Planar Biped Locomotion: Background & Modeling

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Outline

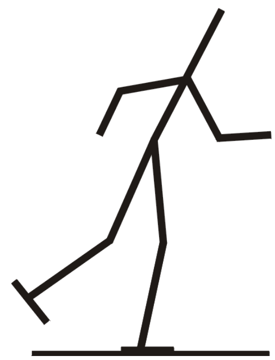
- Why is bipedal locomotion a hard problem
- Heuristics and the ZMP principle
- What is RABBIT & why this morphology
- Systematic derivation of planar models
  - Lagrangian dynamics for SS (continuous portion)
  - Rigid impact model for DS (discrete portion)
  - System with impulse effects
- Automating model computations within MATLAB
  - Files available online at :

www.eecs.umich.edu/~grizzle/CDC2003Workshop/

Why biped locomotion is hard

Inherent difficulties:

1. High DOF system with low DOF task



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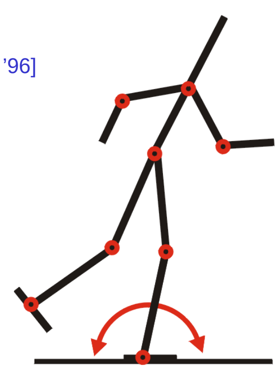
2. Effectively underactuated [Kajita '96]



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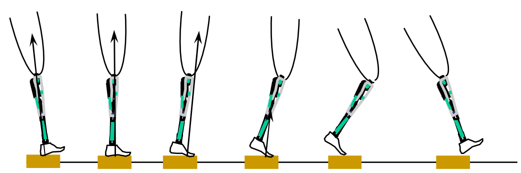

Normal gait human gait has underactuated phases

Why biped locomotion is hard

Inherent difficulties:

1. High DOF system with low DOF task
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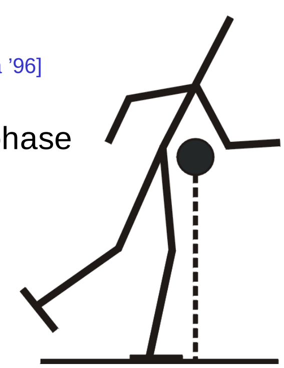
3. Static instability during swing phase



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Inherent difficulties:

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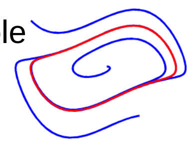
4. Desire periodic motions that are stable



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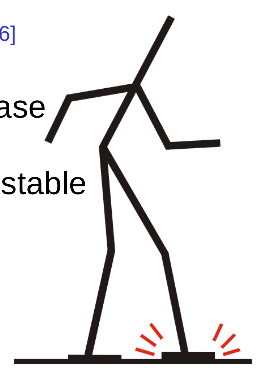
Why biped locomotion is hard

Inherent difficulties:

1. High DOF system with low DOF task
2. Effectively underactuated [Kajita '96]
3. Static instability during swing phase
4. Desire periodic motions that are stable
5. Impacts and/or nontrivial double support phase



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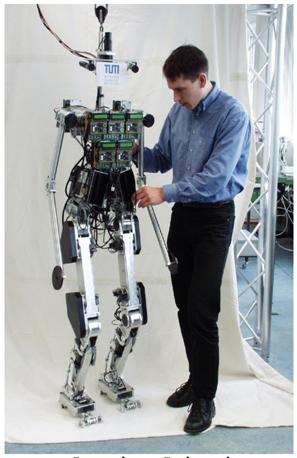
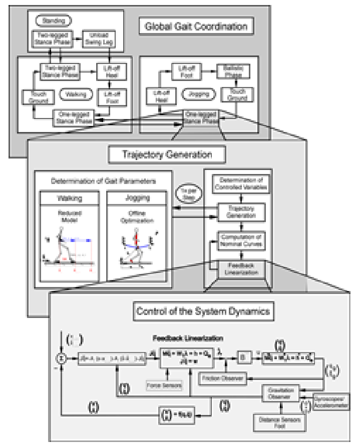
Consequence: Impressive Advances in Mechanisms, but *status quo* in Control Notions



