Density of States
Fermi Statistics

EECS 320

Introduction to Semiconductor Devices

Electronic states

Start with a piece of semiconductor material

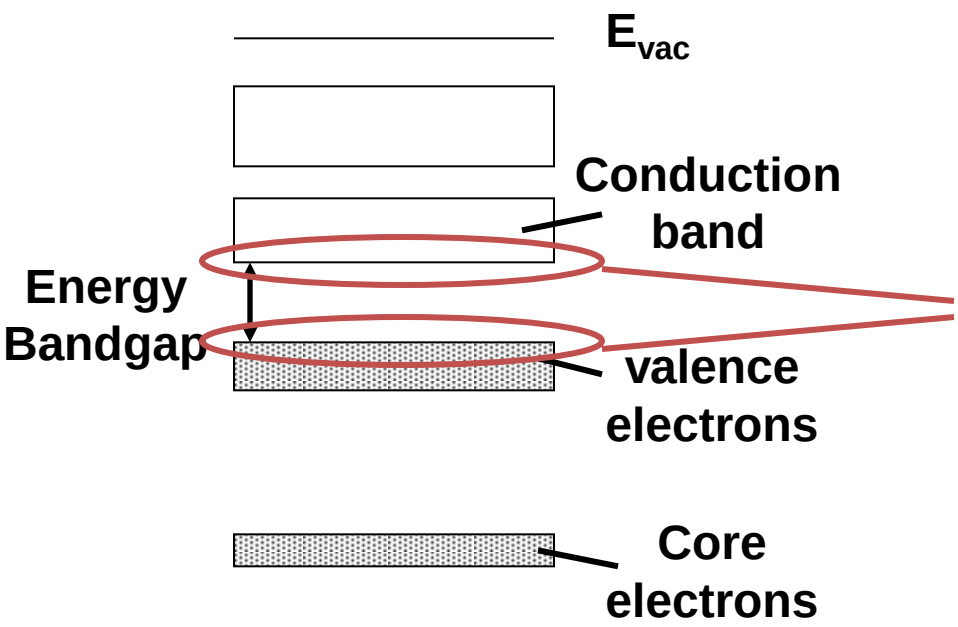
How would you define the number of electron states?

