAST-BASED INFERENCE OF NETWORK LINK DELAY DISTRIBUTIONS USING MIXED FINITE MIXTURE MODELS

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ABSTRACT

As telecommunication networks grow larger and more complex, it is important to monitor internal link characteristics for operation, monitoring, and diagnosis purposes. Since router link monitoring is not practical due to high communication overhead, there has been considerable interest in monitoring from edge (end-to-end) observations. This paper focuses on the estimation of internal link delay distributions from edge measurements. Discrete and continuous delay models are introduced and we propose a new mixed finite mixture model for link delay probability density functions(p.d.f.). When collecting end-to-end unicast packet delays from edge nodes, we are able to estimate internal link delay distributions using the EM algorithm. Simulation results are given to illustrate our method.

1. INTRODUCTION

Network tomography is a new area concerned with inference of internal statistics of a network based on end-to-end measurements at edge nodes. Tomography provides a vital tool for characterizing network behaviors without cooperating internal nodes. This is especially useful when either internal parameters are inaccessible or direct measurement of data traffic statistics are not supported by internal switches and routers [1, 2, 3, 5].

Link-level packet delays are important particulars for optimizing network performance and evaluating quality of service(QoS). While transmission and propagation delays are usually considered as constant factors, the queueing delays contribute random packet delays. The inference of link delay distributions from end-to-end measurements was first proposed by Presti et al[4]. They applied a discrete (binned) delay model and derived a sample-average based algorithm using multicast probes. Coates and Nowak[5] suggested an alternative unicast probing scheme using packet pairs and developed a maximum likelihood(ML) inference technique based on the EM algorithm. A unicast approach for estimating cumulant generating functions of internal link delays was introduced in [6].

In this paper, our focus is on a mixed discrete and continuous model for unicast end-to-end packet delays for inference of internal link delay distributions. First we review identifiability conditions for discrete and continuous delay models. Under a mixed finite mixture model for packet link delay p.d.f.'s, we show that it is possible to reconstruct consistent estimates of the link delay p.d.f.'s using single packet unicast. An EM-based algorithm is used to approximate ML estimates of a Gaussian mixture with a Dirac delta at zero delay. Computer simulations are reported which demonstrate the promise of our approach.

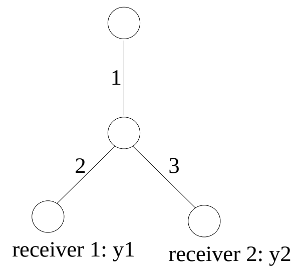

Fig. 1. Two-leaf network.

2. PROBLEM FORMULATION

As in [5], we consider the network topology as a logical tree $T = (V, E)$, where V is the set of nodes and E is the set of links. Assume the root node has only one child node. Assume a total of L links and number them from $1, \ldots, L$. Associate the only link connecting the root node as link 1. Each node in the tree has two or more child nodes, except the root and leaf nodes.

Time stamped unicast packets are sent from the root node to one of the leaf nodes. The leaf nodes, considered as packet receivers, collect end-to-end delays for those packets. The purpose is to identify the packet delay distribution for each individual internal link from the edge observations at the packet receivers. Assume there are R receivers and R possible routes from root to leaf nodes in the network. Note that all the packet routes include link 1.

Let X_l, $l = 1, \ldots, L$ be the packet delay at link l, and Y_i, $i = 1, \ldots, R$ be the end-to-end delay of a packet along the path destinated to receiver i. We make the following independence and stationarity assumptions: (A1) *Spatial Independence*: Packet delays at different links are statistically independent, i.e., X_i and X_j are independent for $i \neq j$. (A2) *Temporal Independence and Stationarity* : For a given link, the delays encountered by different packets at that link are statistically independent and identically distributed (i.i.d.).

3. DELAY MODELS

3.1. Discrete Delay Model

In the discrete delay model, a universal bin size q is used to discretize link delays at all links. Assume that discrete packet delays on each link fall in the set $\{0, q, 2q, \cdots, Dq\}$, where D is a positive integer and ∞ is possibly included to denote lost packets or large delays which are out of range. Probability mass function(p.m.f.) $P_l = \{p_{l,d} : d = 0, \ldots, D\}$ is assigned to delays at link l, where $p_{l,d} = P(X_l = dq)$ is unknown and is to be estimated. For a path containing i links, the end-to-end packet delay can have values from 0 to $(iD)q$.

