

Energy Management Strategy for a Parallel Hybrid Electric Truck

Chan-Chiao Lin¹, Jun-Mo Kang², J.W. Grizzle², and Huei Peng¹

¹Dept. of Mechanical Engineering, University of Michigan, MI 48109-2125
chancl@umich.edu, hpeng@umich.edu

²Dept. of Electrical Engineering and Computer Science, University of Michigan, MI 48109-2122
junmo@eecs.umich.edu, grizzle@umich.edu

Abstract

Due to the complex nature of hybrid electric vehicles, control strategies based on engineering intuition frequently fail to achieve satisfactory overall system efficiency. This paper presents a procedure for improving the energy management strategy for a parallel hybrid electric truck on the basis of dynamic optimization over a given drive cycle. Dynamic Programming techniques are utilized to determine the optimal control actions for a hybrid powertrain in order to minimize fuel consumption. By carefully analyzing the resulting optimal policy, new rules can be ascertained to improve the basic control strategy. The resulting new control strategy is shown to achieve better fuel economy through simulations on a detailed vehicle model.

1. Introduction

With the growing demand from the world community to reduce the emission of carbon dioxide, and after a decade of intense research, hybrid electric vehicles (HEV) suddenly appear more viable and necessary than ever before. These vehicles either reduce or eliminate the reliance on fossil fuels. Owing to their dual on-board power sources and regenerative braking, HEVs offer unprecedented possibilities to pursue higher fuel economy, particularly if a parallel HEV configuration is employed. To realize fuel economy benefits, the power management function of these advanced vehicles must be carefully designed. By power management, we mean the development of a higher-level control algorithm that determines the total amount of energy to be generated, and its split between the two power sources.

Most of the control strategies developed for parallel HEVs can be classified into three categories. The first type employs intelligent control techniques such as rules/fuzzy logic/NN for estimation and control algorithm development ([1],[2]). The second approach is based on static optimization methods. Generally speaking, electric energy is translated into an equivalent amount of fuel to calculate the energy cost ([3],[4]). The optimization scheme then figures out proper energy and/or power split between the two energy sources under steady-state operation. Because of its relatively simple point-wise optimization nature, it is possible to extend such optimization schemes to solve the simultaneous fuel economy and emission optimization problem [5]. The basic idea of the third type of HEV control algorithm takes into account the dynamic nature of the system when performing the optimization ([6],[7]). Furthermore, the optimization is with respect to a time horizon, rather than for a fixed point in time. In general, a power split algorithm resulting from dynamic optimization will be more accurate under transient conditions.

In this paper, we apply dynamic programming to solve the minimum fuel optimal control problem for a hybrid electric truck. A dynamic optimal solution to the energy management problem

over a driving cycle is developed. The resulting feedback laws from the dynamic programming algorithms are not implementable due to their preview nature and heavy computational requirement. They are, on the other hand, a good design tool and a benchmark against which a basic control strategy can be compared and improved. We then study the behavior of the dynamic programming solution carefully, and extract simple, implementable rules. These rules are then used to augment a simple, intuition-based control algorithm. It was found that the performance of the intuition (rule) based algorithm can be enhanced significantly through this design procedure.

The paper is organized as follows: In Section 2, the configuration of the hybrid electric truck is briefly described, followed by the description of the preliminary rule-based control strategy. Next, dynamic programming is introduced and the optimization result for minimum fuel consumption is given in Section 3. Section 4 discusses how to design a better rule-based strategy using the results of the dynamic programming algorithm. Conclusions are presented in Section 5.

2. Hybrid-Electric Vehicle System (HE-VESIM)

2.1 System Configuration

The baseline vehicle studied here is the International 4700 series truck, a 4X2 Class VI diesel truck produced by Navistar. The original diesel engine was downsized from the V8 (7.3L) to a V6 (5.5L) and a 49 KW electric motor has been selected as the second power source. The vehicle system in this study is configured as a parallel hybrid with the electric motor positioned after the transmission. A schematic of the vehicle and the propulsion system is given in Figure 1. The engine is connected to the torque converter (TC), whose output shaft is then coupled to the transmission (Trns). The transmission and the electric motor can be linked to the propeller shaft (PS), differential (D) and two driveshafts (DS), coupling the differential with the driven wheels. Basic vehicle specifications are given in the Appendix.

