Use of DSP concepts in System Identification

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***Abstract*— This work focuses on friction and table mass identification using signal processing techniques. This report describes the system, data collection process and analysis of the data.**

Keywords—Friction Curve Identification, Table Mass Identification, Least Squares, Sampling Frequency, Butterworth Filter

I. Introduction

Digital Signal Processing (DSP) tools are used in a wide range of applications. The focus of this work is to use DSP concepts to identify parameters (friction curve and mass of table) of electromechanical system shown in Fig. 1.

Section II discusses the theoretical aspects of the identification process. Section III describes the system identification process. Section IV concludes the paper.

II. Theory

A. Equation of Motion

The table can be actuated using Linear Motor (LM) and Rotary Motor (RM) as shown in Fig. 1. In this work, only LM is considered. In the absence of RM, the only forces acting on the table include LM force and friction between the table and guide ways. The equation of motion of the table is given by (1).

$$F_{LM} - F_f(v) = m_t a \quad (1)$$

F_{LM} is the force exerted by LM on table, $F_f(v)$ is the force on the table due to friction between table and guide ways, m_t is the mass of the table and a is acceleration of the table.

B. Stribeck Friction Model

The frictional force between the table and guide ways is a function of velocity of the table. From the measured data in Fig. 6 it is observed that the relationship between friction force and velocity follows Stribeck friction model given by (2) and (3). The variation of friction force with velocity for a typical Stribeck friction model is shown in Fig. 2.

$$F_f^+(v) = F^+_{stat}exp(-v/v_1^+) + F^+_{coul}(1-exp(-v/v_2^+)) + B^+v \quad (2)$$

$$F_f^-(v) = F^-_{stat}exp(-v/v_1^-) + F^-_{coul}(1-exp(-v/v_2^-)) + B^-v \quad (3)$$

The terms on the right hand side of (2) and (3) denote static, coulomb and viscous friction respectively. Plus superscript corresponds to motion along the positive direction whereas minus corresponds to opposite direction.

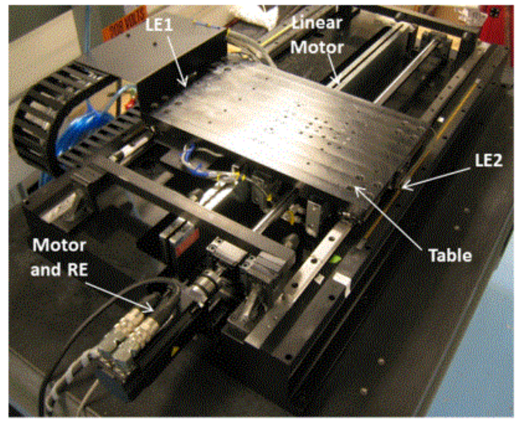

Figure 1: Electromechanical System

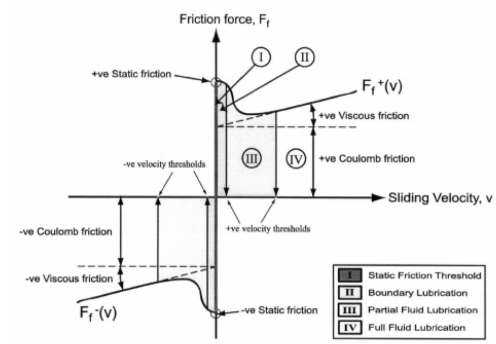

Figure 2: Stribeck Friction Model (Source: [1])

C. Least Squares

Least Squares is a standard technique used to approximate solution of overdetermined systems (when the number of equations is more than number of unknowns). The most common application of this technique is fitting data. This method estimates the parameters of a model ($\boldsymbol{\alpha}$) by minimizing the sum (S) of residuals ($\boldsymbol{r}$) between experimental data ($\boldsymbol{y}$) and data obtained ($f(\boldsymbol{x},\boldsymbol{\alpha})$) using the model as formulated in (4) and (5). The minimum is obtained by setting the gradient of S with respect to each parameter α_i to be zero. The technique can be classified as linear and non-linear.

$$S = \sum_{i=1}^{n} r_i^2 \quad (4)$$

$$r_i = y_i - f(x_i, \alpha) \quad (5)$$

If the model can be characterized as a linear combination of model parameters then linear least squares is used. The parameters of the model can be obtained using (6). X is a matrix of coefficients of linear combination and Y is vector of experimental data y.

$$\alpha = \left(\left(X^T X\right)^{-1} X^T\right)Y \tag{6}$$

Non-linear least squares have no closed-form solution. Iterative numerical algorithms are used to solve for the parameters. Most algorithms rely on initial value of parameters for accurate solution.

