Exam 2 Takehome Solutions

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1. a)

$$\rho(x) = \begin{cases} -qN_A & (-x_p \le x \le 0) \\ +q(1\times10^{21}\,cm^{-4})x & (0 \le x \le 0.5\mu m) \\ \quad = +qmx & \\ +qN_D & (0.5\mu m \le x \le x_n) \end{cases}$$

$N_A = 1\times10^{16}\,cm^{-3}$

$N_D = 5\times10^{16}$

$$\mathcal{E}(x) = \int_{-x_p}^{x} \frac{\rho}{\epsilon_s}\,dx$$

$$= \begin{cases} -\dfrac{qN_A}{\epsilon_s}(x_p + x) & (-x_p \le x \le 0) \\ -\dfrac{qN_A}{\epsilon_s}x_p + \dfrac{qmx^2}{2\epsilon_s} & (0 \le x \le 0.5\mu m) \\ \left\{\left[-\dfrac{qN_A}{\epsilon_s}x_p + \dfrac{qm(0.5\mu m)^2}{2\epsilon_s}\right] + \dfrac{qN_D}{\epsilon_s}(x - 0.5\mu m)\right\} & (0 \le x \le 0.55\mu m) \\ = \dfrac{qN_D}{\epsilon_s}(x - 0.55\mu m) & \end{cases}$$

$$V(x) = -\int \mathcal{E}\, dx$$

$$= V_p + \frac{qN_A}{\epsilon_s}\left(x_p x + \tfrac{1}{2}x^2\right)\Big|_{-x_p}^{x}$$

$$= V_p + \frac{qN_A}{2\epsilon_s}(x + x_p)^2 \quad (-x_p \le x \le 0)$$

$$V(0) = V_p + \frac{qN_A}{2\epsilon_s} x_p^2$$

$$V(x) = V(0) + \int_0^x \left(\frac{qN_A x_p}{\epsilon_s}\right) - \frac{qmx^2}{2\epsilon_s}\, dx$$

$$= V(0) + \left(\frac{qN_A x_p}{\epsilon_s}\right)x - \frac{qmx^3}{6\epsilon_s} \quad 0 \le x \le 0.5\mu m$$

$$V(0.5\mu m) = V(0) + \left(\frac{qN_A x_p}{\epsilon_s}\right)(0.5\mu m) - \left(\frac{qm(0.5\mu m)^3}{6\epsilon_s}\right)$$

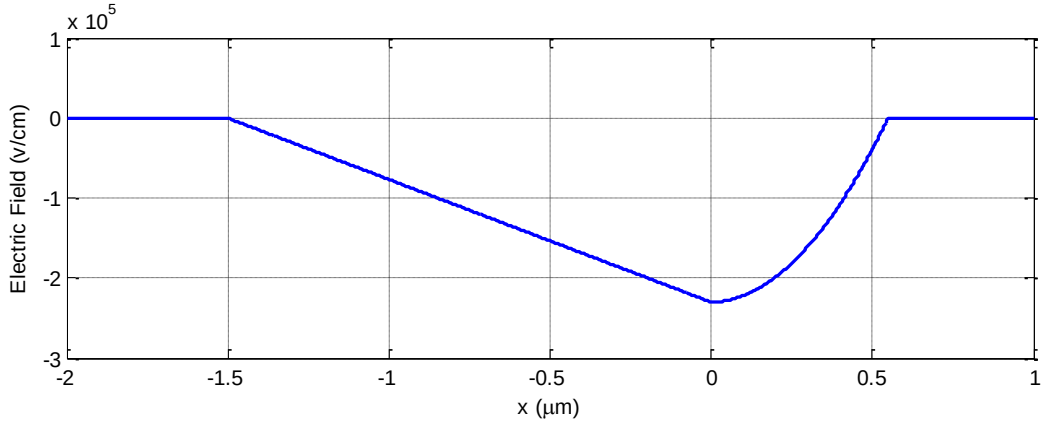
$$V(x) = V(0.5\mu m) + \int_{0.5\mu m}^{x} \frac{qN_D}{\epsilon_s}(\overset{x_n}{0.55\mu m} - x)\, dx$$