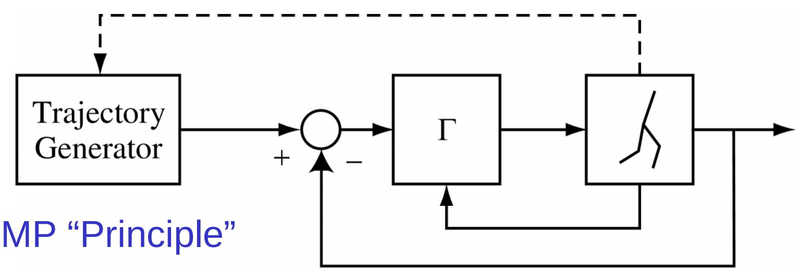
Jogging Johnnie



Control Architecture

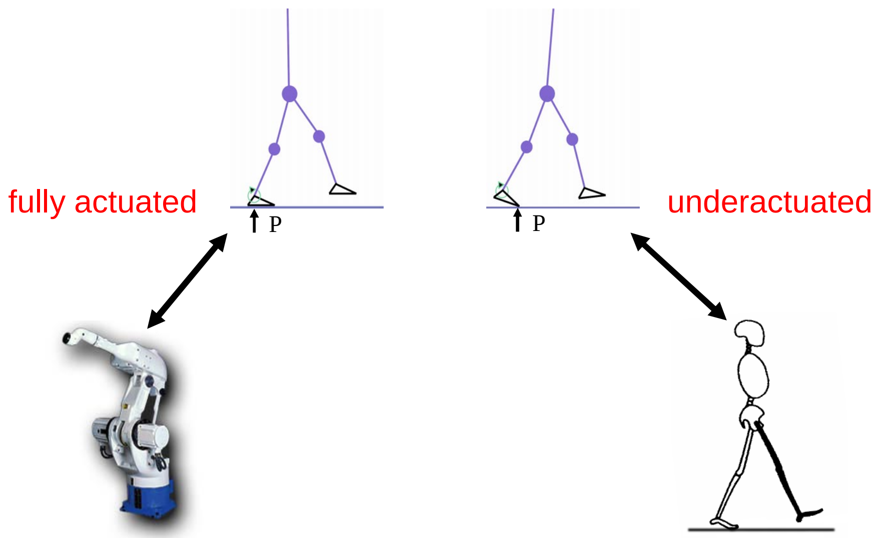
Control Block Diagram

ZMP: "Stability" is determined by the trajectory generator and NOT the within-stride feedback loop.



ZMP "Principle"

ZMP Principle (Heuristic)

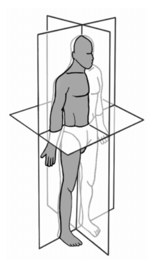


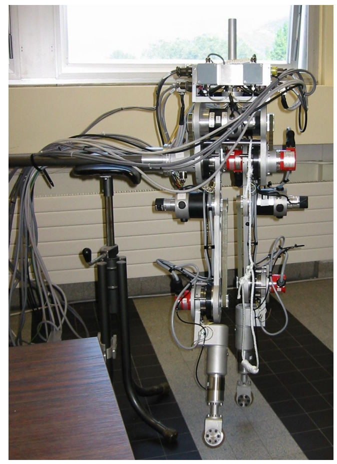
Minority View: Inherent Complexity does NOT Render Analysis Impossible!

- Carefully develop low level behaviors
- Obtain complex behaviors via composition of simpler behaviors
  - Koditschek (Michigan)
  - Krishnaprasad (Maryland)
  - The present workshop
- See also Workshop M-4: “Motion Description Languages for Multi-Modal Control”, CDC 2003

RABBIT: Simplest Mechanism Capable of Quasi-Anthropomorphic Gait

- Sagittal plane dynamics
- Two legs, knees, torso
- No feet = No ZMP = Need control theory!





ROBEA: **Robotique et Entités Artificielles (1997)**

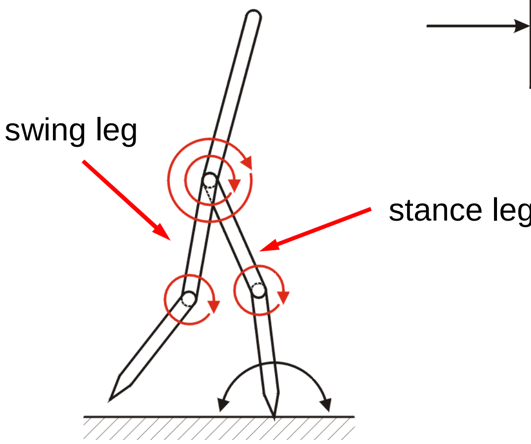
– Supported by **CNRS** and links seven French laboratories
  - Gabriel Abba, Carlos Canudas-de-Wit, C. Chevallereau, ….

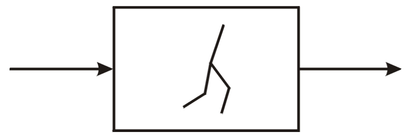
– Michigan joined in late 1998 through a sabbatical in Strasbourg; **NSF** support came in September 2000

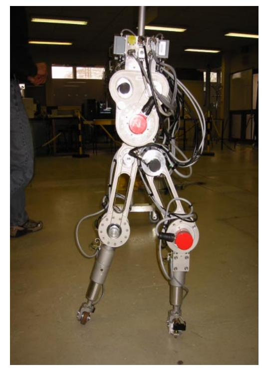
– Themes:
  - modeling of underactuated systems, systems with impacts
  - dynamic gaits (fast walking and running)
  - optimal motions
  - control of underactuated & hybrid systems
  - experimentation

– Robot named RABBIT:
  - LAG: Laboratoire Automatique de Grenoble
  - See Carlos Canduas-de-Wit for details! →

RABBIT Prototype





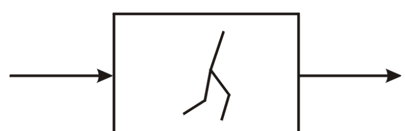


4 actuators and 5 links

32 kg mass and 1.425 m tall

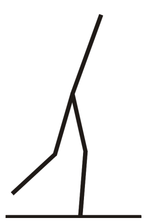
Robot Model: *SS* + *DS* = Hybrid

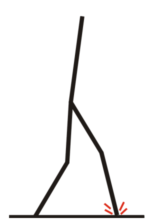
Normal walking:



