Passivity Based Control of Bipedal Walking Robots

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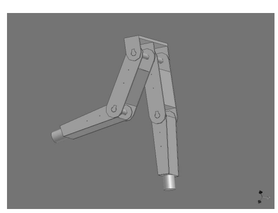
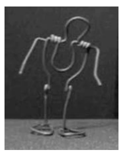
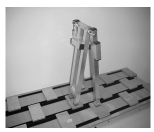
Outline of Presentation



- Passive Walking in 2D and 3D
- Group Actions and Invariance
- Controlled Symmetry and Potential Energy Shaping
- Passivity Based Nonlinear Control
- Simulations

Passive Walking in 2D and 3D

- It is well known that locomotion of simple mechanisms is achievable passively $\rightarrow$ i.e., without actuation
- McGeer, T., "Passive Dynamic Walking," International Journal of Robotics Research, 1990.
- Goswami, et.al., "A Study of the Passive Gait of a Compass-like Biped Robot: Symmetry and Chaos," International Journal of Robotics Research, 1998.
- Such mechanisms can exhibit stable walking down a constant incline, turning gravitational potential energy into kinetic energy of motion.
- 3D Wirewalker 
- 2D Passive Walker 

3-D Passive Walking

Passive Walking in 3-D has been investigated in

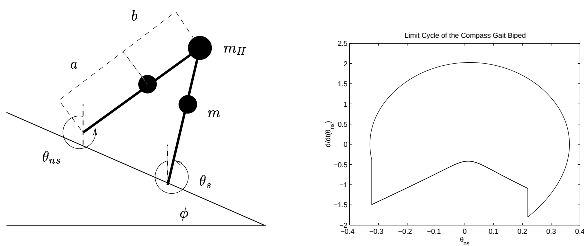
- Kuo, A. D., "Stabilization of Lateral Motion in Passive Dynamic Walking," Int. Journal of Robotics Research, 1999.
- Collins, SH, Wisse, M, Ruina, A„ "A 3-D passive-dynamic walking robot with two legs and knees", International Journal of Robotics Research, 2001.
- 3D Walker of Collins, Wisse, & Ruina 

  videos courtesy of Martijn Wisse, Delft University of Technology, http://mms.tudelft.nl/Dbl/

How Does This Work?

Walking involves the interaction among
—Kinetic Energy
—Potential Energy
—Impacts

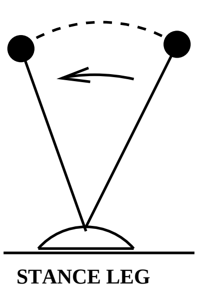
Impacts (foot/ground, knee strike) cause jumps in velocity $\longrightarrow$ a loss of Kinetic Energy
A passive limit cycle results when the loss of kinetic energy equals the change in potential energy during the step.



The Compass Gait Biped

Human Walking

- In fact, in human walking the leg muscles are active primarily during the beginning of the Swing Phase after which they shut off and allow the swing leg to swing through like a Free Pendulum
- in the Stance Phase the stance leg acts as an inverted pendulum.
- Thus evolution has taught us to exploit passive dynamics which may help to account for the efficiency of biological locomotion.



Properties of Passive Gaits

In this Compass Gait example:

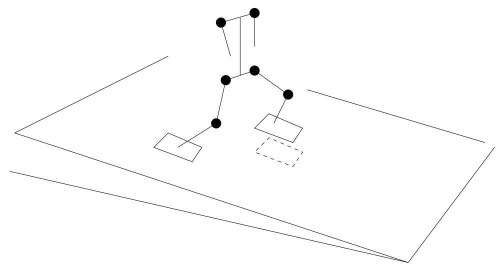
- The limit cycle is extremely sensitive to slope angle
- As the slope is increased period doubling bifurcations leading to eventual chaos occur
- The basin of attraction of the limit cycle is very small

These issues can be addressed by the addition of feedback control. In this talk we will show how some concepts from geometric mechanics and nonlinear control can be used to generate stable walking that exploits the passive walking idea and overcomes the above limitations.

Our Results

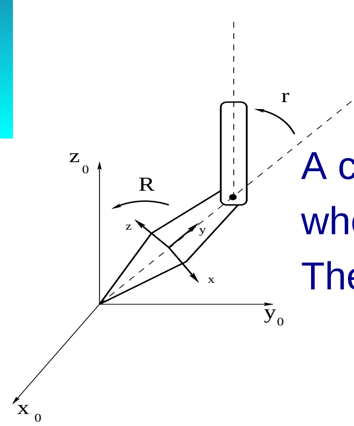
- We will show how feedback control can completely remove the sensitivity to ground slope — specifically, we will make the passive limit cycle "slope invariant" via active control.
- These results rely on some symmetry properties in the Lagrangian dynamics of mechanical systems together with so-called *Potential Energy Shaping*
- We will also discuss *Passivity-Based Control* to increase the basin of attraction of the stable limit cycle.