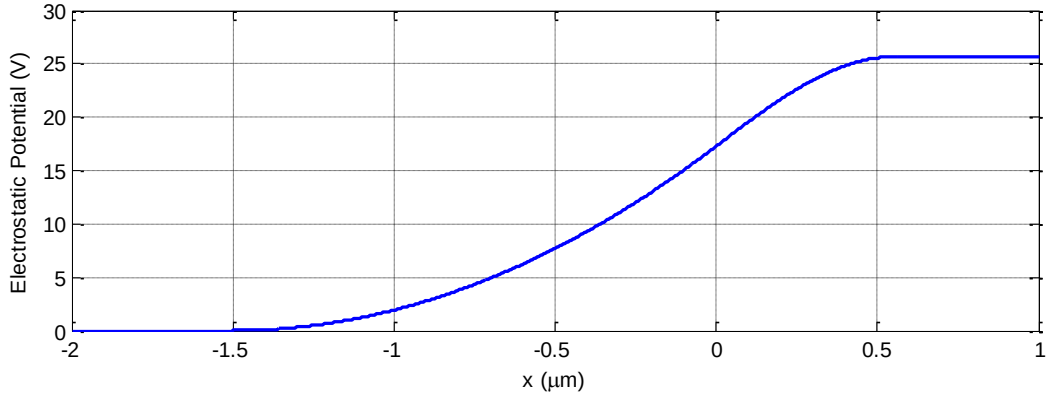
$$= V(0.5\mu m) + \frac{qN_D}{\epsilon_s}\left(x_n x - \tfrac{1}{2}x^2\right)\Big|_{0.5\mu m}^{x}$$

$$= V(0.5\mu m) + \frac{qN_D}{\epsilon_s}\Big[x_n x - \tfrac{1}{2}x^2 - x_n(0.5\mu m) + \tfrac{1}{2}(0.5\mu m)^2\Big]$$

