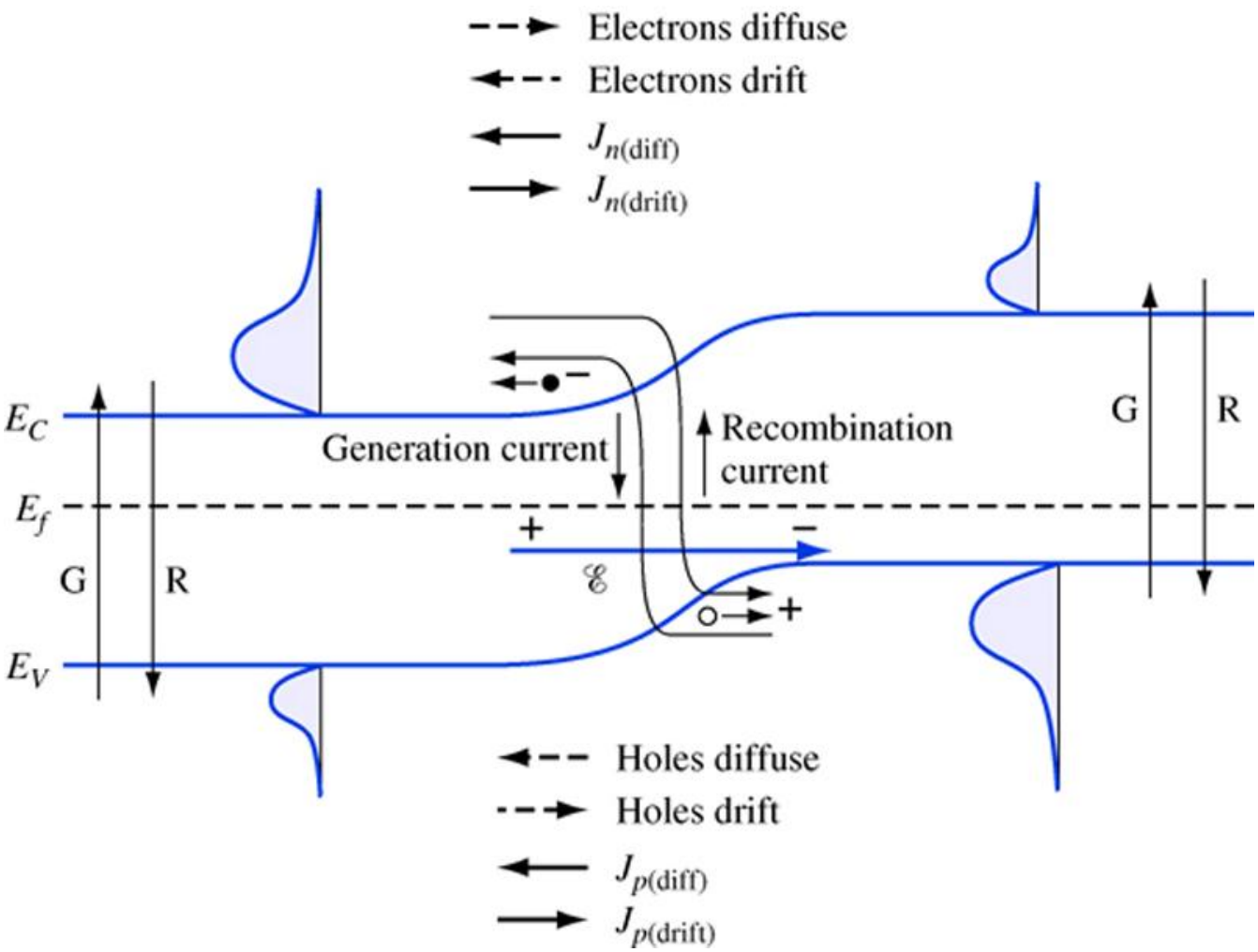
Diode Current-Voltage Characteristics

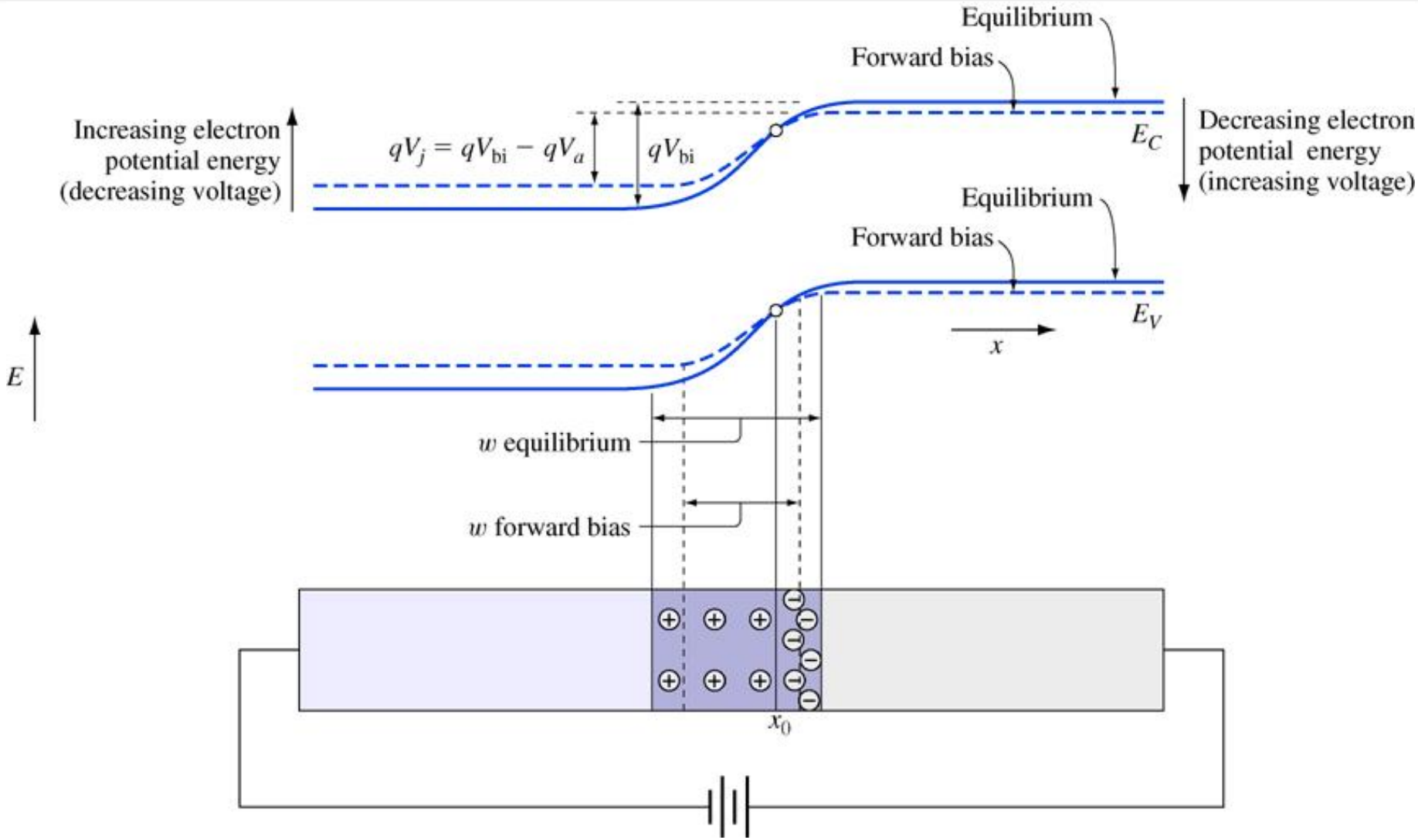
EECS 320

Introduction to Semiconductor Devices

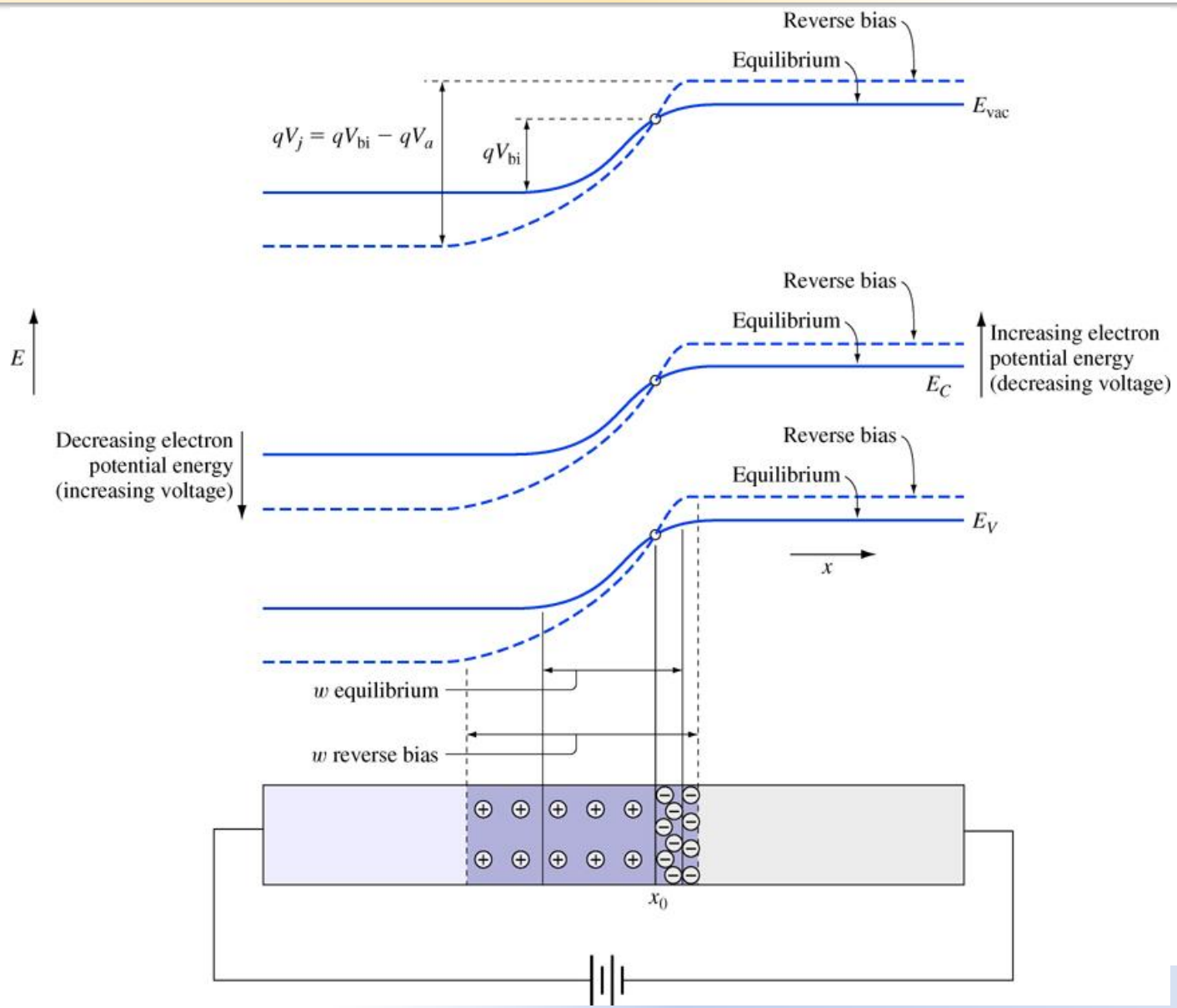
Current in P-N Junction (Equilibrium)



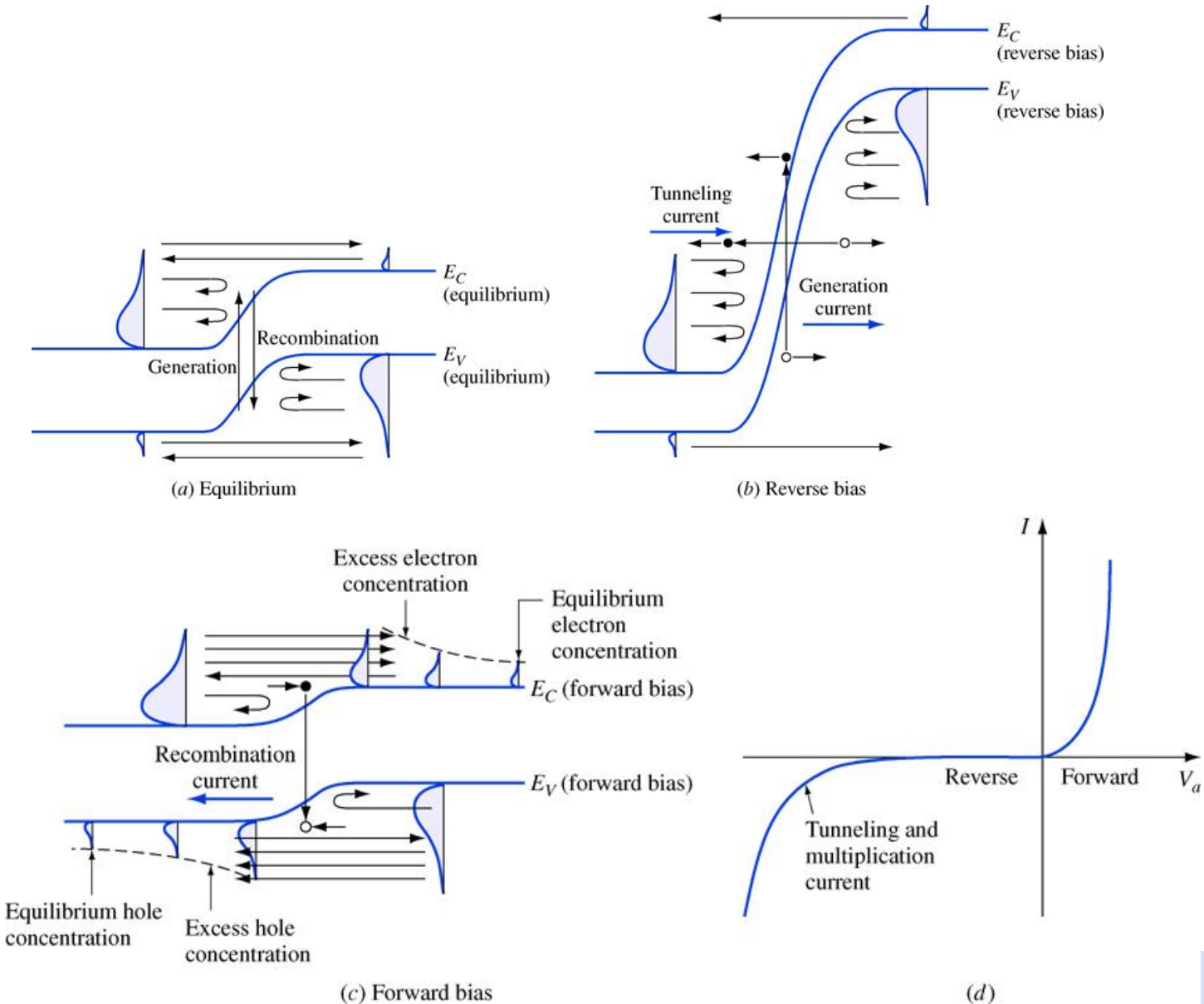
Current in P-N Junction (Forward Bias)