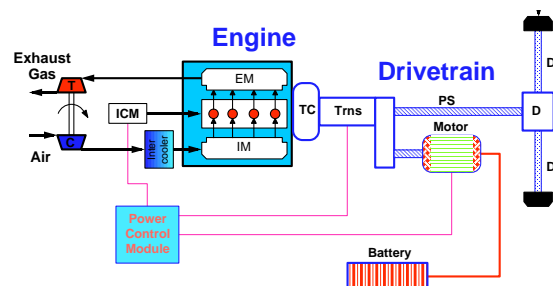


Figure 1: Schematic diagram of the hybrid electric truck

Our Hybrid Vehicle-Engine Simulation (HE-VESIM) model is based on the high-fidelity conventional vehicle simulator VESIM previously developed at the University of Michigan [8]. VESIM has been validated against measurements for a Class VI truck, and proven to be a very versatile tool for mobility, fuel economy and drivability studies. To construct a hybrid-vehicle simulator, some of the main modules required modifications, e.g. reduction of the engine size/power, and the integration of electric component models into the system. The model is implemented in the MATLAB/SIMULINK software environment, as presented in Figure 2. Since the detailed vehicle/chassis models have been presented in ([8],[9]), they are not reviewed here.

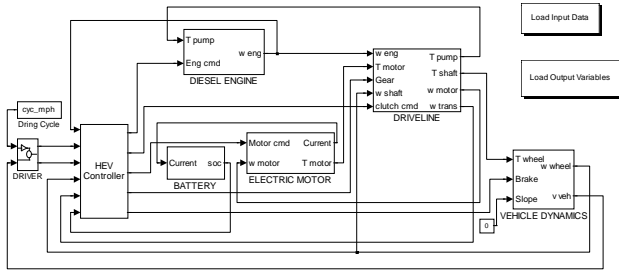


Figure 2: Hybrid-electric vehicle simulation in SIMULINK

2.2 Rule Based Control Strategy

The final HEV controller that will be implemented will be rule-based. The energy management strategy will only use current and past vehicle states and driver commands to calculate a proper (hopefully, close to optimal) control signal. The rule-based energy management strategy used as a starting point here was developed on the basis of engineering intuition and simple analysis of component efficiency tables/charts [9,10]. The design process starts from interpreting the driver pedal signal as a power request, P_{req} . According to the power request, the operation of this controller is divided into three control modes: Braking Control, Power Split Control or Recharging Control. If the power request is negative, Braking Control will be applied to decelerate the vehicle. If the power request is positive, either Power Split Control or Recharging Control will be applied according to a charge-sustaining policy. The charge-sustaining strategy assures that the battery state of charge (SOC) stays within preset lower and upper bounds. A 55-60% SOC range is chosen for efficient battery operation as well as to prevent battery depletion or damage in an extreme situation. In a normal propulsive driving condition, the Power Split Control determines the power flow in the hybrid powertrain. Whenever the SOC drops below the lower limit (55%), the controller will switch to the Recharging Control mode until the SOC reaches the upper limit (60%), and then Power Split Control will resume. The basic logic of each control mode is briefly described in the following.

Power Split Control: Based on the engine efficiency map shown in Figure 3, a pre-selected “engine on” power line, $P_{e_{on}}$, and “motor assist” power line, $P_{m_{a}}$, are chosen to avoid engine operation in inefficient areas. If the total power request is less than the “engine on” power level, the electric motor will supply the requested power. Beyond $P_{e_{on}}$, the engine replaces the motor to provide the total power request. Once the power request

exceeds what the engine can efficiently generate, $P_{m_{a}}$, the motor is activated to supply the additional power ($P_{req} - P_{m_{a}}$).