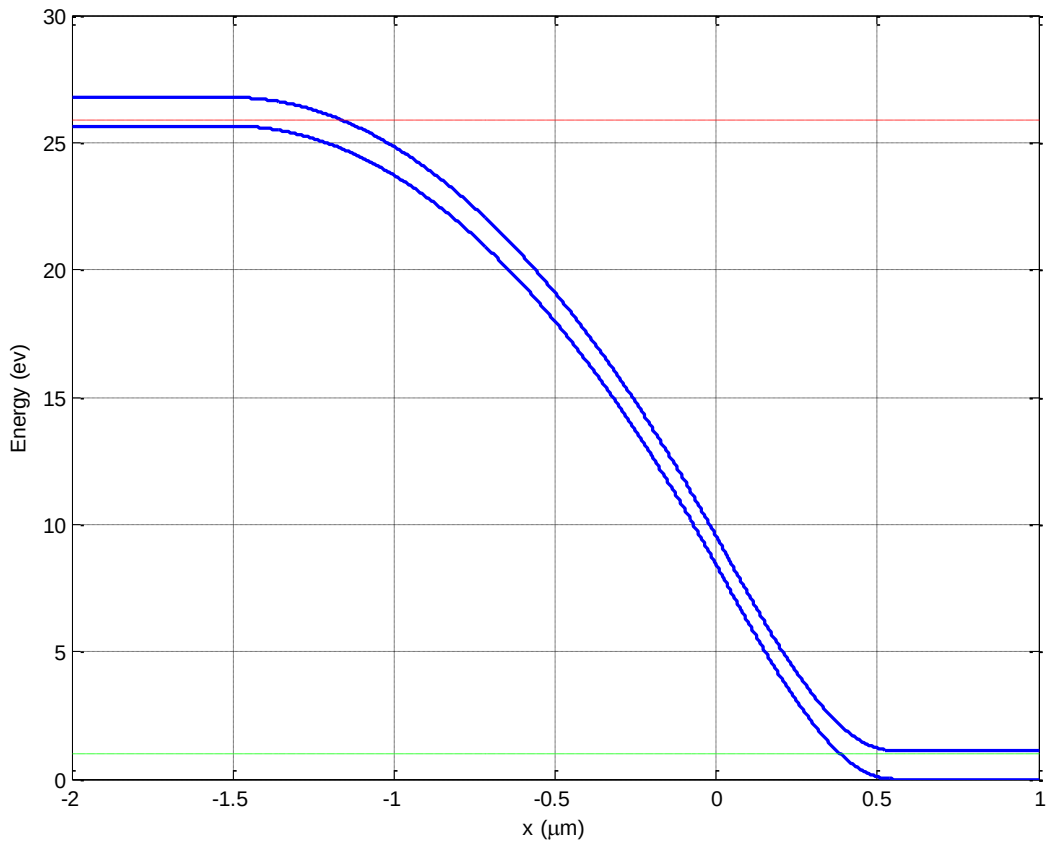
$$0.5\mu m \le x \le x_n = 0.55\mu m$$

1. Fields and Potential

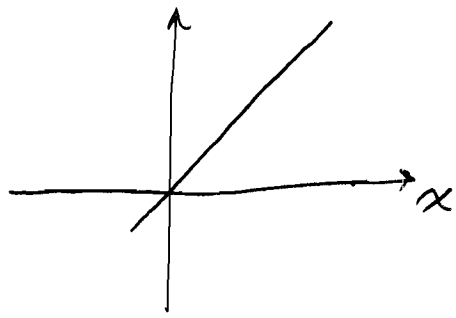




1. Bands E_{FN} Green E_{FP} Red



1 c)



$$n \approx N_D = mx \qquad (m = 1\times10^{21}\ cm^{-4} \text{ here})$$

But $J_n = q n \mu_n \vec{\mathcal{E}} + q D_n \frac{\partial n}{\partial x} = 0$

$$= q\left[n \mu_n \vec{\mathcal{E}} + \frac{kT}{q}\mu_n \frac{\partial n}{\partial x}\right] = 0$$

$$\Rightarrow \quad \vec{\mathcal{E}} = -\frac{kT}{q}\frac{1}{n}\frac{\partial n}{\partial x}$$

$$= -\frac{kT}{q}\left(\frac{1}{mx}\right) m$$

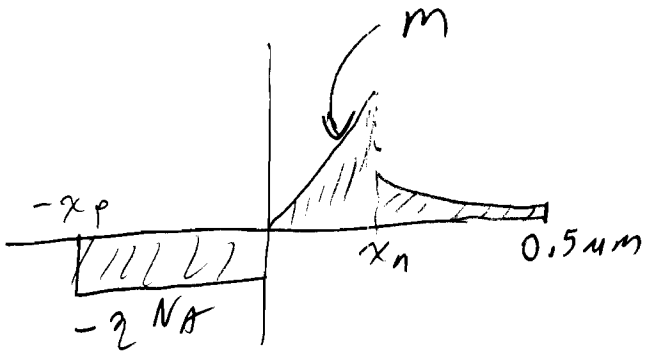
$$= -\frac{kT}{q}\frac{1}{x} \quad \text{here for } 0 \le x \le 0.5\mu m$$

Note that this implies a non-zero charge in the quasi-neutral region

$$\frac{\partial \mathcal{E}}{\partial x} = \frac{\rho}{\epsilon_s} \Rightarrow \rho = \epsilon_s \frac{\partial \mathcal{E}}{\partial x}$$

$$= +\epsilon_s\left(\frac{kT}{q}\right)\left(\frac{1}{x^2}\right)$$

1 d)



Charge neutrality dictates that

$$qN_A x_p = \frac{q}{2} m x_n + \int_{x_n}^{0.5\mu m} \epsilon_s \left(\frac{kT}{q}\right) \frac{1}{x^2}\, dx$$

$$= \frac{q}{2} m x_n + \epsilon_s \left(\frac{kT}{q}\right)\left(\frac{1}{x_n} - \frac{1}{0.5\mu m}\right)$$

This links x_p & x_n

We will still have a total $V_{bi} = \frac{kT}{q} \ln\left(\frac{N_D N_A}{n_i^2}\right)$

since we need to equalize the Fermi levels in equilibrium

Thus, integrating the charge profile twice to get V vs x_n and setting $V = V_{bi}$ will yield x_n & x_p. Messy but possible