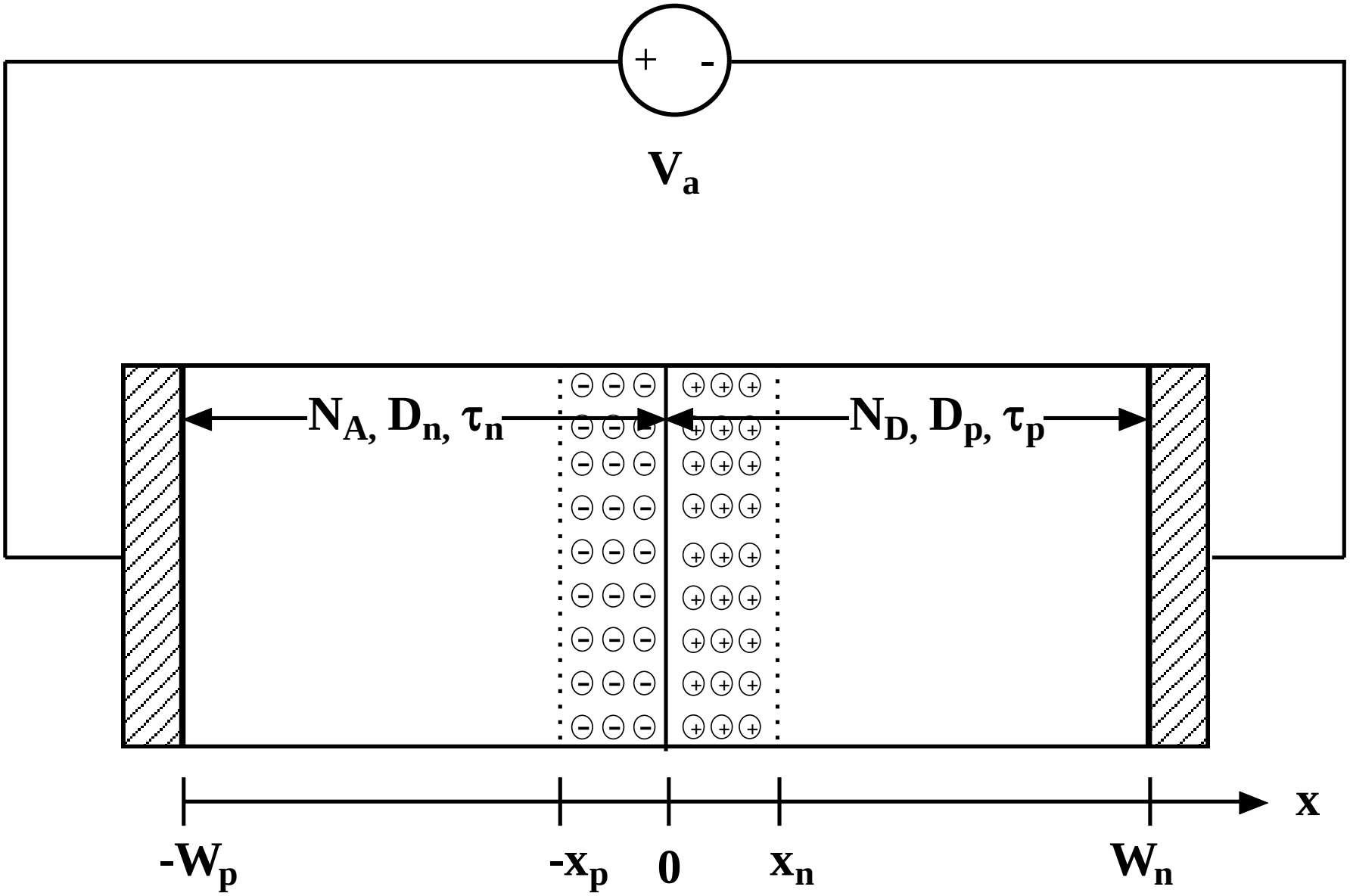


Current in P-N Junction (Reverse Bias)



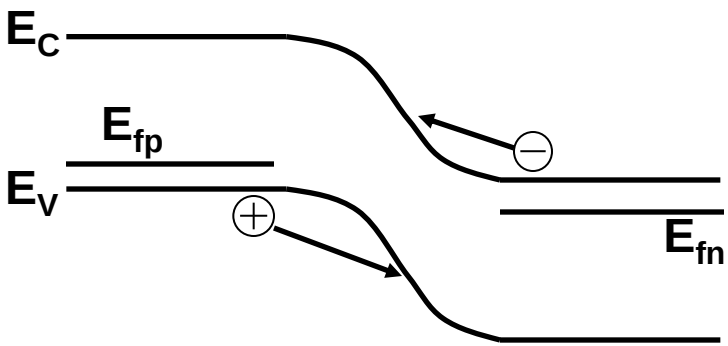
Current-Voltage Characteristics




+
-
V_a
N_A, D_n, τ_n
N_D, D_p, τ_p
x
$-W_p$
$-x_p$
0
x_n
W_n

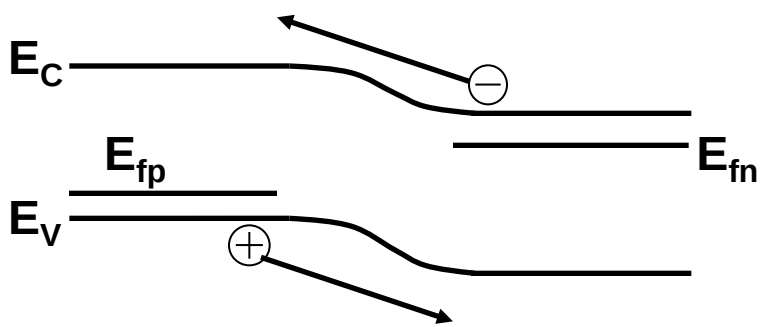
Applied Bias – Current Flow

$V_a < 0$ (reverse bias)



In reverse bias, larger barrier for diffusion current

$V_a > 0$ (forward bias)



In forward bias, lower barrier for diffusion current

How is drift current affected by bias?

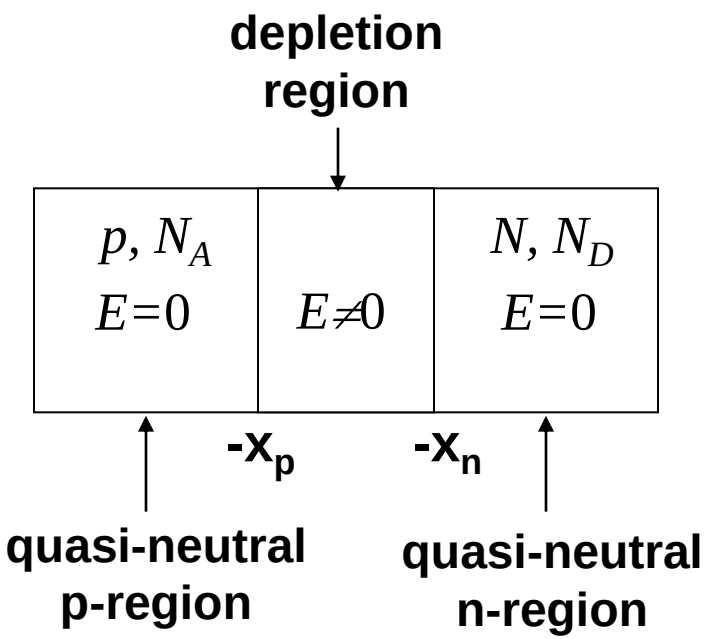
Current – Voltage Relationships

Determine current flow assuming steady state, low-level injection

$$I = AJ \qquad J = J_n + J_p$$

$$J_n = nq\mu_n E + qD_n \frac{\partial n}{\partial x} = n\mu_n \frac{\partial E_{fn}}{\partial x}$$

$$J_p = pq\mu_p E - qD_p \frac{\partial p}{\partial x} = p\mu_p \frac{\partial E_{fp}}{\partial x}$$



Need to determine carrier densities, Solve continuity equations

Three regions of interest

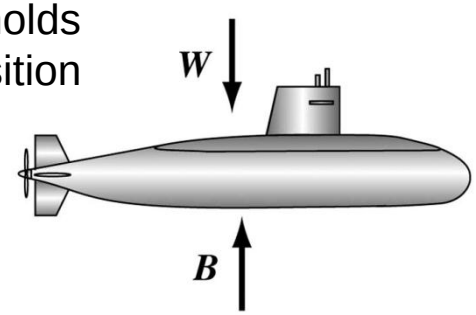
Current-Voltage Relationship

- Minority Current Flow in Quasi-Neutral Regions is dominated by Diffusion Current
  - Finding this current is "easy" if we know minority concentrations at depletion layer edges
  - Shockley-Fletcher Boundary Conditions on Quasi-Fermi Levels
- Assume Current flows are constant across depletion region
- Add to get total current flow

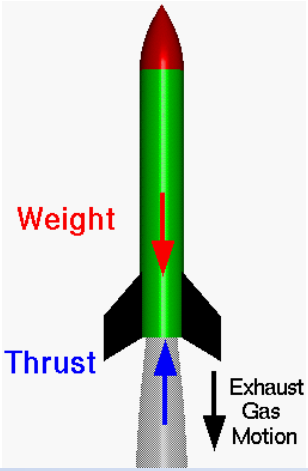
Non-Equilibrium Operation

- Equilibrium: All forces balanced
- Non-Equilibrium: net force acting on system
- Example: Applied voltage, electric field
- Equilibrium: Fermi-level is constant
- Non-Equilibrium: Fermi-level can vary with position, common to define *quasi-Fermi levels* for electrons, holes

Equilibrium:
Submarine holds
constant position

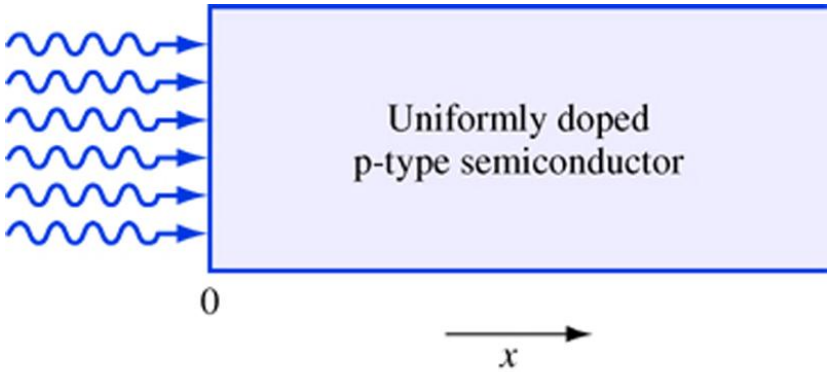


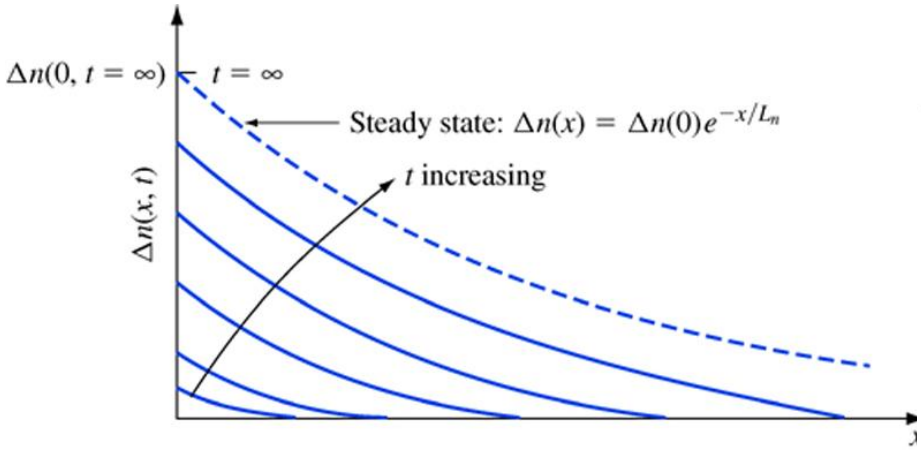
Non-equilibrium:
Rocket launches



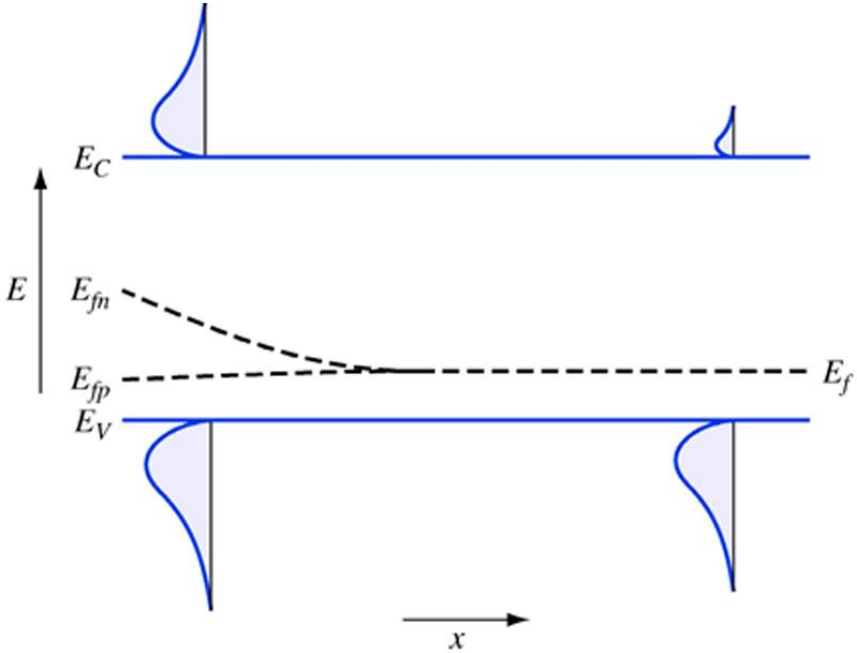
Quasi-Fermi Levels

Under non-equilibrium conditions, quasi-Fermi levels may be used to describe carrier concentration