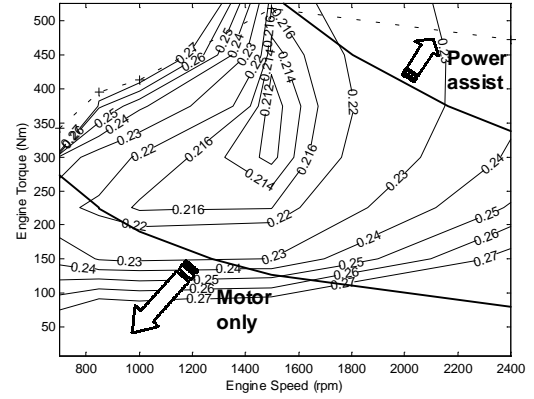


Figure 3: Power Split Control strategy

Recharging Control: The engine is the prime mover in this mode. In addition to powering the vehicle, the engine has to provide additional power for charging the battery. A pre-selected recharge power level, P_{ch} , is added to the driver’s power request, and the motor power command is forced to become negative in order to recharge the battery ($P_m = -P_{ch}$). One exception is that when the total power request is less than the “engine on” power level, $P_{e_{on}}$, the motor alone will still propel the vehicle to prevent the engine from operating in this inefficient region. The other exception is that when total power request is greater than the maximum engine power, the motor power will become positive to assist the engine.

Braking Control: The regenerative braking is activated to absorb the braking power. However, when the braking power request exceeds the regenerative braking capacity $P_{m_{min}}$, the hydraulic brakes will be activated to assist in vehicle deceleration ($P_b = P_{req} - P_{m_{min}}$).

The hybrid electric truck with this preliminary rule-based controller was tested through simulation over the EPA Urban Dynamometer Driving Schedule for Heavy-Duty Vehicles (UDDSHDV) in order to evaluate the fuel economy. Table 1 compares the resulting fuel economy with that of the conventional diesel engine truck.

Table 1: Fuel economy comparison: conventional, and rule-based (RB)

	RB	Conventional
MPG	12.56	10.63

3. Dynamic Optimization Problem

Contrary to the rule-based algorithm, the dynamic optimization approach usually relies on a model to compute the best control strategy. The model can be either analytical or numerical; in other words, it can work with numerical black boxes like HE-VESIM. For a given driving cycle, the optimal operating strategy to deliver

the best fuel economy can be obtained by solving a dynamic optimization problem. A numerical dynamic programming approach will be applied to solve this finite horizon optimization problem.

3.1 Problem Formulation

In the discrete-time format, a model of the hybrid electric vehicle can be expressed as:

$$x(k+1) = f(x(k), u(k)) \quad (1)$$

where $u(k)$ is the vector of control variables such as fuel injection rate to the engine (kg/cycle), desired output torque from the motor (Nm), and gear shift command to the transmission, and $x(k)$ is the vector of state variables of the system. The sampling time has been selected to be one second.

The goal of the optimization scheme is to find the optimal control input, $u(k)$, which minimizes the total fuel consumption over a driving cycle. This defines the cost function to be minimized as follows:

$$J = \text{fuel} = \sum_{k=0}^{N-1} L(x(k), u(k)) \quad (2)$$

where N is the time length of the driving cycle, and L is the instantaneous fuel consumption rate.

During the optimization procedure, it is necessary to impose certain inequality constraints on the states and control to ensure they remain within their corresponding bounds:

$$\begin{aligned} \omega_{e-\min} &\leq \omega_e \leq \omega_{e-\max} \\ SOC_{\min} &\leq SOC \leq SOC_{\max} \\ T_{m-\min}(\omega_m, SOC) &\leq T_m \leq T_{m-\max}(\omega_m, SOC) \end{aligned} \quad (3)$$

where ω_e is the engine angular speed and T_m is the motor torque. In addition, equality constraints are imposed so that the vehicle always meets the speed and load demands of the specific driving cycle.

