MOS Capacitor Charges & The Threshold Voltage Approximation

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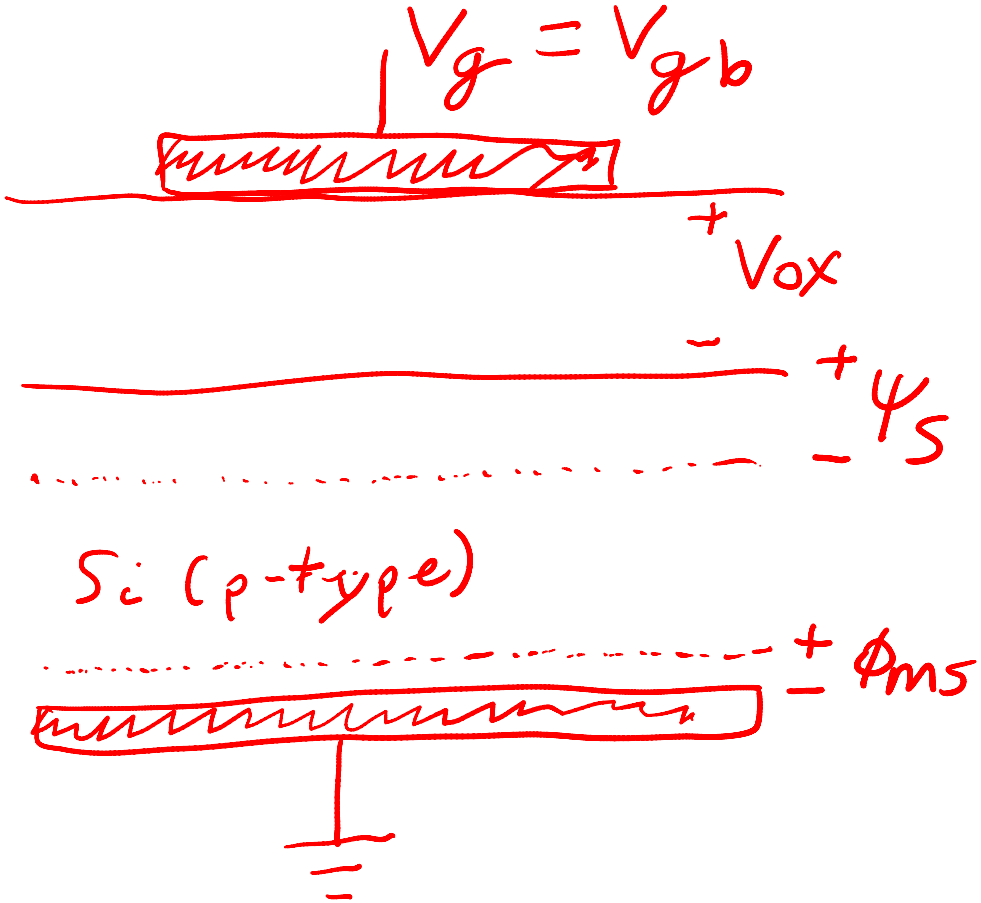
EECS 320 SP 12

