Generation and Recombination Continuity Equations

EECS 320

Introduction to Semiconductor Devices

Generation and Recombination

Generation: creation of conducting electrons and holes

Recombination: elimination of conducting electrons and holes

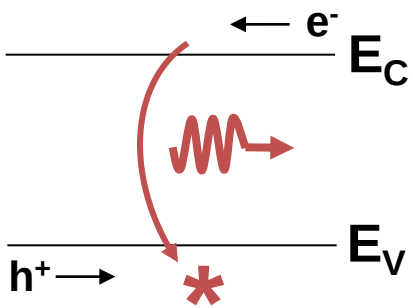
There are several physical processes where G-R may occur

Big Principles

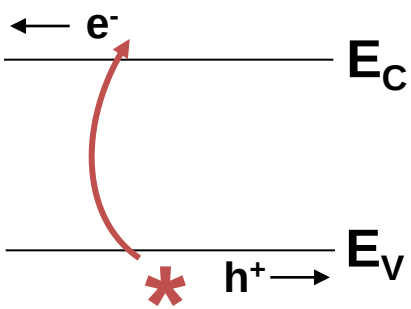
- Generation and Recombination Mechanisms come in balanced pairs
  - Principle of Detailed Balance: At equilibrium every event is balanced by an equal reverse event
- G-R Mechanisms interact through the carrier populations (n,p) but are otherwise considered to be independent
- G-R event happen in pairs (e^-,h^+)
- One of the big considerations in G-R physics is where to get the energy (G) or how to get rid of the energy (R)

Generation and Recombination

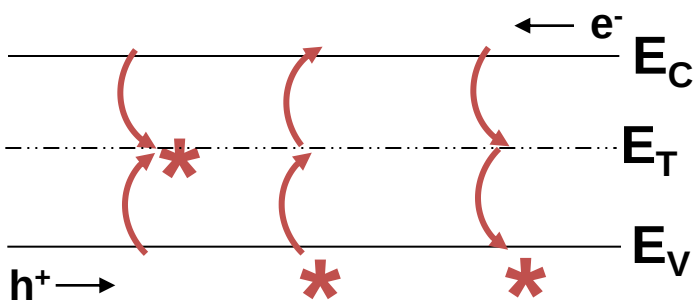
Band-to-Band Recomb.



Photogeneration



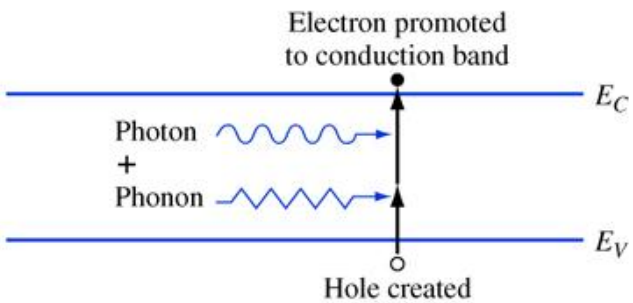
G-R through Centers (Traps)



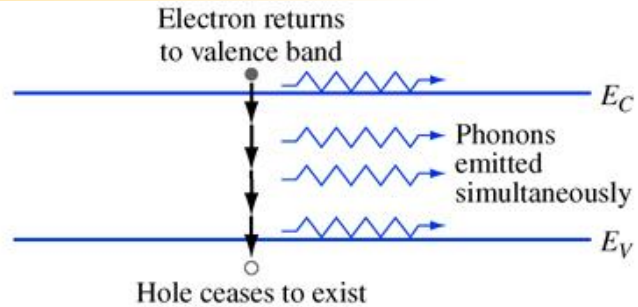
Other Methods:

- Auger
- Impact Ionization

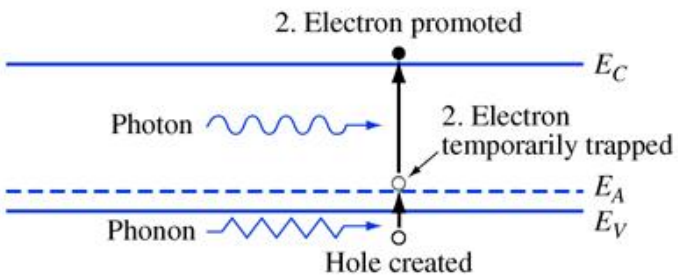
Generation and Recombination Examples



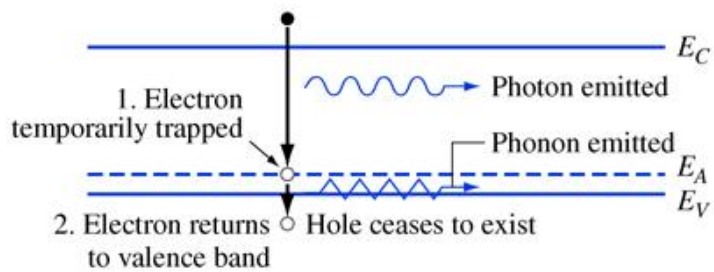
(*a*) Band-to-band generation via simultaneous absorption of a phonon and a photon (for example)