Example for semiconductor with surface illumination



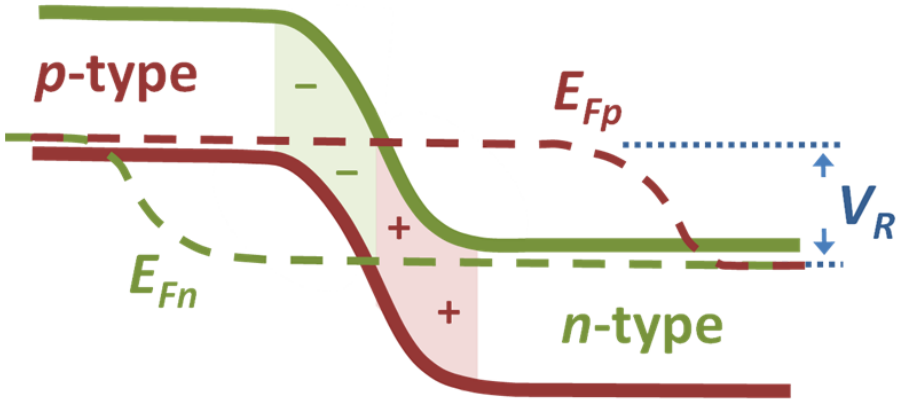
Diode Under Applied Bias

Energy band diagram change with applied bias

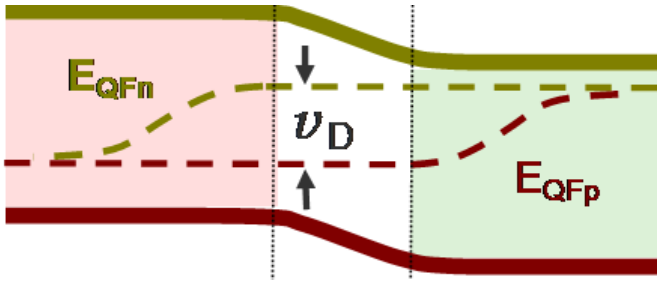
$$n = n_i \exp\left(\frac{E_{fn} - E_i}{kT}\right) \qquad p = n_i \exp\left(\frac{E_i - E_{fp}}{kT}\right) \qquad E_{fn} - E_{fp} = qV_a$$

$$n = n_0 \exp\left(\frac{E_{fn} - E_f}{kT}\right) \qquad p = p_0 \exp\left(\frac{E_f - E_{fp}}{kT}\right)$$

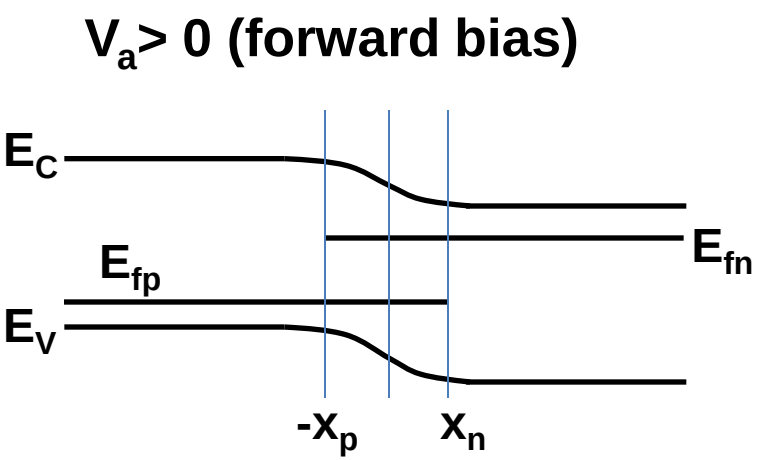
V_a< 0 (reverse bias)

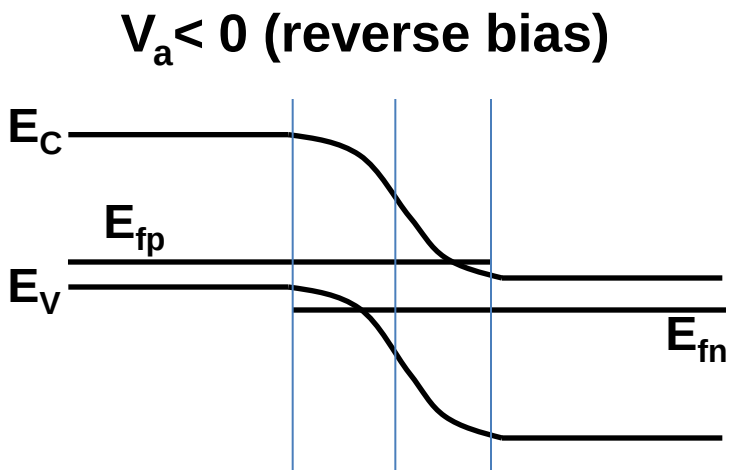


V_a> 0 (forward bias)



Shockley-Fletcher Boundary Conditions

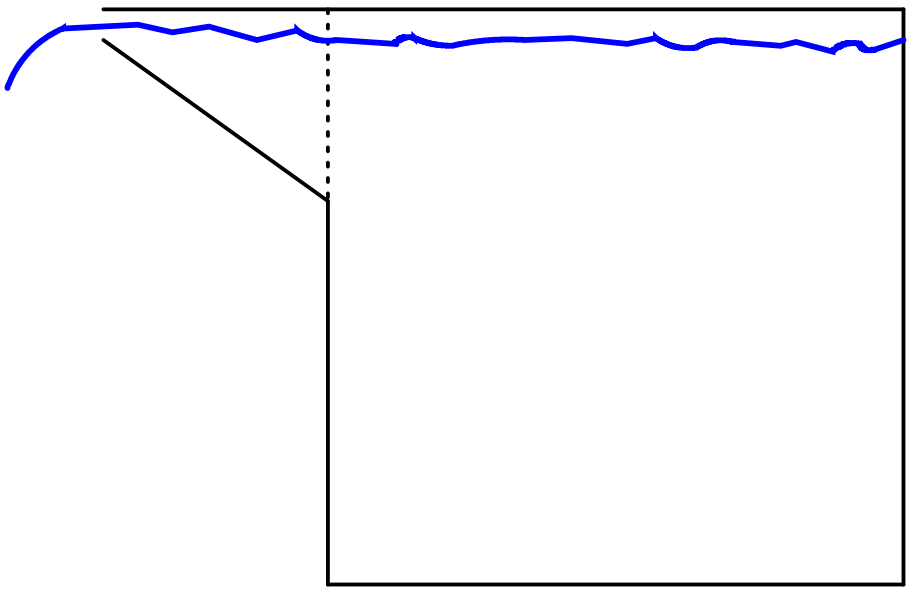




- Extend E_{fn}, E_{fp} flat to opposite site of depletion region
- Yields $n_p(-x_p) = n_{p0}\exp\left(\frac{qV_A}{kT}\right)$ *and* $p_n(x_n) = p_{n0}\exp\left(\frac{qV_A}{kT}\right)$
- Why should we believe it?

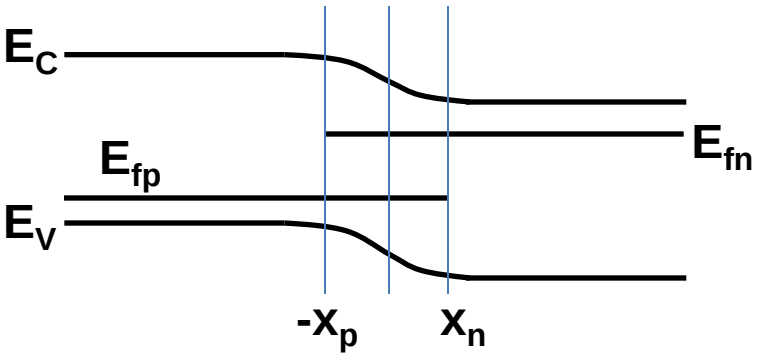
Slightly Leaky Bucket Idea

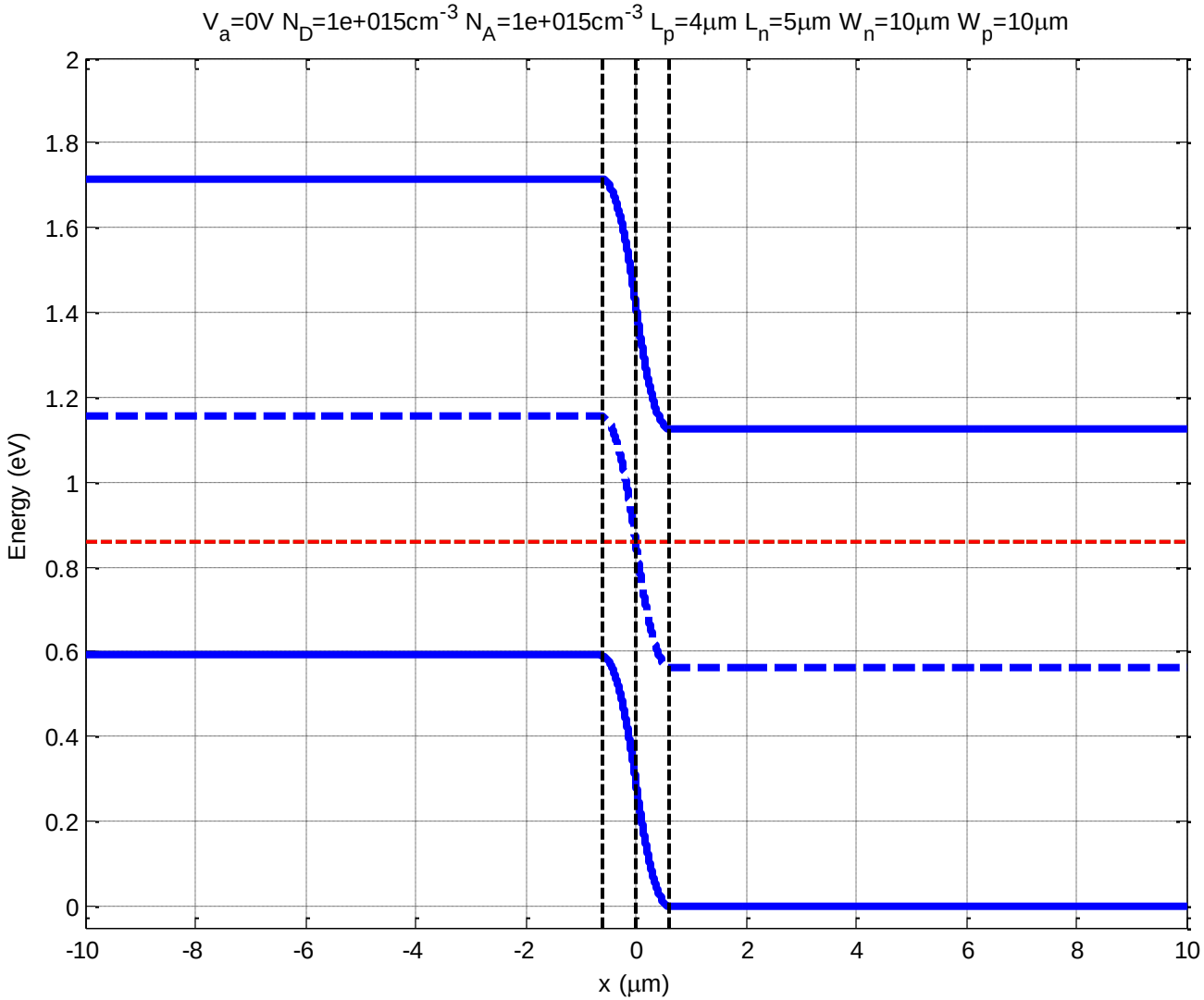
- Level of water in the bucket is not significantly affected if hole is small
- Small system attached to large reservoir
- Formalized in statistical mechanics