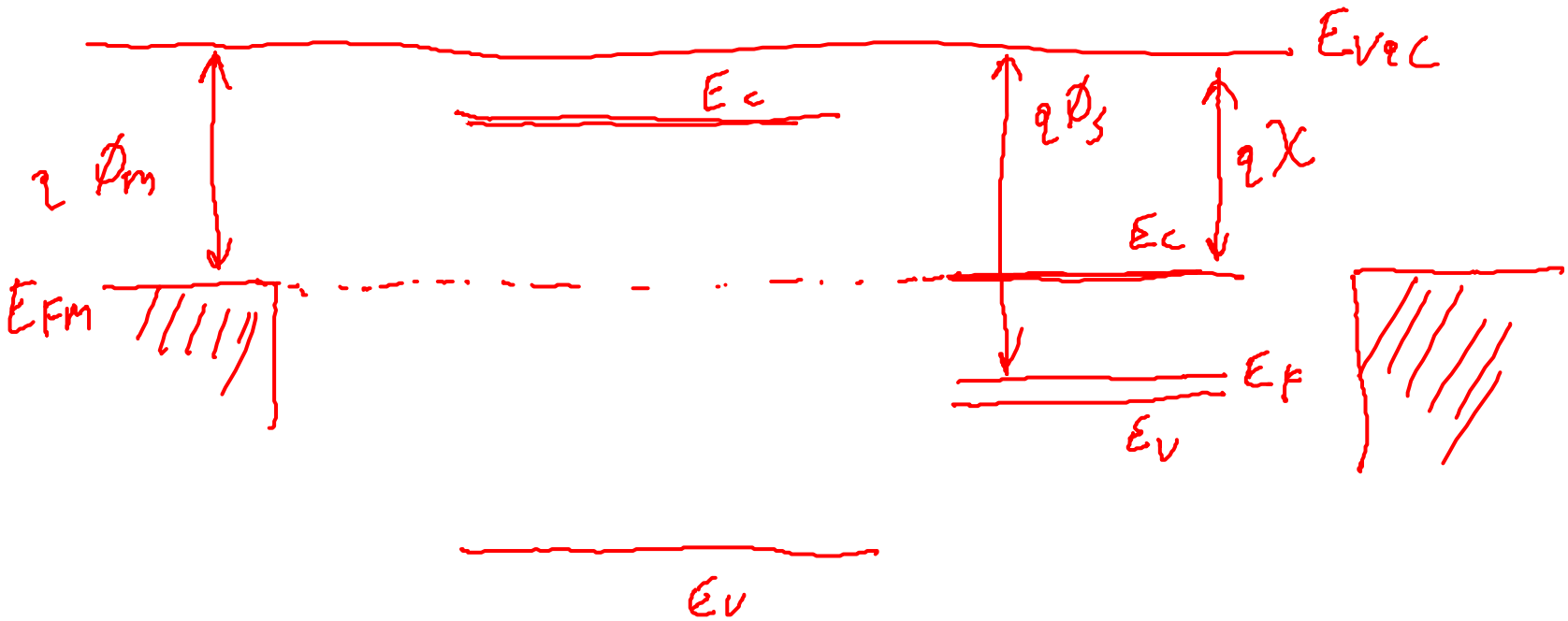
MOS Capacitor Voltage Drops



$$V_g = V_{ox} + \psi_s + \phi_{ms}$$

) Backside Depletion very thin for <u>good</u> ohmic contact

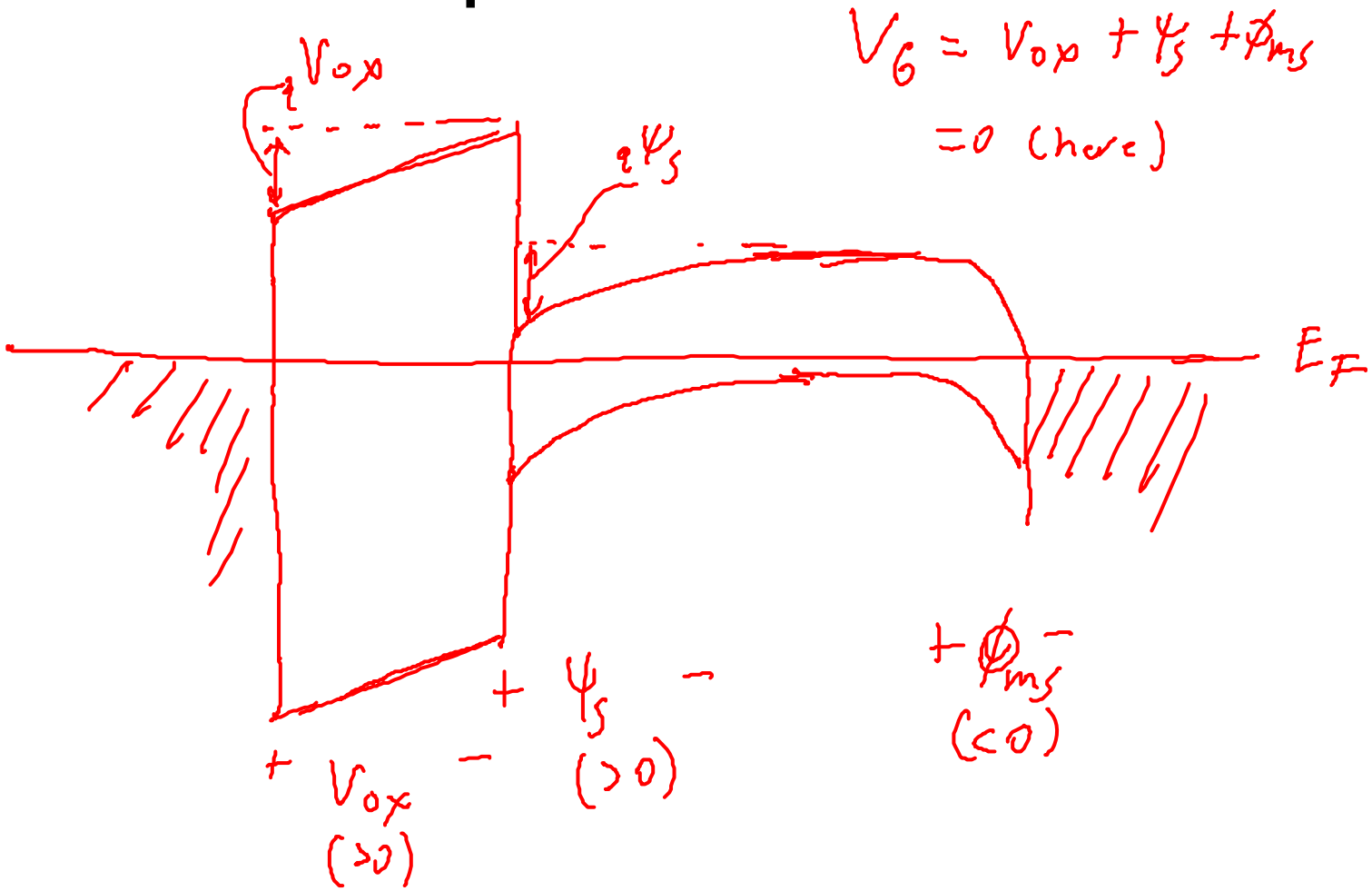
Band Diagram Example



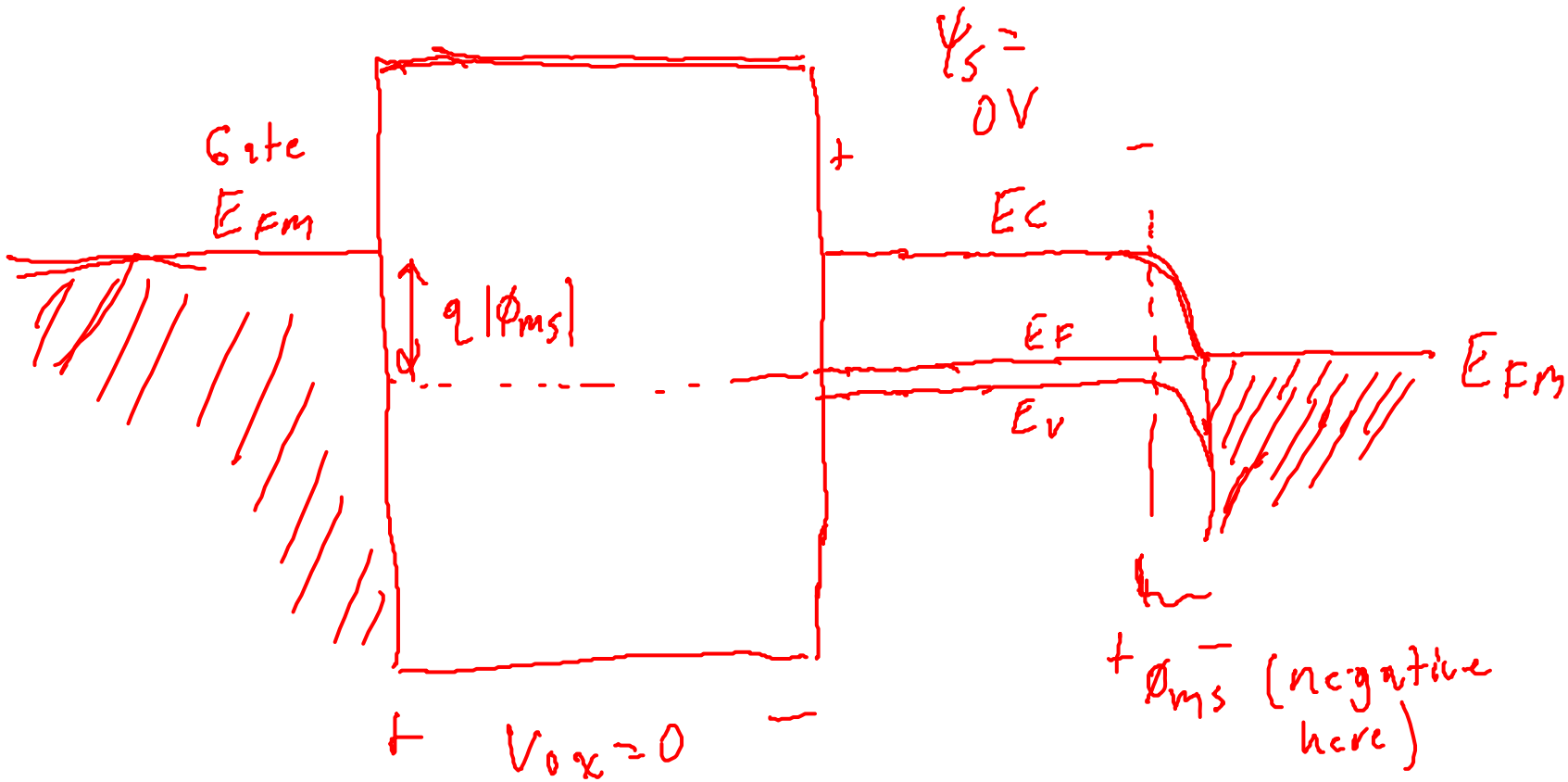
Seperated Materials $\phi_{ms} = (\phi_m - \phi_s) < 0$

$\chi = \phi_m$ for example

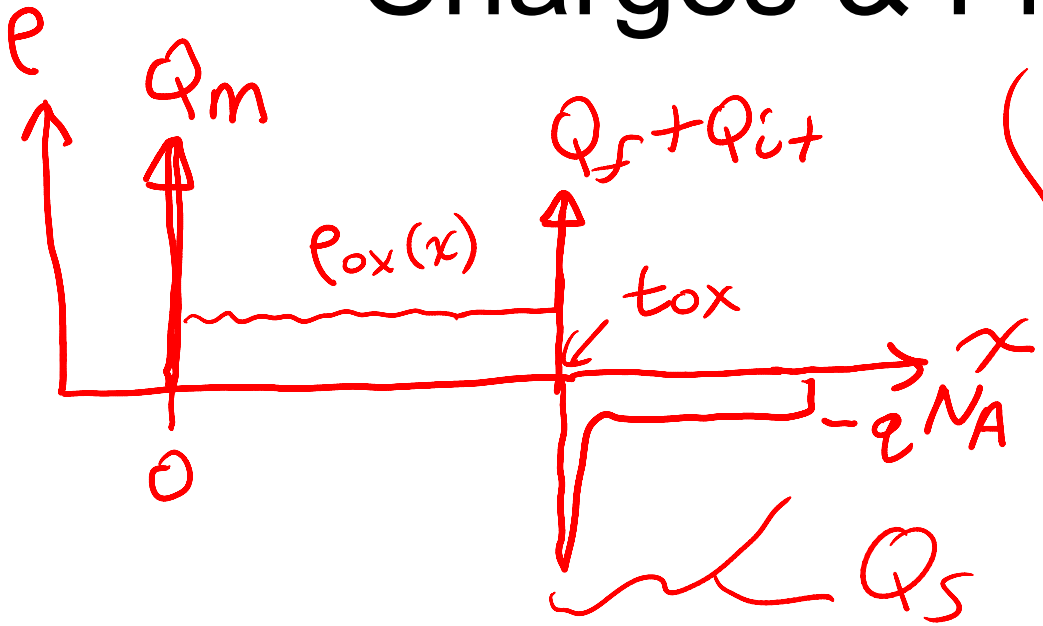
Band Diagram Example at Equilibrium



Band Diagram Example at Flatband



Charges & Fields



ρ_{ox} = bulk ox charge

Q_f = fixed charge

Q_{it} = interface traps

General

$$\mathcal{E}_{ox}(x) = \frac{Q_m}{\epsilon_{ox}} + \int_0^x \frac{\rho(x)}{\epsilon_{ox}}\,dx$$