The identifiability of the p.m.f.'s from edge measurements can be easily studied for the simple case of a two-leaf tree network, as shown in Fig.1, with only two bins ($D = 1$) in each link delay distribution. We say that the p.m.f.'s are not identifiable if the Fisher information matrix is singular.

Lemma 1. *The delay p.m.f. with two bins at each link is uniquely identifiable from end-to-end packet delays, except when the delay p.m.f.'s at all links are identical.*

proof: Let the true delay p.m.f.'s for link 1 to 3 be $P_1 = \{p_1, 1 - p_1\}$, $P_2 = \{p_2, 1 - p_2\}$, and $P_3 = \{p_3, 1 - p_3\}$, respectively. Define the parameter vector $\theta = \{p_1, p_2, p_3\}$. Let **y1**, **y2** be the sets of end-to-end packet delays observed at receiver 1 and 2, respectively. Assume both receivers have n observations. The Fisher information matrix $\mathbf{A} = E[\nabla_\theta \log P(\mathbf{y1}, \mathbf{y2}; \theta)]$ is shown in next page, where $Q_1 = p_1(1 - p_2) + p_2(1 - p_1)$, and $Q_2 = p_1(1 - p_3) + p_3(1 - p_1)$. **A** becomes singular when $p_1 = p_2 = p_3$.(Q.E.D.)

The identifiability conditions of delay p.m.f.'s with more bins can also be derived from corresponding Fisher information matrix. For a network topology containing more than two leaf nodes, every pair of packet routes can be viewed as a two-leaf tree and conclusions of Lemma 1 can be accordingly extended.

Discrete delay model has two main drawbacks. First, the proper bin size should be known before application of such algorithms, which is impossible when statistics of internal links are not available. Second, a universal bin size may not be suitable in practice due to large variation of packet delay ranges among different links. Although in [4] it was proposed to adopt different bin sizes for different links, those bin sizes still need to be chosen in advance.

3.2. Continuous Delay Model

To avoid the use of binning, an alternative would be to assume link delays are continuous random variables. Although queueing delay distributions have been derived for single link queues such as M/M/1, there are no known forms of probability distributions suitable for end-to-end queueing delays in most of today's telecommunication networks. Furthermore, for the internet, the simple M/M/1 queue is an inadequate model [8]. An alternative is the class of Gaussian mixture densities, which describes arbitrary shapes of the link delay distributions [9]. Let $f_l(x)$ be the link delay p.d.f. at link l for $l = 1, \ldots, L$. The Gaussian mixture model

$$f_l(x) = \sum_{m=1}^{k_l} \alpha_{l,m} \phi(x; \theta_{l,m}), \tag{1}$$

where k_l denotes the number of mixture components, $\alpha_{l,m}$, $m = 1, \ldots, k_l$, denotes the mixing probability for the mth component with $0 < \alpha_{l,m} < 1$, $\sum_{m=1}^{k_l} \alpha_{l,m} = 1$, and $\phi(x; \theta_{l,m})$ is the Gaussian density function given by mean and variance $\theta_{l,m} = \{\mu_{l,m}, \sigma_{l,m}^2\}$.

However, the use of a pure continuous Gaussian mixture density function causes a serious identifiability problem. Consider the simple two-leaf tree of Fig.1 as before and assume that $k_1, k_2, k_3 = 1$. The joint p.d.f. of (Y_1, Y_2) is $f(y_1, y_2) = \phi(y_1; \{\mu_1 + \mu_2, \sigma_1^2 + \sigma_2^2\})\phi(y_2; \{\mu_1 + \mu_3, \sigma_1^2 + \sigma_3^2\})$. Even if the p.d.f's were exactly known, their parameters give only 4 equations for 6 unknowns ($\mu_1, \mu_2, \mu_3, \sigma_1, \sigma_2, \sigma_3$) and hence no unique solution exists.

One can also consider the packet pair/stripe schemes suggested in [5] and [7], in which a pair or a stripe of unicast packets with distinct destinations are sent back-to-back from the root node. These packets experience the same (or highly correlated) delays on shared links among their paths. As shown in [10], this kind of schemes allows identification of variances or higher order moments of internal link delays with branching ratio larger than 2 in the case of packet stripes. However, packet stripes cannot identify the link delay means.