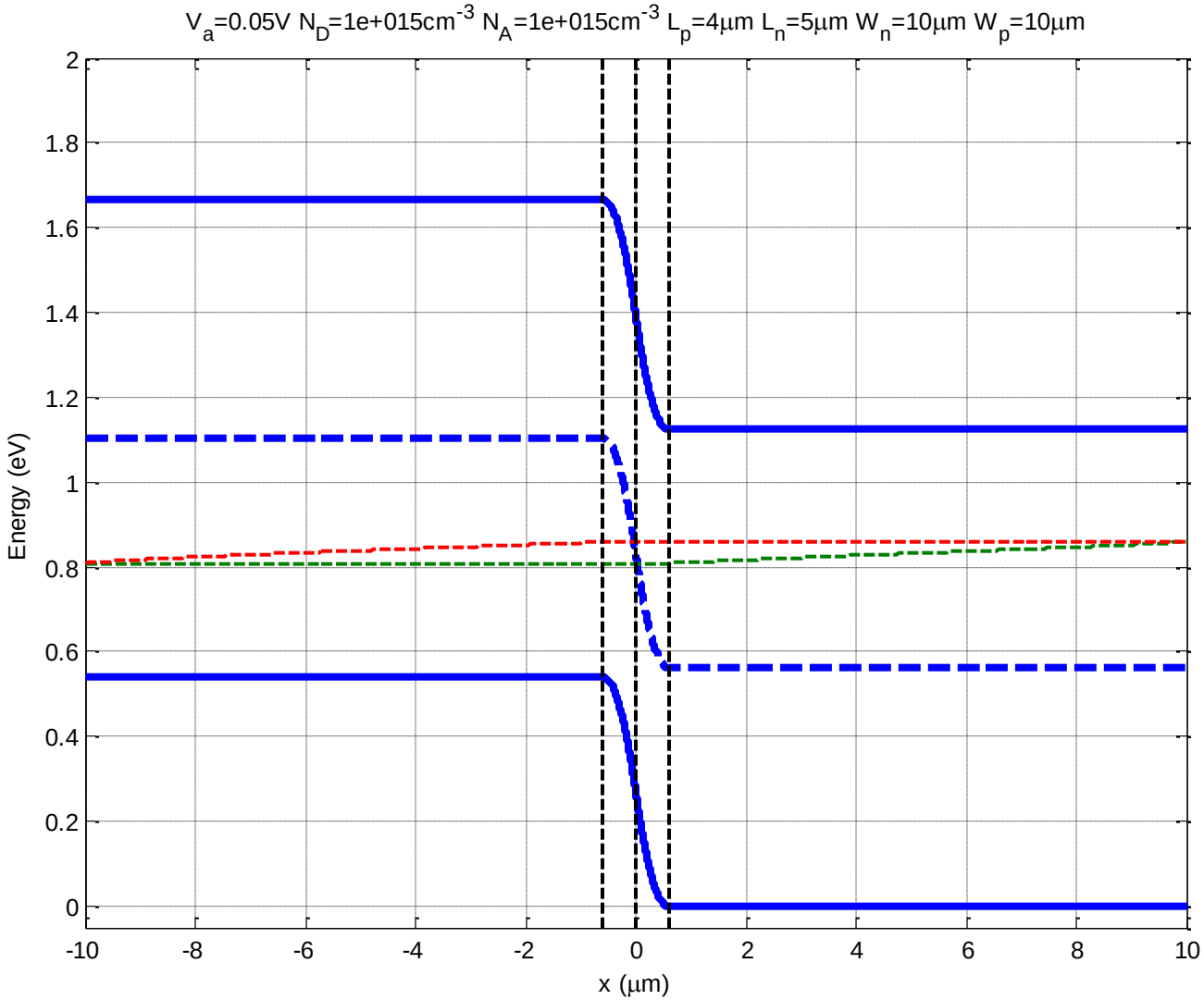


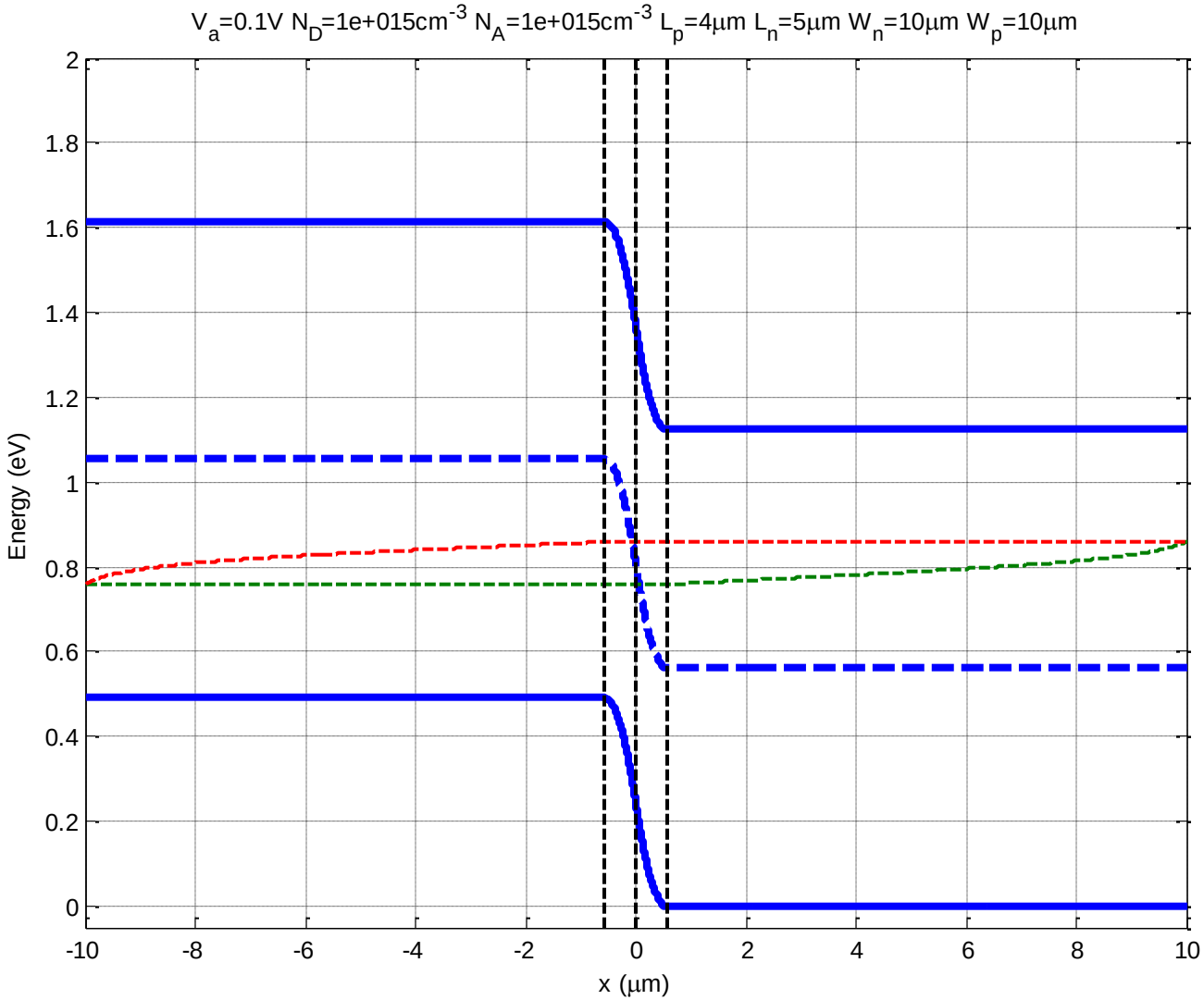
Shockley-Fletcher Boundary Conditions

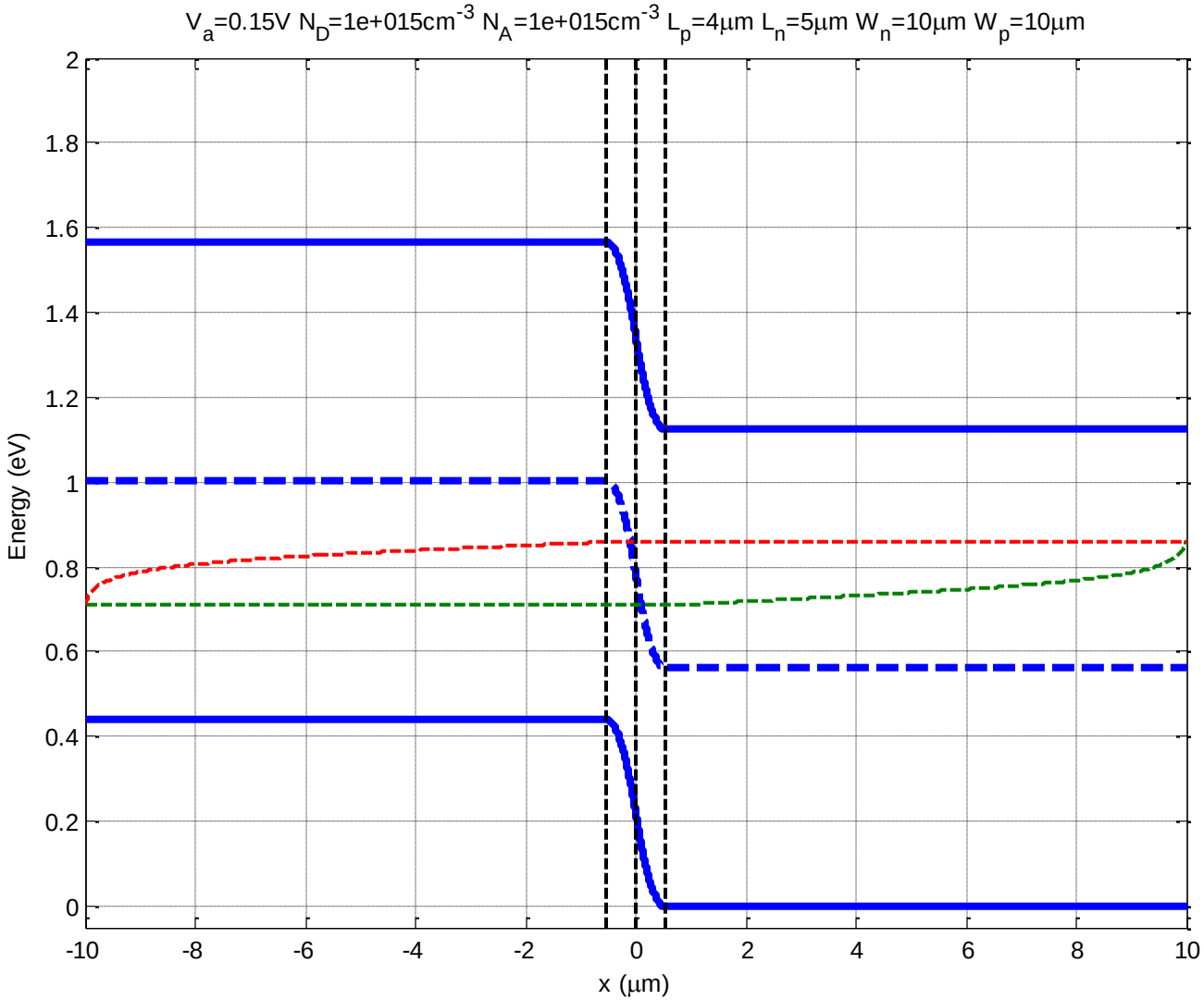
$V_a > 0$ (forward bias)

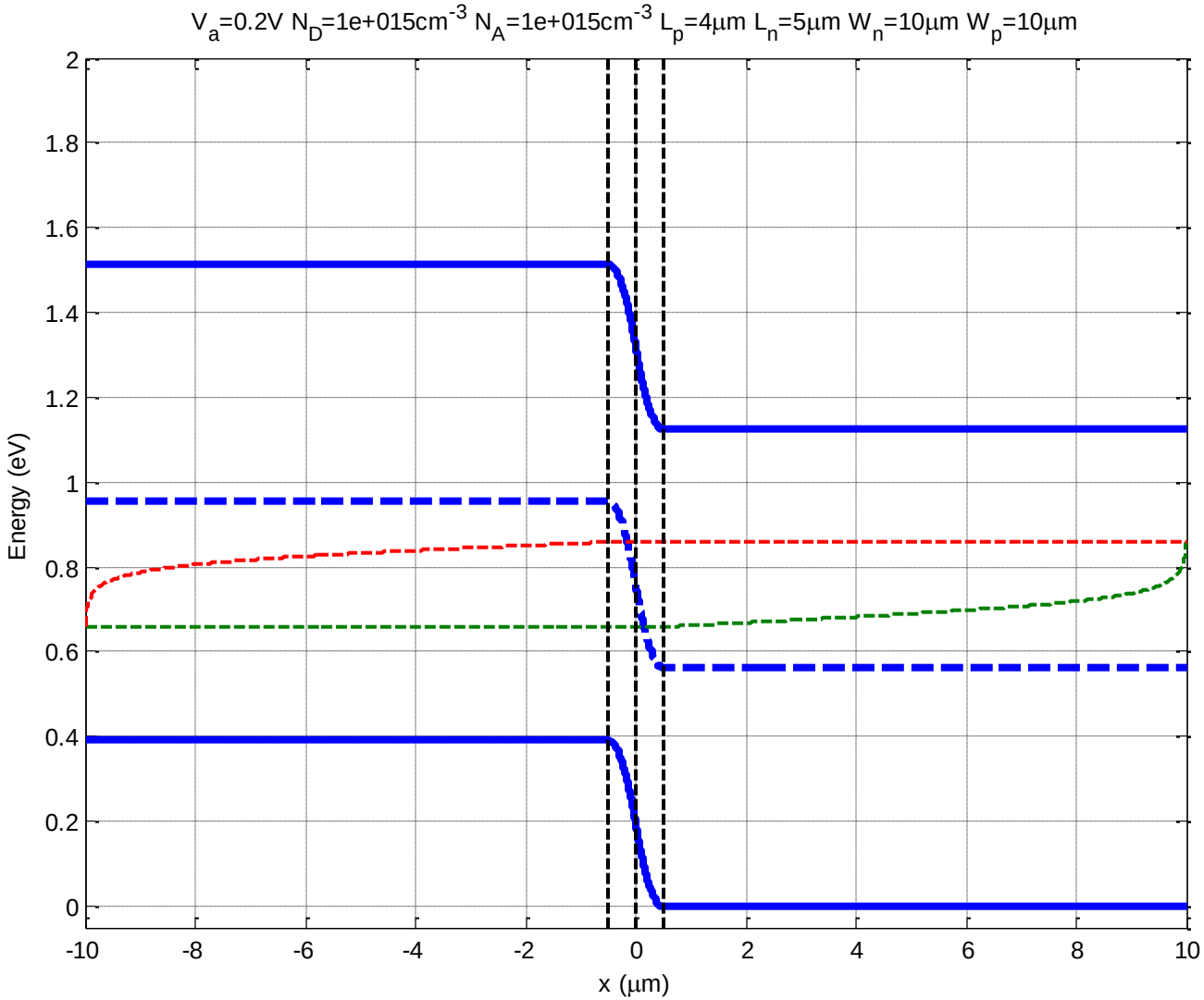


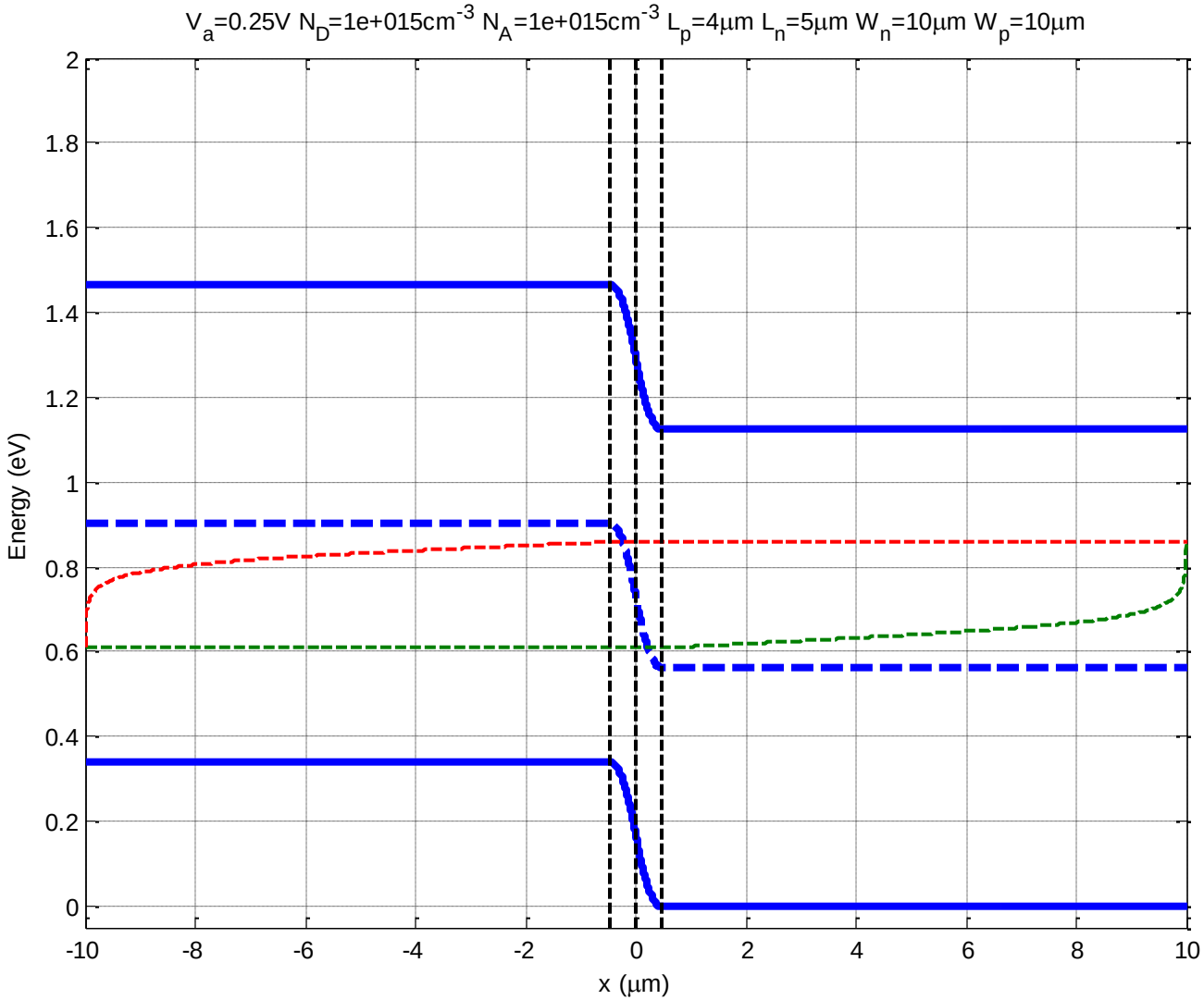

V_a=0V N_D=1e+015cm^{-3} N_A=1e+015cm^{-3} L_p=4μm L_n=5μm W_n=10μm W_p=10μm
Energy (eV)
x (μm)
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-10
-8
-6
-4
-2
0
2
4
6
8
10