2. a) $D_n = 64.626\ cm^2/s$ $\tau_n = 5ns$

$L_n = 5.6845\ \mu m$

$D_p = 9.306\ cm^2/s$ $\tau_n = 5ns$

$L_p = 2.1571\ \mu m$

$$\Delta n_p(x) = n_{p0} \frac{\sinh\left(\frac{x+W_p}{L_n}\right)}{\sinh\left(\frac{W_p - x_p}{L_p}\right)} \left(e^{qV_A/kT} - 1\right)$$

$$\Delta p_n(x) = p_{n0} \frac{\sinh\left(\frac{W_n - x}{L_p}\right)}{\sinh\left(\frac{W_n - x_n}{L_p}\right)} \left(e^{qV_a/kT} - 1\right)$$

at $V_a = 0.4V$

$x_n = 11.209\ nm$ $x_p = 20.757\ nm$

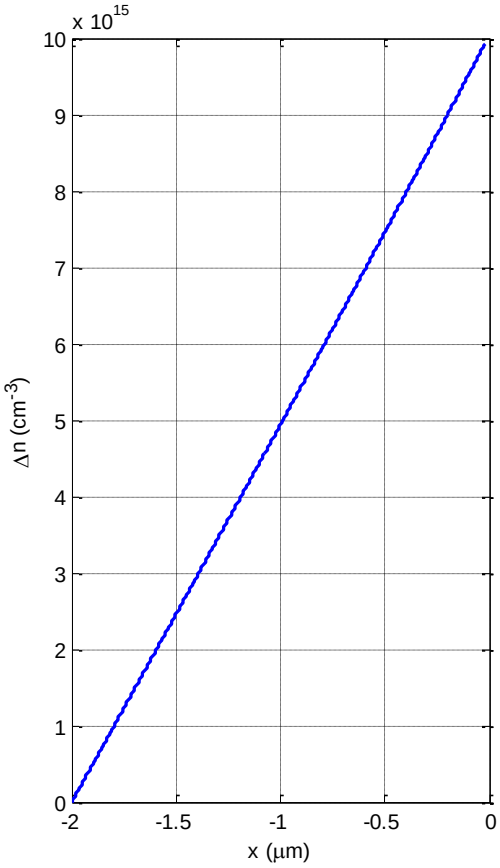
$n_{p0} \simeq 1.8917 \times 10^9\ cm^{-3}$ $p_{n0} \simeq 1.0215 \times 10^9\ cm^{-3}$

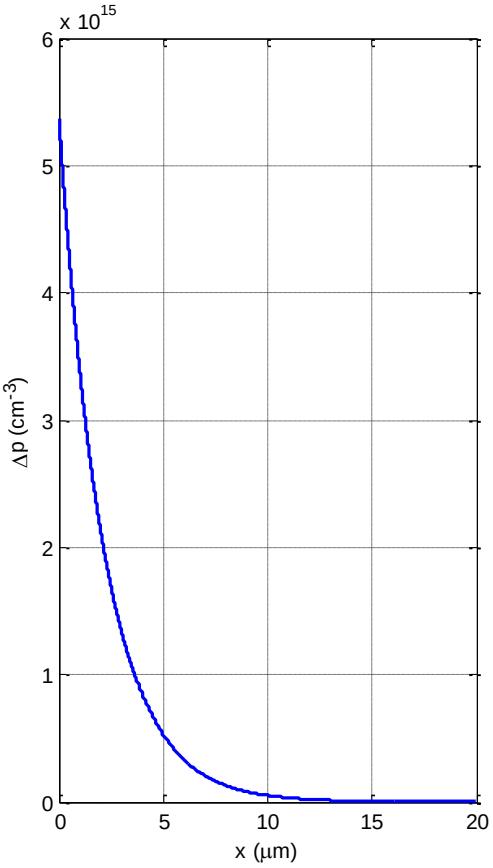
$\left(\frac{L_n}{W_p - x_p}\right) \simeq 2.872$ region is ~short but shows some recombination loss

$\left(\frac{L_p}{W_n - x_n}\right) \simeq 0.1079$ Region long

~ 1.1[illegible] $\times 10$ [illegible]/cm^2

2. Forward Bias Minority Carrier Densities





2. b) Depletion capacitance

$$C_j = \frac{\epsilon_s}{W} = \frac{\epsilon_s}{x_n + x_p} = 4.4317 \times 10^{-7} \ F/cm^2$$