Since the above problem formulation does not impose a charge sustaining policy, the optimization algorithm tends to deplete the battery in order to attain minimal fuel consumption. Hence, a final state constraint on SOC should be imposed to account for maintaining the energy of the battery and to achieve a fair comparison of fuel economy. A soft terminal constraint on SOC (quadratic penalty function) is added to the cost function as follows:

$$J = \sum_{k=0}^{N-1} L(x(k), u(k)) + G(x(N)) \quad (4)$$

where $G(x(N)) = \alpha(SOC(N) - SOC_f)^2$ represents the penalty associated with the error in the terminal SOC; SOC_f is the desired SOC at the final time; and α is a weighting factor.

3.2 Model Simplification

The detailed HE-VESIM model is not suitable for the purpose of dynamic optimization because its complexity leads to low computation efficiency. Dynamic Programming is well-known to require computations that grow exponentially with the number of

states. A simplified vehicle model is thus developed for optimization purposes. The engine, torque converter, differential, and electric motor are reduced to static models with look-up tables for I/O mapping and efficiencies. Since the gear shifting duration is about one second, the automatic transmission was approximated to be a gearbox with gear number as the state. For this reason, the control to the transmission is constrained to take on the values of -1 , 0 , and 1 for downshift, no shift and upshift, respectively. The other state left is the battery SOC that is dynamically updated by the battery current. The simplified model was found to approximate well the complex model except under rapid transients.

3.3 Dynamic Programming (DP) Solution

A powerful algorithm to solve the above optimization problem is to use Dynamic Programming (DP). Based on Bellman's principle of optimality, the DP algorithm is presented as follows [11]:

Step $N-1$:

$$J_{N-1}^*(x(N-1)) = \min_{u(N-1)} [L(x(N-1), u(N-1)) + G(x(N))] \quad (5)$$

Step k , for $0 \leq k < N-1$

$$J_k^*(x(k)) = \min_{u(k)} [L(x(k), u(k)) + J_{k+1}^*(x(k+1))] \quad (6)$$

The recursive equation is solved backwards from step $N-1$ to 0 in order to find the optimal control policy. Each of the minimizations is performed subject to the constraints imposed by (3) and the driving cycle.

The standard method to solve a Dynamic Programming problem numerically is to use quantization and interpolation ([11],[12]). The state and control values are first quantized into finite grids. At each step of the DP algorithm, the function $J_k(x(k))$ is evaluated only at the grid points. If the next state, $x(k+1)$, does not fall exactly on to a quantized value, then function interpolation is used to determine the values of $J_{k+1}^*(x(k+1))$ in (6) as well as $G(x(N))$ in (5).

Despite the use of a simplified model, the long horizon of the UDDSHDV driving cycle makes the direct application of the above algorithm computationally infeasible for today's technology. Several approaches have been adopted to accelerate the computational speed [12]. From the velocity profile of the driving cycle, the vehicle model can be replaced by a finite set of operating points parameterized by wheel torque and speed. Pre-computed look-up tables are constructed for recording next states and instantaneous cost as a function of quantized states, control inputs, and operating points. Once these tables are built, they can be used to update (6) in a very efficient manner [12].

The dynamic programming procedure produces an optimal, time-varying, state-feedback control policy that is stored in a table for each of the quantized states and time stages, i.e., $u^*(x(k), k)$; this function is then used as a state feedback controller in the simulations. It should be noted that dynamic programming creates a family of optimal paths for all possible initial conditions. In our case, once the initial SOC is given, the optimal policy will find an optimal way to bring the final SOC back to the terminal value (SOC_f) while achieving the minimal fuel consumption.

3.4 Simulation Results

Since the control policy determined by the dynamic programming algorithm is generated on the basis of the simplified model, the control policy should be verified on the original complex model. Therefore, the optimal control policy found by DP was applied to the original HE-VESIM model. The same driving cycle (UDDSHDV) is used to evaluate the fuel economy. The terminal SOC constraint was selected as 0.57 and the initial SOC in the simulation is chosen to be 0.57 as well for the purpose of calculating fuel economy. Dynamic trajectories of the vehicle under the optimal control policy for the UDDSHDV cycle are shown in Figure 4. The difference between the desired vehicle speed (UDDSHDV) and the actual vehicle speed is within 2 mph. The SOC trajectory starts at 0.57 and ends around 0.57 with a small quantization error. Consequently, we have confidence that the optimal solutions based on the simplified model are reliable. The fuel economy of the DP-optimized hybrid truck is 13.63 (MPG). Significant improvement has been achieved by the DP algorithm as compared with values shown in Table 1.