Recall: Pauli Exclusion Principle

Electronic states

Electron states distributed in energy

Semiconductor
Valence band filled with electrons

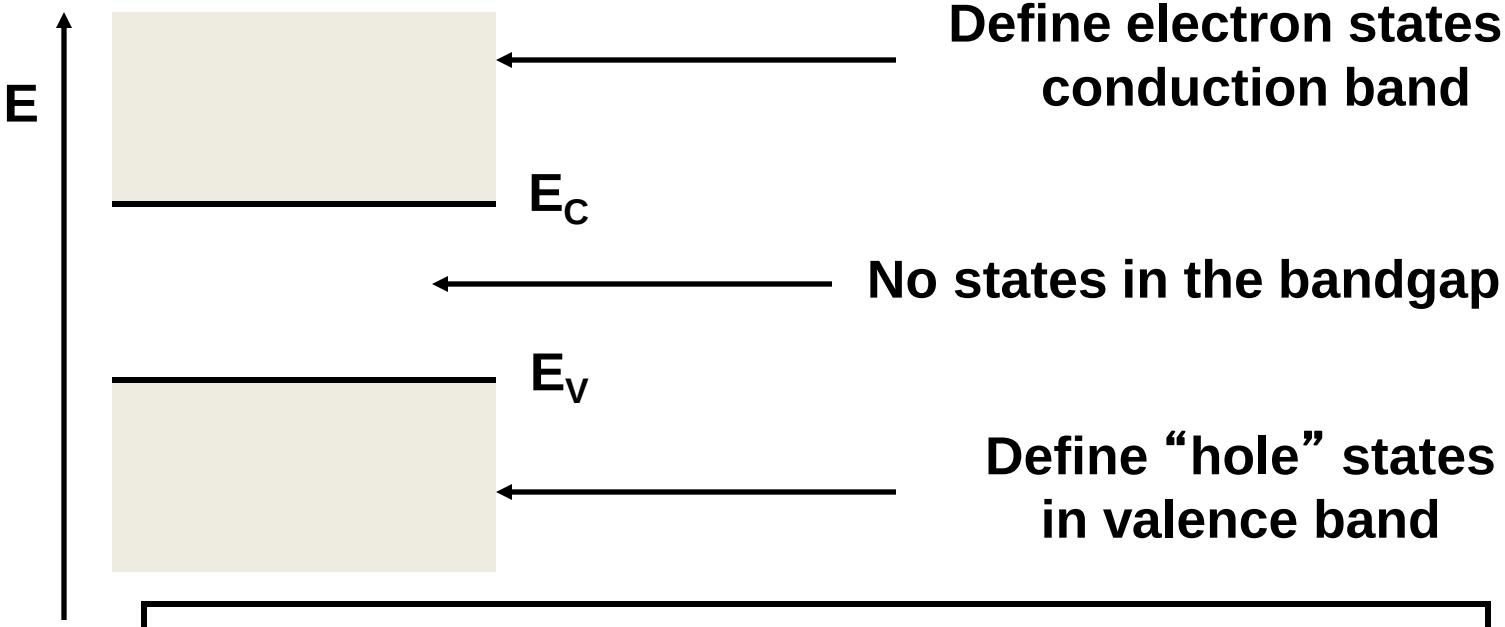


Electron states are not uniformly distributed in energy

We are primarily concerned with states near E_C and E_V

States In Conduction/Valence Band

Recall Conduction and Valence Band

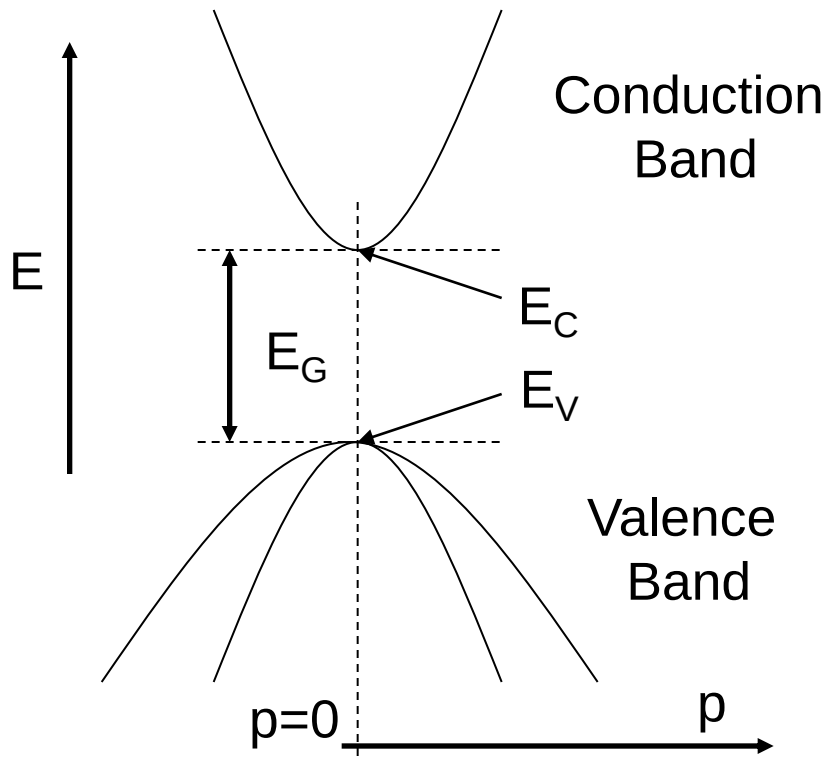


$$\text{Density Of States} = \frac{\text{Number of states}}{\text{Unit volume x Unit energy}}$$