D. *Butterworth Filter*

Butterworth lowpass filters are defined by the property that the magnitude response is maximally flat in the passband and monotonic in passband and stopband [2]. As shown in Fig. 3, Butterworth filter rolls off more slowly around the cutoff frequency than the Chebyshev filters or elliptic filter. Also, it does not produce ripples. Hence, Butterworth filter is used in this work.

III. SYSTEM IDENTIFICATION

Two sets of data are collected, one for friction curve identification and another for table mass identification.

A. Friction Curve Identification

If the table moves at constant velocity, the friction force at that velocity is equal to the force exerted by the linear motor on the table according to (1). The table is commanded to move according to a trapezoidal velocity profile (constant velocity 500 mm/s) as shown in Fig. 4. The average of the force corresponding to constant velocity is equal to friction force at that constant velocity.

The LM acts as a current amplifier. The force on the table is the force constant (57 N/A) times the current through the motor. The position of the table is measured using the optical linear encoder (LE2) shown in Fig. 1. The velocity and current (force) data are discrete in nature and sampled at a frequency of 2 kHz. The data is stored in the form of a Microsoft Excel csv file. This data can be analyzed with ease using MATLAB.

The LM force profile corresponding to velocity waveform in Fig. 4 is shown in Fig. 5. Fig. 4 and 5 are interpolated forms of the data. Force is measured for different constant velocities. The force needed to just start motion of drive in either direction corresponds to zero velocity. The data is plotted as shown in Fig. 6.

Using least squares techniques, the friction curve described by (2) and (3) is obtained as shown in Fig. 7. The static friction constants F^{+}_{stat} and F^{-}_{stat} in (2) and (3) are the forces required to just start the motion of the table in either direction. The linear portion of the friction curve is considered (Fig. 2) by fitting the high velocity points (linear portion) in Fig. 6 using the curve fitting tool in MATLAB 8. The curve fitting tool can fit data into linear, exponential, quadratic, cubic, higher order polynomials, splines or any other curve desired by the user. The polynomial (degree 1) option is used as the desired segment is a straight line. This fitting tool uses linear least squares described in Section II.C. The slope and intercept are the parameters which form $\boldsymbol{\alpha}$ vector. The slopes of the linear

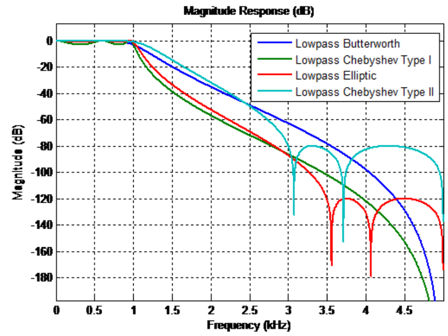

Figure 3: Lowpass Filters (5th Order)

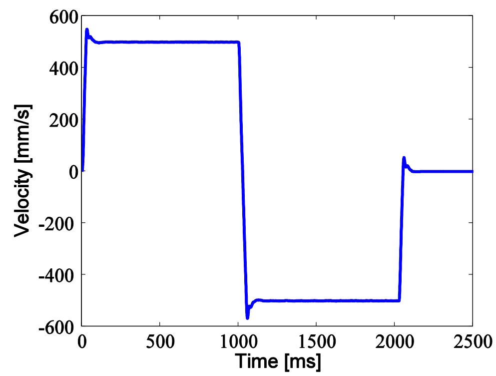

Figure 4: Trapezoidal Velocity Profile (500mm/s)