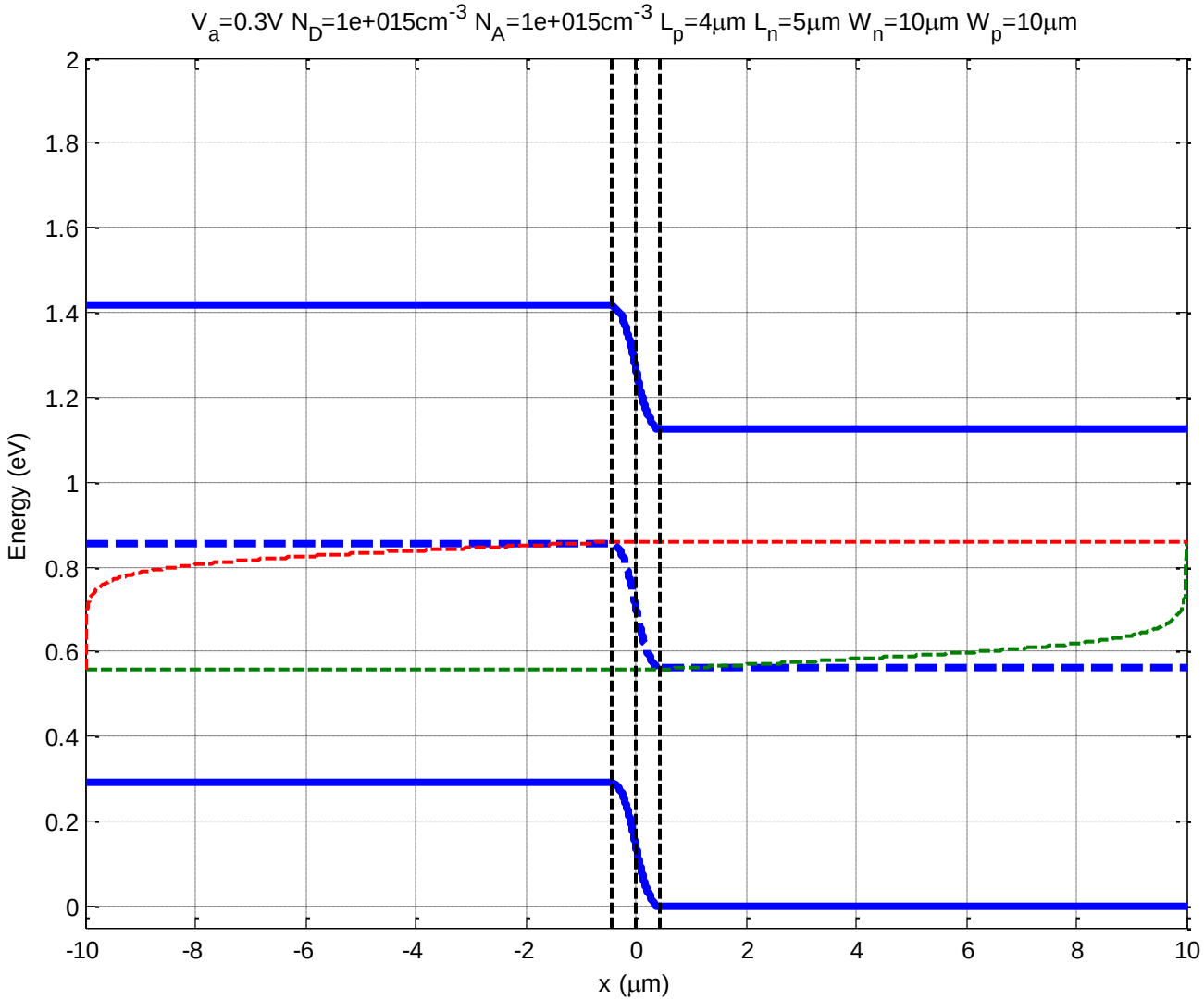

V_a=0.05V N_D=1e+015cm^{-3} N_A=1e+015cm^{-3} L_p=4μm L_n=5μm W_n=10μm W_p=10μm
Energy (eV)
x (μm)
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-10
-8
-6
-4
-2
0
2
4
6
8
10


V_a=0.1V N_D=1e+015cm^{-3} N_A=1e+015cm^{-3} L_p=4μm L_n=5μm W_n=10μm W_p=10μm
Energy (eV)
x (μm)
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-10
-8
-6
-4
-2
0
2
4
6
8
10


V_a=0.15V N_D=1e+015cm^{-3} N_A=1e+015cm^{-3} L_p=4μm L_n=5μm W_n=10μm W_p=10μm
Energy (eV)
x (μm)
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-10
-8
-6
-4
-2
0
2
4
6
8
10


V_a=0.2V N_D=1e+015cm^{-3} N_A=1e+015cm^{-3} L_p=4μm L_n=5μm W_n=10μm W_p=10μm
Energy (eV)
x (μm)
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-10
-8
-6
-4
-2
0
2
4
6
8
10


V_a=0.25V N_D=1e+015cm^{-3} N_A=1e+015cm^{-3} L_p=4μm L_n=5μm W_n=10μm W_p=10μm
Energy (eV)
x (μm)
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-10
-8
-6
-4
-2
0
2
4
6
8
10


V_a=0.3V N_D=1e+015cm^{-3} N_A=1e+015cm^{-3} L_p=4μm L_n=5μm W_n=10μm W_p=10μm
Energy (eV)
x (μm)
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-10
-8
-6
-4
-2
0
2
4
6
8
10

Minority Carrier Concentration

Relate hole concentration on n and p-side

$$\frac{p_p}{p_n} = e^{qV_{bi}/kT}$$

Under bias

$$\frac{p(-x_p)}{p(x_n)} = e^{q(V_{bi}-V_A)/kT}$$

Relating above

$$\frac{p(x_n)}{p_{n0}} = e^{qV_A/kT}$$

$$\Delta p_n(x_n) = \frac{n_i^2}{N_D}\left(e^{qV_A/kT} - 1\right)$$

Similarly for electrons

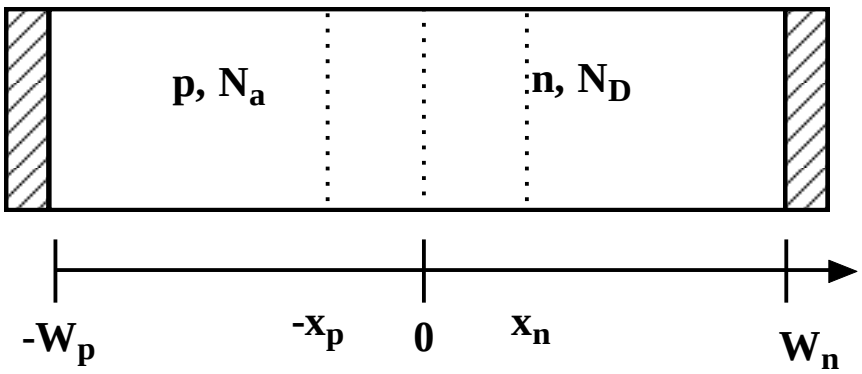
$$\frac{n(-x_p)}{n_{p0}} = e^{qV_A/kT}$$

$$\Delta n_p(-x_p) = \frac{n_i^2}{N_A}\left(e^{qV_A/kT} - 1\right)$$

These are boundary conditions for continuity equation

Minority Currents: Diffusion Equation

- Next Steps: Find $J_n(-x_p)$ and $J_p(x_n)$
- Assume Minority Current is Dominated by Diffusion
- Solve Quasi-Neutral Region Diffusion Equation
  - Special Cases of Long & Short Regions



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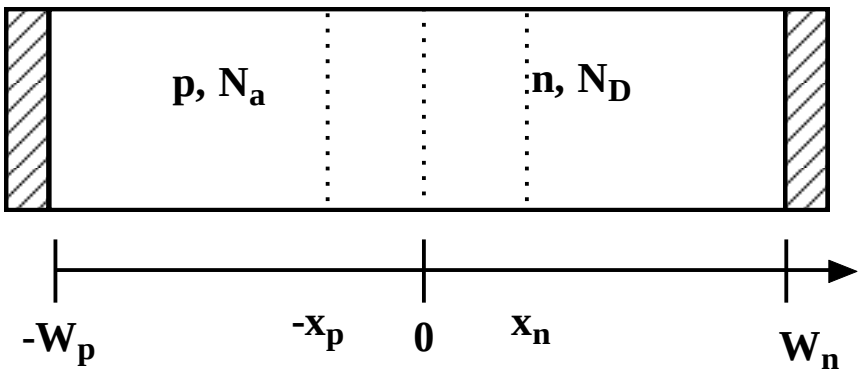

Minority Currents: Diffusion Equation

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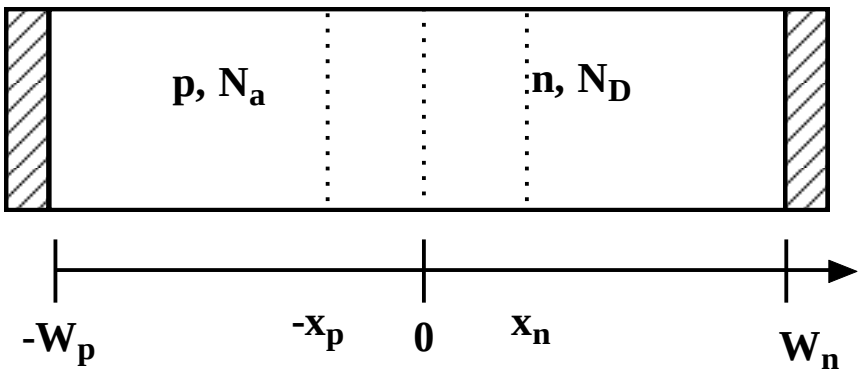
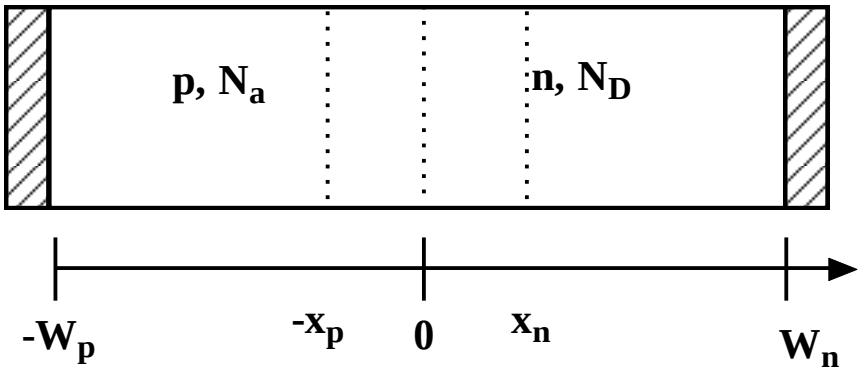
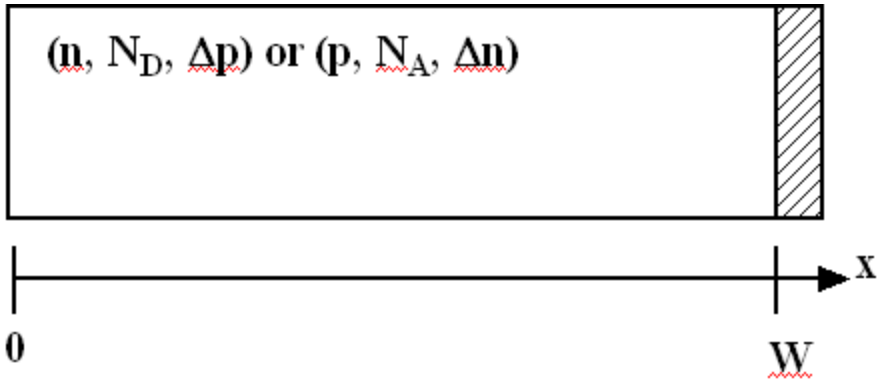
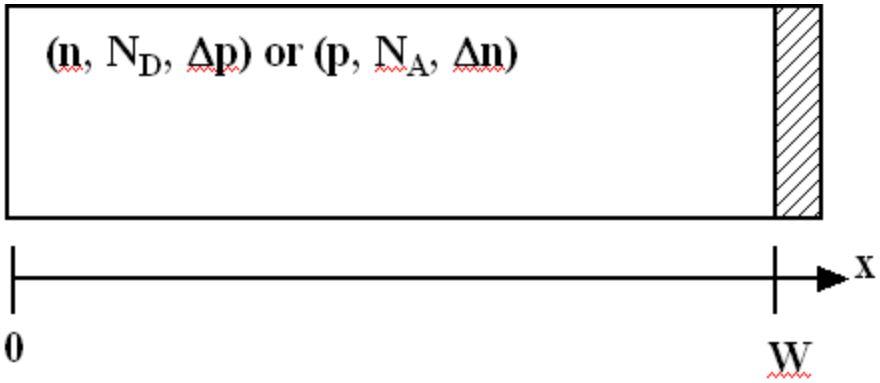


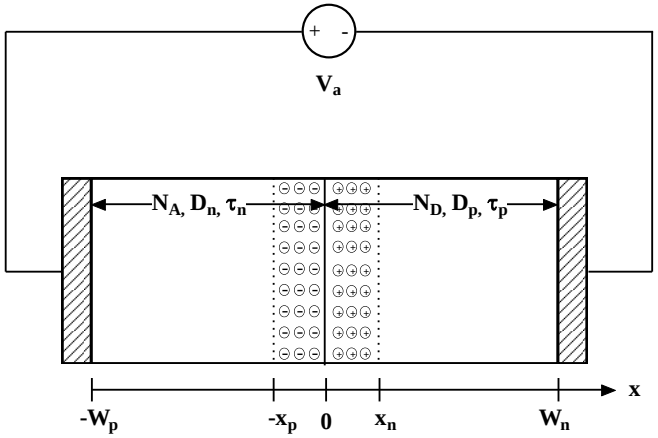

p, N_a
n, N_D
$-W_p$
$-x_p$
0
x_n
W_n


(n, N_D, Δp) or (p, N_A, Δn)
x
0
W


(n, N_D, Δp) or (p, N_A, Δn)
x
0
W

"General Solution"

- Assumptions
  - Uniform, Abrupt Doping
  - Uniform Diffusivity, Lifetime
  - Low Level Injection
  - No Recombination in Depletion Region
  - Perfect Ohmic Contacts
  - Minority Current flow in neutral regions by diffusion only



Results

$$\left.\begin{aligned}\Delta n_p(x) &= \left[\frac{\sinh\left(\frac{W_p+x}{L_n}\right)}{\sinh\left(\frac{W_p-x_p}{L_n}\right)}\right] n_{p0}\left(\exp\left(qV_a/kT\right)-1\right)\\ J_n(x) &= \frac{qD_n}{L_n}\left[\frac{\cosh\left(\frac{W_p+x}{L_n}\right)}{\sinh\left(\frac{W_p-x_p}{L_n}\right)}\right] n_{p0}\left(\exp\left(qV_a/kT\right)-1\right)\end{aligned}\right\} -W_p \le x \le -x_p$$