… *SS*, *DS*, *SS*, *DS*, …

SS — Single Support *DS* — Double Support





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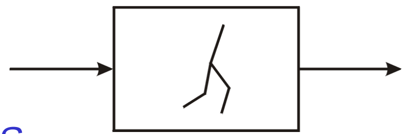

… SS, DS, SS, DS, …

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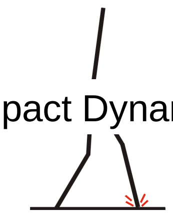
Lagrangian Dynamics



Impact Dynamics

Lagrangian Dynamics

Lagrange's Equation

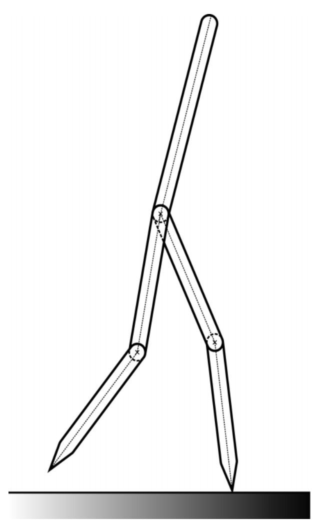
$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q} = \Gamma \qquad L = K - V$$

K = Kinetc Energy
V = Potential Energy

Γ = Forces & Torques

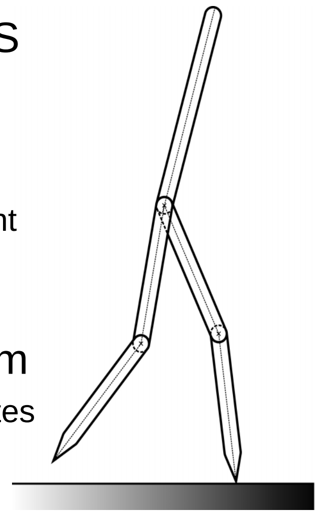
Typical Form in Robotics

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = Bu$$



Computing the Lagrangian

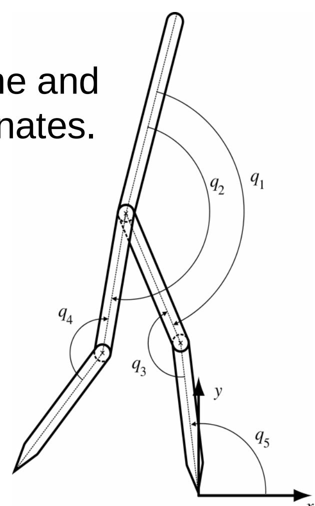
- Basic Assumptions in SS
  - Stance leg acts as a pivot
  - Swing leg is not interacting with the ground
  - Rigid links, rigid joints, no joint friction,…
- Five Degrees of Freedom
  - need 5 generalized coordinates



Computing the Lagrangian

Define a reference (world) frame and a set of five generalized coordinates.

$$q = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_5 \end{bmatrix}$$



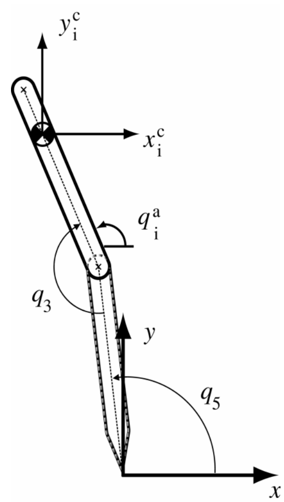
Computing the Lagrangian

- For each link, assign coordinates to
  - the center of mass

    $$\begin{bmatrix} x_i^c \\ y_i^c \end{bmatrix}$$

  - and the angle of the link with respect to the world frame

    $$q_i^a$$



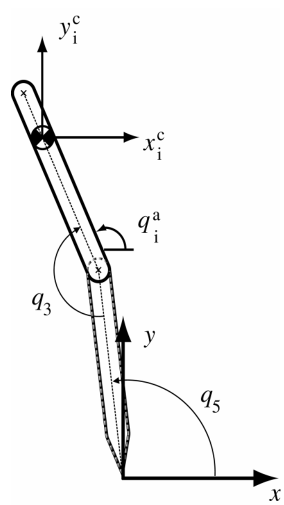
Computing the Lagrangian

- And express these as a function of the generalized coordinates, q
  - the center of mass

    $$\begin{bmatrix} x_i^c \\ y_i^c \end{bmatrix} = \begin{bmatrix} x_i^c(q) \\ y_i^c(q) \end{bmatrix}$$