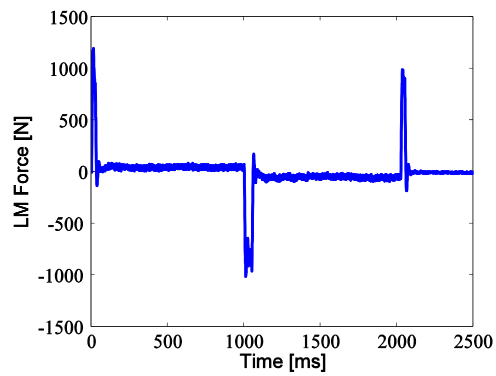

Figure 5: Linear Motor Force corresponding to 500mm/s

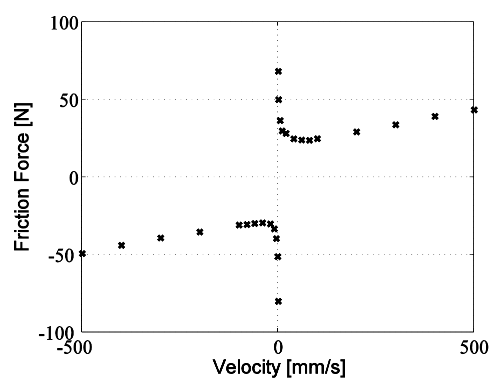

Figure 6: Friction Force for different velocity values

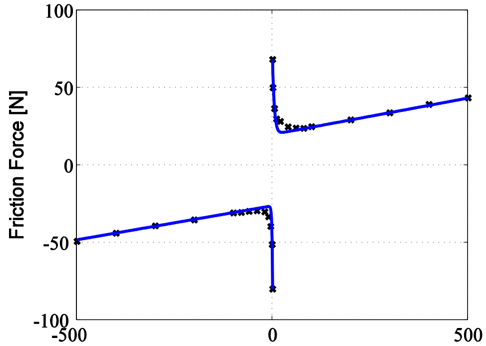

Figure 7: Friction Curve fitting

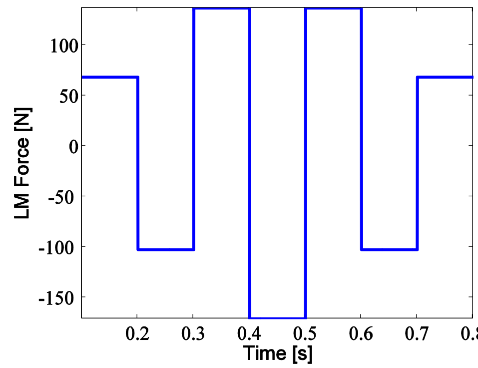

Figure 8: Linear Motor Force

portions are the constants B^+ and B^- and the intercepts are F^+_{coul} and F^-_{coul}. The constants v_1^+, v_1^-, v_2^+ and v_2^- are calculated using *lsqcurvefit* function in MATLAB 8. This function solves for the constants using non-linear least-squares data fitting technique discussed in Section II.C according to (7). In (7), $F_{f,data}$ is obtained from Fig. 6 and $F_f^{+/-}(v_{data})$ is obtained by substituting velocity data from Fig. 6 in (2) or (3). In this work, Trust-Region-Reflective Optimization Algorithm with initial condition 0.5 for all parmeters is used to find the parameters.

$$minimize \; ||F_{f,data} - F_f^{+/-}(v_{data})||^2 \; over \; all \; v_i^{+/-} \qquad (7)$$

B. Table Mass Identification

For a given LM force, the velocity and acceleration are obtained by differentiating the data obtained from LE2. These are shown in Fig. 8 to 10. The interpolated data in Fig. 8 to 10 is collected at a sampling frequency of 10 kHz. A finite-difference approximation is used and hence, differentiating the experimental data amplifies the noise [3]. This is more clearly understood if the signal is considered to be superposition of desired signal and noise. The desired signal of the original data might have high amplitude but low frequency whereas the noise signal has low amplitude and high frequency (amplitude of desired signal is much larger than noise). Amplitude of differentiated signal is product of amplitude and frequency. Hence, the amplitude of differentiated noise signal can exceed that of desired signal. Differentiating the data (Fig. 9) amplifies noise but its amplitude is still lower than desired signal. Further differentiation (Fig. 10) results in higher noise amplification such that the amplitude of noise is larger than that of desired signal.