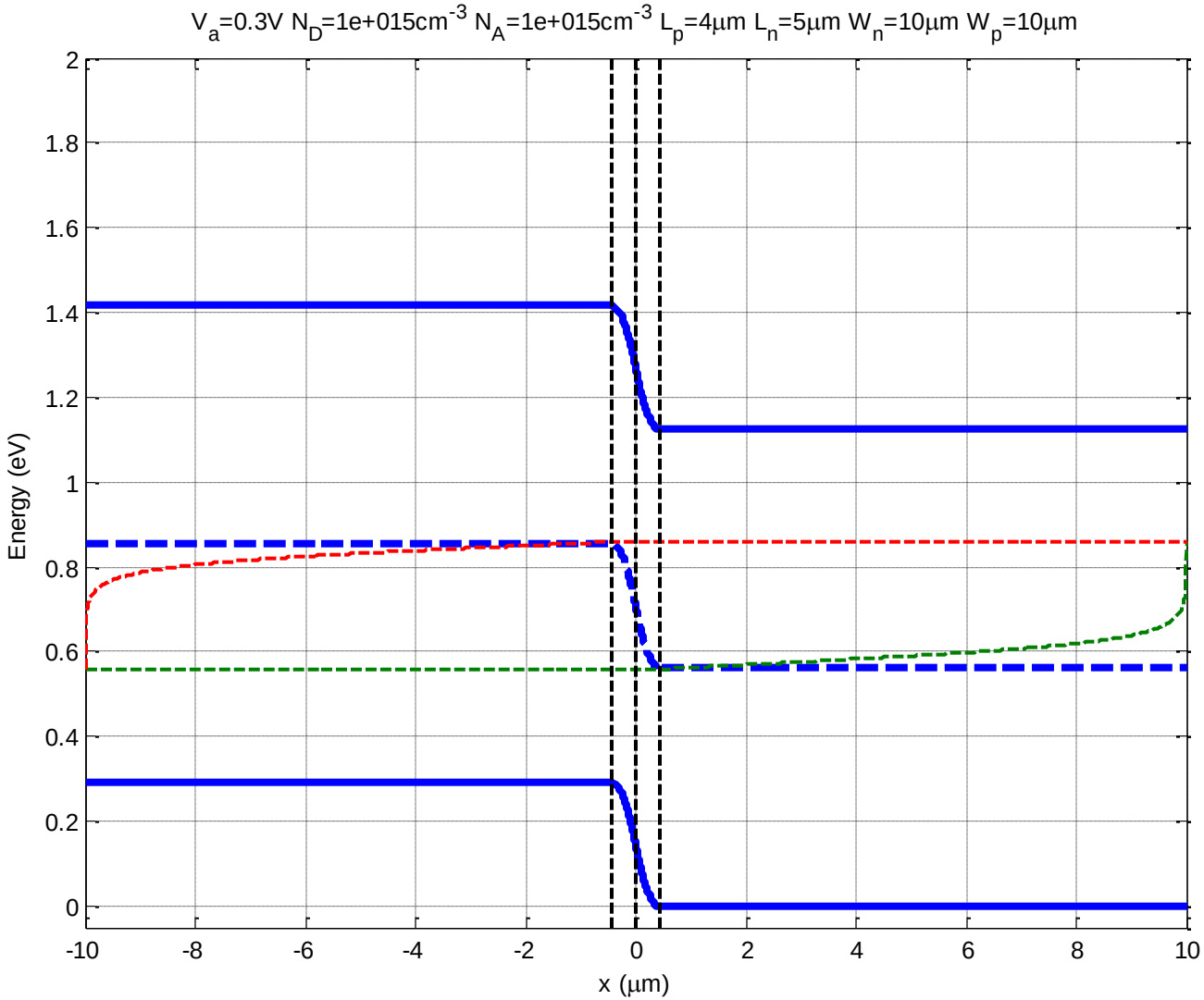
$$J_n(x) = J_n(-x_p) = \frac{qD_n}{L_n}\left[\coth\left(\frac{W_p-x_p}{L_n}\right)\right] n_{p0}\left(\exp\left(qV_a/kT\right)-1\right) \qquad \left\{-x_p \le x \le x_n\right\}$$

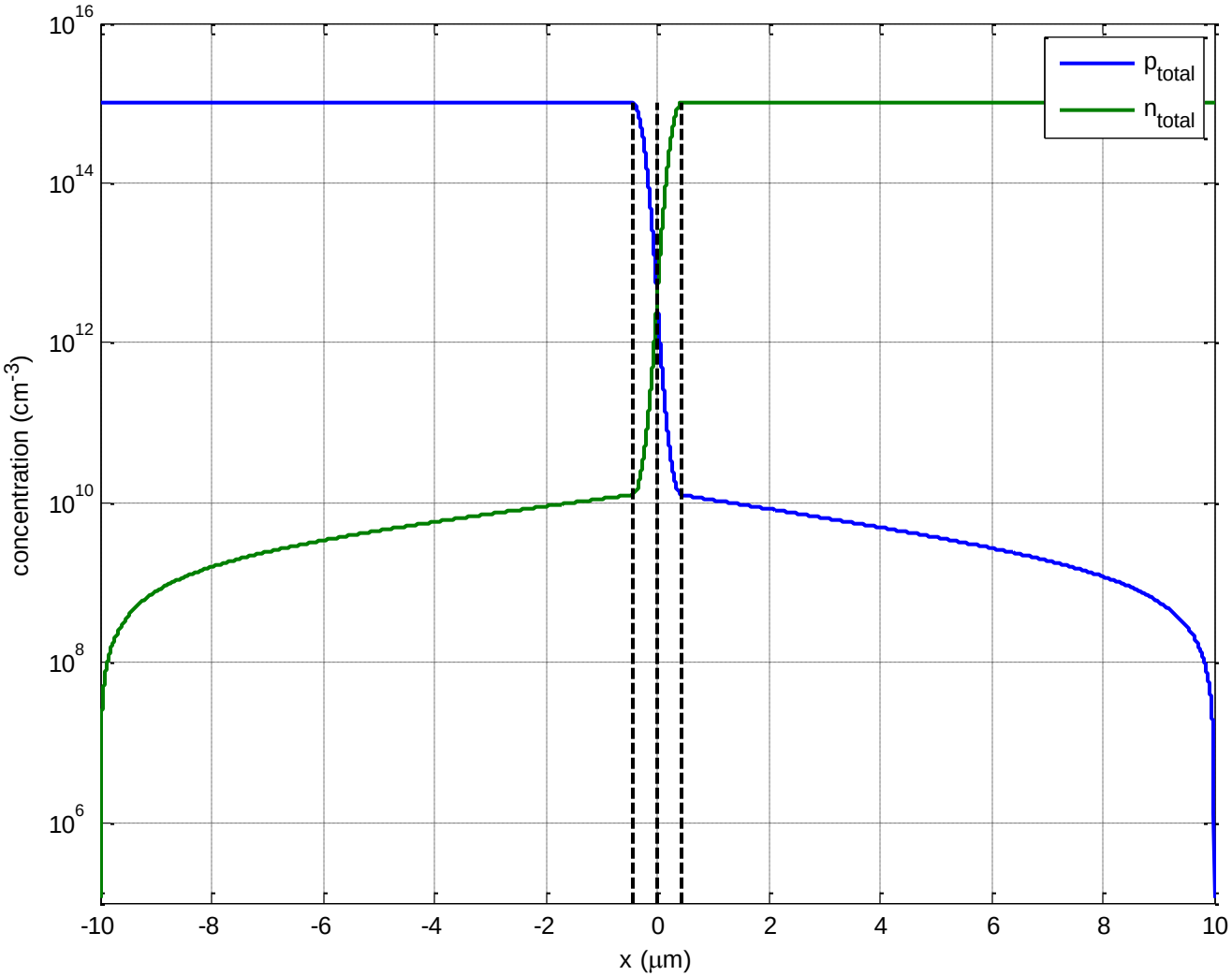
$$\left.\begin{aligned}\Delta p_n &= \left[\frac{\sinh\left(\frac{W_n-x}{L_p}\right)}{\sinh\left(\frac{W_n-x_n}{L_p}\right)}\right] p_{n0}\left(\exp\left(qV_a/kT\right)-1\right)\\ J_p(x) &= \frac{qD_{pn}}{L_p}\left[\frac{\cosh\left(\frac{W_n-x}{L_p}\right)}{\sinh\left(\frac{W_n-x_n}{L_p}\right)}\right] p_{n0}\left(\exp\left(qV_a/kT\right)-1\right)\end{aligned}\right\} x_n \le x \le W_n$$

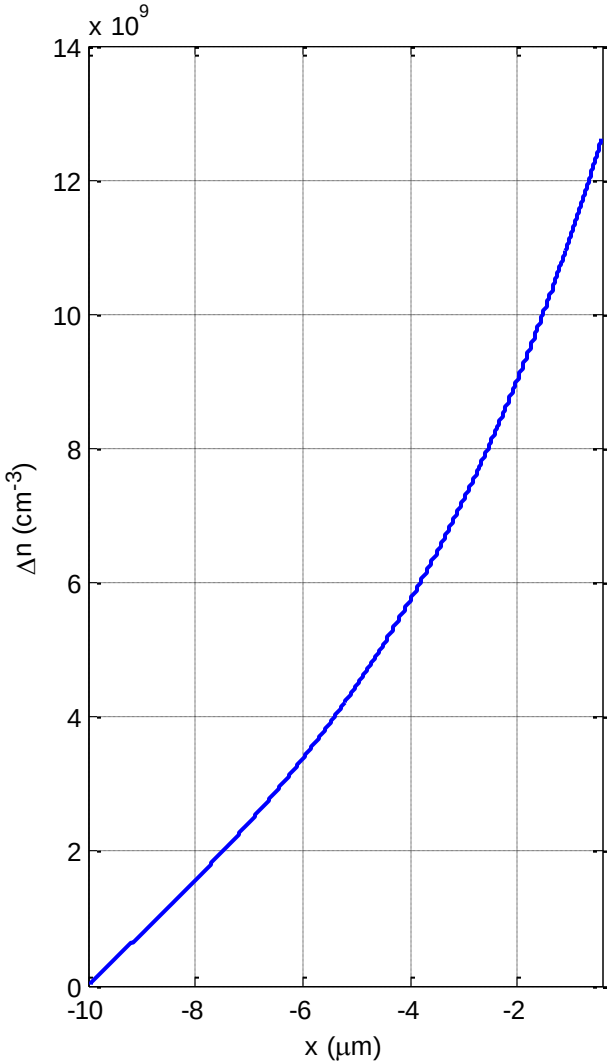
$$J_p(x) = J_p(x_n) = \frac{qD_{pn}}{L_p}\left[\coth\left(\frac{W_n-x_n}{L_p}\right)\right] p_{n0}\left(\exp\left(qV_a/kT\right)-1\right) \qquad \left\{-x_p \le x \le x_n\right\}$$

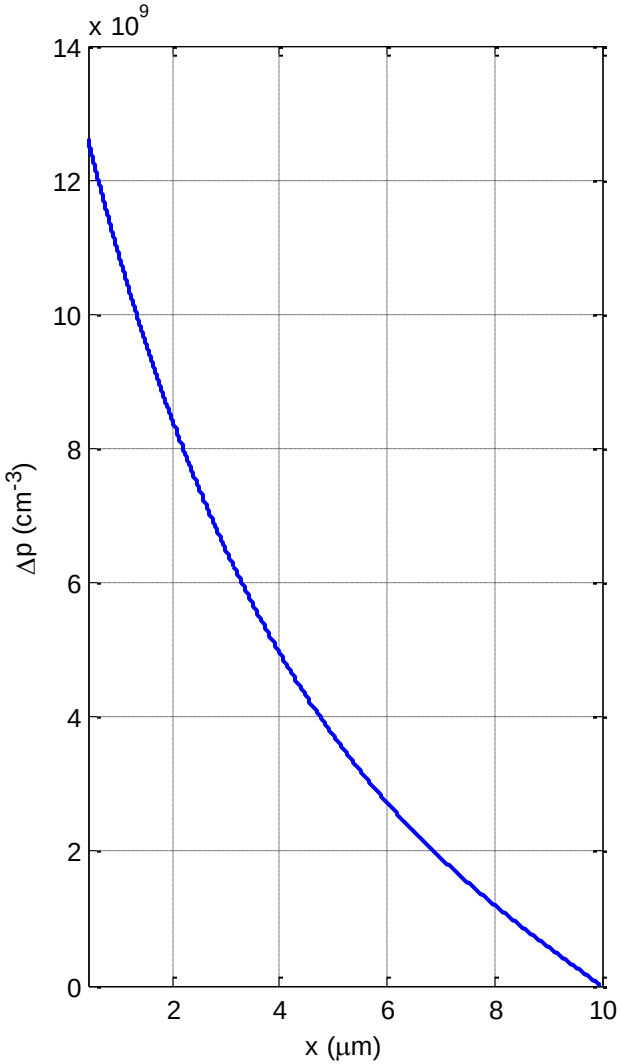
$$J_{sat} = \frac{qD_{pn}}{L_p}\left[\coth\left(\frac{W_n-x_n}{L_p}\right)\right] p_{n0} + \frac{qD_n}{L_n}\left[\coth\left(\frac{W_p-x_p}{L_n}\right)\right] n_{p0}$$