Diffusion capacitance

$$Q_n = q \int_{-W_p}^{-x_p} \Delta n(x)\, dx$$

$$= \frac{q n_{p0}(e^{qV_A/kT} - 1)}{\sinh\left(\frac{W_p - x_p}{L_n}\right)} \int_{-W_p}^{-x_p} \sinh\left(\frac{x + W_p}{L_n}\right) dx$$

$$= \frac{q L_n n_{p0}(e^{qV_A/kT} - 1)}{\sinh\left(\frac{W_p - x_p}{L_n}\right)} \cosh\left(\frac{x + W_p}{L_n}\right)\Bigg|_{-W_p}^{-x_p}$$

$$= \frac{q L_n n_{p0}}{\sinh\left(\frac{W_p - x_p}{L_n}\right)} \left(\cosh\left(\frac{W_p - x_p}{L_n}\right) - 1\right)\left(e^{qV_A/kT} - 1\right)$$

$$C_n^{Diff} = \left|\frac{\partial Q_n}{\partial V_a}\right| = \left(\frac{q}{kT}\right) \frac{q L_n n_{p0}}{\sinh\left(\frac{W_p - x_p}{L_n}\right)} \left(\cosh\left(\frac{W_p - x_p}{L_n}\right) - 1\right) e^{qV_a/kT}$$

$$\approx 6.0301 \times 10^{-6} \ F/cm^2$$

Similarly

$$C_p^{Diff} = \left(\frac{q}{kT}\right) \frac{q L_p p_{n0}}{\sinh\left(\frac{W_n - x_n}{L_p}\right)} \left(\cosh\left(\frac{W_n - x_n}{L_p}\right) - 1\right) e^{qV_a/kT}$$

$$\approx 7.1680 \times 10^{-6} \ F/cm^2$$

$$C = C_j + C_p^{Diff} + C_n^{Diff} \approx 1.3641 \times 10^{-5} \ F/cm^2$$

2. c) $R-G = U = \dfrac{np - n_i^2}{(n+n_1)\tau_p + (p+p_1)\tau_n}$

In the depletion region $np = n_i^2 \exp(qV_a/kT)$
Here $E_T = E_i \Rightarrow n_1 = p_1 = n_i$
& $\tau_n = \tau_p = \tau = 5ns$
So

$$U = \frac{n_i^2(e^{qV_a/kT} - 1)}{(n+p+2n_i)\tau}$$

From the Shockley-Fletcher conditions

$$n = n_{p0} e^{+qV/kT} \quad \& \quad p = p_{p0} e^{-q(V_a - V_{bi} - V)/kT}$$

If $V(x)$ = electrostatic potential in junction
& $V(x_n) = V_n = 0$
$V(-x_p) = V_p = V_a - V_{bi}$

We can get $V(x)$ from the abrupt junction depletion approximation

$$V(x) = V_p + \frac{qN_A}{2\epsilon_s}(x_p + x)^2 \quad -x_p \le x \le 0$$
$$= V_a - V_{bi} + \left(\frac{qN_A}{2\epsilon_s}\right)(x_p + x)^2 \quad -x_p \le x \le 0$$