(Note we have 4-D, 3-D in space + energy)

Density Of States

Density of states may be determined if bandstructure is known



Near bandedge (E_C and E_V), approximate using effective mass m_n^* and m_p^*

Density Of States

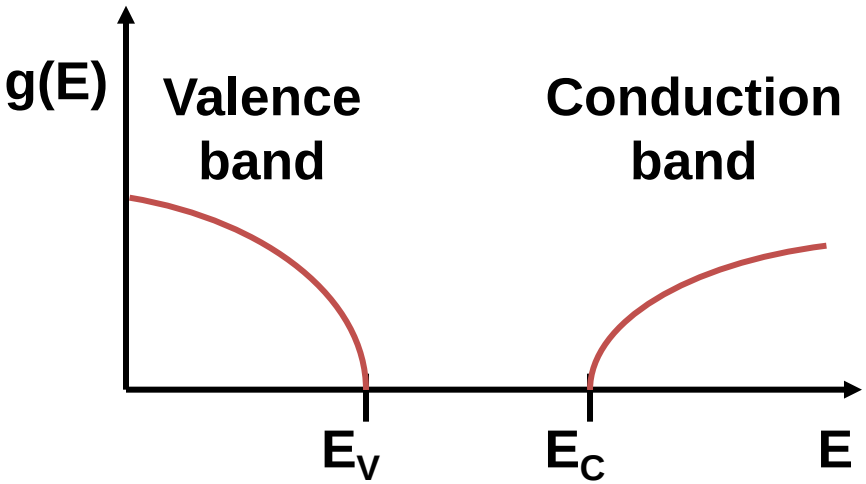
(Insert quantum mechanics + E-k + geometry)

(You will see this in detail in 420, or in office hours)

Density Of States

$$g_c(E) = \frac{\sqrt{2}m_n^{*3/2}(E - E_C)^{1/2}}{\pi^2\hbar^3}$$

$$g_v(E) = \frac{\sqrt{2}m_p^{*3/2}(E_V - E)^{1/2}}{\pi^2\hbar^3}$$



Density Of States

Number of electrons per unit volume per unit energy ($cm^{-3}eV^{-1}$)

Density of states determined by effective mass, square root dependence on energy relative to E_C, E_V

Calculating Density Of States

Tricky units, be careful!

$$g_c(E) = \frac{\sqrt{2}m_n^{*3/2}(E - E_C)^{1/2}}{\pi^2\hbar^3}$$

$$J = mass \times accel \times dist = \frac{kg \cdot m^2}{s^2}$$

$$\hbar = 1.055 \times 10^{-34} J \cdot s$$

$$1kg = \frac{J \cdot s^2}{m^2} = 10^{-4}\frac{J \cdot s^2}{cm^2}$$

$$m_0 = 9.11 \times 10^{-31} kg$$

$$1eV = 1.60 \times 10^{-19} J$$

E in eV

$$g \text{ in } \frac{kg^{3/2}eV^{1/2}}{J^3 s^3} = \frac{\left(10^{-4}\right)^{3/2} J^{3/2} s^3 eV^{1/2}}{cm^3 J^3 s^3} = \frac{\left(10^{-4}\right)^{3/2} eV^{1/2}}{cm^3 J^{3/2}}$$

$$\boxed{g = \frac{\sqrt{2}m_n^{*3/2}(E - E_C)^{1/2}}{\pi^2\hbar^3} \times \left(10^{-4}\right)^{3/2}\left(1.6 \times 10^{-19}\right)^{3/2} cm^{-3} eV^{-1}}$$