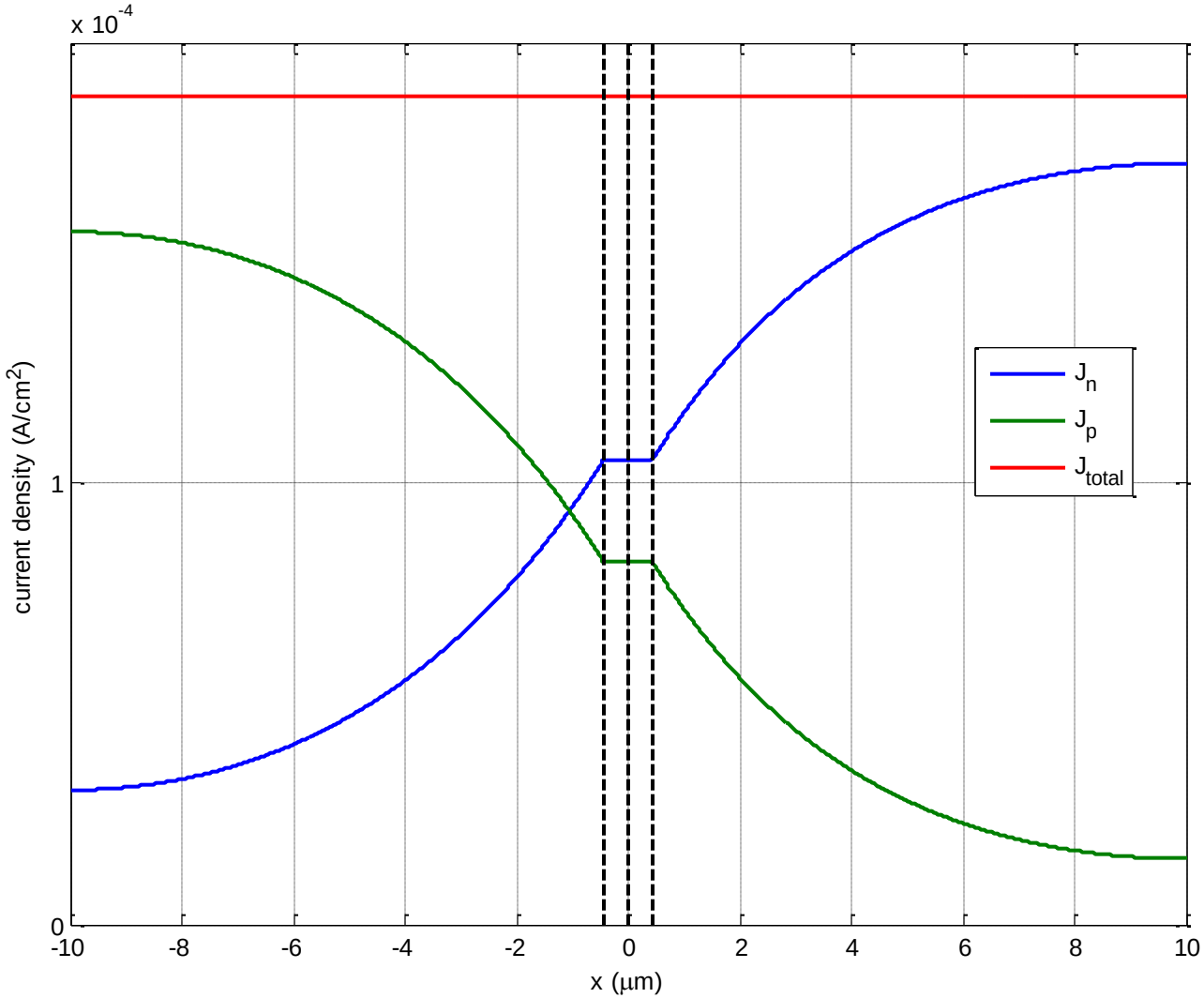
$$J_{total} = J_{sat}\left(\exp\left(qV_a/kT\right)-1\right)$$


V_a=0.3V N_D=1e+015cm^{-3} N_A=1e+015cm^{-3} L_p=4μm L_n=5μm W_n=10μm W_p=10μm
Energy (eV)
x (μm)
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-10
-8
-6
-4
-2
0
2
4
6
8
10


10^{16}
10^{14}
10^{12}
10^{10}
10^{8}
10^{6}
concentration (cm^{-3})
-10
-8
-6
-4
-2
0
2
4
6
8
10
x (μm)
p_{total}
n_{total}


x 10^9
14
12
10
8
6
4
2
0
Δn (cm^-3)
-10
-8
-6
-4
-2
x (μm)

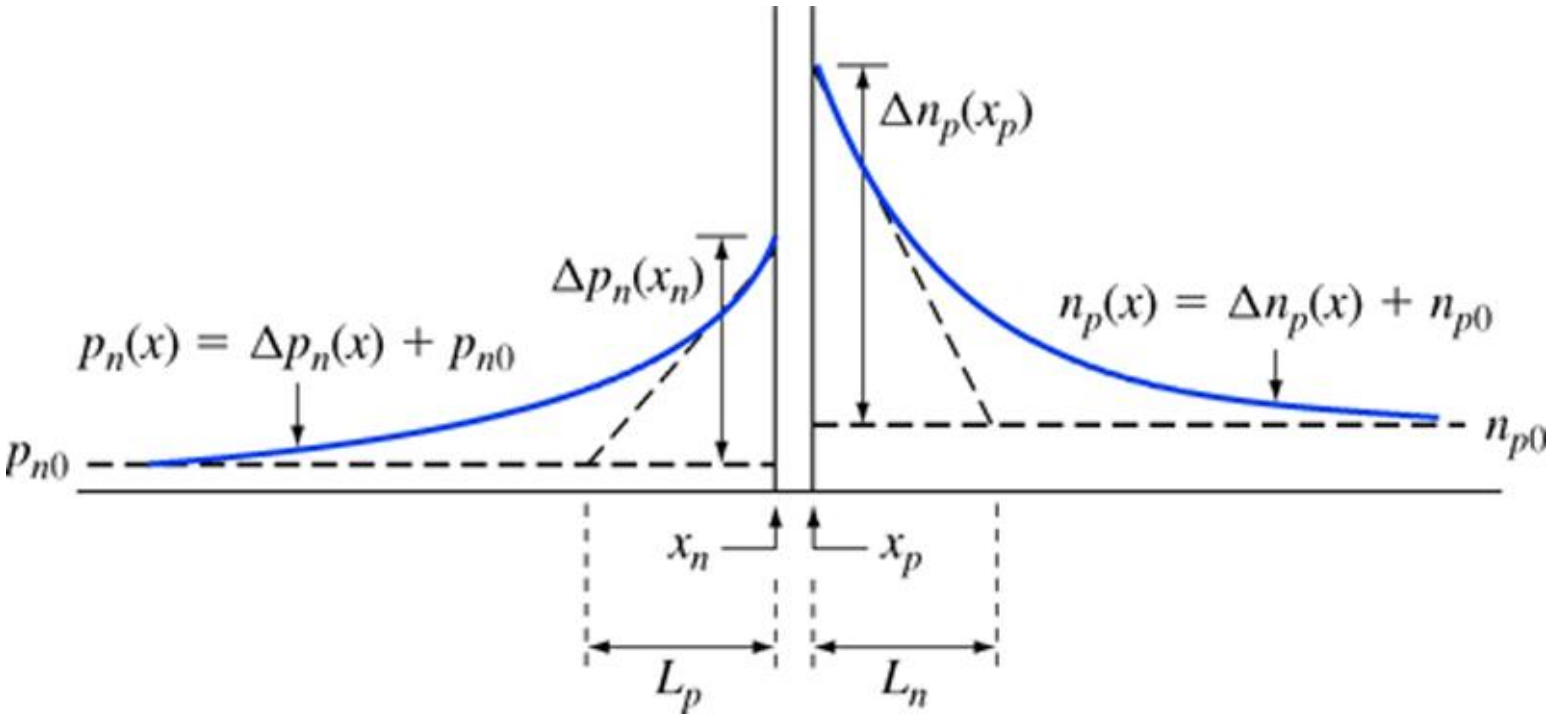

x 10^9
14
12
10
8
6
4
2
0
Δp (cm^-3)
2
4
6
8
10
x (μm)


x 10^-4
current density (A/cm^2)
1
0
-10
-8
-6
-4
-2
0
2
4
6
8
10
x (μm)
J_n
J_p
J_total

Two Limiting Special Cases

- Long regions $(W-x) >> L_{min}$
- Short regions $(W-x) << L_{min}$

Minority Carrier Concentration: Long Diode



$$\Delta p_n(x) = \frac{n_i^2}{N_D}\left(e^{qV_A/kT} - 1\right)e^{(x+x_n)/L_p}$$

$$\Delta n_p(x) = \frac{n_i^2}{N_A}\left(e^{qV_A/kT} - 1\right)e^{-(x-x_p)/L_n}$$

$$L_p = \sqrt{D_p \tau_p}$$

$$L_n = \sqrt{D_n \tau_n}$$

Minority Carrier Densities

Solution To Minority Carrier Diffusion Equation

$$\Delta n_p(x) = \frac{n_i^2}{N_A}\left(e^{qV_A/kT} - 1\right)e^{(x+x_p)/L_N}$$

$$\Delta p_n(x) = \frac{n_i^2}{N_D}\left(e^{eV_A/kT} - 1\right)e^{-(x-x_n)/L_P}$$

$$L_N = \sqrt{D_N \tau_N}$$

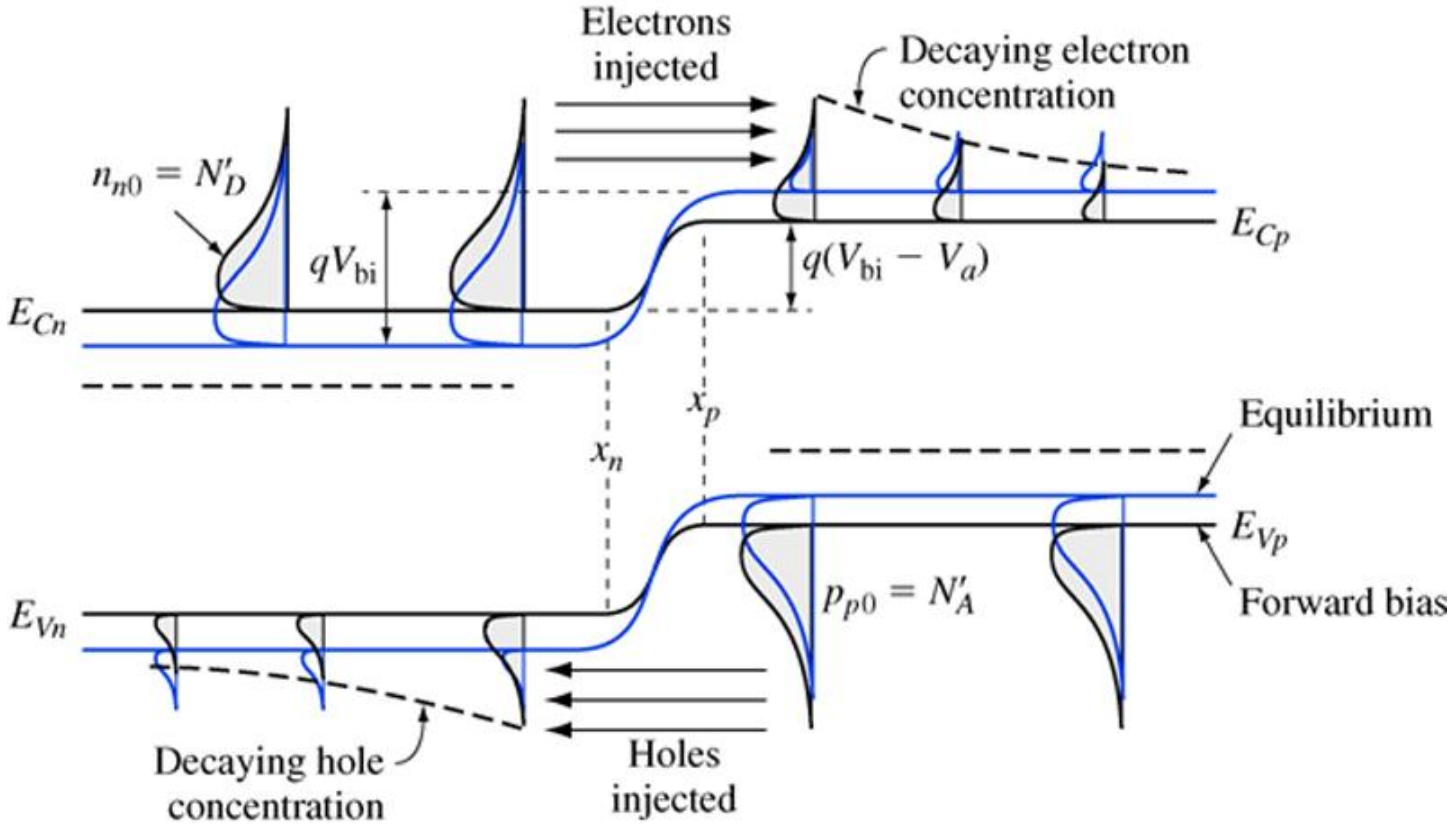
$$L_P = \sqrt{D_P \tau_P}$$

Source of J_N on p-side

Source of J_P on n-side

Write expressions for $J_N(x)$ on p-side and $J_P(x)$ on n-side

Diffusion Current



$$J_p = \frac{qD_p}{L_p}\Delta p_n(x_n)e^{(x+x_n)/L_p}$$

$$J_n = \frac{qD_n}{L_n}\Delta n_p(x_p)e^{-(x-x_p)/L_n}$$

Ideal Diode Equation

Assumption: G-R negligible in depletion region

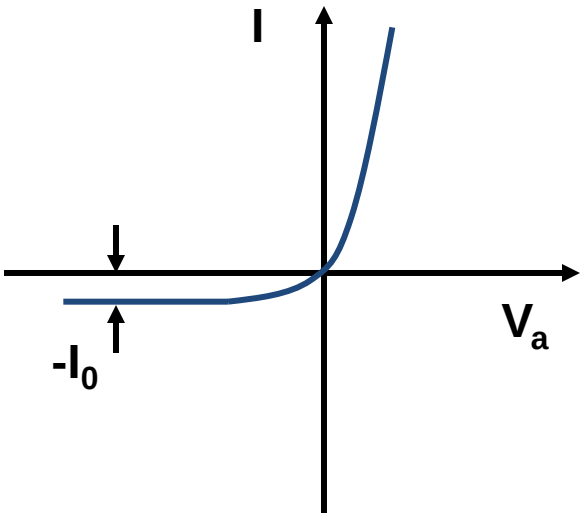
Result: J_N and J_P constant in depletion region