  - angle of the link with respect to the world frame

    $$q_i^a = q_i^a(q)$$



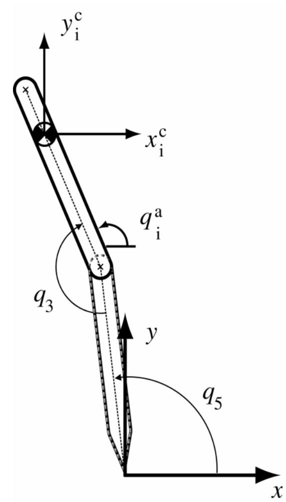
Computing the Lagrangian

- By the chain rule
  - velocity of center of mass

$$\begin{bmatrix} \dot{x}_i^c \\ \dot{y}_i^c \end{bmatrix} = \begin{bmatrix} \frac{\partial x_i^c(q)}{\partial q} \\ \frac{\partial y_i^c(q)}{\partial q} \end{bmatrix} \dot{q}$$

  - angular velocity of link

$$\dot{q}_i^a = \frac{\partial q_i^a(q)}{\partial q} \dot{q}$$



Computing the Lagrangian

Kinetic Energy of i-th Link

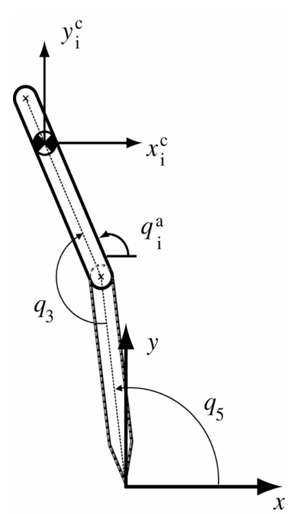
$$K_i = \frac{1}{2} m_i \left((\dot{x}_i^c)^2 + (\dot{y}_i^c)^2 \right) + \frac{1}{2} I_i (\dot{q}_i^a)^2$$

$$K_i = \frac{1}{2} \dot{q}^T D_i(q) \dot{q}$$

Potential Energy of i-th Link

$$V_i = m_i g y_i^c$$

$$V_i = m_i g y_i^c(q)$$



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Computing the Lagrangian

Total Kinetic Energy

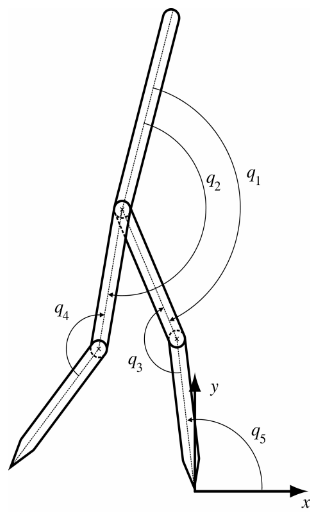
$K = \sum_{i=1}^{5} K_i = \frac{1}{2}\dot{q}^T D(q)\dot{q}$

Total Potential Energy

$V = \sum_{i=1}^{5} V_i$

Lagrangian

$L = K - V$



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Dynamic Model in SS

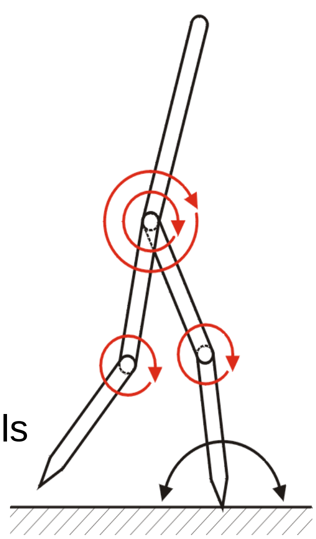
Lagrange's Equation

$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q} = \Gamma$

Form of the Equations

$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = Bu$

Underactuated: 5 DOF & 4 Controls



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A Property of the SS Model

Coordinates: 4 relative and 1 absolute

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_k} - \frac{\partial L}{\partial q_k} = \begin{cases} u_k & k = 1, \cdots, 4 \\ 0 & k = 5 \end{cases}$$

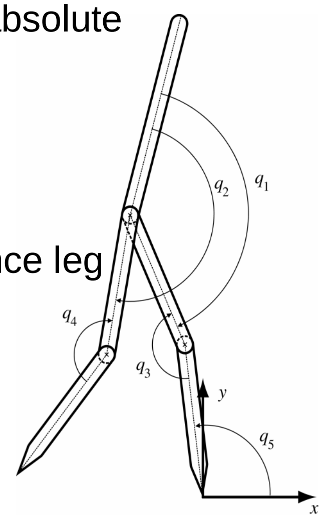
$$\frac{\partial K}{\partial q_5} = 0$$ cyclic coordinate

Angular momentum about stance leg

$$\sigma = \frac{\partial L}{\partial \dot{q}_5}$$

$$\sigma = \sum_{k=1}^{5} d_{5,k}(q_1, \cdots, q_4)\dot{q}_k$$

$$\dot{\sigma} = -\frac{\partial V}{\partial q_5}(q)$$



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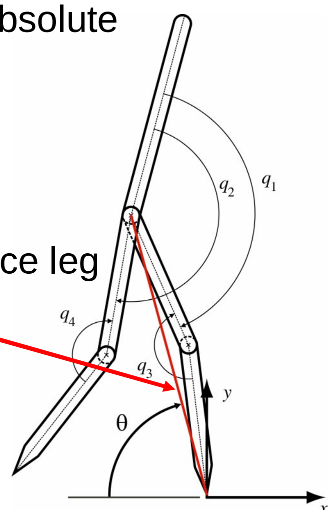
$q_5 = \theta$

