Controlling Planar Biped Locomotion

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Common Control Approaches to Biped Walking

Tracking trajectories and modifications thereof

- Algorithms: PID, Computed torque, Sliding modes
- Modifications:
  Zero moment point [Honda], Intuitive control [Pratt et al. '01], Track center of mass location [Kajita et al. '96, Mitobe et al.'95, Fujimoto and Kawamura '98, …], Use of angular momentum [Sano and Furusho '90, Aoustin and Formal'sky '99, Furusho and Masubuchi '86]
- Trajectories obtained via:
  Analogy with biological systems [Honda, Vukobratovic '90], Analogy with simpler (passive) robots [McGeer '90, Thuilot et al. '97, Linde '98], van der Pol, etc. [Katoh and Mori '84], Optimization [Cabodevilla et al. '96, Chealllereau et al. '01, Rostami and Bessonnet '01, Hasegawa et al. '00, Hardt '99, …]

None of these has led to a stability proof!

Control Presentation Highlights

Introduce a common framework for the systematic design, analysis, and performance enhancement of controllers that induce stable walking motions.

Outline of Presentation

- Feedback design approach
  - walking at a fixed rate
  - walking at multiple rates
- Experiments

The Model: System with Impulse Effects

$$\begin{aligned} \dot{x}(t) &= f(x(t)) & x^-(t) \notin S \\ x^+(t) &= \Delta(x^-(t)) & x^-(t) \in S \end{aligned}$$

$x^+(t) := \lim_{\tau \searrow t} x(\tau)$ $\qquad$ $x^-(t) := \lim_{\tau \nearrow t} x(\tau)$

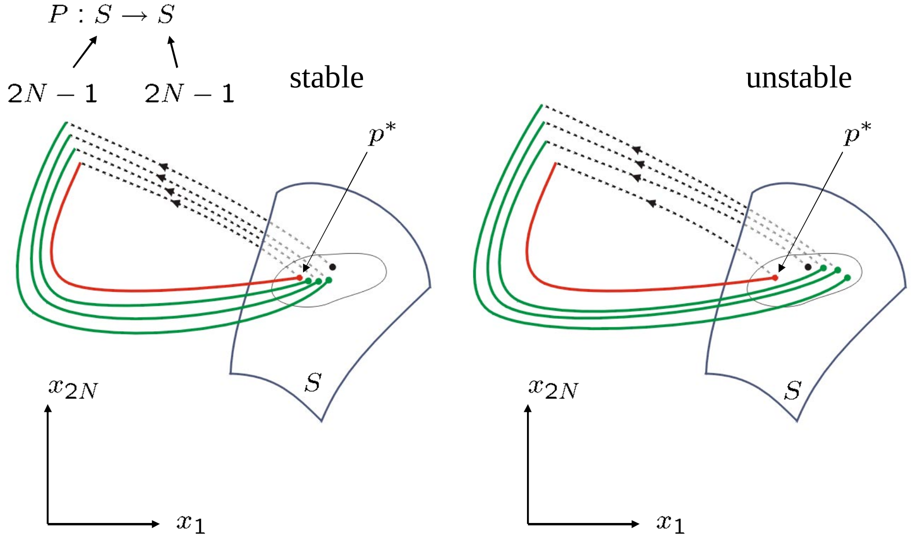
$S = \{x \in \mathcal{X} \mid \text{swing leg contacts the ground}\}$

Stable walking

$\Leftrightarrow$

Asymptotically stable limit cycle (transversal to S)

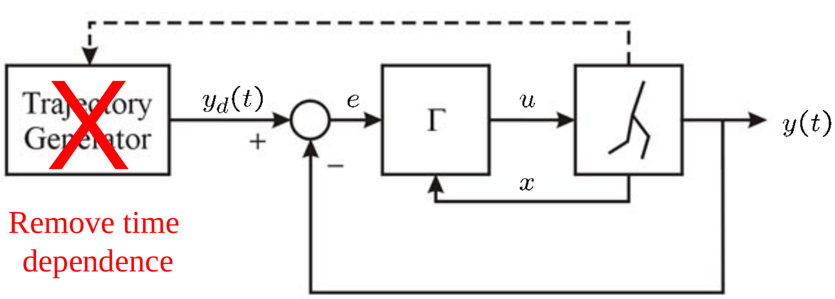
A Stability Analysis
(Poincaré)



Recall Standard Feedback Design

$$e = y_d(t) - y(t)$$
$$e \equiv 0 \;\Rightarrow\; y(t) = y_d(t)$$

robot state slaved to time

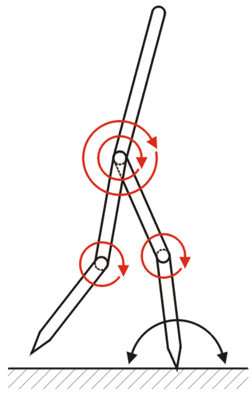


Remove time dependence

Virtual Constraints

Idea:

slave the actuated variables to the unactuated variable



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Virtual Constraints

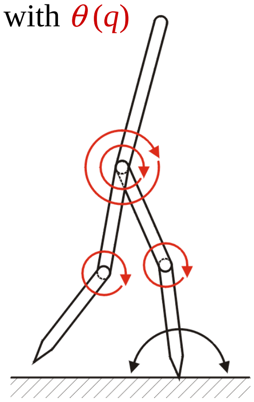
Idea:

slave the actuated variables to the unactuated variable

Impose virtual constraints—replace time with $\theta(q)$

$\rightarrow$

reduction of dimension



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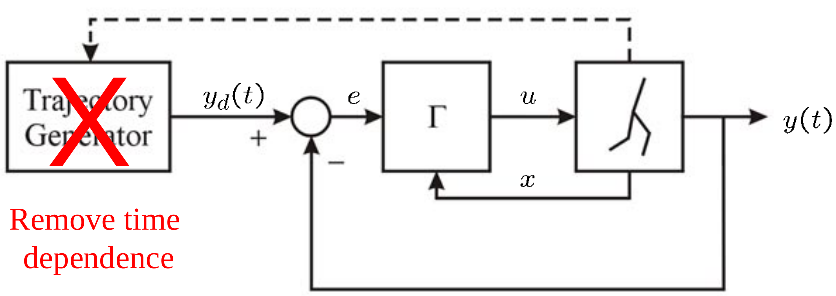
(do virtual constraint demo now)

Recall Standard Feedback Design

$e = y_d(t) - y(t)$

$e \equiv 0 \;\Rightarrow\; y(t) = y_d(t)$

robot state slaved to time



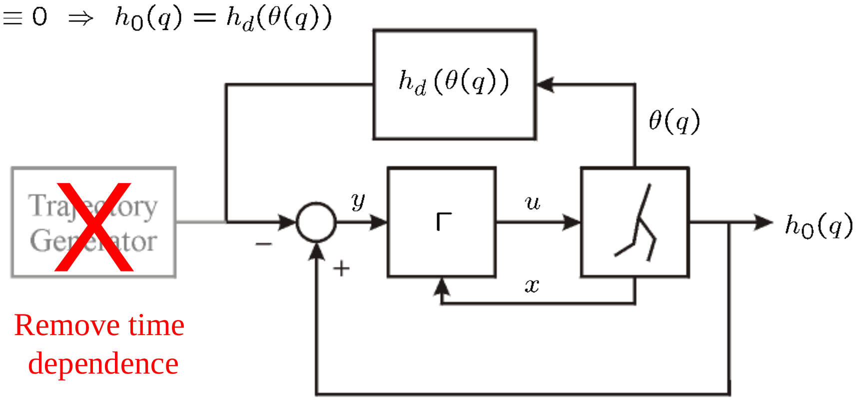
Remove time dependence

Our Approach to Feedback Design
(controlling for analytical tractability)

$y = h_0(q) - h_d(\theta(q))$

$y \equiv 0 \;\Rightarrow\; h_0(q) = h_d(\theta(q))$

robot state slaved to the robot state



Remove time dependence

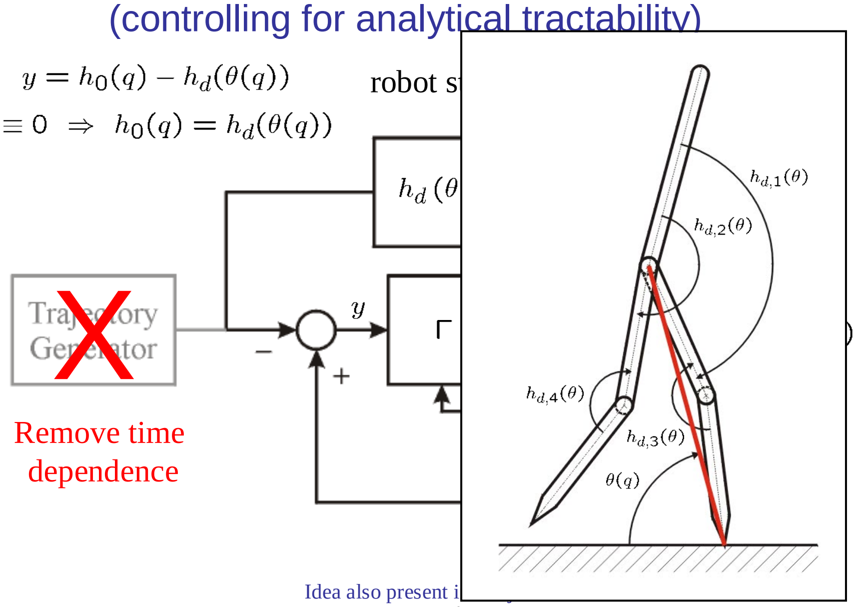
Idea also present in [Kajita et al., '92; Hurmuzlu, '93; Ono '01]

Our Approach to Feedback Design
(controlling for analytical tractability)