Assume for now $\rho_{ox}(x) = 0$

$$\Rightarrow \boxed{\mathcal{E}_{ox}(x) = Q_m/\epsilon_{ox}}$$ Constant in x

V_{ox}

$$V_{ox} = V(0) - V(t_{ox})$$

$$= -\int_{t_{ox}}^{0} \mathcal{E}_{ox}(x)\,dx = \int_{0}^{t_{ox}} \mathcal{E}_{ox}\,dx$$

$$= \frac{Q_m}{\epsilon_{ox}} t_{ox} = \frac{Q_m}{C_{ox}}$$

$$C_{ox} \equiv \frac{\epsilon_{ox}}{t_{ox}}$$ capacitance/unit area

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Charge Neutrality

$$Q_m + \underbrace{\cancel{Q_{ox}}}_{0} + Q_{it} + Q_f + Q_s = 0$$

V_{ox}

$$Q_m = -(Q_s + Q_f + Q_{it})$$

$$V_{ox} = -\left[\frac{Q_s}{C_{ox}} + \frac{Q_f}{C_{ox}} + \frac{Q_{it}}{C_{ox}}\right]$$

Q_s function of ψ_s

Q_{it} function of ψ_s & Defects

Q_f fixed vs. ψ_s, Defects

Drop Q_{it} & Q_f for now

V_{gb}

$$V_g = \phi_{ms} + \psi_s + \underbrace{\frac{-Q_s}{C_{ox}}}_{V_{ox}}$$

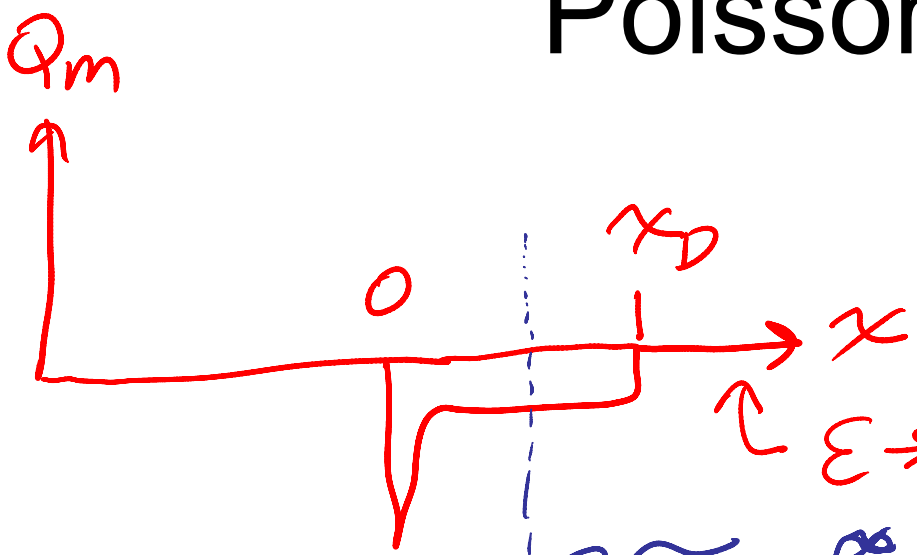
ignoring oxide & interface defects

Q_s vs ψ_s?

General Poisson Eqn:

$$\frac{\partial \mathcal{E}}{\partial x} = \frac{\rho}{\epsilon} = -\frac{\partial^2 \psi}{\partial x^2}$$

Poisson



$\mathcal{E} \to 0$ in Si bulk

$$Q = \int_x^{\infty} \rho \, dx = \epsilon_{Si} \mathcal{E}(x)$$

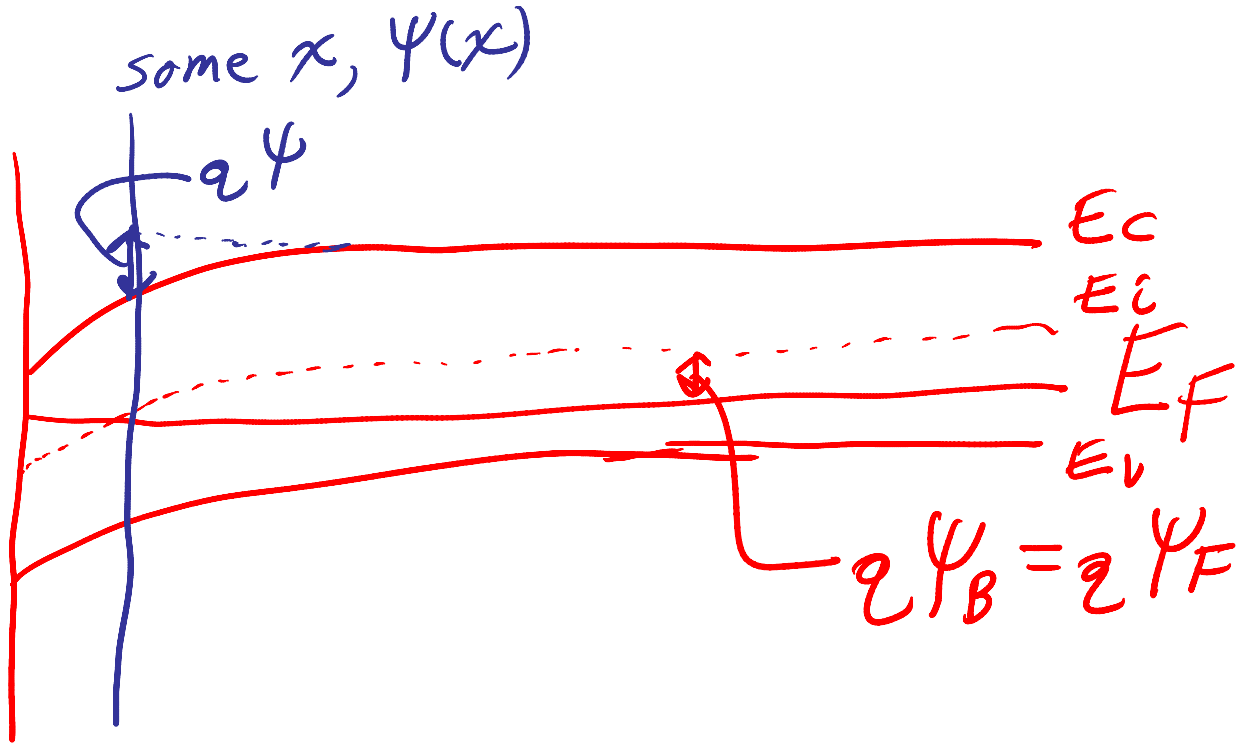
$$\rho = q\left[N_D^+ - N_A^- + p - n\right] \quad \text{in Si}$$

take $\left.\begin{array}{l} N_D^+ \simeq N_D \\ N_A^- \simeq N_A \end{array}\right\}$ 100% ionization

Assume Boltzmann Approximation

$$n = n_i e^{q(E_F - E_i)/kT}$$

$$= n_0 e^{q\psi/kT}$$



$$n = n_0 e^{q\psi/kT} \qquad p = p_0 e^{-q\psi/kT}$$

So

$$\rho = q\left[N_D - N_A + p_0 e^{-q\psi/kT} - n_0 e^{q\psi/kT}\right]$$

In the bulk $\psi \to 0$ & $\rho \to 0$

So

$$0 = q\left[N_D - N_A + p_0 - n_0\right]$$

$$\Rightarrow N_D - N_A = n_0 - p_0$$

$$\Rightarrow \rho = q\left[p_0\left(e^{-q\psi/kT} - 1\right) - n_0\left(e^{+q\psi/kT} - 1\right)\right]$$