$$V(x) = -\frac{qN_D}{2\epsilon_s}(x - x_n)^2 \quad 0 \le x \le x_n$$

Thus we can get $U(x)$

Finally $J_{G-R} = +q \int_{-x_p}^{x_n} U \, dx$

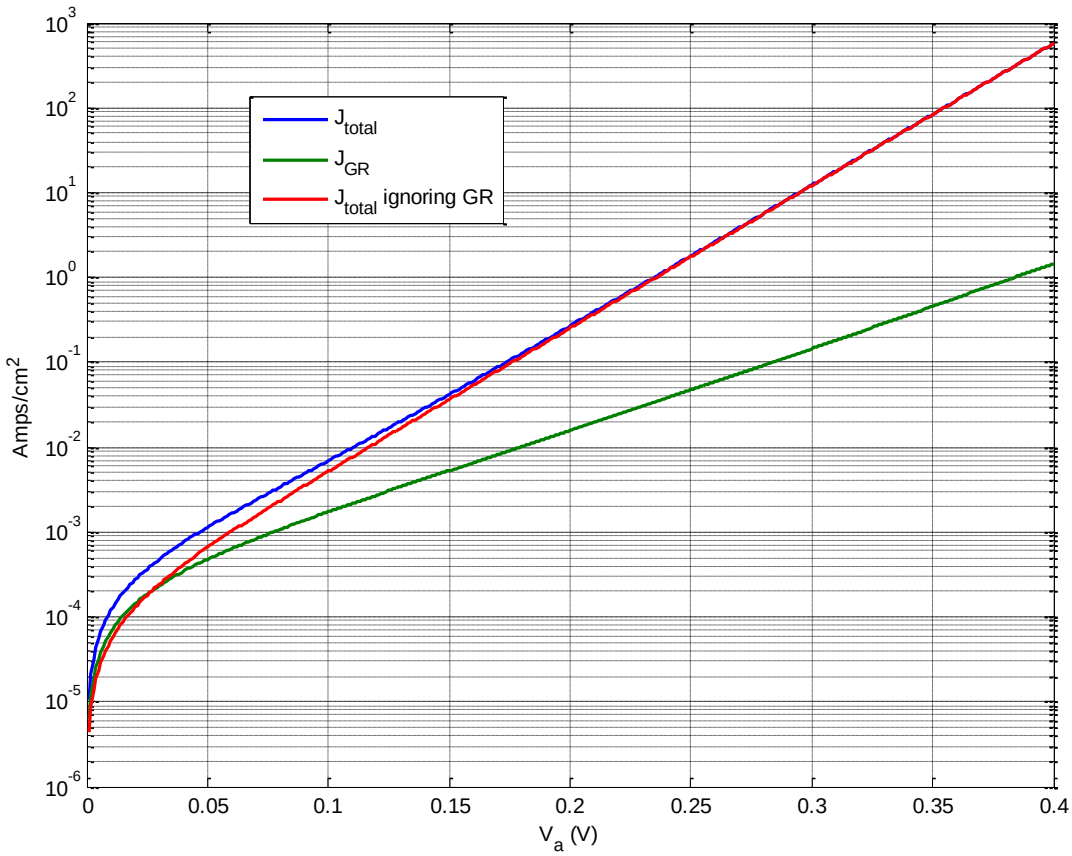
$$J_n(-x_p) = \frac{q D_n n_{p0}}{L_n} \coth\left(\frac{W_p - x_p}{L_n}\right)\left(e^{qV_A/kT} - 1\right)$$

$$J_n(x_n) = J_n(-x_p) + J_{G-R}$$

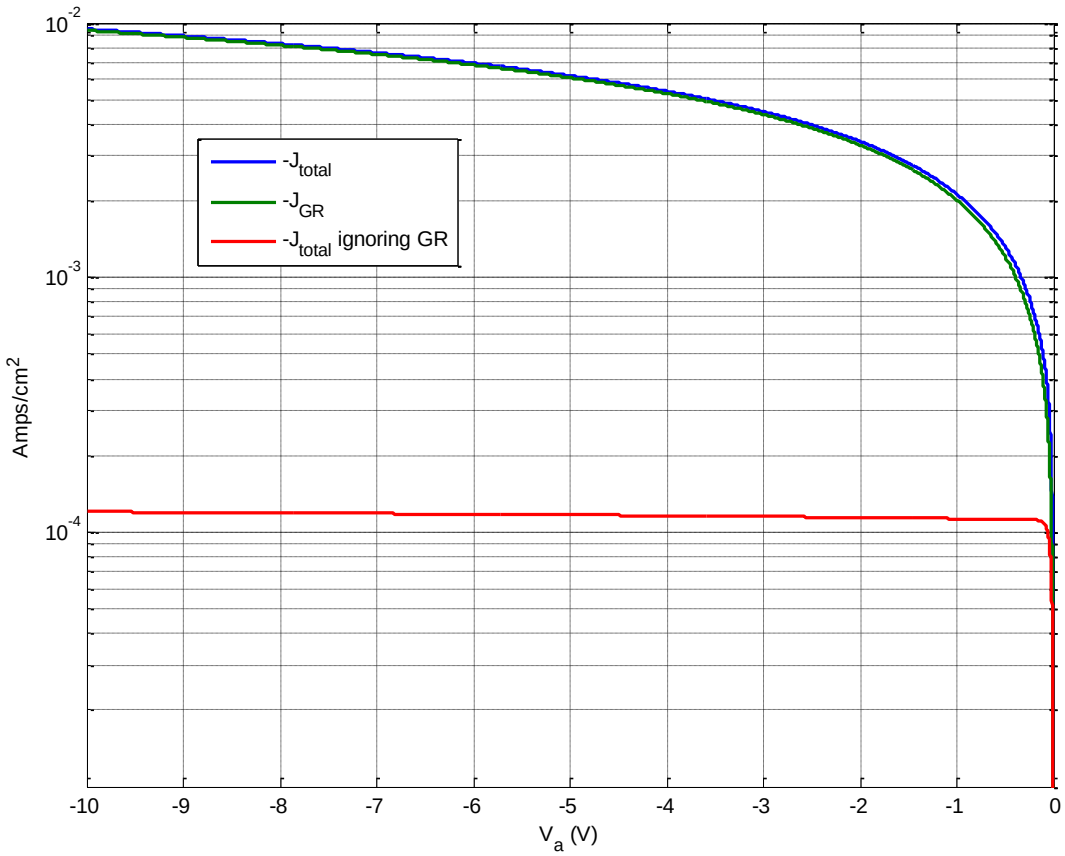
$$J_p(x_n) = \frac{q D_p p_{n0}}{L_p} \coth\left(\frac{W_n - x_n}{L_p}\right)\left(e^{qV_A/kT} - 1\right)$$