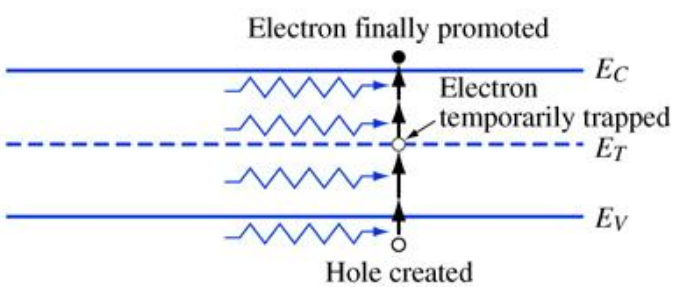
(*b*) Band-to-band recombination via emission of several phonons



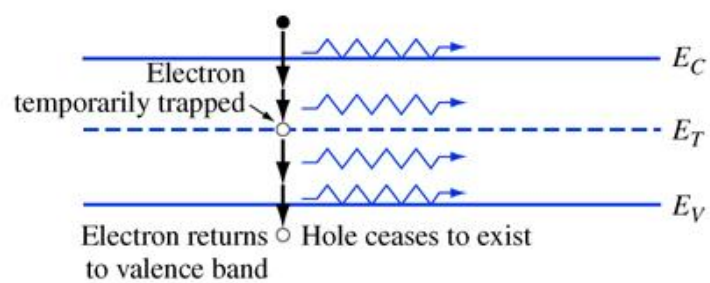
(*c*) Two-step generation process involving acceptor state



(*d*) Two-step recombination process involving acceptor state



(*e*) Two-step generation process involving trap state

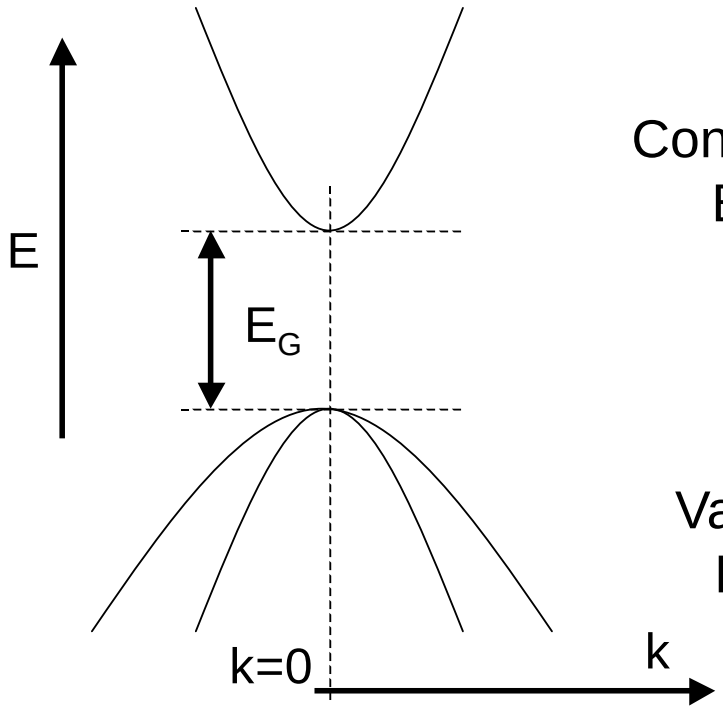


(*f*) Two-step recombination process involving trap states

Conservation Of Energy/Momentum

Energy and momentum needs to be conserved for generation and recombination events