$y = h_0(q) - h_d(\theta(q))$

$y \equiv 0 \;\Rightarrow\; h_0(q) = h_d(\theta(q))$

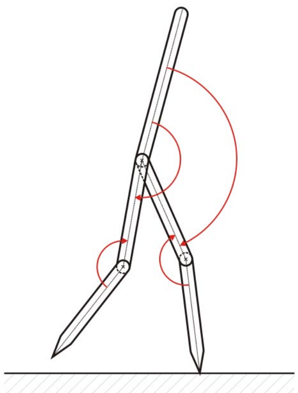
robot s

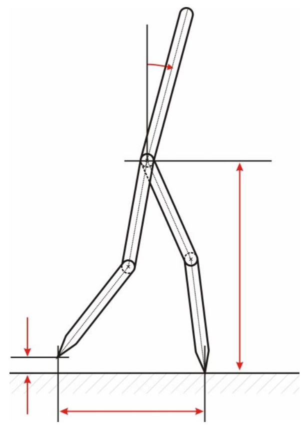


Remove time dependence

Idea also present i

Choosing what to Control?





angular vs. "Cartesian"

Control Design Summary: Imposing Constraints via Feedback

$$\begin{aligned} \dot{x} &= f(x) + g(x)u \\ y &= h(q) = h_0(q) - h_d(\theta(q)) \end{aligned}$$

$$\begin{aligned} \dot{y} &= L_f h \\ \ddot{y} &= L_f^2 h + \underbrace{L_g L_f h}_{\text{decoupling matrix}} u \end{aligned}$$

Design: $y = h(q)$ such that $y = 0$ encodes the desired posture as a function of $\theta(q)\ldots$

…AND so that the decoupling matrix is invertible.

Thm: $L_g L_f h$ invertible $\Rightarrow$ can use computed torque control:

$$\begin{aligned} u &= \left(L_g L_f h\right)^{-1}\left(v - L_f^2 h\right) \\ \ddot{y} &= v \end{aligned}$$

(back to demo, now restriction dynamics)

Constraints Yield Single Support Zero Dynamics

achieving control objective

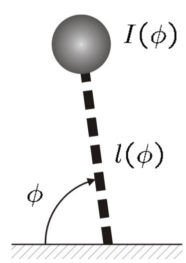
$y \equiv 0 \;\Rightarrow\; h_0(q) = h_d(\theta(q))$

$\Leftrightarrow$

imposing VIRTUAL constraints

$\rightarrow$

reduction of dimension



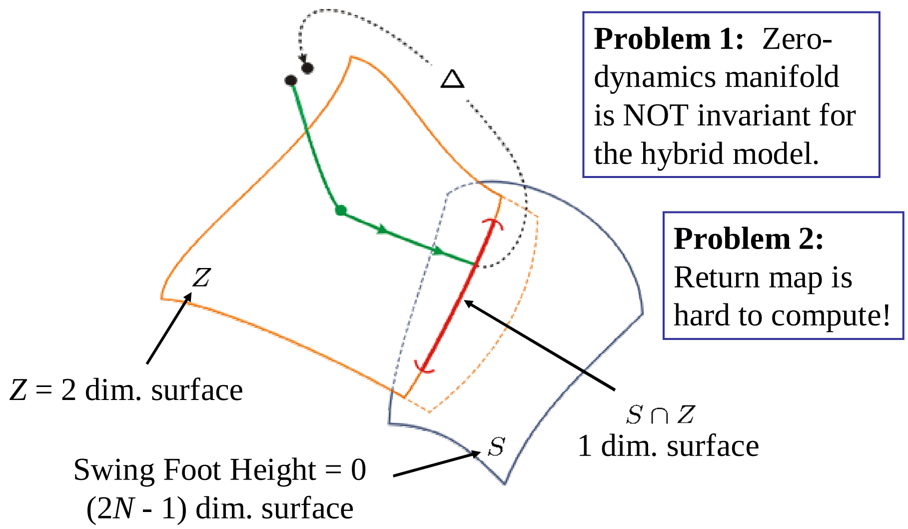
1 dof

Working definition of Isidori-Moog 1988

"Largest internal dynamics compatible with $y(t) \equiv 0$"

$$\begin{aligned} Z &= \{x_0 \in TQ \mid \exists u \text{ such that } y(t, x_0, u) = 0\} \\ &= \Big\{x_0 \in TQ \mid h(x) = 0,\; L_f h(x) = \tfrac{\partial h}{\partial x}\dot{x} = 0, \\ &\qquad L_g L_f h \text{ invertible}\Big\} \end{aligned}$$

The Geometry



Analytical Development: Zero Dynamics Form

[Westervelt et al. TAC Jan. '03]

$$\begin{aligned} \dot{x} &= f(x) + g(x)u \\ y &= h(x) \equiv 0 \end{aligned}$$

$L = K - V =$ Lagrangian of the swing phase model

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\theta}} - \frac{\partial L}{\partial \theta} = 0 \quad \Rightarrow \quad \begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} = \begin{bmatrix} \kappa_1(z_1) z_2 \\ \kappa_2(z_1) \end{bmatrix} \qquad \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} \theta \\ \gamma \end{bmatrix}$$

Theorem: $\kappa_1(z_1)$ and $\kappa_2(z_1)$ have the following forms

$$\kappa_1(z_1) := \frac{\partial \theta}{\partial q} \begin{bmatrix} \frac{\partial h}{\partial q} \\ \gamma_0 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \Bigg|_Z \qquad \kappa_2(z_1) := -\frac{\partial V}{\partial q_N}\Bigg|_Z$$