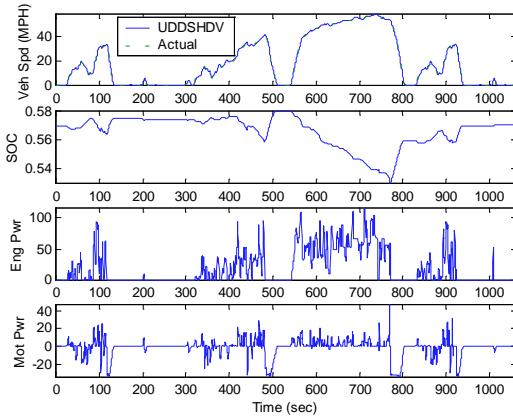


Figure 4: Simulation result of UDDSHDV cycle. The engine and motor power are given in kW

4. Improved Rule-Based Control Strategy

Although the dynamic programming approach provides an optimal solution for minimizing fuel consumption, the resulting control policy is not implementable in real driving conditions because the optimal policy requires knowledge of the future speed and load profile of the vehicle. Nonetheless, analyzing optimal policies determined through dynamic programming can provide insight into how the fuel economy improvement is achieved. An improved rule-based control algorithm is proposed in this section based on the investigation of the dynamic programming results.

4.1 Gear Shift Control

Determining the gear shift strategy is crucial to the fuel economy of hybrid electric vehicles [13]. In the dynamic programming scheme, gear shift is one of the control inputs to the system. It is interesting to find out how the DP solution chooses the gear position to improve fuel economy. From the optimization results, the gear operation points are expressed on the engine power demand vs. wheel speed plot (Figure 5). It can be seen that four gear positions are separated into four regions and the boundary between two adjacent regions seem to represent better gear shifting thresholds. After adding a hysteresis function to the

shifting thresholds, a new gear shift map determining when an upshift or downshift event occurs was developed. It should be mentioned that the optimal gear shift map for minimum fuel consumption can also be constructed through static optimization ([10],[14]). Given an engine power and wheel speed, the best gear position for minimum fuel consumption can be chosen based on the steady-state engine fuel consumption map. It is found that the steady-state gear map nearly coincides with Figure 5. This is not surprising since the electric motor is positioned after the transmission, which means that the engine efficiency will dominate the gear shifting policy. Finally, we apply the new gear shift logic (Figure 5) to the original rule-based control strategy. Fuel economy is improved to 13.02 MPG as shown in Table 4.

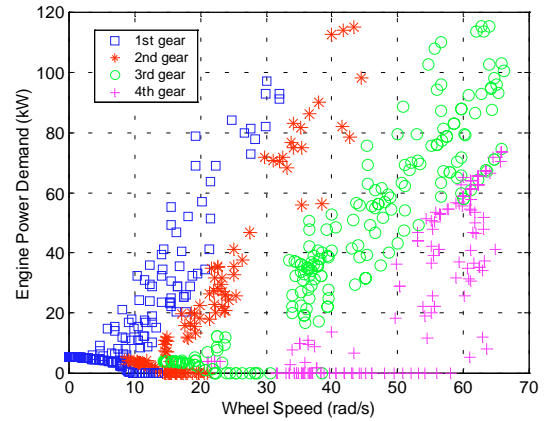


Figure 5: Gear operating points of DP optimization

4.2 Power Split Control

In this section, we explore how Power Split Control of the preliminary rule-based strategy can be improved on the basis of dynamic programming. In Power Split Control, there are four possible operating modes of splitting the power demand between the engine and motor: motor only mode, engine only mode, hybrid mode (both the engine and motor), and recharge mode (the engine offers additional power to charge the battery). Rules for switching between the different modes will be established by examining the optimization results obtained from Section 3. The operating points displaying different operating modes are presented in the transmission input speed and power demand plane (see Figure 6).