Shape Variables and Configuration Space



- Consider an n-DOF biped in $3D$ during the single support phase.
- We assume that the stance leg is rigidly attached to the ground (no slipping) and that the first link has 3-degrees-of-freedom with respect to the ground.
- Each subsequent link allows a single degree-of-freedom (a rotation)

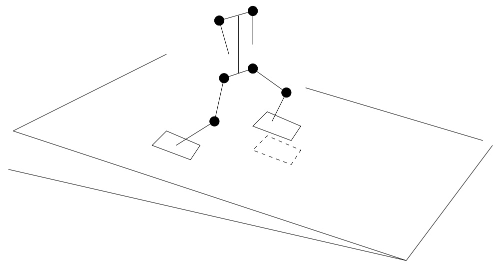
Shape Variables and Configuration Space



A configuration is then an ordered pair $q = (R, r)$
where $R \in \mathrm{SO}(3)$ and $r \in T^{n-1}$ is the $n-1$-torus
The Configuration Space is $Q = \mathrm{SO}(3) \times T^{n-1}$

R is the orientation of the first link, and r is the shape of the multi-body chain, for example the angle of each link referenced to the previous link.

Shape Variables and Configuration Space



- The advantage of this convention is that only the first degree of freedom is reference to an absolute or world frame.
- The remaining joint variables are then invariant under a change of basis of the world frame.
- Configuration spaces that can be written as the Cartesian product of a Lie group and a shape space are referred to as Principal Bundles.
- In the case of a planar mechanism $Q = \mathrm{SO}(2) \times S$ and, in this case, we may identify Q with T^n since elements of $\mathrm{SO}(2)$ can be represented by scalars (angles).

Lagrangian Dynamics

Recall that the Lagrangian is $\mathcal{L}(q,\dot q) = \mathcal{K}(q,\dot q) - \mathcal{V}(q)$ is the difference between the kinetic and potential energies.
We define the Euler-Lagrange Operator, $L(t,q,\dot q)$, as

$$L(t,q,\dot q) = \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot q} - \frac{\partial \mathcal{L}}{\partial q}$$

The Euler-Lagrange equations can thus be written as

$$L(t,q,\dot q) = u(t)$$

where u is the control input. $u = 0$ in the case of passive walking.
In local coordinates we can write

$$M(q)\ddot q + C(q,\dot q)\dot q + g(q) = u(t)$$

With regard to the foot/ground impact, we make the standard assumptions, namely,

- impacts are perfectly inelastic (no bounce)
- transfer of support between swing and stance legs is instantaneous
- there is no slipping at the stance leg ground contact

Under these assumptions each (impulsive) impact results in an instantaneous jump in velocities, whereas the position variables are continuous through the impact.

The change in velocity at impact is found by integrating the Euler-Lagrange equations over the (infinitesimally small) duration of the impact event and leads to:

$$\dot{q}(t^+) - \dot{q}(t^-) = M(q)^{-1} F(q,t)$$

where $F(q,t) = \int_{t^-}^{t^+} \delta F(q,\tau) d\tau$ represents the contact forces over the impact event $[t^-, t^+]$.
Let $h(q) = 0$ represent a constraint defining the ground impact.
The function h is determined, for example, by the forward kinematics of the robot, i.e., the impact event occurs when the end of the swing leg contacts the ground surface.
The contact force F is then aligned with the force of constraint $dh \in T^*Q$ so that there exists a scalar function $f(t)$, i.e., the magnitude of the contact force during the impact, such that $F(q,t) = f(t) dh(q)$.
Thus

$$\dot{q}(t^-) = \dot{q}(t^+) - f(t) M(q)^{-1} dh.$$

$$\dot{q}(t^-) = \dot{q}(t^+) - f(t)M(q)^{-1}dh.$$

Also note that the evolution after the impact is such that $h(q) = 0$, and therefore $\dot{q}(t^+)$ must be in the annihilator of the force of constraint dh, that is $dh \cdot \dot{q}(t^+) = 0$.
Geometrically this means that $\dot{q}(t^+)$ is perpendicular with respect to the M-inner product to the vector $M(q)^{-1}dh$.
In summary, the impact dynamics may therefore be represented as