Hybrid Zero Dynamics

$$\begin{aligned} \dot{x} &= f(x) + g(x)u \\ y &= h(x) \equiv 0 \end{aligned} \quad + \quad x^{+} = \Delta(x^{-}) \quad = \quad ?$$

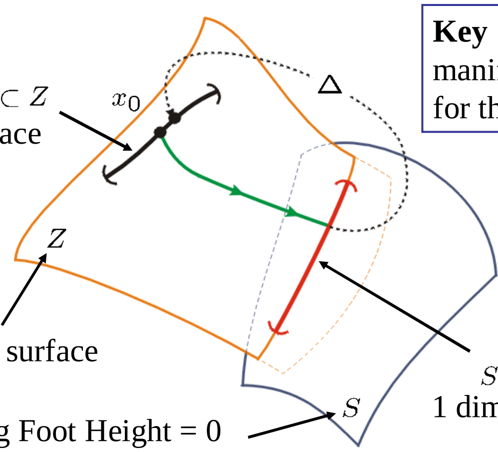
Working definition of Isidori-Moog 1988
"Largest internal dynamics compatible with $y(t) \equiv 0$"

Zeroing the output should lead to a zero dynamics.

How does the impact map show up?

Stability Analysis with Invariance

[Westervelt et al. TAC Jan. '03]



Key 1: Zero-dynamics manifold is invariant for the hybrid model.

Key 2: Return map is scalar & LTI!

HZD: An "Equivalent" Hybrid System
[Westervelt et al. TAC Jan. '03]

- **Thm:** Condition to have a zero dynamics for the full (hybrid) model:

$$\Delta(S \cap Z) \subset Z \Leftrightarrow \begin{array}{rcl} h \circ \Delta|_{S\cap Z} & = & 0 \\ L_f h \circ \Delta|_{S\cap Z} & = & 0 \end{array}$$

- Under this condition, solutions of the zero dynamics are also solutions of the complete model compatible with $y \equiv 0$.

$$\begin{array}{rcll} \dot{z} & = & f_{\text{zero}}(z) & z^- \notin S \cap Z \\ z^+ & = & \Delta(z^-) & z^- \in S \cap Z \end{array}$$

HZD: An "Equivalent" Hybrid System
[Westervelt et al. TAC Jan. '03]

$$\begin{array}{rcll} \dot{x} & = & f(x) + g(x)u & x^- \notin S \\ x^+ & = & \Delta(x^-) & x^- \in S \\ y & = & h(q) \equiv 0 & \end{array} \quad + \quad \Delta(S \cap Z) \subset Z \quad =$$

(an N dof hybrid system) (invariance)

$$\begin{array}{rcll} \dot{z} & = & f_{\text{zero}}(z) & z^- \notin S \cap Z \\ z^+ & = & \Delta(z^-) & z^- \in S \cap Z \end{array}$$

(1 dof hybrid system)

HZD Integration

[Westervelt et al. TAC Jan. '03]

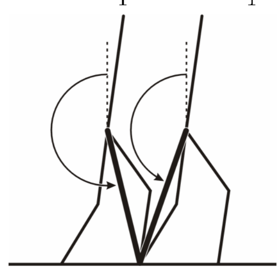
Montonicity of $z_1(t)$ implies

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} = \begin{bmatrix} \kappa_1(z_1) z_2 \\ \kappa_2(z_1) \end{bmatrix} \Rightarrow \frac{dz_2}{dz_1} = \frac{\kappa_2(z_1)}{\kappa_1(z_1) z_2}$$

$$\Rightarrow \begin{cases} z_2 dz_2 = \dfrac{\kappa_2(z_1)}{\kappa_1(z_1)} dz_1 \\ \Updownarrow \\ \dfrac{(z_2^-)^2}{2} = \dfrac{(z_2^+)^2}{2} - \displaystyle\int_{z_1^+}^{z_1^-} \dfrac{\kappa_2(s)}{\kappa_1(s)} ds \end{cases}$$

(almost the Poincaré map for the HZD)

$\theta^+ = z_1^+$ $\theta^- = z_1^-$



HZD Integration $\rightarrow$ HZD Poincaré Map

[Westervelt et al. TAC Jan. '03]

With invariance $P : S \rightarrow S$ becomes $\rho : S \cap Z \rightarrow S \cap Z$

Theorem: $\rho(\zeta_2^-) = \delta_{\mathrm{zero}}^2 \, \zeta_2^- - V_{\mathrm{zero}}(z_1^-)$

$$\frac{(z_2^-)^2}{2} = \frac{(z_2^+)^2}{2} - \int_{z_1^+}^{z_1^-} \frac{\kappa_2(s)}{\kappa_1(s)} ds$$

$$\delta_{\mathrm{zero}}^2 := \gamma_0(q^+) \, \Delta_{\dot{q}}(q_0^-) \, \sigma_{\dot{q}}(q_0^-)$$

- $\gamma_0(q^+)$: last row of inertial matrix
- $\Delta_{\dot{q}}(q_0^-)$: velocity impact map
- $\sigma_{\dot{q}}(q_0^-)$: velocity on $S \cap Z$

$$V_{\mathrm{zero}}(z_1) := -\int_{z_1^+}^{z_1} \frac{\kappa_2(s)}{\kappa_1(s)} ds$$

$$\zeta_2^- := \tfrac{1}{2}(z_2^-)^2$$

$$\zeta_2^+ := \delta_{\mathrm{zero}}^2 \, \zeta_2^-$$

HZD Integration → HZD Poincaré Map
[Westervelt et al. TAC Jan. ’03]