Example: Density of States

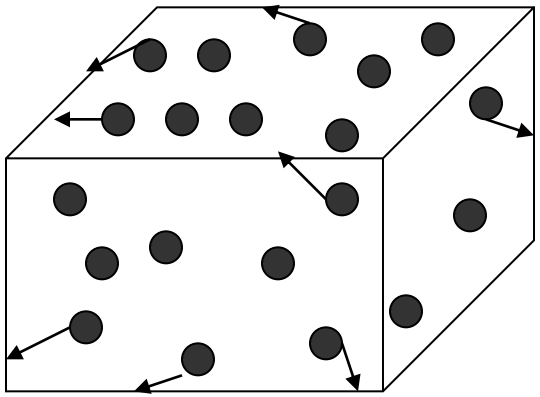
Calculate the density of states for electrons and holes in silicon (in terms of energy relative to conduction and valence band edge)

Effective masses: $m_n^*=1.18m_0$, $m_p^*=0.81m_0$

Statistical Mechanics

Consider a system with a large number of particles

Example: gas in a container



Pressure in container due to collisions of gas molecules with wall of container

Q: Do we care about each individual collision?
A: No, use statistical behavior of group as a whole.

Statistical Laws

Need to define laws that the particles obey

Some common distribution laws:

Maxwell Boltzmann: **particles distinguishable by number, no limit on number of particles in each energy state**

Bose-Einstein: **particles are indistinguishable, no limit on number of particles in each energy state**

Fermi-Dirac: **particles are indistinguishable, only one particle in each energy state**

Which would you expect to apply for electrons in a semiconductor?

Fermi-Dirac Probability Function

In a semiconductor, one electron per state (Pauli Exclusion Principle)

Number of ways to arrange particles in states

For g_i states and N_i particles, there are g_i ways of placing first particle in a state, g_i-1 ways to place second particle, …

The total number of ways to arrange all particles is product for each particle

$$g_i(g_i - 1)\ldots(g_i - (N_i - 1)) = \frac{g_i!}{(g_i - N_i)!}$$

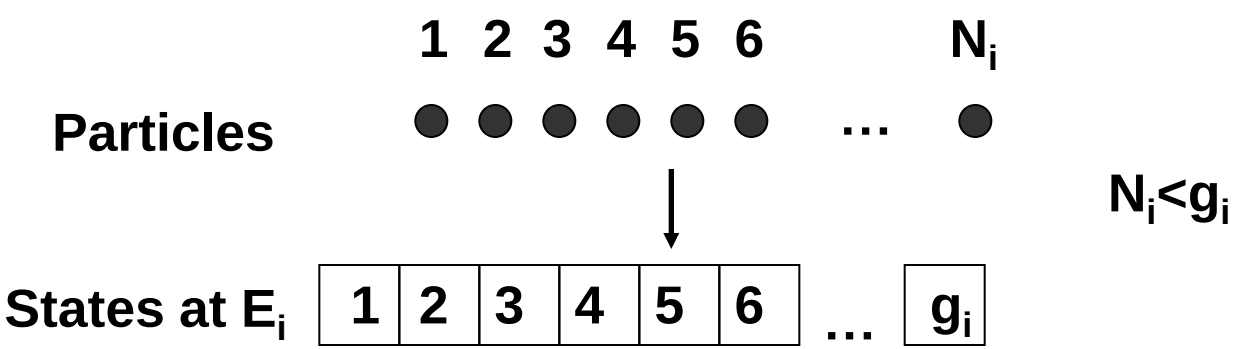
Fermi-Dirac Probability Function

For Fermi-Dirac, particles indistinguishable

The interchange of any two electrons does not count as a new arrangement!

Actual number of ways to arrange particles divided by N_i!

$$W_i = \frac{g_i!}{N_i!(g_i - N_i)!}$$



Fermi-Dirac Probability Function

Determine for all energy levels

Number of arrangements for energy level *i*

$$W_i = \frac{g_i!}{N_i!(g_i - N_i)!}$$

We know electron/hole states distributed in energy (density of states)

Take product of W_i for all energy levels

$$W = \prod_{i=1}^{n} \frac{g_i!}{N_i!(g_i - N_i)!}$$

What does W represent?