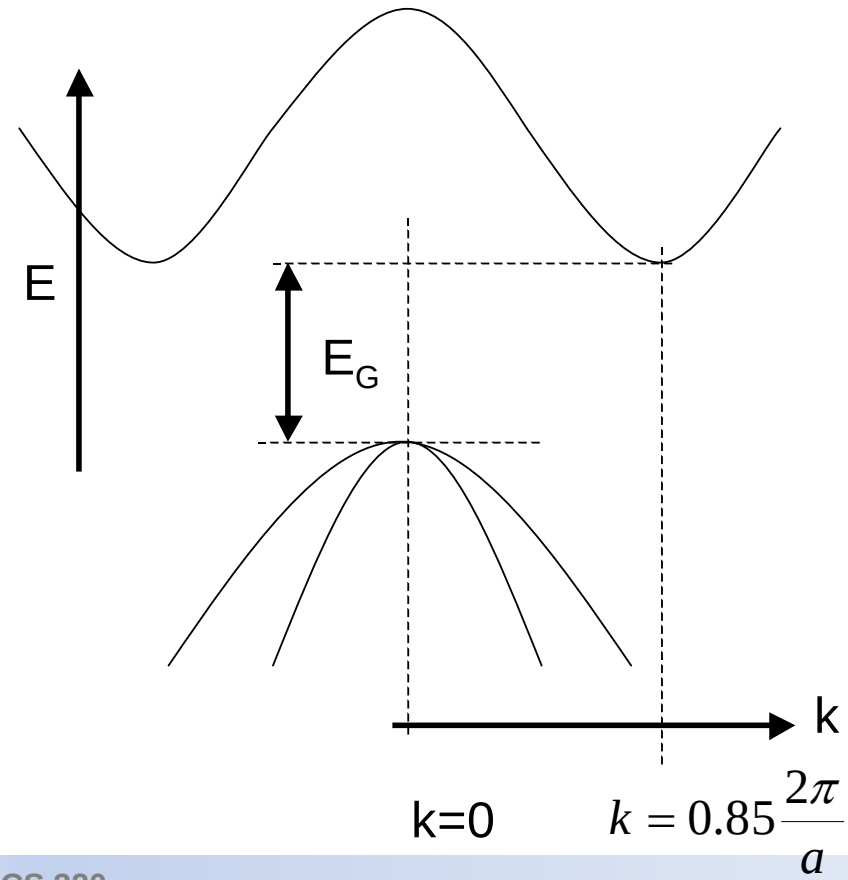
Direct Bandgap (e.g. GaAs)



Conduction Band

Valence Band

Indirect Bandgap (e.g. Si)



Conservation Of Energy/Momentum

Silicon is an indirect bandgap semiconductor

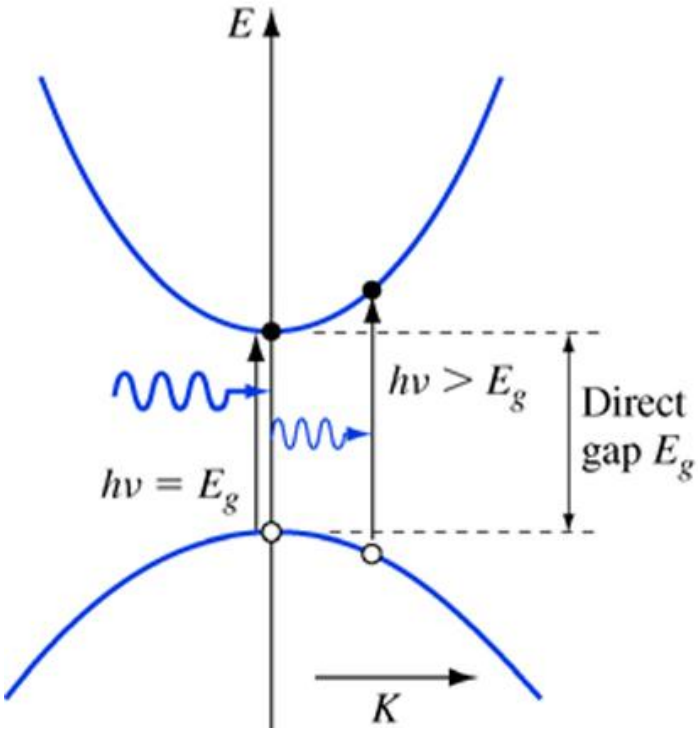
What is the value of k associated with a photon?

Would you expect radiative recombination (light emission) to be significant in silicon?