Alternate choice of the absolute coordinate



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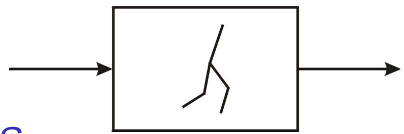

… *SS*, *DS*, *SS*, *DS*, …

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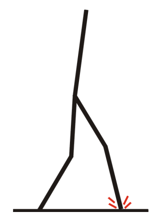
$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = Bu$$

joint angles & velocities   motor torques



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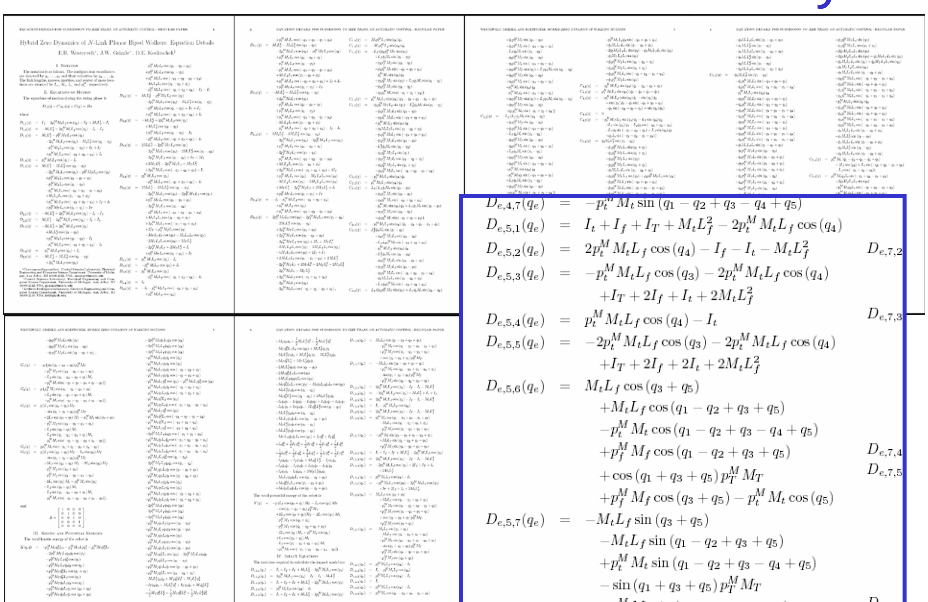
Terms in the Model…Oh my!



$$
\begin{aligned}
D_{e,4,7}(q_e) &= -p_t^M M_t \sin(q_1 - q_2 + q_3 - q_4 + q_5) \\
D_{e,5,1}(q_e) &= I_t + I_f + I_T + M_t L_f^2 - 2p_t^M M_t L_f \cos(q_4) \\
D_{e,5,2}(q_e) &= 2p_t^M M_t L_f \cos(q_4) - I_f - I_t - M_t L_f^2 \\
D_{e,5,3}(q_e) &= -p_t^M M_t L_f \cos(q_3) - 2p_t^M M_t L_f \cos(q_4) \\
&\quad + I_T + 2I_f + I_t + 2M_t L_f^2 \\
D_{e,5,4}(q_e) &= p_t^M M_t L_f \cos(q_4) - I_t \\
D_{e,5,5}(q_e) &= -2p_t^M M_t L_f \cos(q_3) - 2p_t^M M_t L_f \cos(q_4) \\
&\quad + I_T + 2I_f + 2I_t + 2M_t L_f^2 \\
D_{e,5,6}(q_e) &= M_t L_f \cos(q_3 + q_5) \\
&\quad + M_t L_f \cos(q_1 - q_2 + q_3 + q_5) \\
&\quad - p_t^M M_t \cos(q_1 - q_2 + q_3 - q_4 + q_5) \\
&\quad + p_f^M M_f \cos(q_1 - q_2 + q_3 + q_5) \\
&\quad + \cos(q_1 + q_3 + q_5)\, p_T^M M_T \\
&\quad + p_f^M M_f \cos(q_3 + q_5) - p_t^M M_t \cos(q_5) \\
D_{e,5,7}(q_e) &= -M_t L_f \sin(q_3 + q_5) \\
&\quad - M_t L_f \sin(q_1 - q_2 + q_3 + q_5) \\
&\quad + p_t^M M_t \sin(q_1 - q_2 + q_3 - q_4 + q_5) \\
&\quad - \sin(q_1 + q_3 + q_5)\, p_T^M M_T \\
&\quad - p_f^M M_f \sin(q_1 - q_2 + q_3 + q_5) \\
&\quad - p_f^M M_f \sin(q_3 + q_5) + p_t^M M_t \sin(q_5)
\end{aligned}
$$

$D_{e,7,2}$ $D_{e,7,3}$ $D_{e,7,4}$ $D_{e,7,5}$ $D_{e,7,6}$ $D_{e,7,7}$

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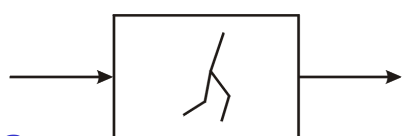
Using MATLAB to Obtain the Model