Fermi-Dirac Distribution Function

Maximize W – Most probable distribution for a given N and E

$$N_i = f_i g_i$$

After much math and application of Thermodynamics, determine f_i

$$\boxed{f(E) = \frac{N(E)}{g(E)} = \frac{1}{1 + \exp\left(\frac{E - E_F}{kT}\right)}}$$

$$k = \text{Boltzmann Constant}$$

$$k = 8.617 \times 10^{-5} eV / K$$

$$kT = 0.026 eV \text{ at } \mathrm{T} = 300\mathrm{K}$$

Fermi-Dirac Distribution Function

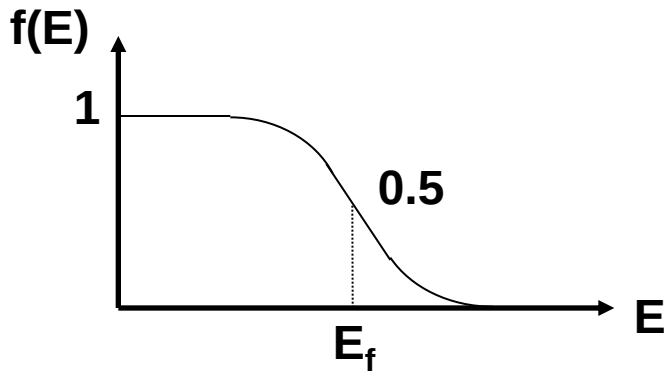
Plot f(E) at T=0 K and T=300 K
(assume arbitrary E_F)

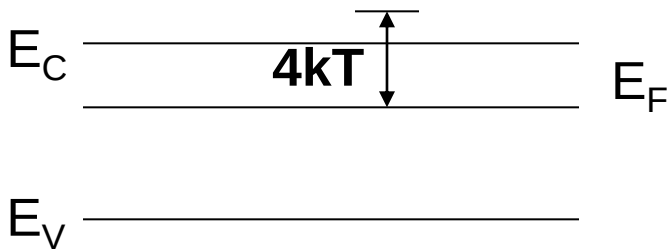
Apply the Fermi-Dirac distribution function to electrons in a semiconductor. If g(E) is the density of states, what does f(E) tell us?

The Fermi Energy

The Fermi energy is a point of reference for f(E)

At E_F, f(E) = 1/2





What is the probability of full electron state at $E=E_F+4kT$?

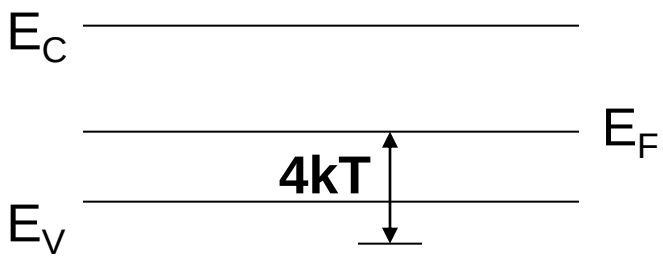
Note that the Fermi function is the probability of occupation regardless of whether or not a state exists!

Fermi-Dirac Statistics For Holes

Does the Fermi-Dirac Distribution hold for holes?

Thusfar, we have said that f(E) represents probability of filling electron state

What does this tell us about holes?



What is the probability of full hole state at $E = E_F - 4kT$?

Approximation to Fermi-Dirac Statistics

For $E-E_F >> kT$

$$f(E) \approx \frac{1}{\exp\left(\frac{E-E_F}{kT}\right)} = \exp\left(-\frac{E-E_F}{kT}\right)$$

This is known as the *Boltzmann Approximation*

Calculate the energy range (in kT) where you could use the Boltzmann approximation, where the difference will be <5% of the Fermi-Dirac function