With invariance $P : S \to S$ becomes $\rho : S \cap Z \to S \cap Z$

Theorem: $\rho(\zeta_2^-) = \delta_{\text{zero}}^2 \, \zeta_2^- - V_{\text{zero}}(z_1^-)$

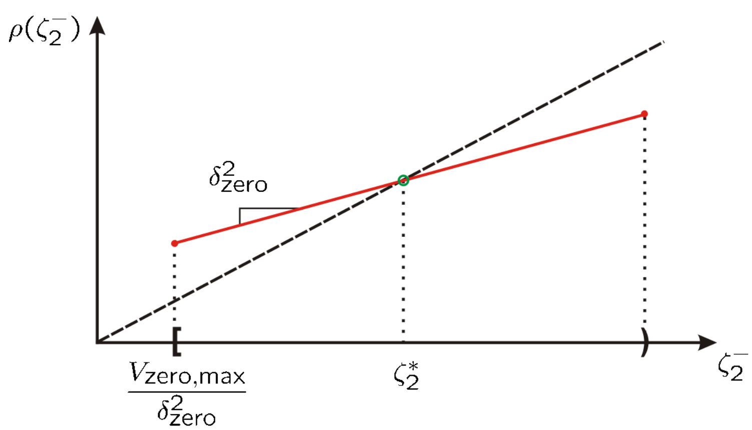
Domain of definition: $\{\zeta_2^- > 0 \mid \delta_{\text{zero}}^2 \, \zeta_2^- - V_{\text{zero,max}} > 0\}$

$$V_{\text{zero,max}} := \max_{\theta^+ \le z_1 \le \theta^-} V_{\text{zero}}(z_1)$$

Fixed point: $\zeta_2^* = -\dfrac{V_{\text{zero}}(z_1^-)}{1 - \delta_{\text{zero}}^2}$

HZD Integration → HZD Poincaré Map
[Westervelt et al. TAC Jan. ’03]

Poincaré map: $\rho(\zeta_2^-) = \delta_{\text{zero}}^2 \, \zeta_2^- - V_{\text{zero}}(z_1^-)$



HZD Poincaré Map Relation to full Hybrid model
[Westervelt et al. TAC Jan. '03]

$$\begin{aligned} \dot{z} &= f_{\text{zero}}(z) & z^- \notin S \cap Z \\ z^+ &= \Delta(z^-) & z^- \in S \cap Z \end{aligned}$$

Theorem: There exists an exponentially stable periodic orbit of the hybrid zero dynamics if, and only if,

a) $\frac{\delta^2_{\text{zero}}}{1-\delta^2_{\text{zero}}} V_{\text{zero}}(z_1^-) + V_{\text{zero,max}} < 0$

b) $\delta^2_{\text{zero}} < 1$

Theorem: Above orbit is exponentially stabilizable for the full-order model.

Using HZD for Controller Design
[Westervelt et al. TAC Jan. '03]

- Finitely parameterize the outputs (we use Bezier Polynomials): $y = h_0(q) - h_d(\theta(q), a)$
- Impose invariance condition: $\Delta(S \cap Z_a) \subset Z_a$
- Stability guaranteed if, and only if, two inequality constraints hold:

a) $\frac{\delta_{\text{zero}}(a)^2}{1-\delta_{\text{zero}}(a)^2} V_{\text{zero}}(z_1^-, a) + V_{\text{zero,max}}(a) < 0$

b) $\delta^2_{\text{zero}}(a) < 1$

Using HZD for Controller Design

[Westervelt et al. TAC Jan. '03]

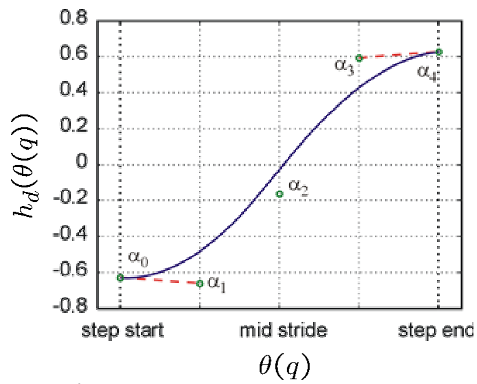
Bezier Polynomial: $b : [0, 1] \to \mathbb{R}$

$$h_d(\theta(q)) := b(s) \circ \bar{\theta}(q)$$

$$b(s) := \sum_{i=0}^{M} \alpha_i \frac{M!}{i!(M-i)!} s^i (1-s)^{M-i}$$

$\theta(q)$ is a normalized to be between 0 and 1:

$$\bar{\theta}(q) := \frac{\theta(q) - \theta^+}{\theta^- - \theta^+}$$



Using HZD for Controller Design

[Westervelt et al. TAC Jan. '03]