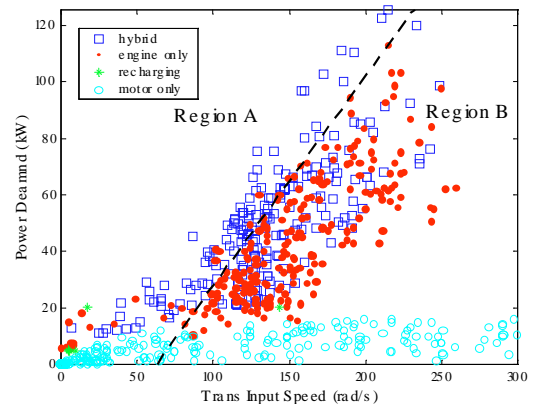


Figure 6: Operating points of DP optimization over UDDSHDV cycle

Some observations can be made as follows:

- Use the motor alone when power demand is less than 15 kW.
- In region A, DP chooses to operate in the hybrid mode.
- The recharge mode rarely happens.

The low number of recharging events may imply that under the current vehicle configuration it is not efficient to use engine power to charge the battery, even if increasing the engine's power would move its operation to a more efficient region. As a result, we will assume there is no recharging during Power Split Control, and recharge will only occur under Recharging Control when SOC is too low. The power distribution between the two prime movers in the hybrid mode is determined next. We wish to extract from the DP solution an optimal motor power model in the hybrid mode, and then determine the engine power demand by subtracting the motor power from the driver power demand. Clearly, optimal motor power may depend on many variables such as wheel speed, engine speed, power demand, SOC, gear ratio, etc. For this reason, a regression-based program was first used to assess which of these variables were the dominant factors in determining motor power. It turned out that power demand, engine speed, and transmission input speed were the critical factors. Motor power, as determined by the DP algorithm, was then fit to these three factors with a Neural Network (NN), using two hidden layers with 3 and 1 neurons, respectively. The basic logic of this improved Power Split Control is summarized in Table 2. After implementing the new Power Split Control rules, the fuel economy was further improved to 13.17 MPG as shown in Table 4.

Table 2: Basic logic rules of new Power Split Control

If $P_{req} \leq 15$ kW,	$P_m = P_{req}$	$P_e = 0$
Else If Region A,	$P_m = Nnet_1(P_{req}, \omega_{trans}, \omega_{eng})$	$P_e = P_{req} - P_m$
If Region B,	$P_m = 0$	$P_e = P_{req}$,
If $P_e > P_{e_max}$,	$P_e = P_{e_max}$	$P_m = P_{req} - P_e$

4.3 Recharging Control

In the modified rule-based control algorithm, the thermostat-like charge sustaining strategy is retained, owing to robustness and safety concerns. The recharging mode will turn on if the battery SOC falls below the lower limit as described in the preliminary rule-based control. However, requiring the engine to provide a constant recharging power level is not necessarily the most efficient way to recharge the battery. For this reason, "when to recharge" and "at what rate to recharge" should be investigated to improve the recharging control policy. Since the engine is rarely used to recharge the battery from the previous optimization result, the dynamic programming procedure was modified in an attempt to observe an optimal recharging policy. First, we turned off the regenerative braking function in the dynamic programming routine. In other words, all the braking power was supplied by the hydraulic braking and hence there was no "free" energy secured from the regenerative braking to recharge the battery. Furthermore, after computing the optimal control policy via DP, the initial SOC was specified to be 0.52 for the purpose of simulating the situation that SOC was too low and the battery needed to be recharged. The simulation result is shown in Figure 7. Note that the above represents the optimal policy for minimum fuel consumption under the condition that the battery SOC has to be recharged

to 0.57 from 0.52. Note also that negative motor power now represents the recharging power supplied by the engine since there is no regenerative braking.