Table velocity and acceleration are filtered using a low pass Butterworth filter. Fig. 11 and 12 show the filtered data for table velocity and acceleration for a 4th order Butterworth filter with normalized cutoff frequency 0.1 (1 kHz for sampling frequency 10 kHz). Mass of the table is estimated using linear least squares approach (Section II.C) and (1). Normalized cutoff frequency of 0.1 (1 kHz) estimates the mass of the table to be 38.93 kg. The actual mass of the table is 39±0.5 kg. If the normalized cutoff frequency is changed to 0.5 the estimated mass is 22.13kg whereas at 0.8 it is 17.53kg. The normalized cutoff frequency is varied from 0 to 1 for data sampled at 10 kHz, 5 kHz, and 2 kHz. The estimated table mass for different cases is shown in Fig. 13. Table 1 lists the

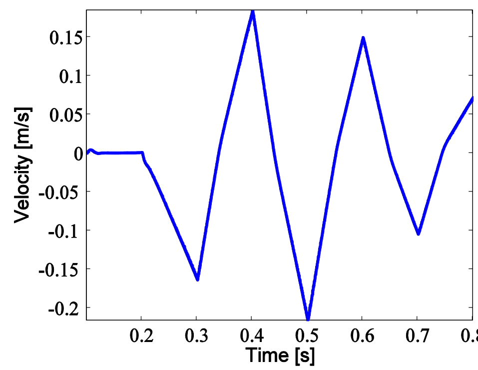

Figure 9: Table Velocity

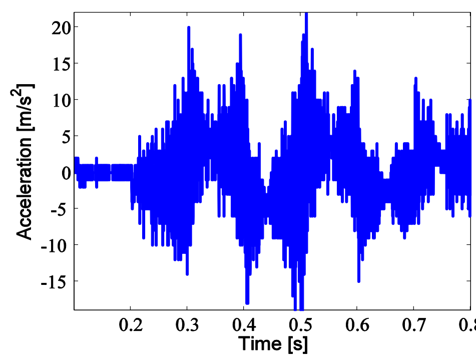

Figure 10: Table Acceleration

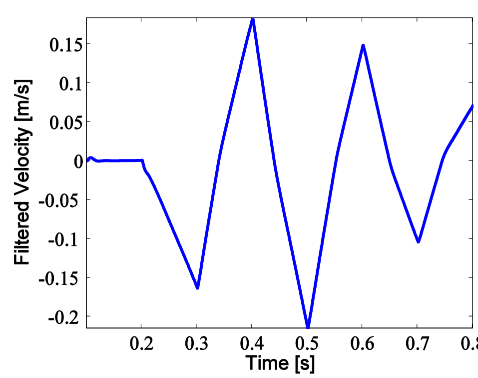

Figure 11: Filtered Velocity (Cutoff frequency 1 kHz for sampling frequency 10 kHz)

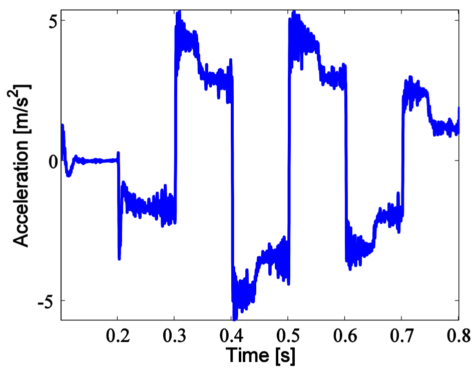

Figure 12: Filtered Acceleration (Cutoff frequency 1 kHz for sampling frequency 10 kHz)