In principle

$$Q_s = \int_0^\infty \rho(x)\,dx$$

Finding $\psi(x)$ not easy

All we want is $Q_s(\psi_s) = -\epsilon_{Si}\mathcal{E}(\psi_s)$

Turns out to be easier to find

$$\mathcal{E}(\psi_s) = -\left.\frac{\partial\psi}{\partial x}\right|_{\psi_s}$$

Trick

Known function

$$\int_0^\psi \frac{\partial^2\psi}{\partial x^2}\,d\psi = \frac{-1}{\epsilon}\int_0^\psi \rho(\psi)\,d\psi$$

$\psi = 0$ as $x \to \infty$ (bulk)

$$\int_0^{\psi} \frac{\partial^2 \psi}{\partial x^2}\, \partial \psi \quad ?$$

let $u = \dfrac{\partial \psi}{\partial x}$ $\qquad \dfrac{\partial u}{\partial x} = \dfrac{\partial^2 \psi}{\partial x} = \dfrac{\partial}{\partial x}\left(\dfrac{\partial \psi}{\partial x}\right)$

$\Rightarrow \partial \psi = u\, \partial x$ $\qquad \Rightarrow \partial u = \partial\left(\dfrac{\partial \psi}{\partial x}\right)$

$\left\{ \left.\dfrac{\partial \psi}{\partial x}\right|_{\psi=0} \right\} = 0$ zero $\mathcal{E}$ field in bulk

so

$$\int_0^{\psi} \frac{\partial^2 \psi}{\partial x^2}\, \partial \psi = \int_0^{\psi} \frac{\partial u}{\partial x}\,(u\, dx)$$

$$= \int_0^{\partial \psi / \partial x} u\, du$$

$$= \left. \frac{1}{2} u^2 \right|_0^{\partial \psi / \partial x}$$

$$\int_0^{\psi} \frac{\partial^2 \psi}{\partial x^2}\, dx = \frac{1}{2}\left(\frac{\partial \psi}{\partial x}\right)^2 = \frac{1}{2}\left(-\mathcal{E}(\psi)\right)^2$$

Evaluating the Integral over $\rho(\psi)$ is straightforward, yielding

$$\frac{1}{2}\mathcal{E}^2(\psi) = \frac{kT}{\epsilon_{Si}}\left\{ n_0\left(e^{\beta\psi} - \beta\psi - 1\right) + p_0\left(e^{-\beta\psi} + \beta\psi - 1\right)\right\}$$

where $\beta \equiv (q/kT)$

We get the sign of $\mathcal{E}(\psi)$ from physics $\mathcal{E} > 0$ if $\psi > 0$

$\mathcal{E} < 0$ if $\psi < 0$

So finally

$$\mathcal{E}(\psi) = \pm \frac{\sqrt{2}\,kT}{qL_D} F(\psi, n_0, p_0) \quad \{+\psi > 0\}$$

$$Q(\psi) = \mp \frac{\sqrt{2}\,kT\epsilon_{Si}}{qL_D} F(\psi, n_0, p_0) \quad \{-\psi > 0\}$$

$$L_D \equiv \left[\frac{kT\epsilon_{Si}}{q^2 p_0}\right]^{1/2}$$ extrinsic Debye Length for p-Si

$$F(\psi, n_0, p_0) \equiv \left\{\left(\frac{n_0}{p_0}\right)\left(e^{\beta\psi} - \beta\psi - 1\right) + \left(e^{-\beta\psi} + \beta\psi - 1\right)\right\}^{1/2}$$

$$Q_D = Q_B \simeq -\frac{\sqrt{2}\, kT\epsilon_{Si}}{qL_D} F(\psi_s, 0, p_0)$$

ignore e^-'s

$$Q_n = Q_s - Q_D$$

electron charge Density
(inversion)

So now we can numerically evaluate

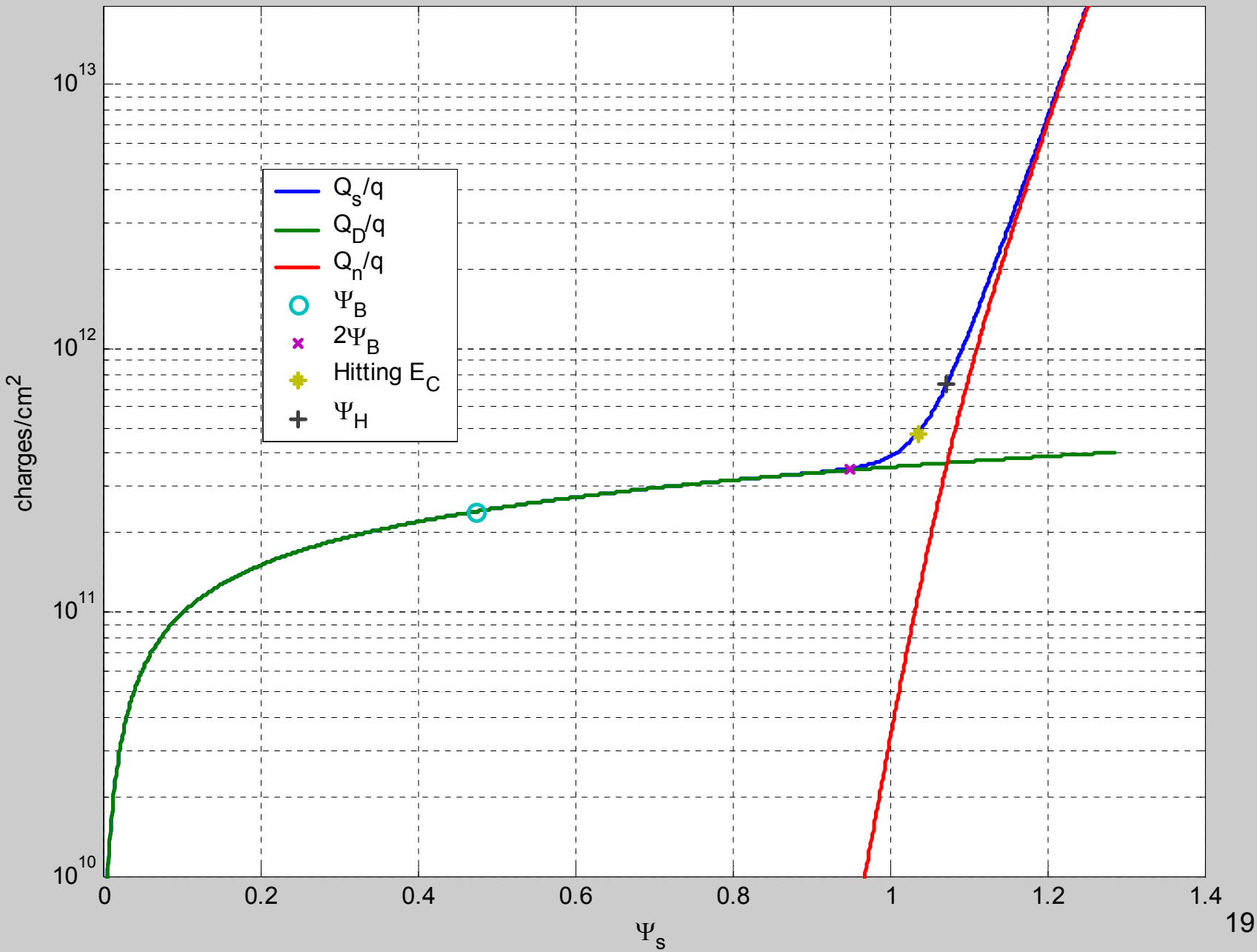
$$V_{gb} = \phi_{ms} + \psi_s + \frac{-Q_s(\psi_s)}{C_{ox}}$$

Example: $\underline{300K}$ (Room Temp)