$$\dot{q}(t^+) = P_q(\dot{q}(t^-)).$$

where the *plastic projection* P_q for the impact occurring at $h(q) = 0$ is the $M(q)$-orthogonal projection of $\dot{q}(t^-)$ onto $\{v \in T_qQ |\ dh(q) \cdot v = 0\}$.
Putting these previous notions together leads to a hybrid dynamical system

$$\begin{array}{rcll} \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{q}} - \frac{\partial \mathcal{L}}{\partial q} & = & u, & \text{for } h(q(t^-)) \neq 0 \\ q(t^+) & = & q(t^-) & \text{for } h(q(t^-)) = 0 \\ \dot{q}(t^+) & = & P_q(\dot{q}(t^-)) & \end{array}$$

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Definition Let Q be a differentiable manifold and G be a Lie group.
G is said to act on Q if there is a mapping $\Phi : G \times Q \to Q$ taking a pair (g, q) to $\Phi(g, q) = \Phi_g(q) \in Q$ and satisfying for all $q \in Q$

- $\Phi_e(q) = q$, where e is the identity element of G, and
- $\Phi_{g_1}(\Phi_{g_2}(q)) = \Phi_{g_1 g_2}(q)$.

Example $SO(3)$ acts on $\Re^3$ via the usual multiplication of a vector by a matrix
Let T_qQ be the linear space of tangent vectors at q, and let g be an element in G.Let $T_q\Phi_g$ be the tangent map to Φ_g mapping T_qQ onto $T_{\Phi_g(q)}Q$. We call $T_q\Phi_g$ the lifted action.

Definition Let $F : M \to N$ be a smooth mapping between manifolds M and N and let $\Phi : G \times M \to M$ be an action of the Lie Group G on M. Then we say that

(i) F is *Invariant* under the group action if $F \circ \Phi = F$, i.e., if, for all $g \in G$ and $m \in M$

$$(F \circ \Phi_g)(m) = F(m)$$

(ii) F is *Equivariant* if there exists an associated group action $\tilde{\Phi} : G \times N \to N$ such that $F \circ \Phi = \tilde{\Phi} \circ F$ in the sense that for all $g \in G$

$$(F \circ \Phi_g)(m) = (\tilde{\Phi}_g \circ F)(m) \text{ for all } m \in M$$

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For example, the distance from the origin is invariant under the action of $SO(3)$ on $\Re^3$

A group action on Q also induces corresponding maps on scalar functions over Q (e.g., potential energy), tangent vectors and vector fields(e.g., the system's instantaneous velocity), and one forms (e.g., the external forces applied to the system)

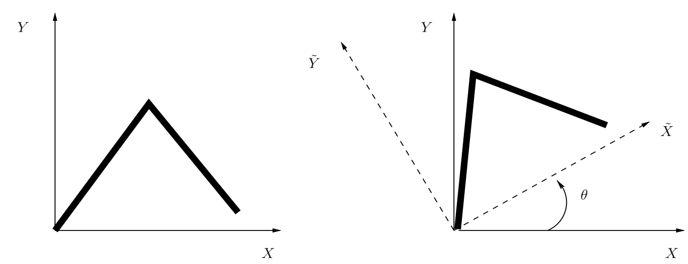
The notion of invariance and equivariance under the group action extends to the induced maps.

Slope Changing Action

- The act of changing the ground slope at the stance leg can be represented by the usual action of $\mathrm{SO}(3)$ on $\Re^3$.
- We may likewise define an action of $SO(3)$ on Q, as a map Φ from $(A, q) = (A, (R, r)) \in \mathrm{SO}(3) \times Q$ into Q such that

$$\Phi(A, (R, r)) = \Phi_A(R, r) = (A \cdot R, r)$$

These group actions are illustrated below in the planar case.



Equivariance of the kinematics and impacts

Let w be the map $w : Q \to \mathbb{R}^3$, that associates to each configuration $q \in Q$ of the biped the coordinates of the tip of the swing leg.

Then it follows, for $A \in SO(3)$ that

$$Aw(q) = Aw(R, r) = w(AR, r) = w(\Phi_A(q))$$

and hence w is equivariant with respect to $SO(3)$.