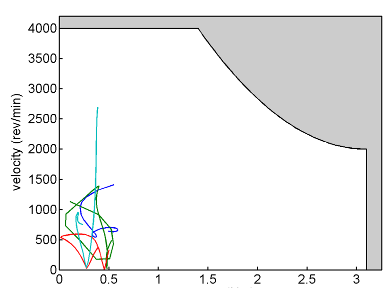
Achieve performance by tuning parameters via optimization on 1 dof model, subject to previous constraints.

$$\begin{aligned} \dot{z} &= f_{a,\text{zero}}(z) & z^- \notin S \cap Z_a \\ z^+ &= \Delta_a(z^-) & z^- \in S \cap Z_a \end{aligned}$$

$$J(a) := \frac{1}{p^h_{2,a}(T^-)} \int_0^{T^-} ||u_a^*(t)||^2 dt$$

Can include:

- dynamic constraints
- kinematic constraints



Using HZD for Controller Design

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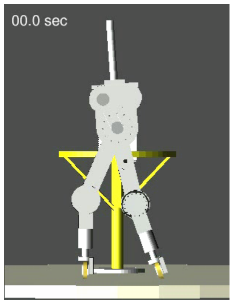
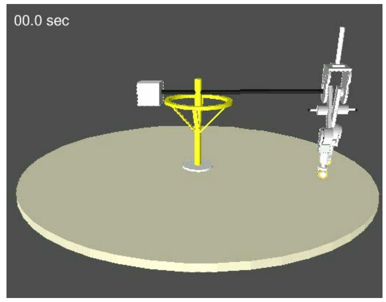




Movie: Walking at a fixed rate 0.7 m/s

RABBIT_walking_0.7ms_no_wheels_HIGH.wmv

Available at:

http://www.mecheng.osu.edu/~westerve/thesis_documentation/movies/March03/RABBIT_walking_0.7ms_no_wheels_HIGH.wmv

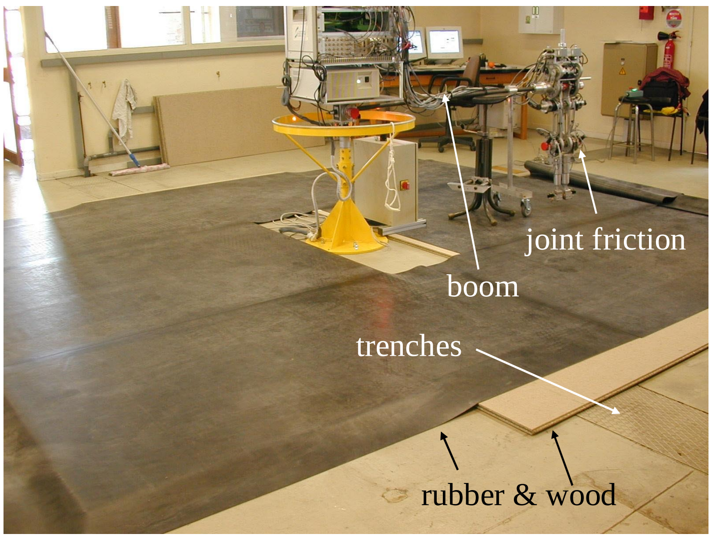
Movie: Robustness demonstration

RABBIT_perturbation_HIGH.wmv

Available at:

http://www.mecheng.osu.edu/~westerve/thesis_documentation/movies/March03/RABBIT_perturbation_HIGH.wmv

Experimental Reality

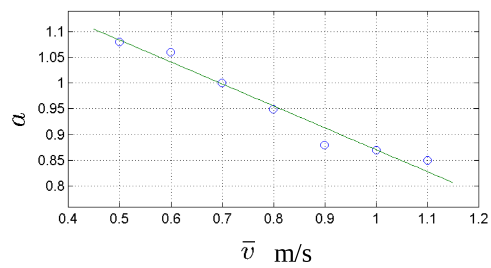


Experimental Reality

- Joint friction & boom dynamics included into model
- Trenches $\rightarrow$ rubber & wood

Poincaré map: $\rho(\zeta_2^-) = \bar{\delta}^2_{\text{zero}}\,\zeta_2^- - V_{\text{zero}}(z_1^-)$

$\bar{\delta}^2_{\text{zero}} := a\delta^2_{\text{zero}} \quad \rightarrow$



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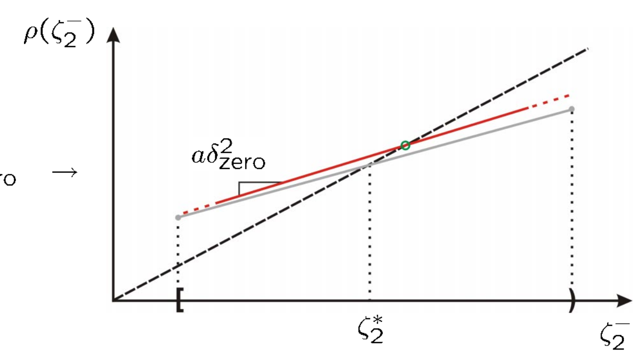


Additional Tools

[Westervelt et al. TAC Feb. '03]