$N_A = 1 \times 10^{16}\ cm^{-3}$ $\qquad \epsilon_{Si} = 11.7\epsilon_0$

$$\psi_B = \psi_F = \frac{kT}{q} \ln\left(\frac{p_0}{n_i}\right) \simeq 0.4745\ V$$

$$2\psi_B \simeq 0.9489\ V$$

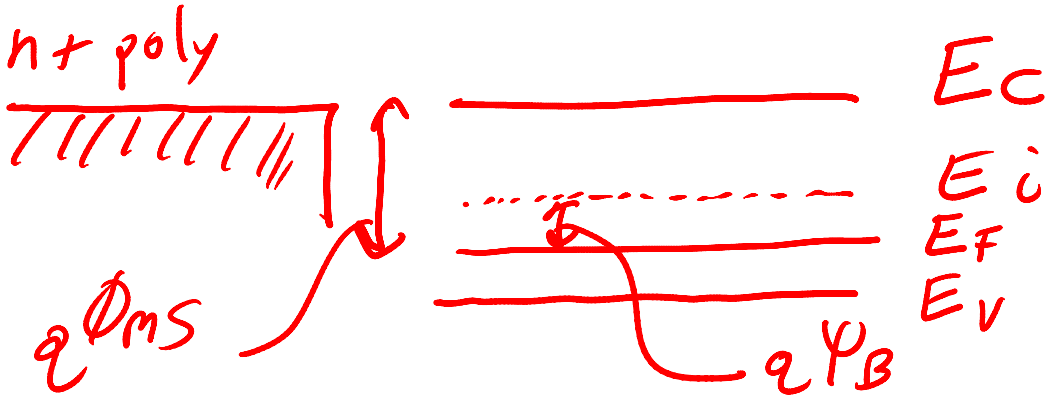

Q_s/q
Q_D/q
Q_n/q
Ψ_B
$2\Psi_B$
Hitting E_C
Ψ_H
charges/cm^2
10^{13}
10^{12}
10^{11}
10^{10}
0
0.2
0.4
0.6
0.8
1
1.2
1.4
Ψ_s

Now add gate & gate oxide

$t_{ox} = 40\ nm \qquad \epsilon_{ox} = 3.9\,\epsilon_0$

$\Rightarrow C_{ox} \simeq 8.633 \times 10^{-8}\ F/cm^2$

Assume n^+ poly-Si gate



Here

$$\phi_{ms} \simeq -(E_c - E_i + q\psi_B)/q$$

$$\simeq -[0.561\,V + \psi_B] \simeq -1.036\ V$$

$$V_g = \phi_{ms} + \psi_s + \frac{-Q_s(\psi_s)}{C_{ox}}$$

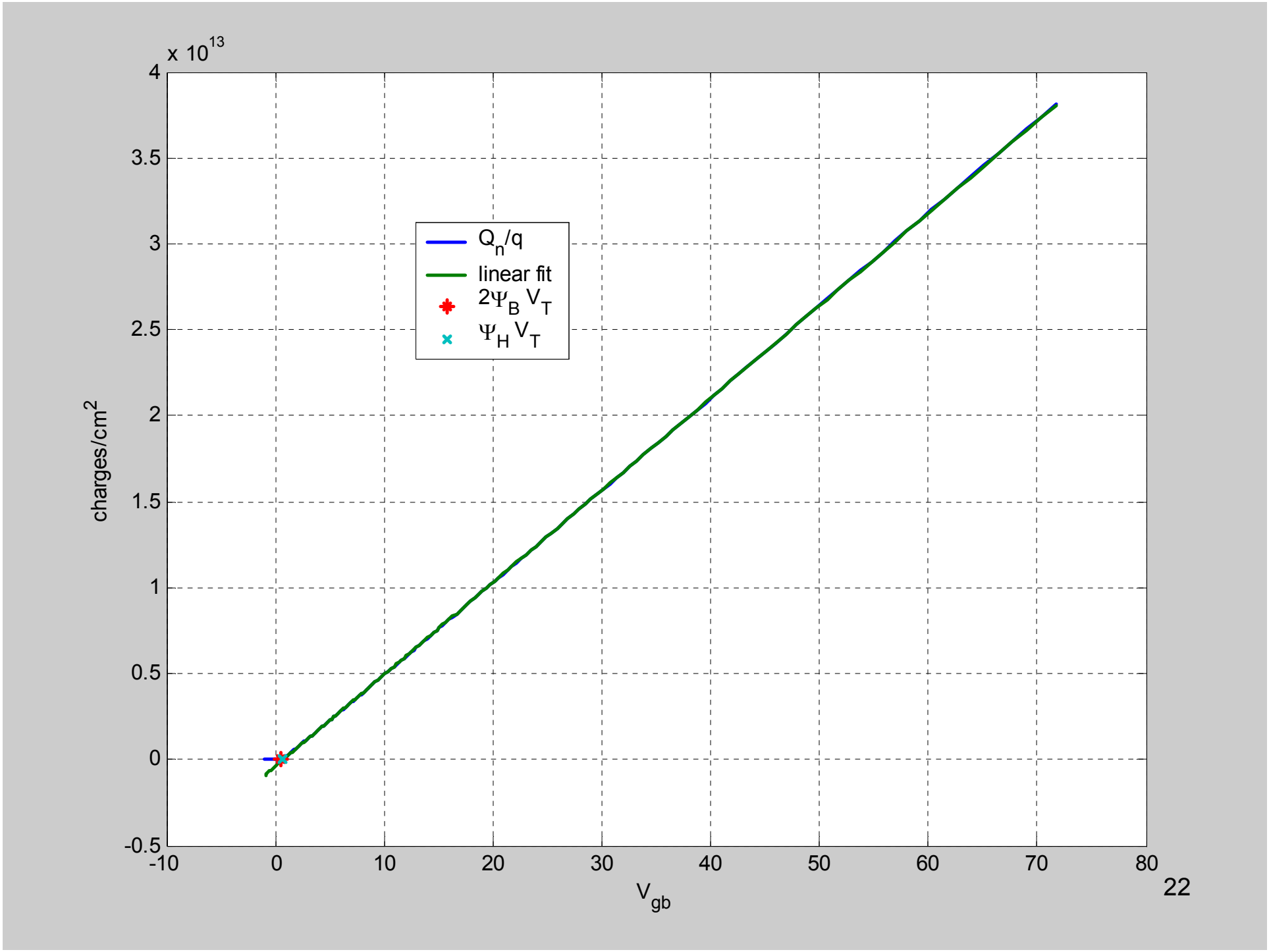
$$Q_s = Q_n + Q_D$$ all functions of ψ_s