Example

Example Let w be the forward kinematics map of a two-link planar robot. Then

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \ell_1 \cos(q_1) + \ell_2 \cos(q_1 + q_2) \\ \ell_1 \sin(q_1) + \ell_2 \sin(q_1 + q_2) \end{bmatrix}.$$

For $A \in SO(2)$ it is easy to see that

$$w(\Phi_A(q)) = \begin{bmatrix} \ell_1 \cos(q_1 + \theta) + \ell_2 \cos(q_1 + \theta + q_2) \\ \ell_1 \sin(q_1 + \theta) + \ell_2 \sin(q_1 + \theta + q_2) \end{bmatrix}.$$

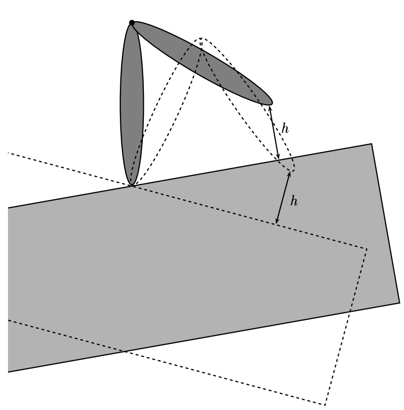
Likewise, a simple calculation using a trigonometric identity, shows

$$\begin{aligned} Aw(q) &= \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} \ell_1 \cos(q_1) + \ell_2 \cos(q_1 + q_2) \\ \ell_1 \sin(q_1) + \ell_2 \sin(q_1 + q_2) \end{bmatrix} \\ &= \begin{bmatrix} \ell_1 \cos(q_1 + \theta) + \ell_2 \cos(q_1 + \theta + q_2) \\ \ell_1 \sin(q_1 + \theta) + \ell_2 \sin(q_1 + \theta + q_2) \end{bmatrix} = w(\Phi_A(q)). \end{aligned}$$

Note that, in this case, the height of the tip of the swing leg is precisely the second component of the forward kinematics

$$h(q) = \ell_1 \sin(q_1) + \ell_2 \sin(q_1 + q_2)$$

Because the forward kinematic maps as well as the ground surface are equivariant, the height of the leg is equivariant.



Invariance Properties

We have the following *(Spong, Bullo, IEEE Transactions on Automatic Control, submitted, August, 2003)*

Proposition

- The kinetic energy $\mathcal{K}$ is invariant under the slope changing action Φ, i.e., for all $A \in \mathrm{SO}(3)$

$$\mathcal{K}(q, \dot{q}) = \mathcal{K}(\Phi_A(q), T_q\Phi_A(\dot{q}))$$

- The velocity changes at impacts, $\dot{q}^+ = P_q(\dot{q}^-)$, are invariant under the slope changing action.

Note that the Potential Energy is not invariant under the slope changing action!

Controlled Symmetries

A Symmetry in a mechanical system arises when the Lagrangian is invariant under a group action Φ, i.e.

$$\mathcal{L}(q, \dot{q}) = \mathcal{L}(\Phi_g(q), T_q\Phi_g(\dot{q})) \quad \text{for all } g \in G$$

Symmetries give rise to conserved quantities, for example, translational symmetry gives rise to conservation of momentum, etc.

Controlled Symmetries

Definition *Controlled Symmetry*
We say that an Euler-Lagrange system has a Controlled Symmetry with respect to a group action Φ if, for every $g \in G$ there exists an admissible control input $u_g(t)$ such that

$$L(t, q, \dot{q}) - u_g(t) = L(t, \Phi_g(q), T_q\Phi_g(\dot{q}))$$

We seek to create symmetries that conserve walking gaits (limit cycle trajectories) on arbitrary slopes
This is an interesting contrast to other work (cf: Bloch, Leonard, Marsden, et.al.) that uses symmetry breaking controls to stabilize equilibria

Potential Shaping

Since the Kinetic Energy is invariant under the slope changing actions, it follows that one need only compensate the Potential Energy in order to make the Lagrangian invariant. We can formalize this result as:
Theorem: Let $A \in \mathrm{SO}(3)$ and define control input u_A as

$$u_A = \frac{\partial}{\partial q}\Big(\mathcal{V}(q) - \mathcal{V}(\Phi_A(q))\Big).$$

Then

$$L(t, q, \dot{q}) - u_A = L(t, \Phi_A(q), T_q\Phi_A(\dot{q}))$$

i.e., u_A defines a Controlled Symmetry for the Lagrangian dynamics of an arbitrary n-DOF biped