1) Provably stable composition of walking motions

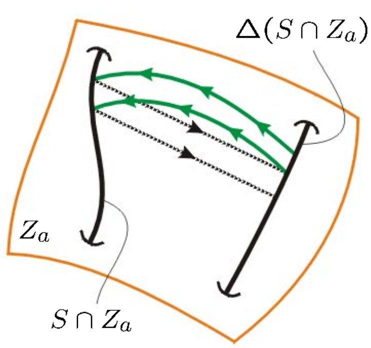
   e.g., switching from 0.75 m/s to 0.85 m/s

2) Walking at a continuum of rates

   e.g., $\bar{v} \in (\bar{v}^* - \delta, \bar{v}^* + \delta)$

Composition of Walking Motions

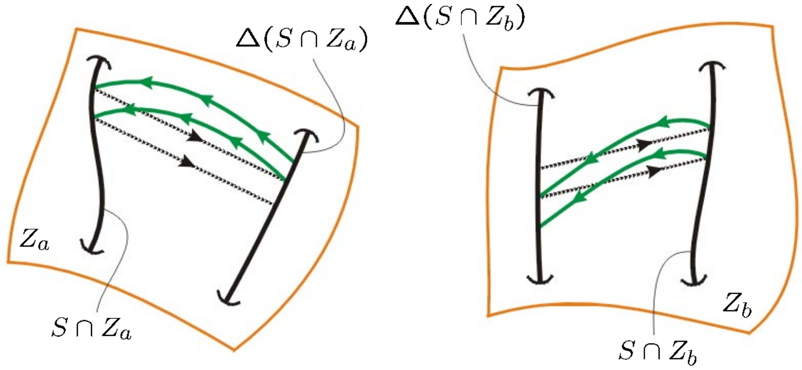
Introduce controller to transition from domain of one Poincaré map to another



(0.75 m/s)

Composition of Walking Motions

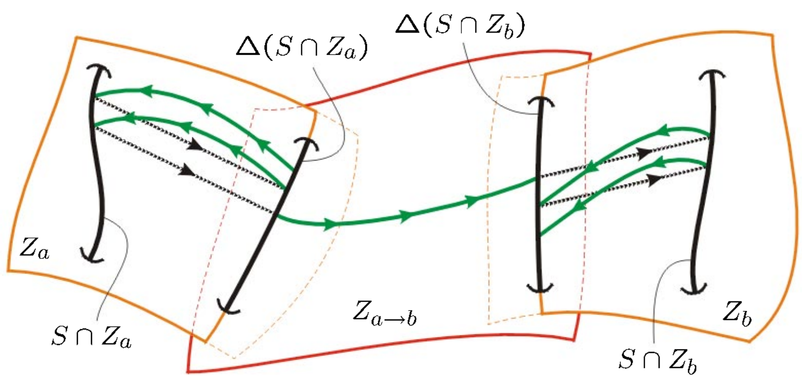
Introduce controller to transition from domain of one Poincaré map to another



(0.75 m/s) (1.0 m/s)

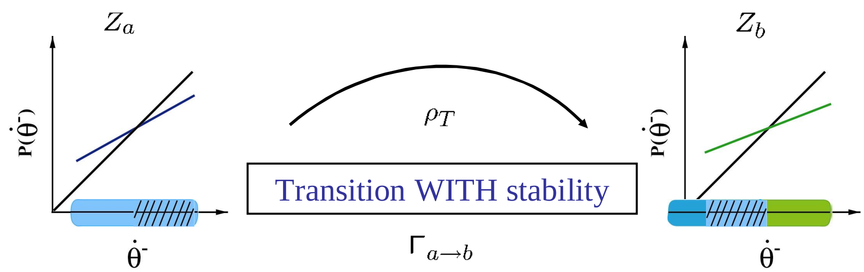
Composition of Walking Motions

Introduce controller to transition from domain of one Poincaré map to another



(0.75 m/s) (1.0 m/s)

Transition Controller & Stability Proof



$$\Gamma_a = \begin{bmatrix} a_0^a \\ a_1^a \\ \vdots \\ a_{M-1}^a \\ a_M^a \end{bmatrix} \quad \Gamma_{a\to b} = \begin{bmatrix} a_0^a \\ a_0^a + \frac{\theta_b^- - \theta_a^+}{\theta_a^- - \theta_a^+}(a_1^a - a_0^a) \\ \frac{a_i^a + a_i^b}{2}, \; i = 2 \ldots M-2 \\ a_M^b + \frac{\theta_b^- - \theta_a^+}{\theta_a^- - \theta_a^+}(a_{M-1}^b - a_M^b) \\ a_M^b \end{bmatrix} \quad \Gamma_b = \begin{bmatrix} a_0^b \\ a_1^b \\ \vdots \\ a_{M-1}^b \\ a_M^b \end{bmatrix}$$