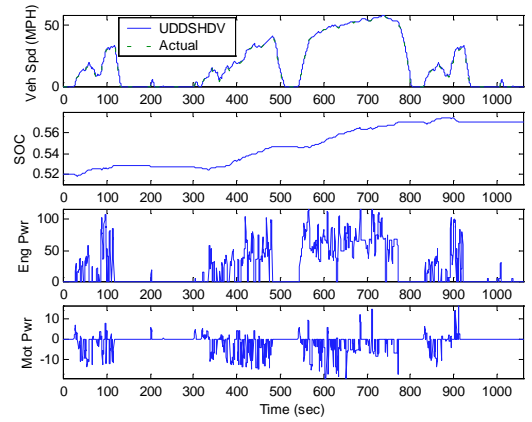


Figure 7: Simulation results of UDDSHDV cycle without regenerative braking

Several rules were extracted from the optimization result as follows (possible reasons are in parentheses):

- Recharging happens only when wheel speed is greater than 10 rad/s (better motor efficiency).
- Battery recharging power is normally smaller than 15kW (better battery charging efficiency).
- The electric motor is the only power source to drive the vehicle when power demand is less than 8 kW (avoid low engine efficiency).

Further rules can be constructed as shown in Figure 8. A threshold line is drawn to divide the plot into two regions. In region C, there are few recharging events and most of the recharging events happen in region D. We extracted all the recharging data in region D in an attempt to determine a function for optimal recharging power, using the method of Section 4.2. A regression program was first used to find which factors should be used to build the model and then a Neural Network was used to fit the function. The basic logic of this improved Recharging Control is summarized in Table 3. As shown in Table 4, fuel economy has been improved to 13.24

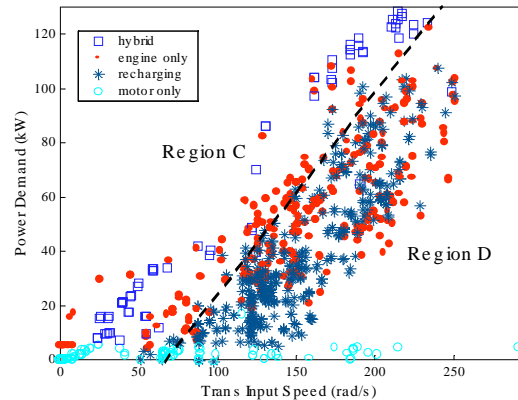


Figure 8: Operating points of DP optimization over UDDSHDV cycle without regenerative braking

MPG under this new Recharging Control policy. The gradual improvements in fuel economy can be seen in Table 4 as the new strategies were added one after another to the preliminary rule-based algorithm.

Table 3: Basic logic rules of new Recharging Control

If $P_{req} \leq 8 \text{ kW}$,	$P_m = P_{req}$	$P_e = 0$
Else If Region C or $\omega_{wheel} < 10$,	$P_e = P_{req}$	$P_m = 0$
If Region D,	$P_m = -P_{ch}$	$P_e = P_{req} + P_{ch}$,
	$P_{ch} = -Nnet_2(P_{req}, \omega_{trans}, \omega_{eng})$	
If $P_e > P_{e_max}$,	$P_e = P_{e_max}$	$P_m = P_{req} - P_e$

Table 4: Fuel economy comparison over UDDSHDV cycle

	Fuel Economy (MPG)
Conventional	10.63
Preliminary Rule-Based	12.56
New Shift Control	13.02
New Power Split Control	13.17
New Recharging Control	13.24
Dynamic Programming	13.63

5. Conclusions

Design of the energy management strategy for a hybrid electric vehicle with the aid of dynamic programming has the advantage of optimizing the overall system efficiency. Dynamic Programming provides engineers with fast quantitative analysis and further understanding of the complex hybrid system. In this paper, the problem of predicting the best fuel economy of a hybrid truck over a driving cycle was investigated. A Dynamic Programming (DP) algorithm based on a simplified vehicle model was developed to determine the optimal policy for hybrid operation. It was found that improvements in fuel economy were derived mainly from optimizing the gear-shifting policy and discharging/charging schedule, and relieving the engine load through more efficient motor/battery operation. By carefully analyzing the optimization results, an improved rule-based control strategy was developed for real driving application.