Relate $J_N(-x_P)$ and $J_P(x_N)$ for total diode current

$$I = AJ_n(-x_p) + J_p(x_n)$$

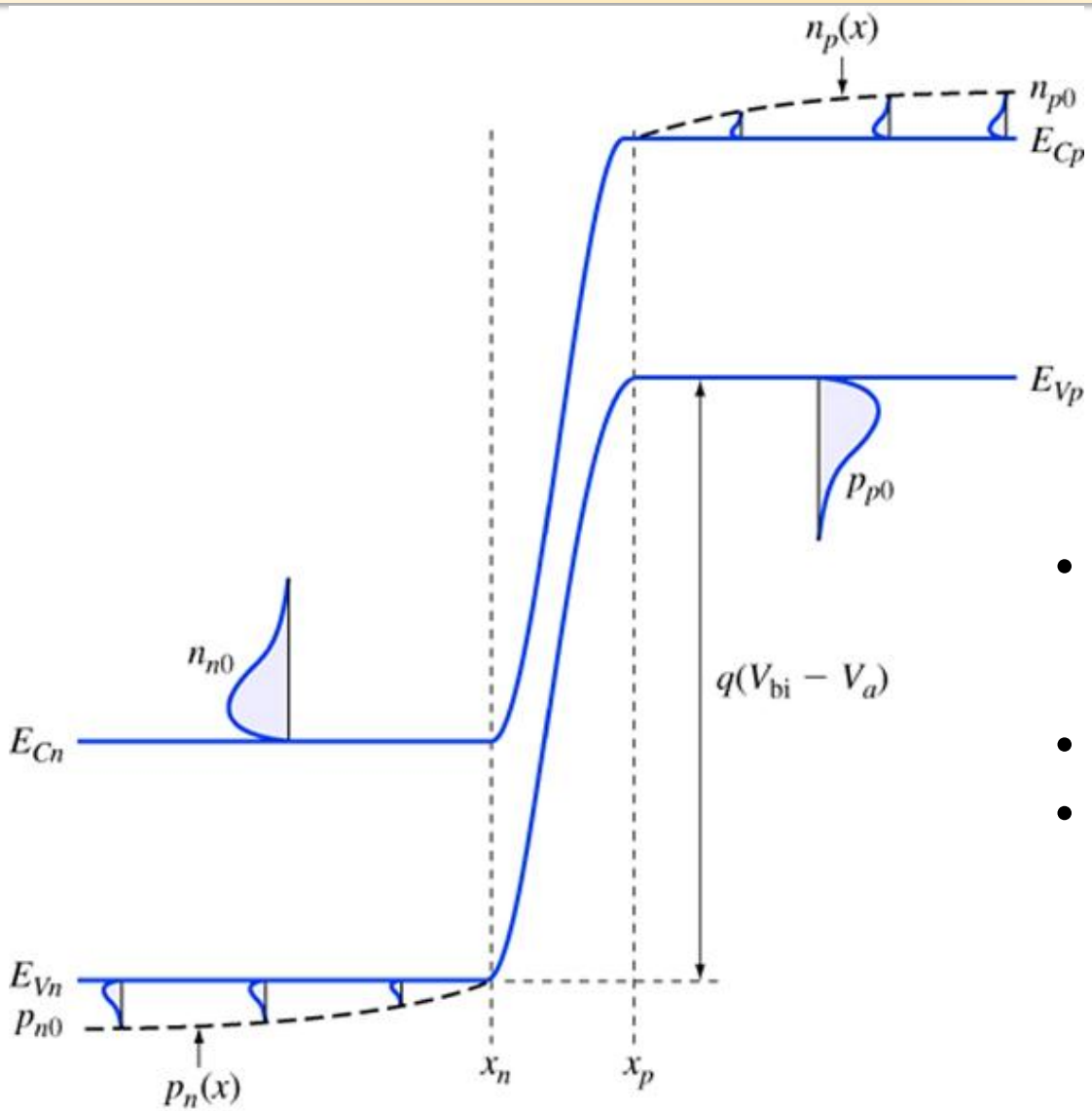
$$\boxed{I = qA\left(\frac{D_N}{L_N}\frac{n_i^2}{N_A} + \frac{D_P}{L_P}\frac{n_i^2}{N_D}\right)\left(e^{qV_A/kT} - 1\right)}$$

$$\boxed{I = I_0\left(e^{qV_A/kT} - 1\right)}$$



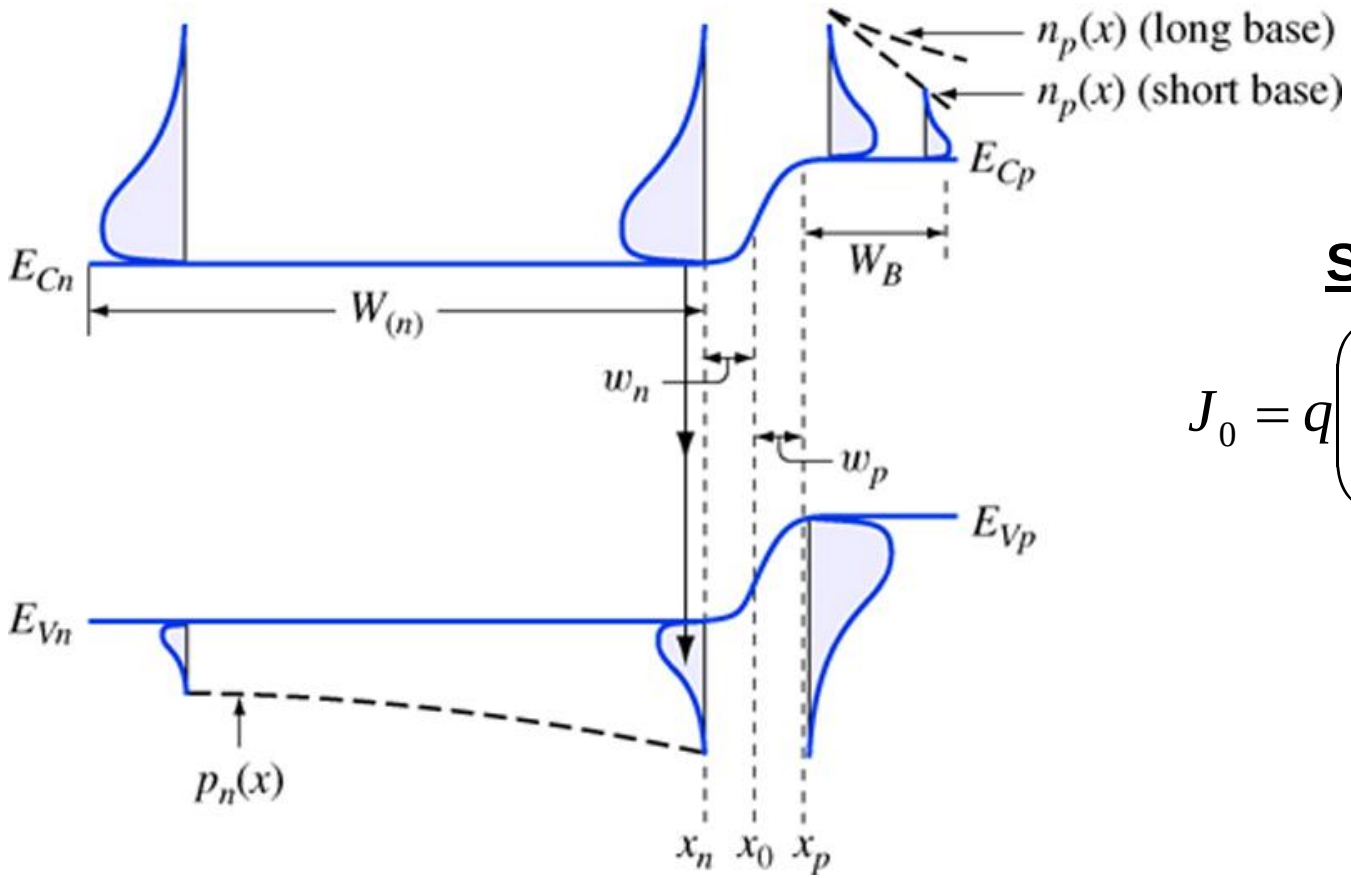
I_0 = Reverse saturation current

Diffusion Current – Reverse Bias



- Carriers at edge of depletion region swept across by electric field
- Carriers diffuse to junction
- Limited by minority carrier generation

Diffusion Current – Reverse Bias, short diode



Short p-side

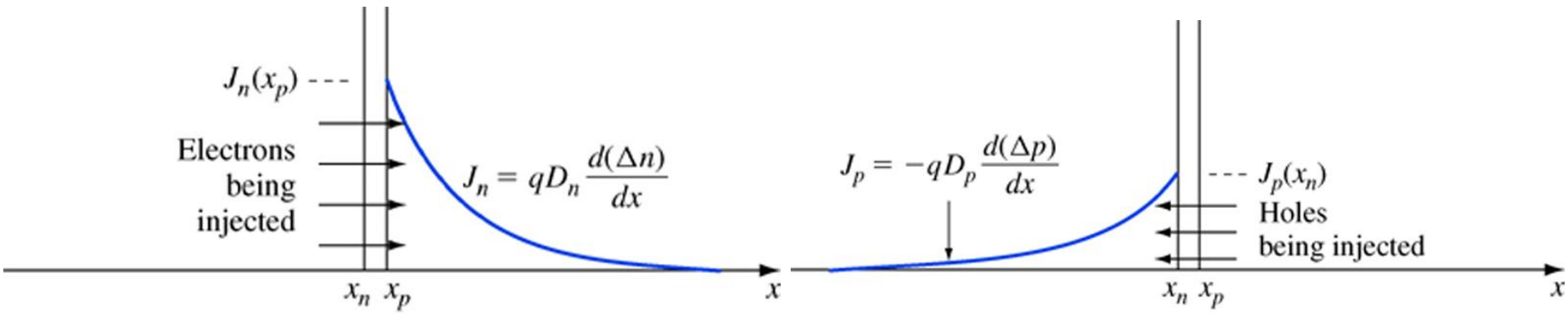
$$J_0 = q\left(\frac{D_n n_{p0}}{W_B} + \frac{D_p p_{n0}}{L_p}\right)$$

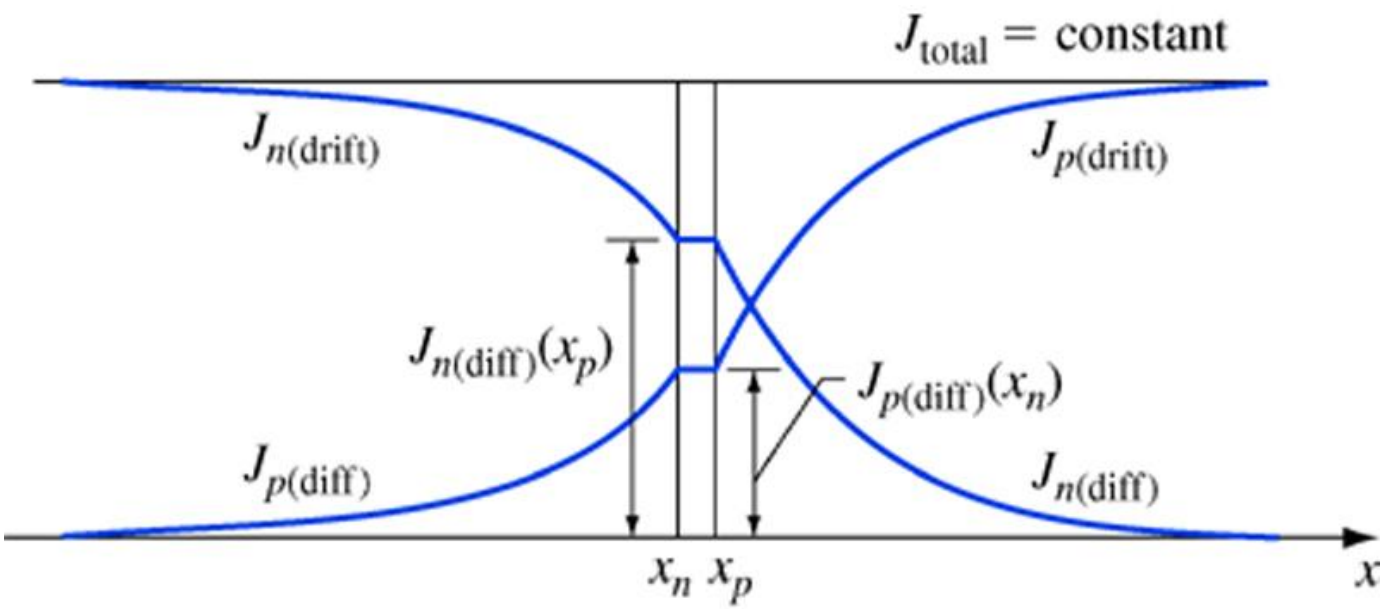
Qualitative Understanding

The equations boil down to a simple expression, but can you explain diode operation in simple terms?

- What is the "depletion region"?
- What is the origin of current flow?
- What happens to an electron once it is injected from the n-type contact?
- Why does current flow differ so much in forward and reverse bias?

Total Current

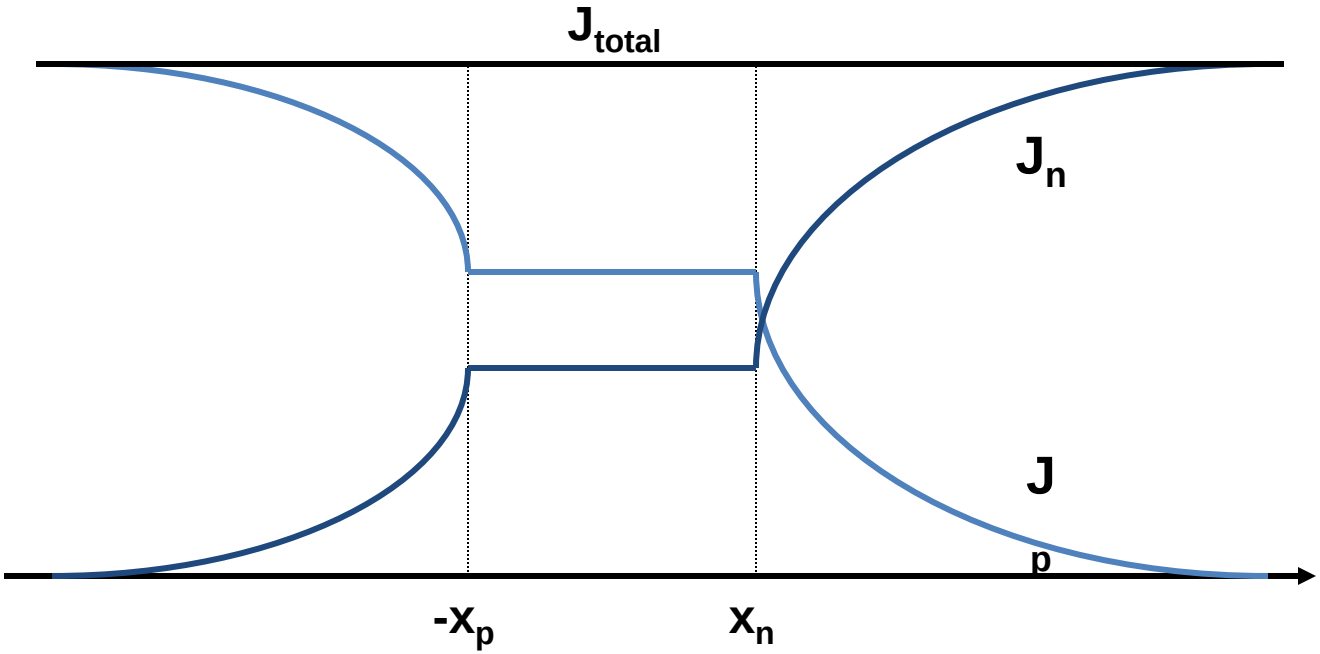




No R-G in depletion region

Electron and Hole Current

$$\text{For all } x,\ J_{total} = J_n + J_p = \text{constant}$$



- For every minority carrier recombination event, a majority carrier is injected from the contact
- $J_{total} > 0$ for $V_a > 0$
- $J_{total} < 0$ for $V_a < 0$

P-N Diode With Asymmetric Doping

Typically, one side of the junction is doped much more heavily

$N_A=10^{18}$ cm^{-3}	$N_D=10^{15}$ cm^{-3}

p^+-n junction

$N_A=10^{15}$ cm^{-3}	$N_D=10^{18}$ cm^{-3}

n^+-p junction

The ideal diode equation may be simplified with assymetric doping, rewrite expressions for p^+-n and n^+-p