- Very convenient to derive model, compute control laws, and perform simulations in a common environment:
  - we have been using MATLAB
  - model derived using SYMBOLIC TOOLBOX
  - m-files for ODE45 or Simulink are automatically generated from the symbolic computations
  - relevant files can be downloaded at

www.eecs.umich.edu/~grizzle/CDC2003Workshop/

Robot Model: *SS* + *DS* = Hybrid

Normal walking:



… *SS, DS, SS, DS,* …

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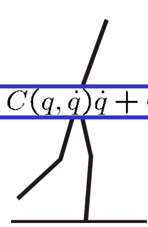
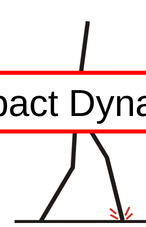


$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = Bu$$

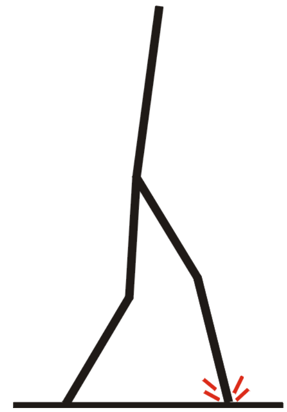


Impact Dynamics

DS: Rigid Impact Model

[Brach-1989, Hurmuzlu-Marghitu-1994]

- Basic Assumptions in DS
  - Impact instantaneous
    - $\Rightarrow$ impulsive contact forces
  - No rebound, no slip at impact
  - Former stance leg releases freely and does not interact with the ground
    - Yields conservation of angular momentum about impact point
    - Positions are continuous but **instantaneous jump** in the **velocities**
  - Relabel coordinates so that previous SS model can be reused



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DS: Rigid Impact Model

[Brach-1989, Hurmuzlu-Marghitu-1994]

$S = \{q \mid y_2(q) = 0, x_2(q) > 0\}$ Impact Surface

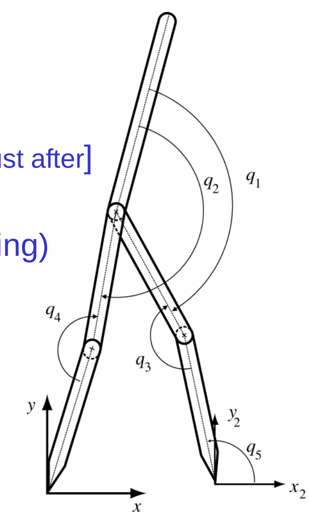
$q^+ = Rq^-$ (relabel states)

[- is just before impact and + is just after]

$\dot{q}^+ = \Delta_{\dot{q}} \dot{q}^-$ (jump in velocities + relabeling)

$$x^+ = \begin{bmatrix} q \\ \dot{q} \end{bmatrix}^+ = \begin{bmatrix} Rq^- \\ \Delta_{\dot{q}} \dot{q}^- \end{bmatrix} = \Delta(x^-)$$

$x^+ = \Delta(x^-)$ (overall effect of impact)



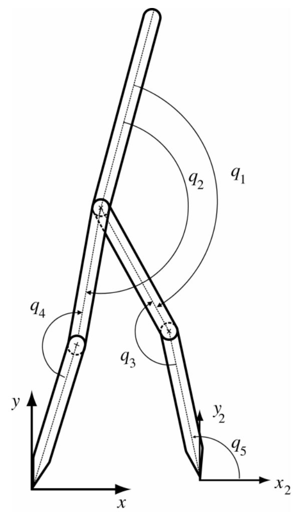
Dec. 7, 2003 Maui, Hawaii Feedback Control of Bipedal Walking Robots, Grizzle, Canudas-de-Wit, Westervelt, and Spong

DS: Rigid Impact Model

[Brach-1989, Hurmuzlu-Marghitu-1994]

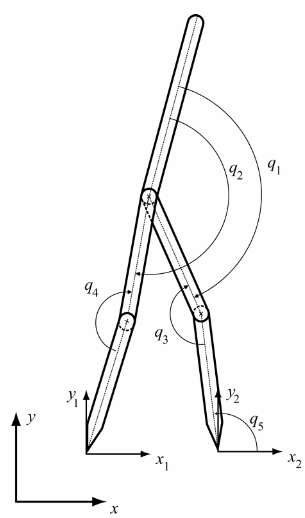
- Derivation is provided in handout, but is NOT covered in oral presentation
- See web site for MATLAB symbolic code

www.eecs.umich.edu/~grizzle/CDC2003Workshop/



DS: Rigid Impact Model

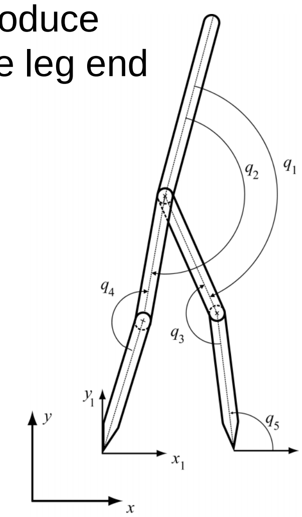
- Standard reference is: Y. Hurmuzlu and D.B. Marghitu, "Rigid Body Collisions of Planar Kinematic Chains with Multiple Contact Points," IJRR, Vol. 13, No. 1, 1194, pp. 82-92.
- Step-by-step derivation in: J.W. Grizzle, G. Abba and F. Plestan, "Asymptotically Stable Walking for Biped Robots: Analysis via Systems with Impulse Effects," *IEEE T-AC*, Volume 46, No. 1, January 2001, pp. 51-64, is easier to read