Thus, if $(q(t), \dot{q}(t))$ is a solution trajectory

$$L(t, q, \dot{q}) = 0$$

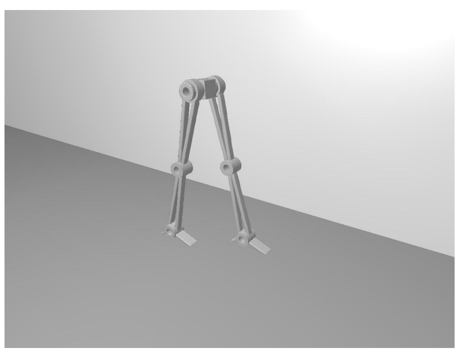
Then $\Phi_A(q)(t), T_q\Phi_A(\dot{q}(t))$ is a solution trajectory for the controlled biped

$$L(t, q, \dot{q}) - u_A = L(t, \Phi_A(q), T_q\Phi_A(\dot{q})) = 0$$

Corollary: Suppose there exists a passive gait on one ground slope, represented by $A_0 \in SO(3)$, and let $A \in SO(3)$ represent any other slope. Then the control input $u_{A^T A_0}$ generates a walking gait on slope A. Moreover the basin of attraction of the passive gait is mapped to the basin of attraction of the controlled gait.

Proof: Follows from the above theorem and the fact that the change of velocities at impacts are invariant.

Examples and Simulations



Finding Passive Limit Cycles

Passive Limit Cycles are investigated via the Poincaré Map.

$$X_{k+1} = P(X_k)$$

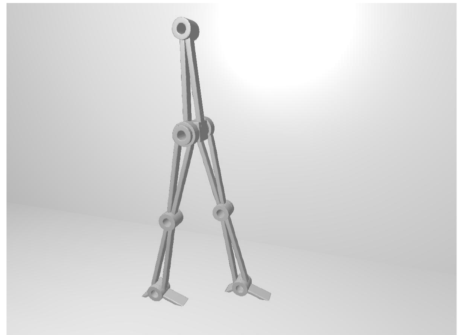
where X_k is the vector of joint positions and velocities at the beginning of step k.

The limit cycle is thus a fixed point of the Poincaré Map. Any initial condition in the basin of attraction of a stable limit cycle for one slope can be used to initialize a controlled gait on any other slope.

The video shows walking on level ground

Extension to Biped with Torso

The next video shows a biped with a torso walking on level ground



How is this done?
A PD-type control is used to stablize the torso in the inverted position. The resulting system is analyzed via the Poincaré map to find a stable limit cycle. The details are omitted.

Part II: Passivity Based Control

- Having made the passive limit cycles slope invariant via potential energy shaping we now investigate total energy shaping for robustness.
- Improving the rate of convergence to the limit cycle and increasing the basin of attraction are needed for robustness to external disturbances, changes in ground slope, etc.
- We now take the control input as

$$u = u_A + \bar{u}$$

  where $\bar{u}$ is to be determined
- In other words we consider the system

$$L(t, \Phi_A(q), T_q\Phi_A(\dot{q})) = \bar{u}$$

- Define the Storage Function $\mathcal{S}$ as

$$\mathcal{S} = \frac{1}{2}(E - E_{ref})^2$$

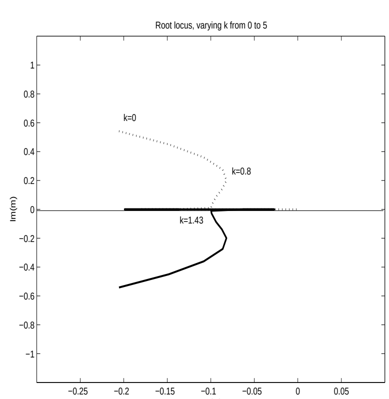
where E_{ref} is the total energy of the biped along the limit cycle trajectory of the system.

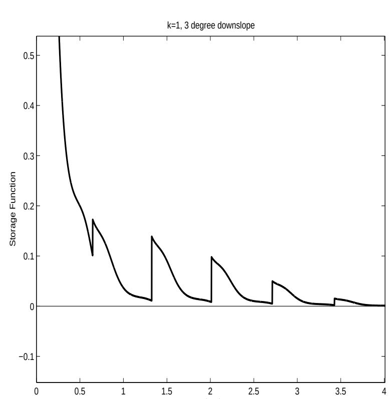
- From the usual passivity property of robot dynamics, $\dot{E} = \dot{q}^T \bar{u}$ and hence
- if we choose the additional control $\bar{u}$ according to

$$\bar{u} = -k\dot{q}(E - E_{ref})$$

- we obtain

$$\dot{\mathcal{S}} \quad = \quad (E - E_{ref})\dot{q}^T \bar{u} = -k||\dot{q}||^2 S$$

- Thus $S(t)$ converges exponentially toward zero during each step[a]
- At impacts, S will experience a jump discontinuity. If the value of S at impact $k+1$ is less than it's value at impact k it follows that $E(t)$ converges to E_{ref}.
- 



- Simulation: Walking on a Varying Slope