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Feedback Control of Dynamic Bipedal Robot Locomotion

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Preface

The objective of this book is to present systematic methods for achieving stable, agile and efficient locomotion in bipedal robots. The fundamental principles presented here can be used to improve the control of existing robots and provide guidelines for improving the mechanical design of future robots. The book also contributes to the emerging control theory of hybrid systems. Models of legged machines are fundamentally hybrid in nature, with phases modeled by ordinary differential equations interleaved with discrete transitions and reset maps. Stable walking and running correspond to the design of asymptotically stable periodic orbits in these hybrid systems and not equilibrium points. Past work has emphasized quasi-static stability criteria that are limited to flat-footed walking. This book represents a concerted effort to understand truly dynamic locomotion in planar bipedal robots, from both theoretical and practical points of view.

The emphasis on sound theory becomes evident as early as Chapter 3 on modeling, where the class of robots under consideration is described by lists of hypotheses, and further hypotheses are enumerated to delineate how the robot interacts with the walking surface at impact, and even the characteristics of its gait. This careful style is repeated throughout the remainder of the book, where control algorithm design and analysis are treated. At times, the emphasis on rigor makes the reading challenging for those less mathematically inclined. Do not, however, give up hope! With the exception of Chapter 4 on the method of Poincaré sections for hybrid systems, the book is replete with concrete examples, some very simple, and others quite involved. Moreover, it is possible to cherry-pick one's way through the book in order to "just figure out how to design a controller while avoiding all the proofs." This is mapped out below and in Appendix A.

The practical side of the book stems from the fact that it grew out of a project grounded in hardware. More details on this are given in the acknowledgements, but suffice it to say that every stage of the work presented here has involved the interaction of roboticists and control engineers. This interaction has led to a control theory that is closely tied to the physics of bipedal robot locomotion. The importance and advantage of doing this was first driven home to one of the authors when a multipage computation involving the Frobenius Theorem produced a quantity that one of the other authors identified as angular momentum, and she could reproduce the desired result in two lines! Fortunately, the power of control theory produced its share of eye-opening moments on the robotic side of the house, such as when days and days of simulations to tune a "physically-based" controller were replaced by a ten minute design of a PI-controller on the basis of a restricted Poincaré map, and the controller worked like a champ. In short, the marriage of mechanics and control is evident throughout the book. The culture of control theory has inspired the hypothesis-definition-theorem-proof-example format of the presentation and many of the mathematical objects used in the analysis, such as zero dynamics and systems with impulse effects, while the culture of mechanics has heavily influenced the vocabulary of the presentation, the understanding of the control problem, the choice of what to control, and ways to render the required computations practical and insightful on complex mechanisms.

Target audience: The book is intended for graduate students, scientists and engineers with a background in either control or robotics—but not necessarily both of these subjects—who seek systematic methods for creating stable walking and running motions in bipedal robots. So that both audiences can be served, an extensive appendix is provided that reviews most of the nonlinear control theory required to read the book, and enough Lagrangian mechanics to be able to derive models of planar bipedal robots comprised of rigid links and joints. Taken together, the control and mechanics overviews provide sufficient tools for representing the robot models in a form that is amenable to analysis. The appendix also contains an intuitive summary of the method of Poincaré sections; this is the primary mathematical tool for studying the existence and stability of periodic solutions of differential equations. The mathematical details of applying the method of Poincaré sections to the hybrid models occurring in bipedal locomotion are sufficiently unfamiliar to both control theorists and roboticists that they are treated in the main part of the book.

Detailed contents: The book is organized into three parts: preliminaries, the modeling and control of robots with point feet, and the control of robots with feet. The preliminaries begin with Chapter 1, which describes particular features of bipedal locomotion that lead to mathematical models possessing both discrete and continuous phenomena, namely, a jump phenomenon that arises when the feet impact the ground, and differential equations (classical Lagrangian mechanics) that describe the evolution of the robot's motion otherwise. Several challenges that this mix of discrete and continuous phenomena pose for control algorithm design and analysis are highlighted, and how researchers have faced these challenges in the past is reviewed. The chapter concludes with an elementary introduction to a central theme of the book: a method of feedback design that uses virtual constraints to synchronize the movement of the many links comprising a typical bipedal robot. Chapter 2 introduces two bipedal robots that are used as sources of examples of the theory, RABBIT and ERNIE. Both of these machines were specifically designed to study the control of underactuated mechanisms experiencing impacts. A mathematical model of RABBIT is used in many of the simulation examples throughout the book. An extensive set of experiments that have been performed with RABBIT and ERNIE is reported in Chapter 8 and Section 9.9.