$$J_T = J_p(x_n) + J_n(x_n) = J_p(x_n) + J_n(-x_p) + J_{G-R}$$

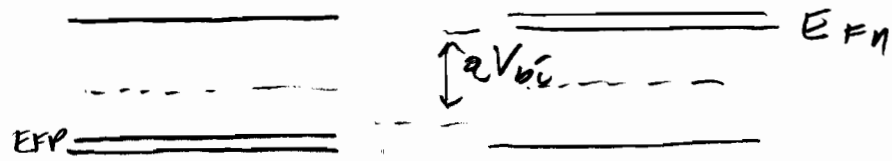
2. Forward Bias Current



2. Reverse Bias Current



3. a) Here it is clear that we will have both partial ionization + degenerate statistics issues



$$qV_{bi} = E_{Fn} - E_{Fp}$$

n-type side $\rho = 0$ condition

$$p - n + N_D^+ - N_A^- = 0$$

$$\left\{ N_v \tilde{\mathcal{F}}_{1/2}\left(\frac{E_v - E_{Fn}}{kT}\right) - N_c \tilde{\mathcal{F}}_{1/2}\left(\frac{E_{Fn} - E_c}{kT}\right) + \frac{N_D}{1 + 2\exp\left(\frac{E_{Fn} - E_D}{kT}\right)} \right\} = 0$$

p-side $\rho = 0$ condition

$$\left\{ N_v \tilde{\mathcal{F}}_{1/2}\left(\frac{E_v - E_{Fp}}{kT}\right) - N_c \tilde{\mathcal{F}}_{1/2}\left(\frac{E_{Fn} - E_c}{kT}\right) - \frac{N_A}{1 + 4\exp\left(\frac{E_A - E_{Fp}}{kT}\right)} \right\} = 0$$

Solving iteratively (Mathematica) yields

$E_{Fn} \cong E_v + 1.08684\,eV$

$E_{Fp} \simeq E_v + 0.0627483\,eV$ (excess digits)

b) $\mathcal{E}_p = 3 \times 10$ V/cm

$$x_p = \frac{\epsilon_s \mathcal{E}_p}{q N_A} = 2.44537 \text{ nm}$$

$$x_n = \frac{\epsilon_s \mathcal{E}_p}{q N_D} = 0.652098 \text{ nm}$$

$$W = x_n + x_p = 3.09747 \text{ nm} = 30.97 \text{ Å}$$

Since $W \lesssim 10$ nm I would expect tunneling breakdown

The calculations assume 100% dopant ionization. This would be ~ true in much of the depletion region

Not requested, but note that

$$V_{bi} - V_a = \frac{q N_A}{2\epsilon_s} x_p^2 + \frac{q N_D}{2\epsilon_s} x_n^2 \simeq 0.0464 \text{ V}$$

This suggests that the device is effectively broken down at ~0 bias.