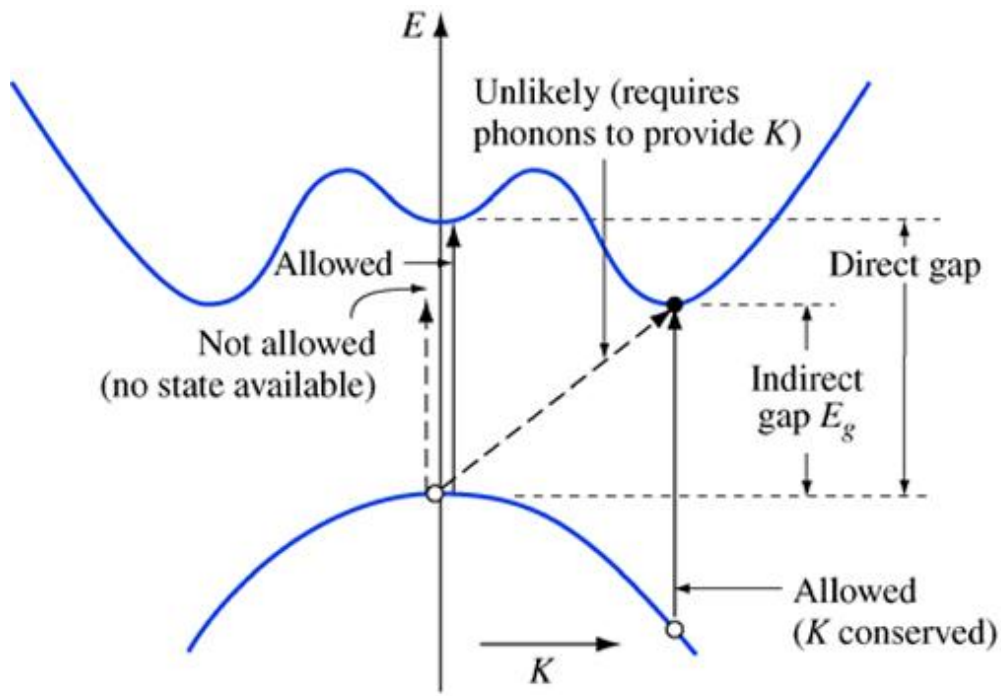
Optical Generation

The momentum of a photon is small

Direct Bandgap (e.g. GaAs)



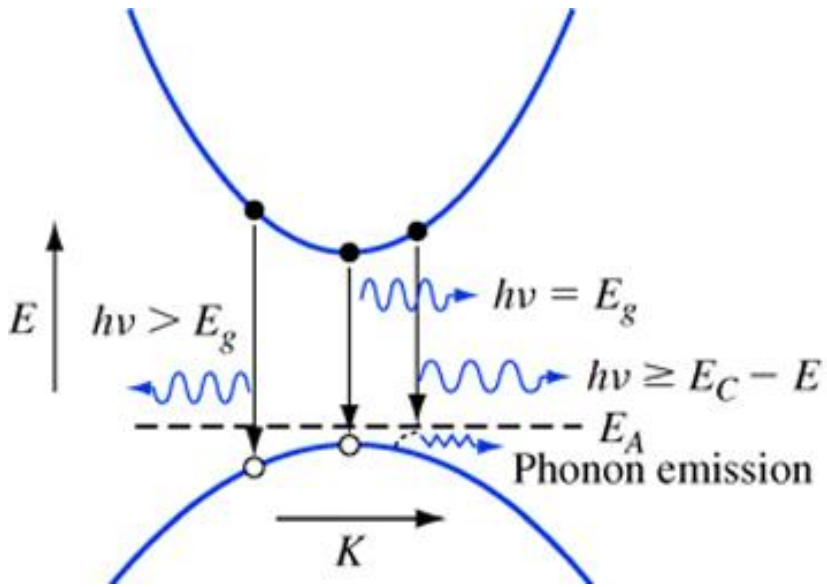
Indirect Bandgap (e.g. Si)



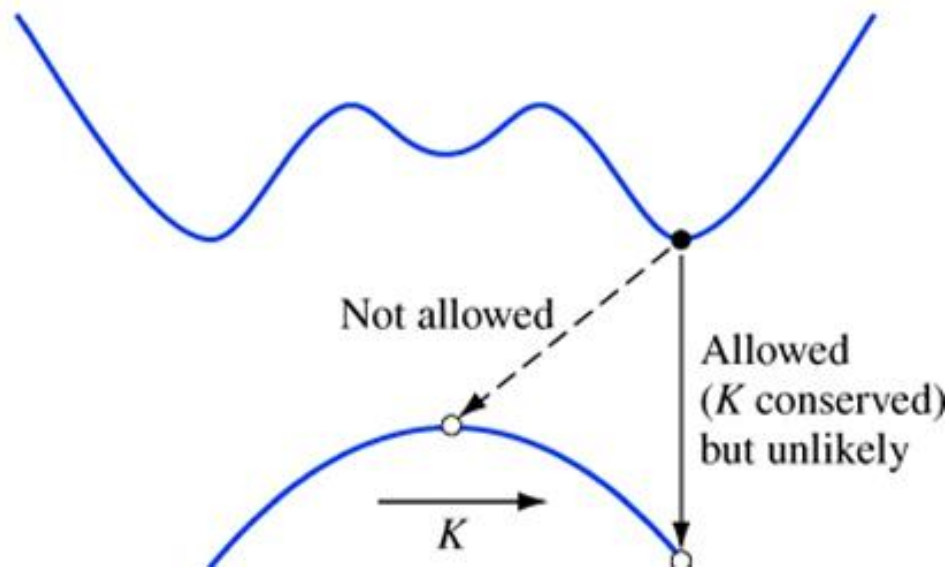
Optical Recombination

The momentum of a photon is small

Direct Bandgap (e.g. GaAs)