DS: Rigid Impact Model

Need to use 7 DOF model. Introduce Cartesian coordinates of stance leg end

$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} \quad \text{and define} \quad q_e = \begin{bmatrix} q \\ x_1 \\ y_1 \end{bmatrix}$$



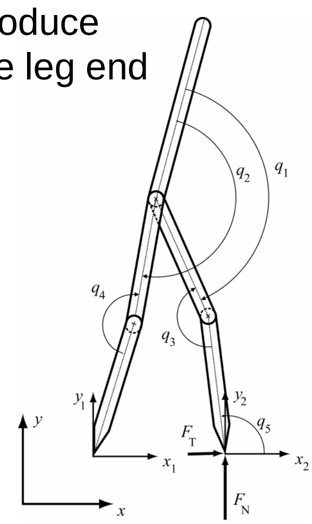
DS: Rigid Impact Model

Need to use 7 DOF model. Introduce Cartesian coordinates of stance leg end

$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} \quad \text{and define} \quad q_e = \begin{bmatrix} q \\ x_1 \\ y_1 \end{bmatrix}$$

Use Lagrange to Compute 7DOF Model

$$D_e(q_e)\ddot{q}_e + C_e(q_e, \dot{q}_e)\dot{q}_e + G_e(q_e) = \Gamma_e$$



DS: Rigid Impact Model

$\dot{q}_e^- =$ velocity just before impact

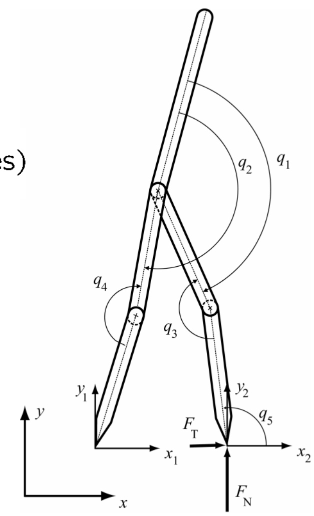
$\dot{q}_e^+ =$ velocity just after impact

$F_e = \begin{bmatrix} F_T \\ F_N \end{bmatrix} =$ impact force (intensities)

$$E = \begin{bmatrix} \frac{\partial x_2(q_e)}{\partial q_e} \\ \frac{\partial y_2(q_e)}{\partial q_e} \end{bmatrix}^T$$

$$\Gamma_e = B_e u + E F_e$$

Assumption: Impact forces are impulses while motor torques are not



DS: Rigid Impact Model

Conservation of momentum about impact point

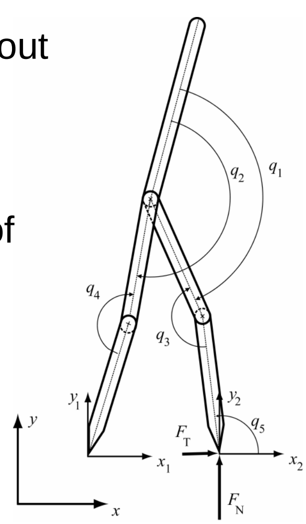
$$D_e \left(\dot{q}_e^+ - \dot{q}_e^- \right) = E F_e$$

No rebound nor slip at impact of swing leg end

$$E^T \dot{q}_e^+ = 0$$

Prior stance leg acted as pivot

$$\dot{q}_e^- = \begin{bmatrix} \dot{q}^- \\ 0 \\ 0 \end{bmatrix}$$

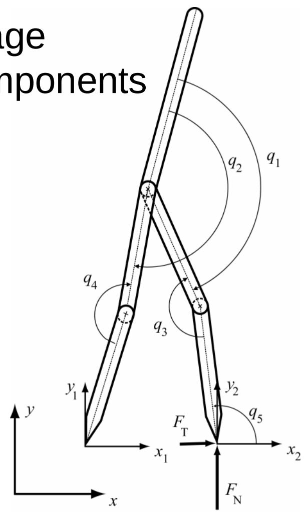


DS: Rigid Impact Model

Solve equations on previous page and then remove Cartesian components to obtain

$$\dot{q}^{+} = \bar{\Delta}_{\dot{q}}\dot{q}^{-}$$

$$F_e = A(q)\dot{q}^{-}$$



DS: Rigid Impact Model

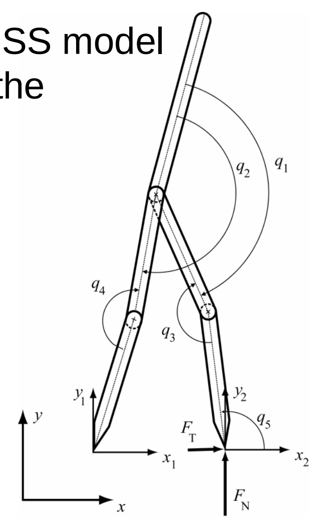
In order to re-use the previous SS model at the next step, must re-label the coordinates

$$q^{+} = Rq^{-}$$

$$\dot{q}^{+} = R\bar{\Delta}_{\dot{q}}\dot{q}^{-} = \Delta_{\dot{q}}\dot{q}^{-}$$