4. MIXED FINITE MIXTURE MODEL AND EM ALGORITHM

4.1. Mixed Finite Mixture Model

In analysis of queueing systems, the utilization factor ρ is an important parameter in describing system behavior. ρ denotes the probability that the system is busy in serving customers. For a stable system, $0 \leq \rho < 1$ (e.g. [11]). As $\rho < 1$ there is a nonzero probability for a packet passing through a link without any appreciable queueing delay. This motivates us to put a delta component at (or near) 0 in the link

$$\mathbf{A} = n \begin{pmatrix} \frac{p_2+p_3}{p_1} + \frac{2-p_2-p_3}{1-p_1} + \frac{(1-2p_2)^2}{Q_1} + \frac{(1-2p_3)^2}{Q_2} & \frac{1}{Q_1} & \frac{1}{Q_2} \\ \frac{1}{Q_1} & \frac{p_1}{p_2} + \frac{1-p_1}{1-p_2} + \frac{(1-2p_1)^2}{Q_1} & 0 \\ \frac{1}{Q_2} & 0 & \frac{p_1}{p_3} + \frac{1-p_1}{1-p_3} + \frac{(1-2p_1)^2}{Q_2} \end{pmatrix}$$

delay mixture model, in addition to the continuous Gaussian mixture components, making link delay a mixed discrete/continuous random variable. The discrete part will be assigned probability mass $\alpha_0 = 1 - \rho$. Hence, similar to (1), we have the modified model

$$f_l(x) = \alpha_{l,0}\delta(x) + \sum_{m=1}^{k_l} \alpha_{l,m}\phi(x; \theta_{l,m}) \qquad (2)$$

with all other parameters defined as in (1), except that in addition $\sum_{m=0}^{k_l} \alpha_{l,m} = 1$, $\alpha_{l,m} \geq 0$. This discrete component not only makes the delay distribution more precisely model the behavior of a link queue, but as shown below also buys us identifiability of the link delay distribution parameters.

For a single path, the end-to-end packet delays are also mixed random variables with continuous Gaussian mixtures. Due to the discrete components in link delay p.d.f.'s, the end-to-end delay distribution includes Gaussian components with exact the same parameters $\theta_{l,m}$ as those in delay p.d.f's for each link in the path. Hence, the identification problem is in fact a problem to uniquely identify such Gaussian components and assign them to individual link p.d.f.'s. Obviously, if the network has only a single path, there is no way to assign the components to specific links. For a logical tree network topology, if the packets are sent through all the R possible routes, a simple sufficient condition for identifiability is the following:

The delay distribution defined in (2) is identifiable from end-to-end measurements if (1) $\alpha_{l,0} > 0$ for all l. (2) All the Gaussian components in link delay distributions have distinct means and variances.

4.2. EM Algorithm

Assume that we have prior knowledge of all the link mixture orders $\{k_l\}$. Let N_i be the number of packets sent from root to receiver i and M_i be the set of links intersecting that path, and let Θ denote the set of unknown parameters. Define $x_l^{(i,n)}$ as the delay at link l encountered by the nth packet sent to receiver i, and the binary vector $\mathbf{z}_l^{(i,n)} = \{z_{l,0}^{(i,n)}, \ldots, z_{l,k_l}^{(i,n)}\}$ as the component indicator vector, where $z_{l,m}^{(i,n)} = 1$ if $x_l^{(i,n)}$ is generated by the mth mixture component and $z_{l,m}^{(i,n)} = 0$ otherwise. Let $\mathbf{x} = \{x_l^{(i,n)}\}$ and $\mathbf{z} = \{\mathbf{z}_l^{(i,n)}\}$ for all l, i, n. $\{\mathbf{x}, \mathbf{z}\}$ is *unobserved data*. Define $y^{(i,n)}$ as the end-to-end delay of the nth packet received by receiver i. The set $\mathbf{y} = \{y^{(i,n)}\}$ is called *incomplete data*, and the set $\{\mathbf{x}, \mathbf{z}, \mathbf{y}\}$ is the *complete data*. As shown in [5], the likelihood of complete data will be proportional to that of the unobserved data, which is

$$\log L(\mathbf{x}, \mathbf{z}|\Theta) = \sum_{l=1}^{L} \sum_{i:l\in M_i} \sum_{n=1}^{N_i} \Big\{ z_{l,0}^{(i,n)} \log \alpha_{l,0} + \sum_{m=1}^{k_l} z_{l,m}^{(i,n)} \left(\log \alpha_{l,m} + \log \phi(x_l^{(i,n)}; \theta_{l,m}) \right) \Big\}. \qquad (3)$$

The EM algorithm is similar to that for a single Gaussian mixture model[12], which is the following:

E-step

Let

$$\begin{aligned} \omega_{l,m}^{(i,n)} &= E\left[z_{l,m}^{(i,n)} | y^{(i,n)}; \hat{\Theta} \right] \\ Q_{l,m}^{(i,n)}(\theta_{l,m}) &= E\left[z_{l,m}^{(i,n)} \log \phi(x_l^{(i,n)}; \theta_{l,m}) | y^{(i,n)}; \hat{\Theta} \right]. \end{aligned}$$

Then

$$E\left[\log L(\mathbf{x}, \mathbf{z}|\Theta) | \mathbf{y}; \hat{\Theta} \right] = \sum_{l=1}^{L} \sum_{i:l\in M_i} \sum_{n=1}^{N_i} \left\{ \sum_{m=0}^{k_l} \omega_{l,m}^{(i,n)} \log \alpha_{l,m} + \sum_{m=1}^{k_l} Q_{l,m}^{(i,n)}(\theta_{l,m}) \right\}$$

M-step

$$\begin{aligned} \alpha_{l,m}^* &= \frac{\sum_{i:l\in M_i} \sum_{n=1}^{N_i} \omega_{l,m}^{(i,n)}}{\sum_{i:l\in M_i} N_i} \\ \theta_{l,m}^* &= \mathbf{argmax}_\theta \sum_{i:l\in M_i} \sum_{n=1}^{N_i} Q_{l,m}^{(i,n)}(\theta) \end{aligned}$$

5. EXPERIMENTAL RESULTS

We simulated of a network with topology shown in Fig.2. Throughout we assume that the number of components k_l's are known. We generated 15000 i.i.d. end-to-end delays by MATLAB for each of the four packet routes and applied EM algorithm to find ML estimates of the Gaussian components and the Dirac delta component at zero. Tab.1 lists the number of Gaussian mixture components for each link and the true/estimated probability $\alpha_{l,0}$ of link delay being 0. Fig.3 compares the estimated Gaussian mixture components to the ideal ones. Both reveal that accurate estimates are obtained from our computer simulation.

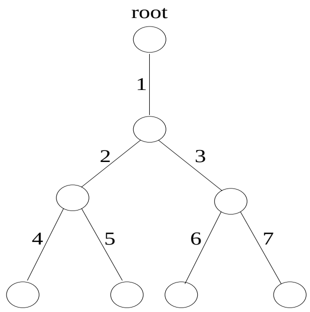

Fig. 2. Network used in simulation.

Table 1. List of numbers of Gaussian mixture components and true/estimated $\alpha_{l,0}$ for simulation in Section 5

Link	1	2	3	4	5	6	7
k_l	3	2	2	2	2	2	2
$\alpha_{l,0}$	0.25	0.3	0.1	0.2	0.15	0.3	0.2
$\hat{\alpha}_{l,0}$	0.253	0.304	0.099	0.199	0.152	0.313	0.201

6. CONCLUSION AND FUTURE WORK

This paper focuses on the estimation of internal link delay distributions from end-to-end unicast packet delay measurements where there is a positive propability of zero queueing delay. We discussed two possible delay models, discrete and continuous, and proposed a new mixed finite mixture model. This new model can describe arbitrary shapes of delay p.d.f.'s and can allow identifiability of link delay p.d.f.'s from edge observations. In a future paper we will apply unsupervised techniques to estimate mixture order k_l's. This technique will also be extended to adaptively capture changes in network statistics.

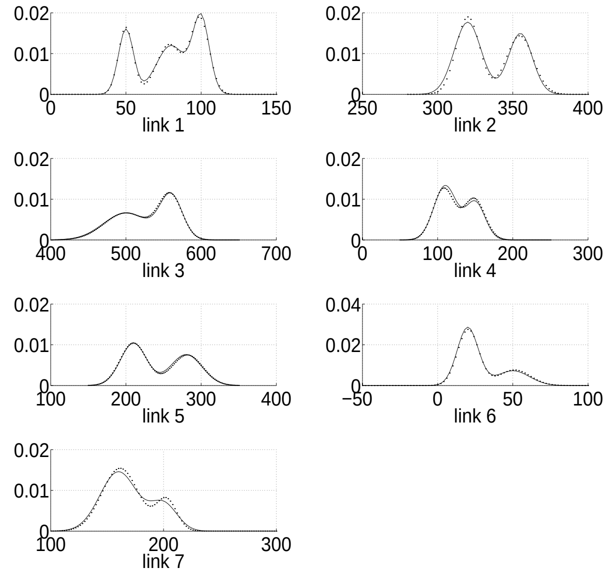

Fig. 3. True(solid) and estimated(dotted) Gaussian mixture components for simulation discussed in Section 5.

7. REFERENCES

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