EECS/IOE 600 WINTER 2021 SYLLABUS,
FUNCTION SPACE METHODS FOR SYSTEMS THEORY

This course will give you fundamental mathematical tools that are used in systems theory (signal processing, control, and optimization). Be prepared for a good time practicing abstract thinking and learning how to construct a beautiful proof!

Instructor: Professor Laura Balzano, girasole@umich.edu

Course time: Tuesdays and Thursdays, 10:30am-12pm US Eastern time

Course location: REMOTE VIA ZOOM. All lectures will be recorded.

Office hours: TBD. Tuesday 12pm US Eastern time for the first two weeks of class only.

Pre-requisites: No particular class is required, though real analysis, matrix analysis, and linear algebra are helpful. In general, a fluency with mathematics at the graduate level (at some level of abstraction) – or a desire to quickly gain this fluency – is required to do well.

Textbook: Optimization by Vector Space Methods by Luenberger.

Grade breakdown: HW 20%, Take-home Midterm Exam 30%, Theorem-Proof Video Project 40%, Participation 10%

Homework: Homework will be due every two weeks (dates TBD depending on office hours). You can work together but you must submit your own ideas and own work. Your lowest homework grade will be dropped.

Exams: This course normally has an in-class midterm and a take-home final. This semester we will instead have a single open-book take-home midterm exam and replace the final exam with a "project".

Theorem-Proof Video Project: You will choose a theorem and proof for which you will prepare a 10-15 minute video that teaches the theorem and its proof. Your video will present the theorem and then go step-by-step through its proof, justifying each step. Ideally you will do it using a tablet and digital writing device of some kind, or if you don't have access to this technology, you can make slides using Latex to go through during the video. The videos will be peer-graded as well as graded by the professor, and grading criteria will include clarity, correctness, level of difficulty, and mathematical precision/rigor.

Participation: There will be three main opportunities for participation: Piazza, Perusall, and iClicker. If you are regularly active and engaged in any one of these formats, you will get a high score for participation. Piazza is used as a message board — a place to ask and answer questions. Perusall is a place to annotate documents and/or videos — I will post my lecture notes *after* class for you to ask questions directly on the document. iClicker is a new technology for me so I will be learning as we go — I will ask questions during class to make sure everyone is following my derivations and proofs.

Honor policy: I take academic integrity very seriously. The College of Engineering is a community in which personal responsibility, honesty, fairness, respect, and mutual trust are maintained. You are expected to practice the highest possible standards of academic integrity. Any deviation from this expectation will result in a minimum academic penalty of your failing the assignment, and will result in additional disciplinary measures. This includes, but not limited to, cheating, using unauthorized material during exams, using or copying another student's work, and any other form of academic misrepresentation or misconduct. In particular, any 600 materials that are found on coursehero, chegg, etc will be immediately reported to the honor council.

For a list of actions that constitute misconduct, and possible sanctions for those actions, please see the Code of Conduct at `http://www.engin.umich.edu/college/academics/bulletin/rules`. I will not hesitate to enforce these standards in 600.

Commitment to equal opportunity: The Faculty of the COE are committed to a policy of equal opportunity for all persons and do not discriminate on the basis of race, color, national origin, age, marital

status, sex, sexual orientation, gender identity, gender expression, disability, religion, height, weight, or veteran status. Please feel free to contact your instructor with any problem, concern, or suggestion. We ask that all students treat each other with respect.

Diversity, Equity, and Inclusion: I consider this classroom to be a place where you will be treated with respect, and I celebrate individuals of all ages, backgrounds, beliefs, ethnicities, genders, gender identities, gender expressions, national origins, religious affiliations, sexual orientations, ability, and other visible and nonvisible differences. All members of this class are expected to contribute to a respectful, welcoming and inclusive environment for every other member of the class. I am dedicated to helping each of you achieve all that you can in this class. I may, either in lecture or smaller interactions, accidentally use language that creates offense or discomfort. Should I do this, please contact me to help me understand and avoid making the same mistake again. If you are someone whose background is underrepresented in electrical engineering or industrial engineering, I want to be your advocate.

Accommodations for Students with Disabilities: If you think you need an accommodation for a disability, please let you instructor know at your earliest convenience. Some aspects of this course may be modified to facilitate your participation and progress. As soon as you make us aware of your needs, we can work with the Services for Students with Disabilities (SSD) office to help us determine appropriate academic accommodations. SSD (734-763-3000; `http://ssd.umich.edu`) typically recommends accommodations through a Verified Individualized Services and Accommodations (VISA) form. Any information you provide is private and confidential and will be treated as such.

Student Mental Health and Wellbeing: University of Michigan is committed to advancing the mental health and wellbeing of its students, as am I. While I am not a mental health expert, I have personal experience with mental health struggles and I support you in identifying these challenges and seeking help. Especially during COVID times it is critical that you monitor and care for your mental health. If you or someone you know is feeling overwhelmed, depressed, and/or in need of support, services are available. For help, contact Counseling and Psychological Services (CAPS) at (734) 764-8312 and `https://caps.umich.edu/` during and after hours, on weekends and holidays, or through its counselors physically located in schools on both North and Central Campus. You may also consult University Health Service (UHS) at (734) 764-8320 and `https://www.uhs.umich.edu/mentalhealthsvcs`, or for alcohol or drug concerns, see `www.uhs.umich.edu/aodresources`. For a listing of other mental health resources available on and off campus, visit: `http://umich.edu/~mhealth/`.

Topics: Tentatively, we will cover the following topics.

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Vector spaces
 - definition
 - subspaces, linear ombinations, span
 - linear independence and basis

Function spaces
 - functions and mappings
 - vector spaces of mappings

Metric spaces
 - topology
 - sequences and convergence in metric spaces
 - fixed points and contraction mappings
 - continuous functions

Normed Vector spaces
 - Banach spaces
 - lp and Lp spaces
 - projections
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Inner Product spaces
 - Hilbert spaces
 - orthogonal complement, orthogonality principle
 - function approximation
 - the projection theorem
 - applications

Linear functions
 - Linear functionals
 - Dual space
 - Riesz Representation theorem
 - Hahn-Banach theorem
 - Linear operators
 - adjoint operators
 - solving linear equations

With time we will cover some subset of the following:

Spectral theory
 - The Spectral theorem
 - eigenvalues and eigenvectors of integral operators

Optimization using Lagrange multipliers
 - sufficient conditions for arbitrary functions
 - convex functions
 - necessary conditions for convex functions
 - Gateaux derivative
 - Kuhn-Tucker sufficiency
 - Frechet derivative

More results in optimization
 - Farkas lemma
 - Karush-Kuhn-Tucker theorem