Part II begins with Chapter 3 on the modeling of bipedal robots for walking and running motions. For many readers, the differential equation portions of the models, which involve basic Lagrangian mechanics, will be quite familiar, but the presentation of rigid impacts and the interest of angular momentum will be new. The differential equations and impact models are combined to form a special class of hybrid systems called nonlinear systems with impulse effects. The method of Poincaré sections for systems with impulse effects is presented in Chapter 4. Some of the material is standard, but much is new. Of special interest is the treatment of invariant surfaces and the associated restricted Poincaré maps, which are the key to obtaining checkable necessary and sufficient conditions for the existence of exponentially stable walking and running motions. Also of interest is the interpretation of a parameterized family of Poincaré maps as a discrete-time control system upon which event-based or stride-to-stride control decisions can be designed. This leads to an effective means of performing event-based PI control, for example, in order to regulate walking speed in the face of model mismatch and disturbances. Chapter 5 develops the primary design tool of this book, the hybrid zero dynamics of bipedal walking. These dynamics are a low-dimensional controlled-invariant subsystem of the hybrid model that is complex enough to retain the essential features of bipedal walking and simple enough to permit effective analysis and design. Exponentially stable periodic solutions of the hybrid zero dynamics are exponentially stabilizable periodic solutions of the full-dimensional hybrid model of the robot. In other words, they correspond to stable walking motions of the closed-loop system. The hybrid zero dynamics is created by zeroing a set of virtual constraints. How to design the virtual constraints in order to create interesting walking gaits is the subject of Chapter 6. An extensive set of feedback design examples is provided in this chapter. The controllers of Chapter 6 are acting continuously within the stride of a walking motion. Chapter 7 is devoted to control actions that are updated on a stride-to-stride basis. The combined results of Chapters 6 and 7 provide an overall hybrid control strategy that reflects the hybrid nature of a bipedal robot. The practical relevance of the theory is verified in Chapter 8, where RABBIT—a reasonably complex mechanism—is made to walk reliably with just a few days of effort, and not the many months of trial and error that is customary. Part II of the book is concluded with a study of running in Chapter 9. A new element introduced in the chapter is, of course, the flight phase, where the robot has no ground contact; the stance phase of running is similar to the single support phase of walking. Chapter 9 develops natural extensions of the notions of virtual constraints and hybrid zero dynamics to hybrid models with multiple continuous phases. An extensive set of design examples is also provided. An initial experimental study of running is described in Section 9.9; the results are not as resoundingly positive as those of Chapter 8.

The stance foot plays an important role in human walking since it contributes to forward progression, vertical support, and initiation of the lifting of the swing leg from the ground. Working with a mechanical model, our colleague Art Kuo has shown that plantarflexion of the ankle, which initiates heel rise and toe roll, is the most efficient method to reduce energy loss at the subsequent impact of the swing leg. Part III of the book is therefore devoted to walking with actuated feet. Chapter 10 addresses a walking motion that allows anthropomorphic foot action. The desired walking motion is assumed to consist of three successive phases: a fully actuated phase where the stance foot is flat on the ground, an underactuated phase where the stance heel lifts from the ground and the stance foot rotates about the toe, and an instantaneous double support phase where leg exchange takes place. It is demonstrated that the feedback design methodology presented for robots with point feet can be extended to obtain a provably asymptotically stabilizing controller that integrates the fully actuated and underactuated phases of walking. By comparison, existing humanoid robots, such as Honda's biped, ASIMO, use only the fully actuated phase (i.e., they only execute flat-footed walking), while RABBIT and ERNIE use only the underactuated phase (i.e., they have no feet, and hence walk as if on stilts). To the best of our knowledge, no other methodology is available for integrating the underactuated and fully actuated phases of walking. Past work that emphasized quasi-static stability criteria and flat-footed walking has primarily been based on the so-called Zero Moment Point (ZMP) or, its extension, the Foot Rotation Indicator (FRI) point. Chapter 11 shows how the methods of the book can be adapted to directly control the FRI point during the flat-footed portion of a walking gait, while maintaining provable stability properties. Importantly, FRI control is done here in such a way that both the fully actuated and underactuated phases of walking are included. For comparison with more standard approaches, a detailed simulation study is performed for flat-footed walking.

Possible paths through the book: This book can be read on many different levels. Most readers will want to peruse Appendix B in order to fill in gaps on the fundamentals of nonlinear control or Lagrangian mechanics. The serious work can then start with the first three sections of Chapter 3, which develop a hybrid model of bipedal walking. The definition of a periodic solution to the hybrid model of walking, the notion of an exponentially stable periodic orbit and how to test for its existence via a Poincaré map are obtained by reading through Section 4.2.1 of Chapter 4. Chapters 5 and 6 then provide a very complete view on designing feedback controllers for walking at a single average speed. If Sections 5.2 and 5.3 seem too technical, then it is advised that the reader skip to Section 6.4, before completing the remainder of Chapter 5. After this, it is really a matter of personal interest whether one continues through the book in a linear fashion or not. A reader whose primary interest is running would complete the above program, read Section 7.3, and finish with Chapter 9, while a reader whose primary interest is walking with feet would proceed to Chapters 10 and 11, for example. For a reader whose interests lie primarily in theory, new results for the control of nonlinear systems with impulse effects are concentrated in Chapters 4 and 5, with several interesting twists for systems with multiple phases given in Chapters 9 and 10; the other parts of the book could be viewed as a simple confirmation that the theory seems to be worthwhile. The numerous worked-out examples and remarks on interesting special cases make it possible for a practitioner to avoid most of the theoretical considerations when initially working through the book. It is suggested to seek out the two-link walker (a.k.a., the Acrobot or compass biped) and three-link walker examples in Chapters 3, 5, and 6, which will provide an introduction to underactuation, hybrid models, the MPFL-normal form, virtual constraints, the swing phase zero dynamics, Bézier polynomials, optimization, and a systematic method to enlarge the basin of attraction of passive gaits. The reader should then be ready to read Chapter 8, with referral to previous chapters as necessary. Further ideas on how to work one's way through the book are given in Appendix A.

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Book webpage: Supplemental materials are available at the following URL:

www.mecheng.osu.edu/~westerve/biped_book/

The webpage includes links to videos of the experiments reported in the book, MATLAB code for several of the book's robot models, a link to submit errors found in the book, and an erratum.

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