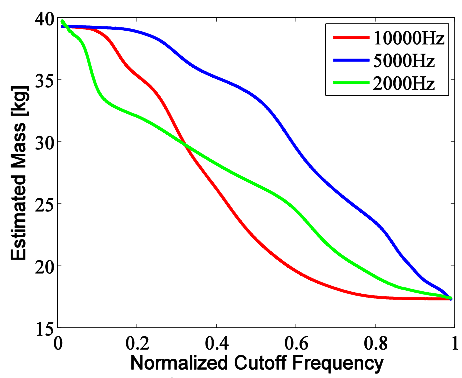

Figure 13: Variation of estimated mass with normalized cutoff frequency for different sampling frequencies

Table 1: Range of Normalized Cutoff Frequencies (where estimated mass ranges from 38.5kg to 39.5kg) for different Sampling Frequencies

Sampling Frequency [kHz]	Normalized Cutoff Frequency	Cutoff Frequency [Hz]
10	<0.1180	<1180
5	<0.2382	<1191
2	0.0162-0.0472	32-94

range of cutoff frequencies corresponding to estimated mass from 38.5kg to 39.5kg for different sampling frequencies.

The mass of the table is also estimated using filters of different orders (2 to 5). Fig. 14 shows the magnitude response of Butterworth filter for different orders. All the filters are maximally flat in passband but the roll-off (slope in stopband) depends on order of the filter. The roll-off increases with increase in order of the filter. Fig. 15 demonstrates effect of order on estimation process. The variation of estimated mass with normalized cutoff frequency is identical for orders 2 to 5. The normalized cutoff frequency (for estimated mass between 38.5kg and 39.5kg) for all orders is less than 0.118 (1180 Hz) which is close to that for 5 kHz as sampling frequency (1191Hz). The same value for sampling frequency 2 kHz is between 32 and 94 Hz (from Table 1).

Fig. 16, 17 and 18 show the Fast Fourier Transform (FFT) representation of acceleration signal for sampling frequencies 10 kHz, 5 kHz and 2 kHz respectively. For each FFT, the first half is considered as the two halves are mirror images.

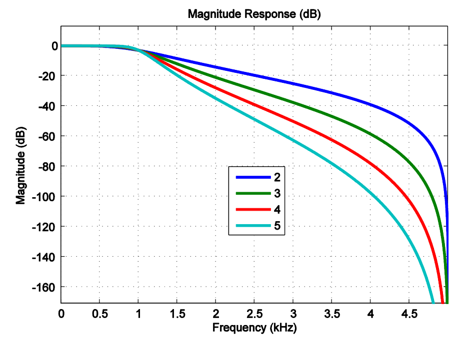

Figure 14: Magnitude response of Butterworth for Orders 2 to 5 for sampling frequency 10 kHz and cutoff frequency 1 kHz

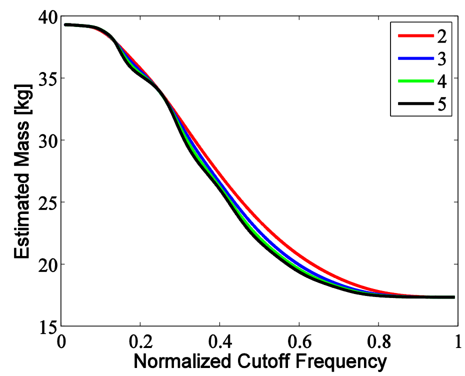

Figure 15: Variation of estimated mass with normalized cutoff frequency for different filter orders