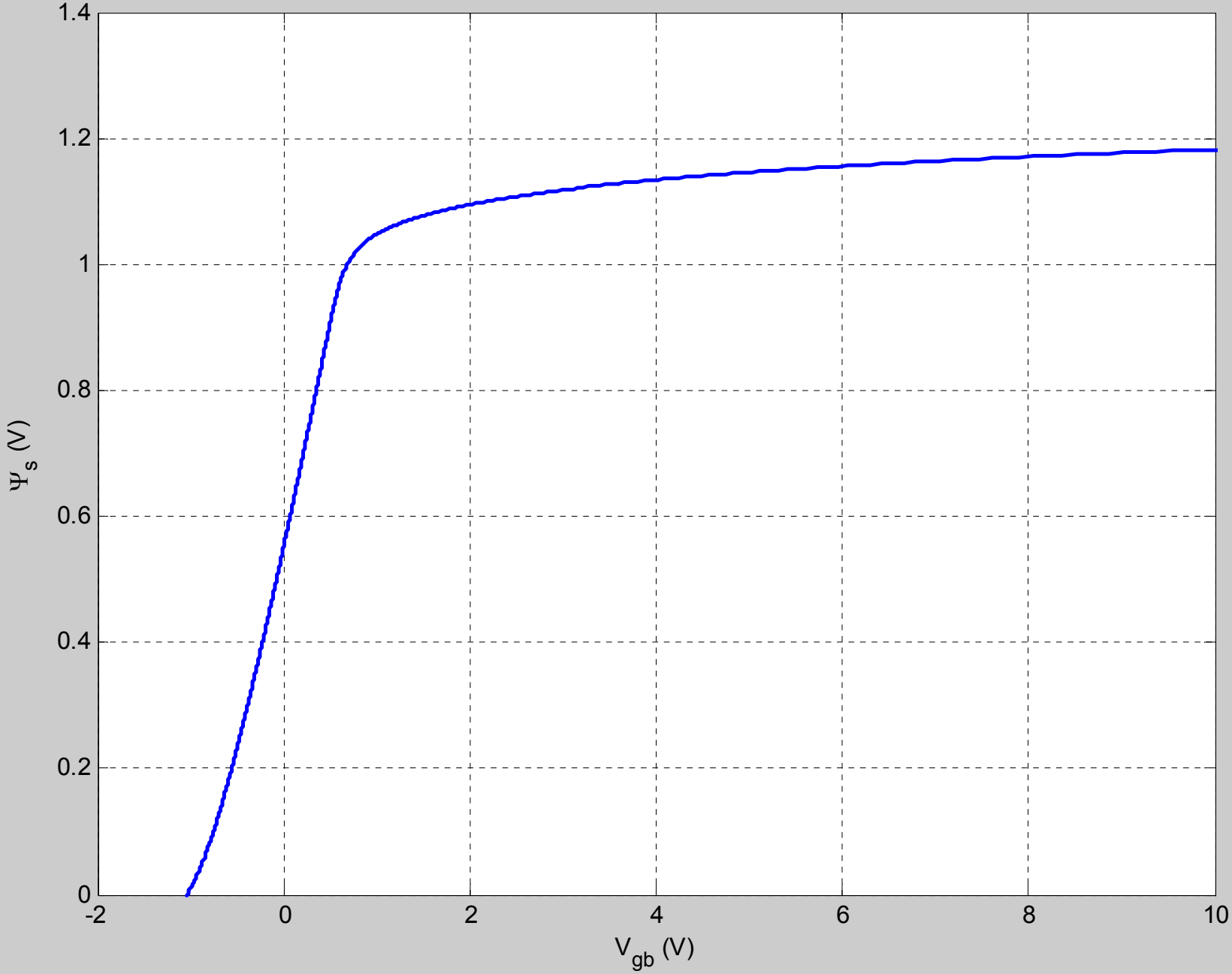
we can plot

V_g vs ψ_s & Q_n vs ψ_s

& thus Q_n vs V_g

with no additional approximations


x 10^13
4
3.5
3
2.5
2
1.5
1
0.5
0
-0.5
charges/cm^2
-10
0
10
20
30
40
50
60
70
80
V_gb
Q_n/q
linear fit
2Ψ_B V_T
Ψ_H V_T


1.4
1.2
1
0.8
0.6
0.4
0.2
0
-2
0
2
4
6
8
10
Ψ_s (V)
V_{gb} (V)

Threshold

- Ideas:
- $\Psi_s \approx$Constant above V_T
- $Q_D \approx$Constant (slowly varying with Ψ_s)
- Q_n cannot be taken as ≈Constant

$$V_g = \phi_{ms} + \psi_s + \frac{-(Q_D + Q_n)}{C_{ox}}$$

$$\approx \phi_{ms} + \psi_H + \frac{-Q_D(\psi_H)}{C_{ox}} + \frac{-Q_n}{C_{ox}}$$

$$V_g \cong \phi_{ms} + \psi_H + \frac{-Q_D(\psi_H)}{C_{ox}} + \frac{-Q_n}{C_{ox}}$$

$$\Rightarrow Q_n \cong -C_{ox}\left[V_g - \underbrace{\left(\phi_{ms} + \psi_H + \frac{-Q_D(\psi_H)}{C_{ox}}\right)}_{V_T}\right]$$

$$-Q_D(\psi_H) \simeq \left[2q\epsilon_{Si}N_A\psi_H\right]^{1/2}$$ Dep. Approx.

$\psi_H \equiv$ Strong Inversion Potential

ψ_H ?