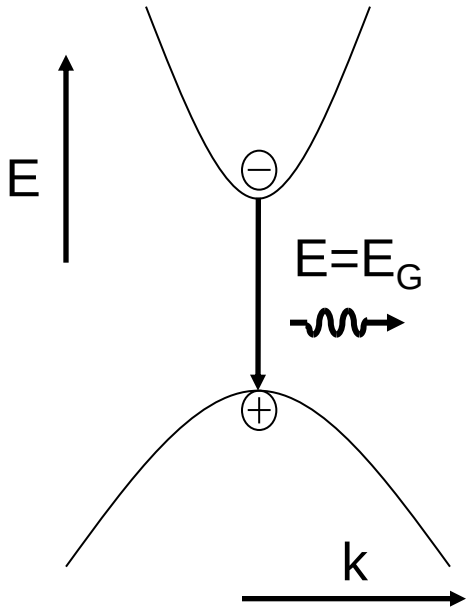


Indirect Bandgap (e.g. Si)

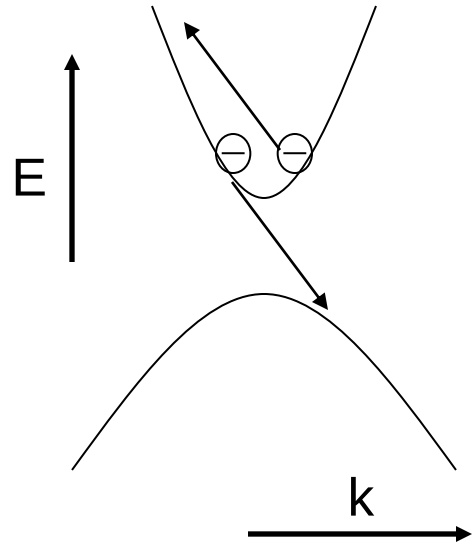


Recombination

Radiative Recombination



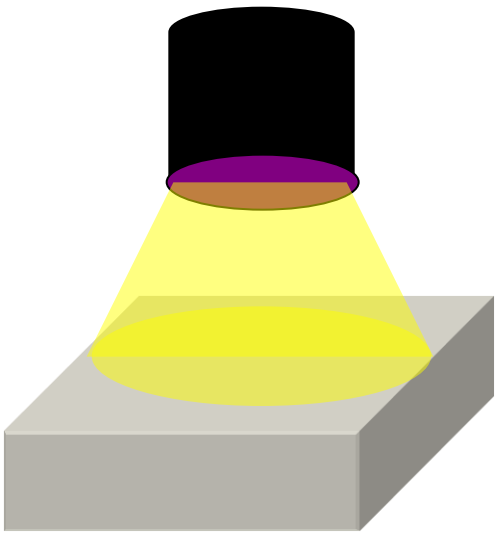
Auger Recombination

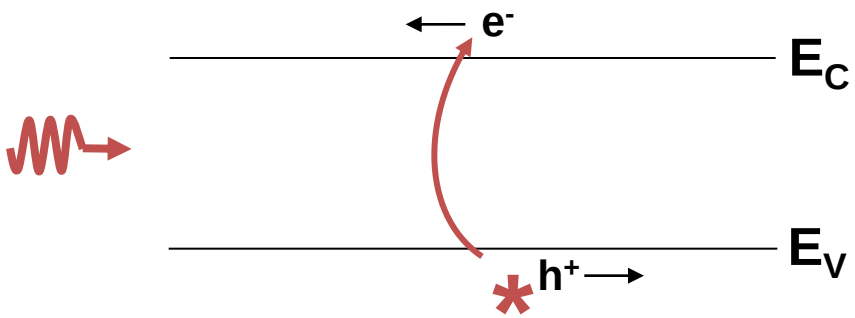


Typically, one mechanism occurs at a much faster rate and is dominant

$$\frac{1}{\tau} = \sum_i \frac{1}{\tau_i}$$

Photogeneration



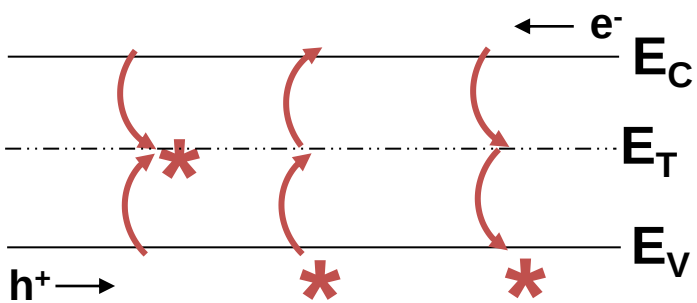


- Intensity of Light in the Semiconductor: $I = I_0 e^{-\alpha(\lambda)x}$
- Generation follows Light Absorption
- Generation Rate: $G_L(x,\lambda) = G_{L0} e^{-\alpha x}$

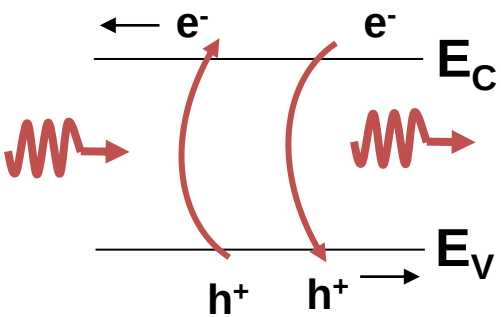
G-R Equations

- Most G-R events involve 1 e^- and 1 h^+
  - Traps: energy exchanged with phonons
  - Optical: energy exchanged with photons
  - Exception is Auger Recombination/Impact Ionization that involves 3 particles (2 e^-,1 h^+ or 1 e^-,2 h^+)

G-R through Centers (Traps)



Photogeneration and Emission



Two Particle G-R

$$U \equiv R - G = Anp - G$$

Where A is rate constant dependent on the physics of the process & material properties

In Equilibrium

$$U = 0 = An_0 p_0 - G$$