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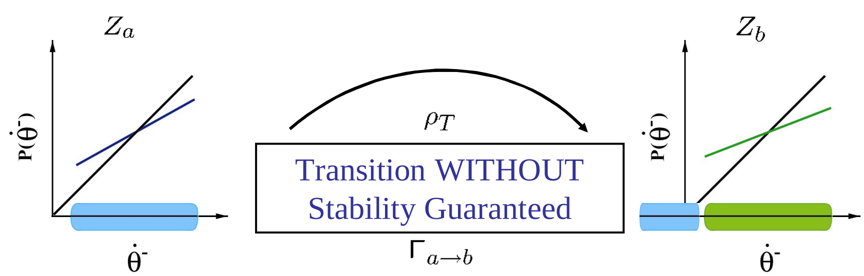
Transition Controller & Stability Proof



$$\Gamma_a = \begin{bmatrix} a_0^a \\ a_1^a \\ \vdots \\ a_{M-1}^a \\ a_M^a \end{bmatrix} \quad \Gamma_{a\to b} = \begin{bmatrix} a_0^a \\ a_0^a + \frac{\theta_b^- - \theta_a^+}{\theta_a^- - \theta_a^+}(a_1^a - a_0^a) \\ \frac{a_i^a + a_i^b}{2}, \; i = 2 \ldots M-2 \\ a_M^b + \frac{\theta_b^- - \theta_a^+}{\theta_a^- - \theta_a^+}(a_{M-1}^b - a_M^b) \\ a_M^b \end{bmatrix} \quad \Gamma_b = \begin{bmatrix} a_0^b \\ a_1^b \\ \vdots \\ a_{M-1}^b \\ a_M^b \end{bmatrix}$$

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Movie: Transitioning

RABBIT_transitioning_HIGH.wmv

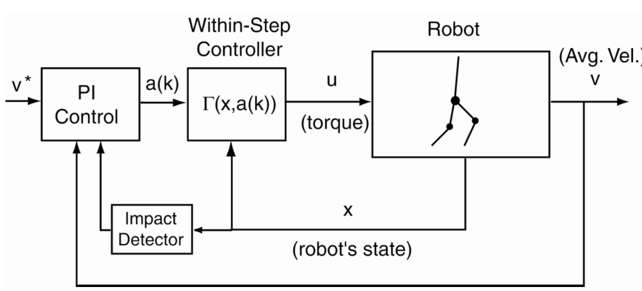
Available at:
http://www.mecheng.osu.edu/~westerve/thesis_documentation/movies/March03/RABBIT_transitioning_HIGH.wmv

Event-Based PI Control

Key Idea:

Use the parameters of the within-step controller as control knobs

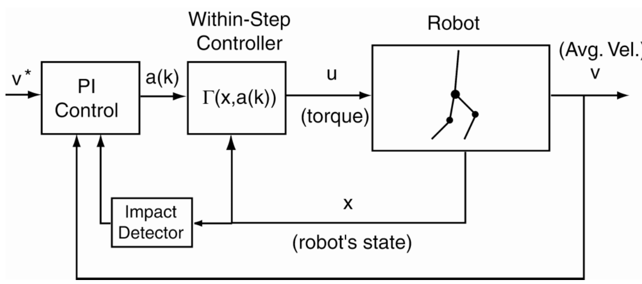
- Do not destroy invariance
- Modify "posture" to change speed



Event-Based PI Control

Well-defined relative degree + exponentially stable equilibrium point yields:

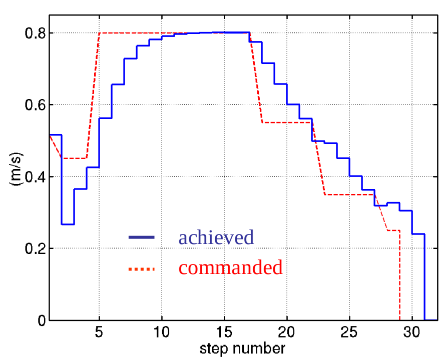
$$\begin{aligned} z(k+1) &= \rho(z(k), a + \delta a w(k)) & e(k+1) &= e(k) + (\eta^* - \eta(k)) \\ \eta(k) &= v(z(k)) & w(k) &= K_P(\eta^* - \eta(k)) + K_I e(k) \end{aligned}$$

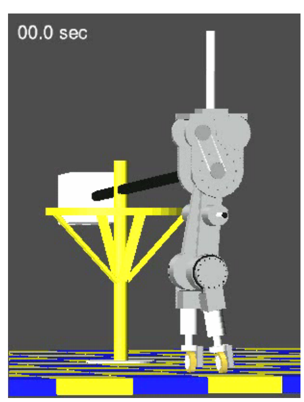


 Eric Westervelt (The Ohio State University) and Jessy Grizzle (University of Michigan)

Putting It All Together: Variable Speed Profile

- Transition control
- Event-based PI control





 Eric Westervelt (The Ohio State University) and Jessy Grizzle (University of Michigan)

Movie: PI, Rejecting a disturbance

RABBIT_perturbation_rejection_HIGH.wmv

Available at:

http://www.mecheng.osu.edu/~westerve/thesis_documentation/movies/March03/RABBIT_perturbation_rejection_HIGH.wmv

Movie: PI, Stopping

RABBIT_stopping_HIGH.wmv

Available at:

http://www.mecheng.osu.edu/~westerve/thesis_documentation/movies/March03/RABBIT_stopping_HIGH.wmv

Conclusions

- Introduced a common framework for the systematic, design, analysis, and performance enhancement of controllers that induce stable walking motions…

  …via hybrid zero dynamics: an invariant sub-dynamic of the hybrid robot model

- Developed tools for the composition of controllers and the modification of fixed points

- Initial experiments illustrate the practicality of the approach