Usual Definition

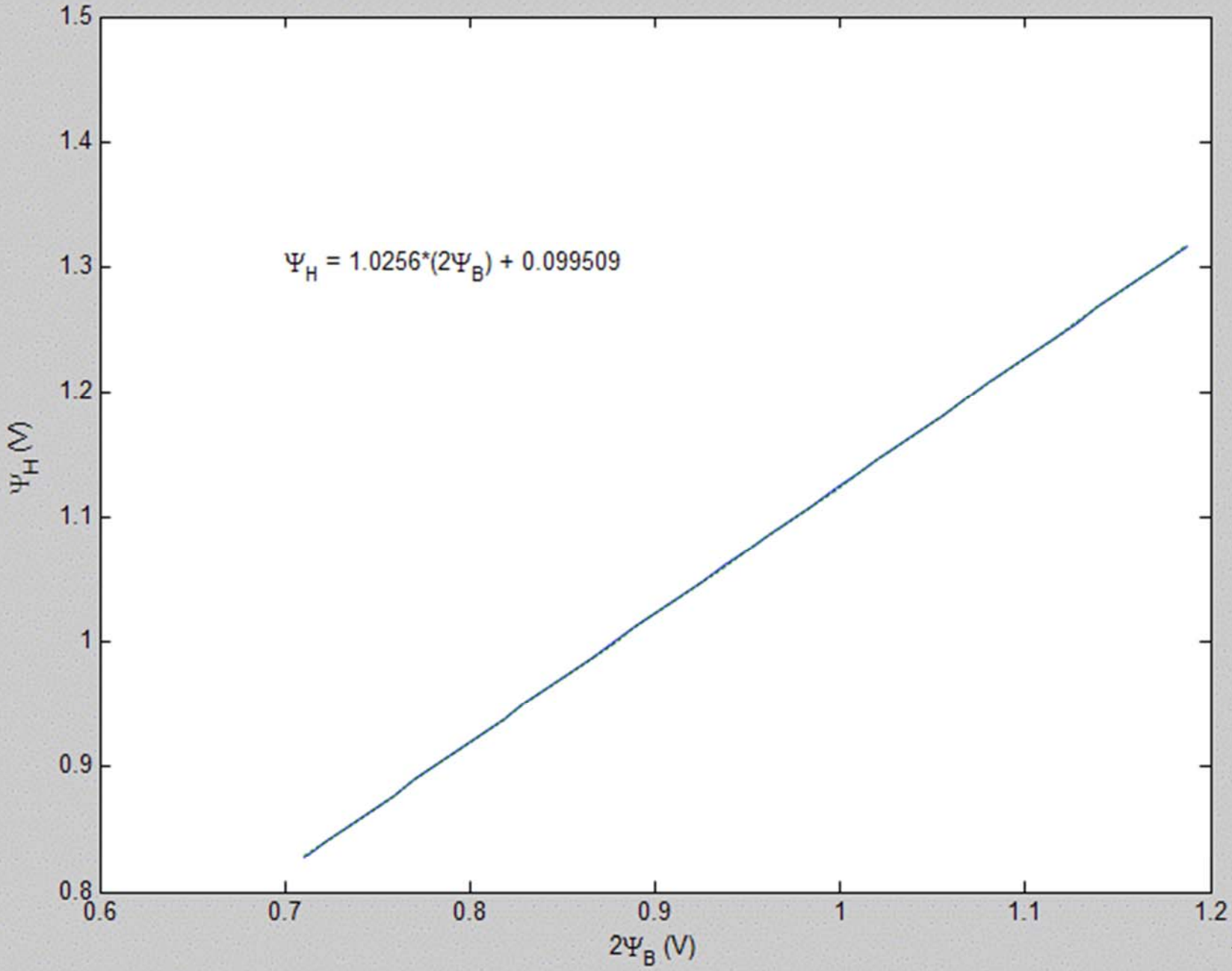
$$\psi_H \simeq 2\psi_B$$

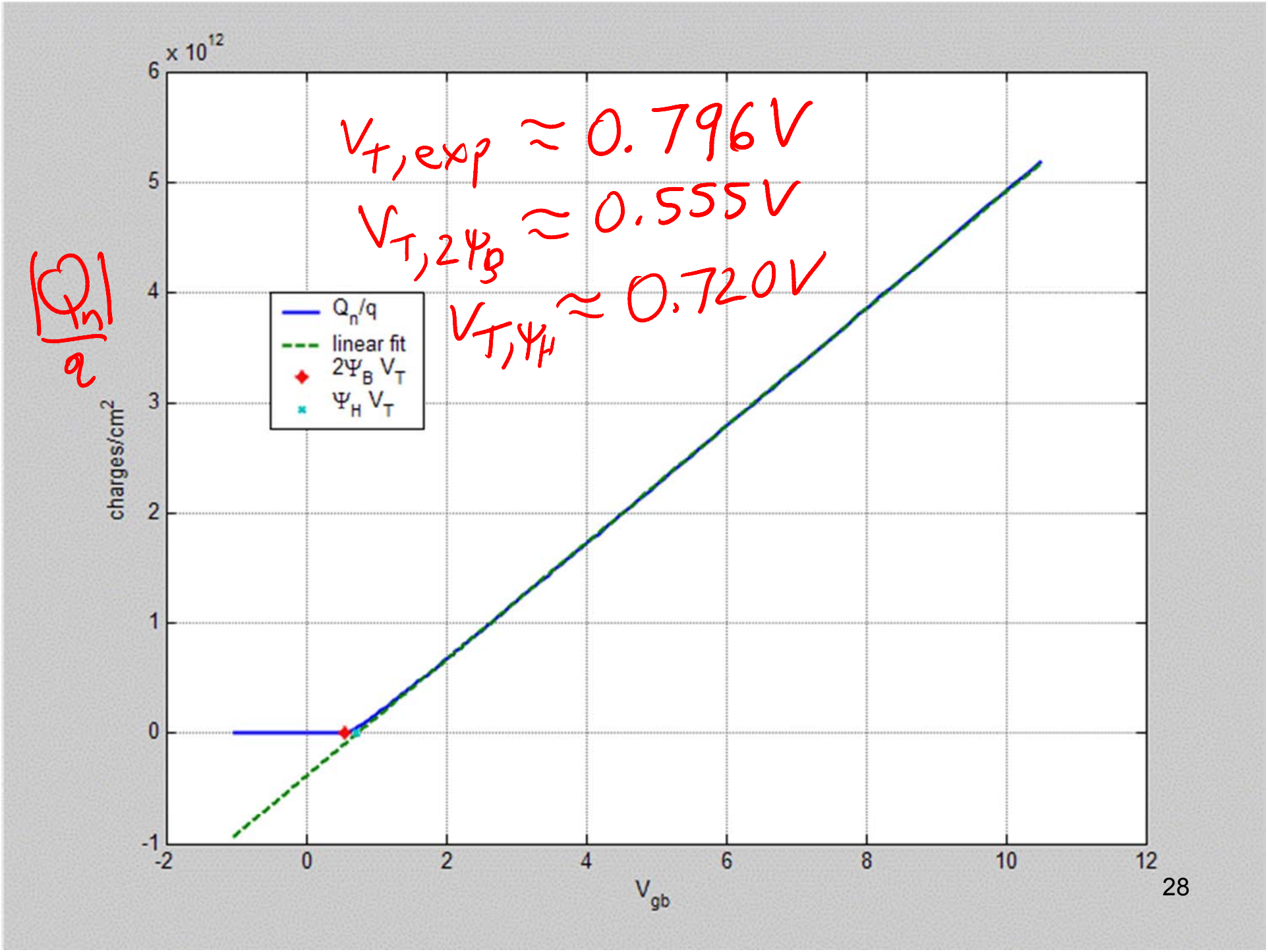
Stronger Condition

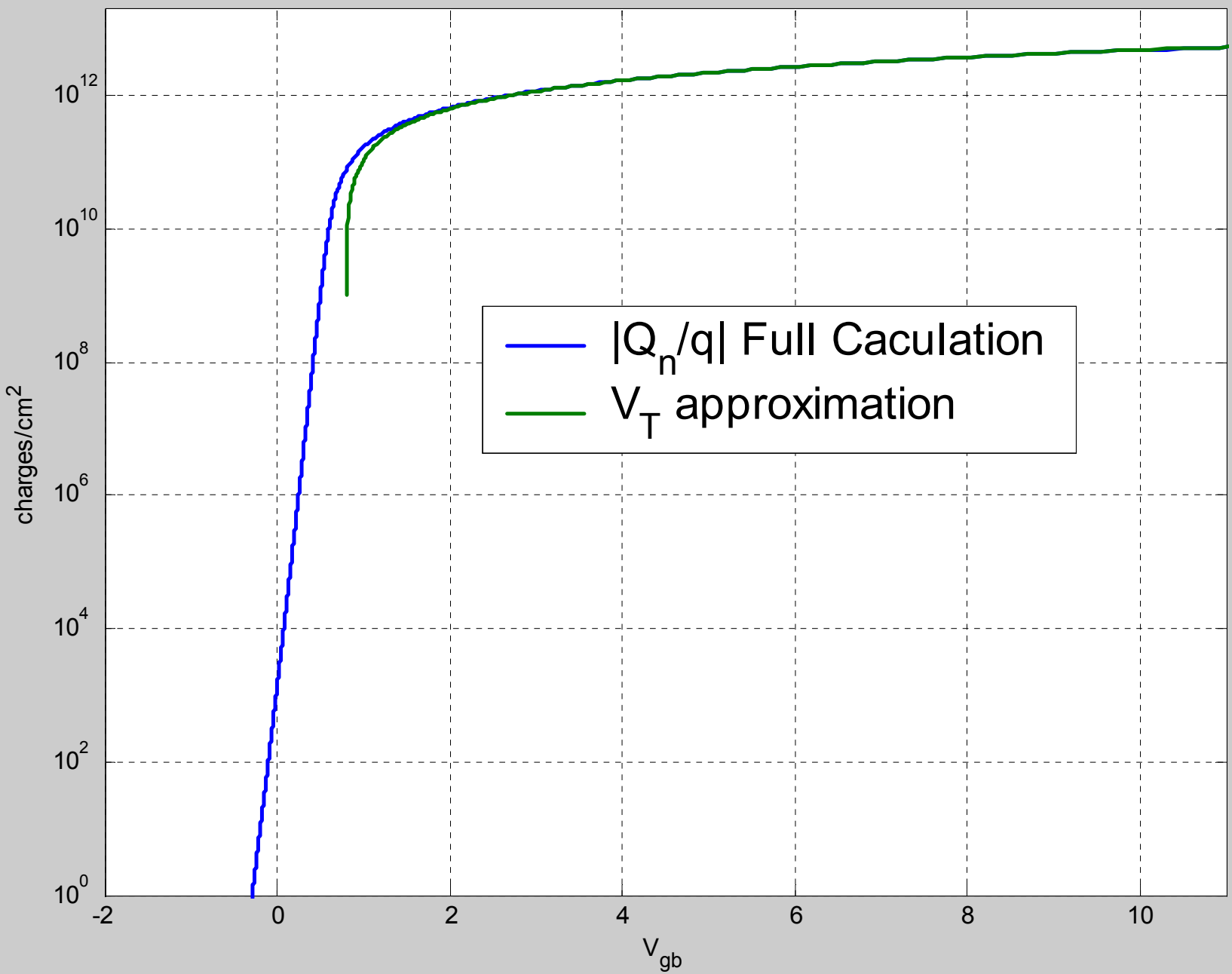
$$Q_D(\psi_H) = Q_n(\psi_H)$$

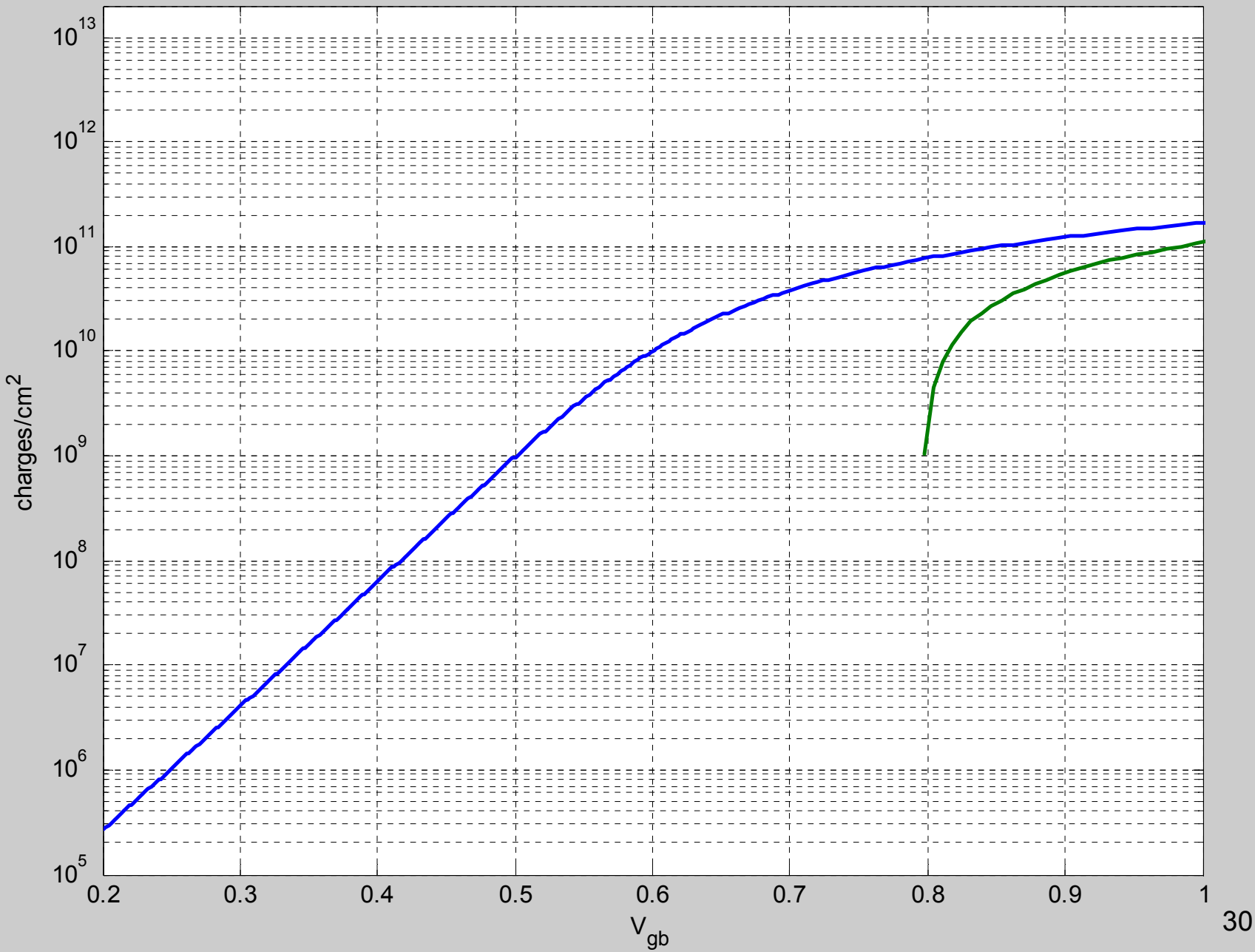
Can be solved numerically but good approximation

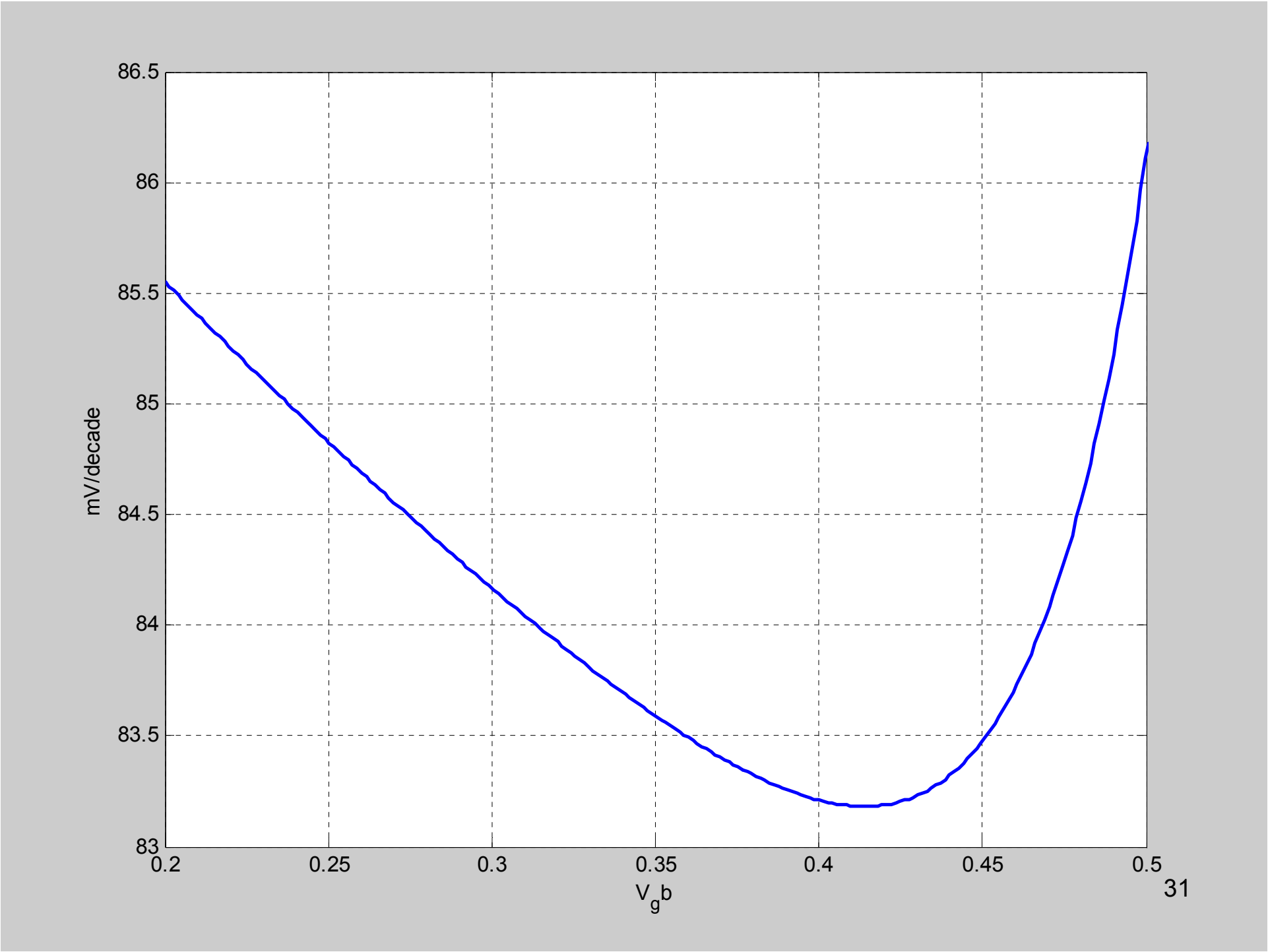
$$\psi_H \simeq [1.2056\,(2\psi_B) + 99.609\text{ mV}]$$


$\Psi_H = 1.0256*(2\Psi_B) + 0.099509$
Ψ_H (V)
$2\Psi_B$ (V)
1.5
1.4
1.3
1.2
1.1
1
0.9
0.8
0.6
0.7
0.8
0.9
1
1.1
1.2


x 10^12
charges/cm^2
V_gb
Q_n/q
linear fit
2Ψ_B V_T
Ψ_H V_T
|Q_n|/q
V_T,exp ≈ 0.796V
V_T,2Ψ_B ≈ 0.555V
V_T,Ψ_H ≈ 0.720V


charges/cm^2
10^{12}
10^{10}
10^{8}
10^{6}
10^{4}
10^{2}
10^{0}
-2
0
2
4
6
8
10
V_{gb}
$|Q_n/q|$ Full Caculation
V_T approximation


charges/cm2
10^13
10^12
10^11
10^10
10^9
10^8
10^7
10^6
10^5
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
Vgb


86.5
86
85.5
85
84.5
84
83.5
83
mV/decade
0.2
0.25
0.3
0.35
0.4
0.45
0.5
$V_g b$