Hence $G = An_0 p_0$

and in general we get

$$U = A\left(np - n_0 p_0\right)$$

$$= A\left(\left(n_0 + \Delta n\right)\left(+\Delta p\right) - n_0 p_0\right)$$

$$= A\left(n_0 p_0 + p_0 \Delta n + n_0 \Delta p + \Delta n \Delta p - n_0 p_0\right)$$

$$= A\left(p_0 \Delta n + n_0 \Delta p + \Delta n \Delta p\right)$$

Important Case: Extrinsic Semiconductor Quasi-neutral Region in Low Level Injection

$$U = A\left(p_0\Delta n + n_0\Delta p + \Delta n\Delta p\right)$$

$\Delta n \approx \Delta p$ in Quasi-neutral region

$$\mathrm{U} \approx A\left(\left(p_0 + n_0\right)\Delta n + \left(\Delta n\right)^2\right)$$

$\Delta n, \Delta p \ll \left(majority\ carrier\ concentration\right)$ Low Level Injection (LLI)

$$\mathrm{U} \approx A\left(\left(p_0 + n_0\right)\Delta n\right)$$

$n_0 \gg p_0$ Extrinsic n-type

$$\mathrm{U} \approx A\left(\left(p_0 + n_0\right)\Delta n\right) \approx An_0\Delta n = \frac{\Delta n}{\tau} = \frac{\Delta p}{\tau}$$

$p_0 \gg n_0$ Extrinsic n-type

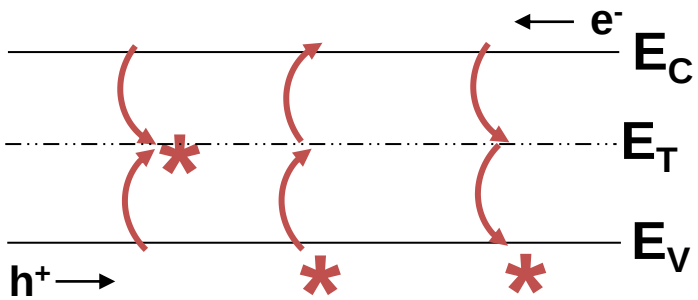
$$\mathrm{U} \approx A\left(\left(p_0 + n_0\right)\Delta n\right) \approx Ap_0\Delta n = \frac{\Delta n}{\tau} = \frac{\Delta p}{\tau}$$