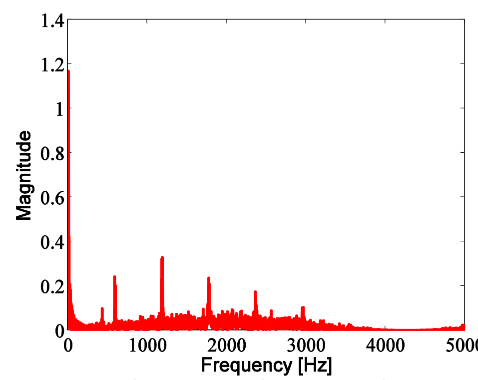

Figure 16: FFT of acceleration for sampling frequency 10 kHz

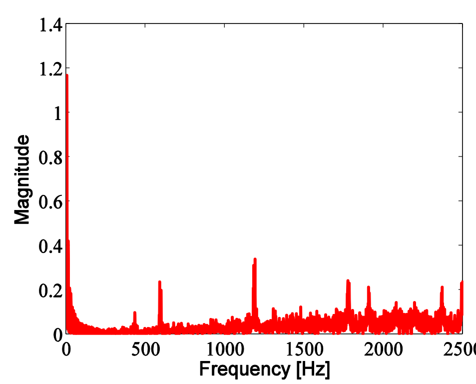

Figure 17: FFT of acceleration for sampling frequency 5 kHz

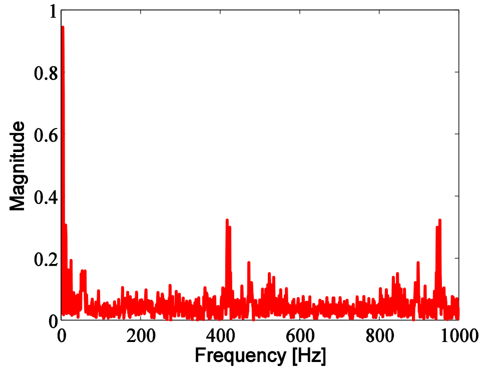

Figure 18: FFT of acceleration for sampling frequency 2 kHz

For analysis, the minimum of the three (i.e. minimum of 5000, 2500 and 1000) is selected as range. Fig. 19 and 20 show FFT for 10 kHz and 5 kHz sampling frequencies for first 1 kHz only.

From Fig. 18 to 20 it is evident that the frequency domain representation for sampling frequency 2 kHz is different from that for 5 kHz and 10 kHz. Sampling frequencies 5 kHz and 10 kHz have similar frequency representations and hence, give accurate estimates below the same cutoff frequency. From Fig. 16 and 17 it is evident that the two sampling frequencies result in similar frequency domain representation up to 1190 Hz which is same as the common value in Table 1. Half of 2 kHz (1 kHz) is less than this frequency value. Hence, information is lost due to aliasing. Aliasing results in a different cutoff frequency for 2 kHz as compared to 5 kHz and 10 kHz.

IV. Conclusion

This work describes and analyzes the different DSP tools used for friction and mass identification process. Tools learnt in-class and out-of-class are used.

This work uses in-class signal processing concepts such as sampling, discrete time differentiation, low pass filter, FFT and aliasing. Out-of-class techniques such as least squares (linear and non-linear) are used to estimate parameters.

A possible future work can be avoiding use of low pass filter by using methods to reduce noise content of differentiated signal as discussed in [3].

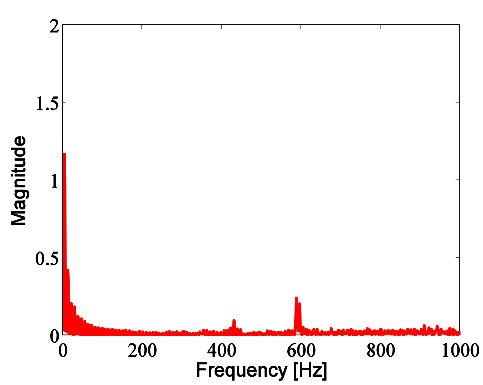

Figure 19: FFT of acceleration for sampling frequency 10 kHz for first 1 kHz

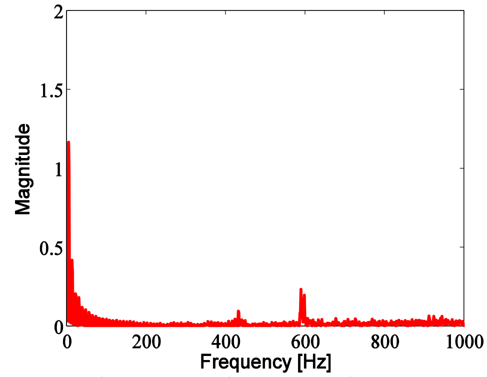

Figure 20: FFT of acceleration for sampling frequency 5 kHz for first 1 kHz

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