Acknowledgments

This research is supported by the U.S. Army TARDEC under the contract DAAE07-98-C-R-L008. The work of J.W. Grizzle was supported in part by NSF contract IIS-9988695.

References

[1]Baumann, B. M., Washington, G. N., Glenn, B. C., and Rizzoni, G., "Mechatronic Design and Control of Hybrid Electric Vehicles," IEEE/ASME Transactions on Mechatronics, v5 n 1 2000. p 58-72, 2000

[2]Farrall, S. D. and Jones, R. P., "Energy Management in an Automotive Electric/Heat Engine Hybrid Powertrain Using Fuzzy Decision Making," Proceedings of the 1993 International Symposium on Intelligent Control, Chicago, IL, 1993

[3]Kim, C., NamGoong, E., and Lee, S., "Fuel Economy Optimization for Parallel Hybrid Vehicles with CVT," SAE Paper No. 1999-01-1148.

[4]Paganelli, G., Ercole, G., Brahma, A., Guezennec, Y. and Rizzoni, G., "A General Formulation for the Instantaneous Control of the Power Split in Charge-Sustaining Hybrid Electric Vehicles." Proceedings of 5th Int'l Symposium on Advanced Vehicle Control, Ann Arbor, MI, 2000.

[5]Johnson, V.H., Wipke, K.B. and Rausen, D.J., "HEV Control Strategy for Real-Time Optimization of Fuel Economy and Emissions," Proceedings of the Future Car Congress, April 2000, SAE Paper No. 2000-01-1543.

[6]Brahma, A., Guezennec, Y. and Rizzoni, G., "Dynamic Optimization of Mechanical/Electrical Power Flow in Parallel Hybrid Electric Vehicles" Proceedings of 5th Int'l Symposium on Advanced Vehicle Control, Ann Arbor, MI, 2000.

[7]Zoelch, U., and Schroeder, D., "Dynamic Optimization Method for Design and Rating of the Components of a Hybrid Vehicle," International Journal of Vehicle Design, v19, n1, p1-13, 1998

[8]Assanis, D.N., Filipi, Z.S., Gravante, S., Grohnke, D., Gui, X., Louca, L.S., Rideout, G.D., Stein, J.L., Wang, Y., "Validation and Use of SIMULINK Integrated, High Fidelity, Engine-In-Vehicle Simulation of the International Class VI Truck," SAE Paper No. 2000-01-0288

[9]Lin, C.C., Filipi, Z.S., Wang, Y., Louca, L.S., Peng, H., Assanis, D.N., and Stein, J.L., "Integrated, Feed-Forward Hybrid Electric Vehicle Simulation in SIMULINK and its Use for Power Management Studies", SAE Paper No. 2001-01-1334

[10]Bowles, P. D., "Modeling and Energy Management for a Parallel Hybrid Electric Vehicle (PHEV) with Continuously Variable Transmission (CVT)," MS thesis, University of Michigan, Ann Arbor, MI, 1999

[11]Bertsekas, D.P., Dynamic Programming and Optimal Control, Athena Scientific, 1995

[12]Jun-Mo Kang, Ilya Kolmanovsky and J.W. Grizzle "Approximate Dynamic Programming Solutions for Lean Burn Engine Aftertreatment," Proceedings of the IEEE Conference on Decision and Control, Phoenix, AZ, December 7-10, 1999

[13]Lee, H.D., Sul, S.K., Cho, H.S., and Lee, J.M., "Advanced Gear Shifting and Clutching Strategy for Parallel Hybrid Vehicle with Automated Manual Transmission," IEEE Industry Applications Conference, v3, 1998

[14]Soltic, P., and Guzzella, L., "Optimum SI Engine Based Powertrain Systems for Lightweight Passenger Cars," SAE Paper No. 2000-01-0827

Appendix

Table 5: Basic vehicle specification

DI Diesel Engine	V6, 5.475L, 157HP/2400rpm
DC Motor	49kW
Lead-acid Battery	Capacity: 18Ah, Number: 25
Automatic Transmission	4 speed, GR: 2.59/1.68/1.06/0.75
Vehicle	Total mass: 7258 kg