Shockley Read Hall Theory: Traps

$$U = \frac{np - n_0 p_0}{(n + n_1)\tau_p + (p + p_1)\tau_n}$$

$$n_1 \equiv n_i e^{(E_T - E_i)/kT}$$

$$p_1 \equiv n_i e^{(E_i - E_T)/kT}$$



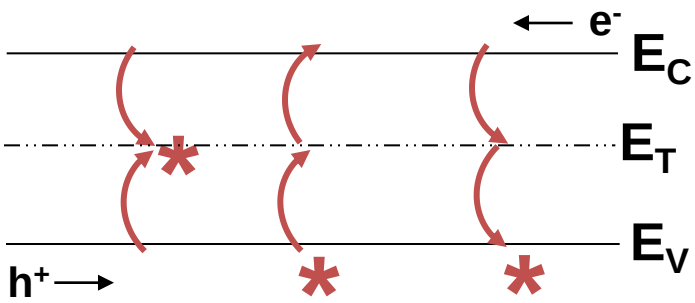
Indirect Thermal G-R

- Low-level Injection:
  - $\Delta p << n_0$ $\quad n \approx n_0$ $\quad$ n-type
  - $\Delta n << p_0$ $\quad p \approx p_0$ $\quad$ p-type
- Steady State Equations:
  - Holes in n-type material

$$\left.\frac{\partial p}{\partial t}\right|_{G-R} = -\frac{\Delta p}{\tau_p}$$

  - Electrons in p-type material

$$\left.\frac{\partial n}{\partial t}\right|_{G-R} = -\frac{\Delta n}{\tau_n}$$



τ_n and τ_p are *minority carrier lifetimes, parameters that we will use often.*

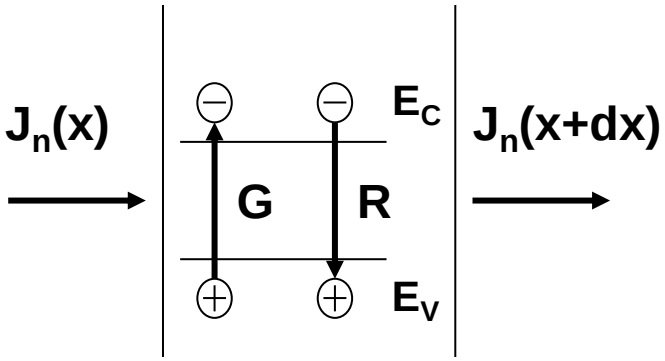
Drift, Diffusion, and G-R

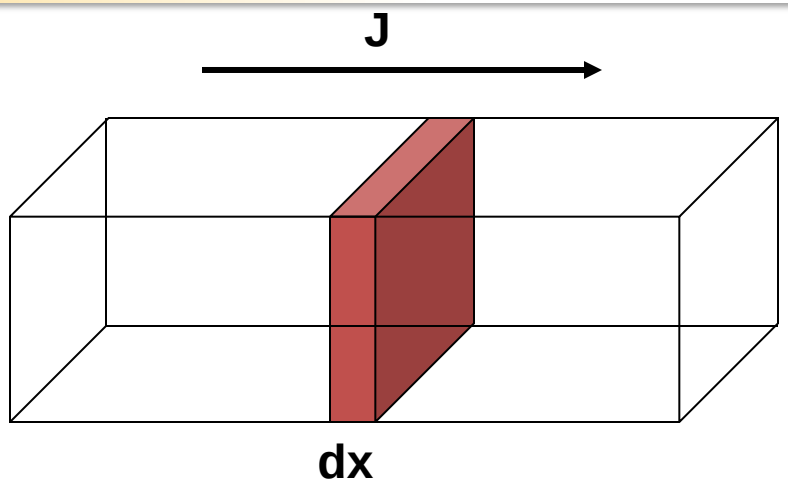
We have determined carrier processes

- Drift
- Diffusion
- Generation and Recombination

The development of relationships between these processes will provide a basis to solve device problems

Continuity Equation





$$\frac{\partial n}{\partial t} A dx = \left[\frac{J_n(x)A}{-q} - \frac{J_n(x+dx)A}{-q} \right] + (G-R)Adx$$

Taylor series Expansion:

$$J_n(x+dx) = J_n(x) + \frac{1}{1!}\frac{\partial J_n}{\partial x}dx + \frac{1}{2!}\frac{\partial^2 J_n}{\partial x^2}(dx)^2 + \ldots$$

Using first two terms:

$$\boxed{\frac{\partial n}{\partial t} = \frac{1}{q}\frac{\partial J_n}{\partial x} + (G-R)}$$

Low-Level Injection

For small perturbation

- Majority carriers remain unchanged (e.g. electrons in n-type material $n=n_0=N_D$)
- Minority carriers do not approach majority carrier density (e.g. holes in n-type material $p = p_0+\Delta p << n$)
- Thermal R-G for minority carriers given by:

Electrons in p-type material

$$\left.\frac{\partial n}{\partial t}\right|_{\substack{thermal \\ R-G}} = -\frac{\Delta n}{\tau_n}$$

Holes in n-type material

$$\left.\frac{\partial p}{\partial t}\right|_{\substack{thermal \\ R-G}} = -\frac{\Delta p}{\tau_n}$$

Diffusion Equations

- Assume uniform p-type doping, LLI, no electric field and look at minority carriers (electrons)

$$\frac{\partial n}{\partial t} = \frac{1}{q}\frac{\partial J_n}{\partial x} + (G - R) \approx \frac{1}{q}\frac{\partial J_n}{\partial x} - \frac{\Delta n_p}{\tau_n}$$

$$J_n = q\mu_n nF + qD_n \frac{\partial n}{\partial x} = qD_n \frac{\partial n}{\partial x}$$

$$\frac{\partial n}{\partial t} = \frac{1}{q}\frac{\partial}{\partial x}\left(qD_n \frac{\partial n}{\partial x}\right) - \frac{\Delta n_p}{\tau_n}$$

$$= D_n \frac{\partial^2 n}{\partial x^2} - \frac{\Delta n_p}{\tau_n}$$

$$n = n_{p0} + \Delta n_p$$

$$\frac{\partial n_{p0}}{\partial t} = 0 \ \& \ \frac{\partial n_{p0}}{\partial x} = 0$$

$$\frac{\partial \Delta n_p}{\partial t} = D_n \frac{\partial^2 \Delta n_p}{\partial x^2} - \frac{\Delta n_p}{\tau_n}$$

Diffusion Equations

- Assume uniform n-type doping, LLI, no electric field and look at minority carriers (holes)

$$\frac{\partial p}{\partial t} = -\frac{1}{q}\frac{\partial J_n}{\partial x} + (G - R) \approx -\frac{1}{q}\frac{\partial J_p}{\partial x} - \frac{\Delta p_n}{\tau_p}$$

$$J_n = q\mu_p pF - qD_p\frac{\partial p}{\partial x} = -qD_p\frac{\partial p}{\partial x}$$

$$\frac{\partial p}{\partial t} = -\frac{1}{q}\frac{\partial}{\partial x}\left(-qD_p\frac{\partial p}{\partial x}\right) - \frac{\Delta p_n}{\tau_p}$$

$$= D_p\frac{\partial^2 p}{\partial x^2} - \frac{\Delta p_n}{\tau_p}$$

$$p = p_{n0} + \Delta p_n$$

$$\frac{\partial p_{n0}}{\partial t} = 0 \ \& \ \frac{\partial p_{n0}}{\partial x} = 0$$

$$\frac{\partial \Delta p_n}{\partial t} = D_p\frac{\partial^2 \Delta p_n}{\partial x^2} - \frac{\Delta p_n}{\tau_p}$$

Minority Carrier Diffusion Equation

For negligible drift and low-level injection conditions

$$\frac{\partial \Delta n_p}{\partial t} = D_n \frac{\partial^2 \Delta n_p}{\partial x^2} - \frac{\Delta n_p}{\tau_n} + G$$

$$\frac{\partial \Delta p_n}{\partial t} = D_n \frac{\partial^2 \Delta p_n}{\partial x^2} - \frac{\Delta p_n}{\tau_p} + G$$

Simplified equations useful for device analysis

Solve differential equations using appropriate boundary conditions, further simplifications may often be used

Summary: Continuity Equations

$$\frac{\partial n}{\partial t} = \frac{1}{q}\nabla \bullet \vec{J}_n + (G - R)$$

$$\frac{\partial p}{\partial t} = -\frac{1}{q}\nabla \bullet \vec{J}_p + (G - R)$$

$1 - D$

$$\frac{\partial n}{\partial t} = \frac{1}{q}\frac{\partial J_n}{\partial x} + (G - R)$$

$$\frac{\partial p}{\partial t} = -\frac{1}{q}\frac{\partial J_n}{\partial x} + (G - R)$$

Low Field, Low Level Injection, Possible Photogeneration

Minority Carrier Diffusion Equations (1-D)

$$\frac{\partial \Delta n_p}{\partial t} = D_n \frac{\partial^2 \Delta n_p}{\partial x^2} - \frac{\Delta n_p}{\tau_n} + G_L$$

$$\frac{\partial \Delta p_n}{\partial t} = D_p \frac{\partial^2 \Delta n_p}{\partial x^2} - \frac{\Delta p_n}{\tau_p} + G_L$$