Final result is:

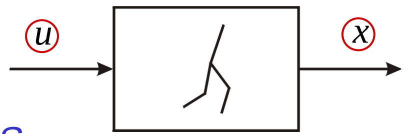
$$x^{+} = \begin{bmatrix} q \\ \dot{q} \end{bmatrix}^{+} = \begin{bmatrix} Rq^{-} \\ \Delta_{\dot{q}}\dot{q}^{-} \end{bmatrix} = \Delta(x^{-})$$



Dec. 7, 2003 Maui, Hawaii Feedback Control of Bipedal Walking Robots, Grizzle, Canudas-de-Wit, Westervelt, and Spong

Robot Model: *SS* + *DS* = Hybrid

Normal walking:

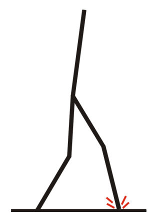


… *SS*, *DS*, *SS*, *DS*, …

SS — Single Support   *DS* — Double Support

$$\dot{x} = f(x) + g(x)u$$

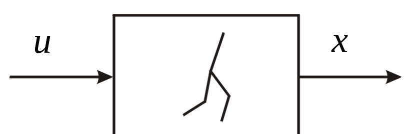
$$x = \begin{bmatrix} q \\ \dot{q} \end{bmatrix}$$



$$x^{+} = \Delta(x^{-})$$

Robot Model: *SS* + *DS* = Hybrid

Normal walking:



… *SS*, *DS*, *SS*, *DS*, …

SS — Single Support   *DS* — Double Support

10 differential equations & impact map

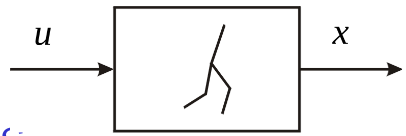
$$\dot{x} = f(x) + g(x)u \qquad SS$$

$$x^{+} = \Delta(x^{-}) \qquad DS$$

(Hybrid Model)

Robot Model: *SS* + *DS* = Hybrid

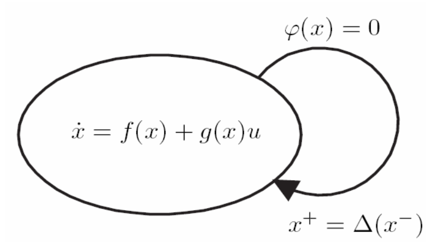
Normal walking:



... *SS*, *DS*, *SS*, *DS*, ...

SS — Single Support   *DS* — Double Support

10 differential equations & impact map



Model as System with Impulse Effects

$$\Sigma : \begin{cases} \dot{x}(t) = f(x(t)) & x^-(t) \notin S \\ x^+(t) = \Delta(x^-(t)) & x^-(t) \in S \end{cases}$$

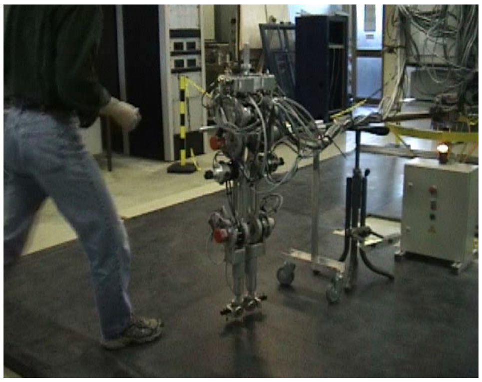
$x^+(t) := \lim_{\tau \searrow t} x(\tau)$ $x^-(t) := \lim_{\tau \nearrow t} x(\tau)$

Good reference: H.Ye, A.N. Michel, and L.Hou, "Stability theory for hybrid dynamical systems," IEEE T-AC,Vol.43(4), 1998, pp. 461—474.

$$S = \{x \in \mathcal{X} \mid \text{swing leg contacts the ground}\}$$

Stable walking ⇔ Asymptotically stable limit cycle (transversal to S)

Is the Model Complete?



- Standard friction compensation is used in the final stage of controller implementation
- Lagrangian model is crucial for inverse pendulum dynamics w.r.t. the support leg!

Conclusions: Background & Modeling

- Bipedal locomotion is a hard problem due to high DOF, impacts, periodic orbits, and underactuation
- Models are necessarily hybrid in nature
  - Lagrangian dynamics for SS (continuous portion)
  - Rigid impact model for DS (discrete portion)
  - System with impulse effects
- RABBIT conceived to enhance understanding
- Systematic derivation of planar model presented
- MATLAB tools shared with participants