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EECS 320

Exam 1

Spring 2012

This exam is closed book. A list of equations and useful constants is provided. More equations and constants are provided than you may need. You are permitted one 8.5x11" page of notes and handheld calculators. Tablets or smartphones with calculator apps are NOT permitted. Notes and/or equations programmed into handheld calculators are forbidden. Please turn off your cellular phone and store them in your bag during the exam. Using or placing cellular on the table during the exam is strictly forbidden.

Show all work and ***state all assumptions*** to receive full credit. Use the space provided to answer questions. ***Use material parameters in the equation sheet provided.***

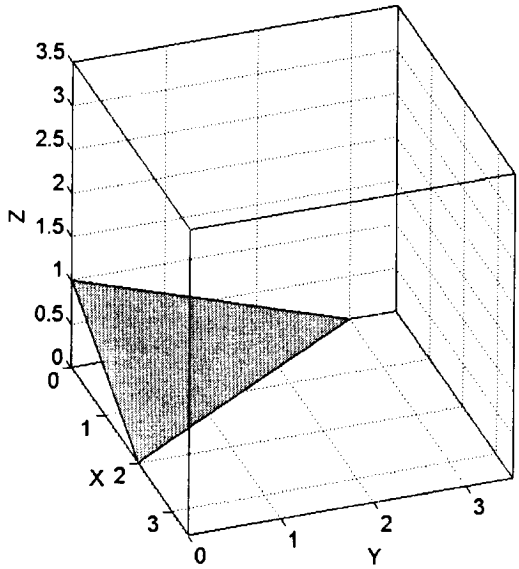
To receive credit for this examination, write the Honor Pledge "I have neither given nor received aid on this examination" and sign below.

Honor pledge:

Signature:_____

Problem 1 Miller Indices (5 points)

Determine the miller indices for the crystalline plane in a cubic lattice in the sketch below. Show your work.



Miller Indices of Plane: (3 2 6)

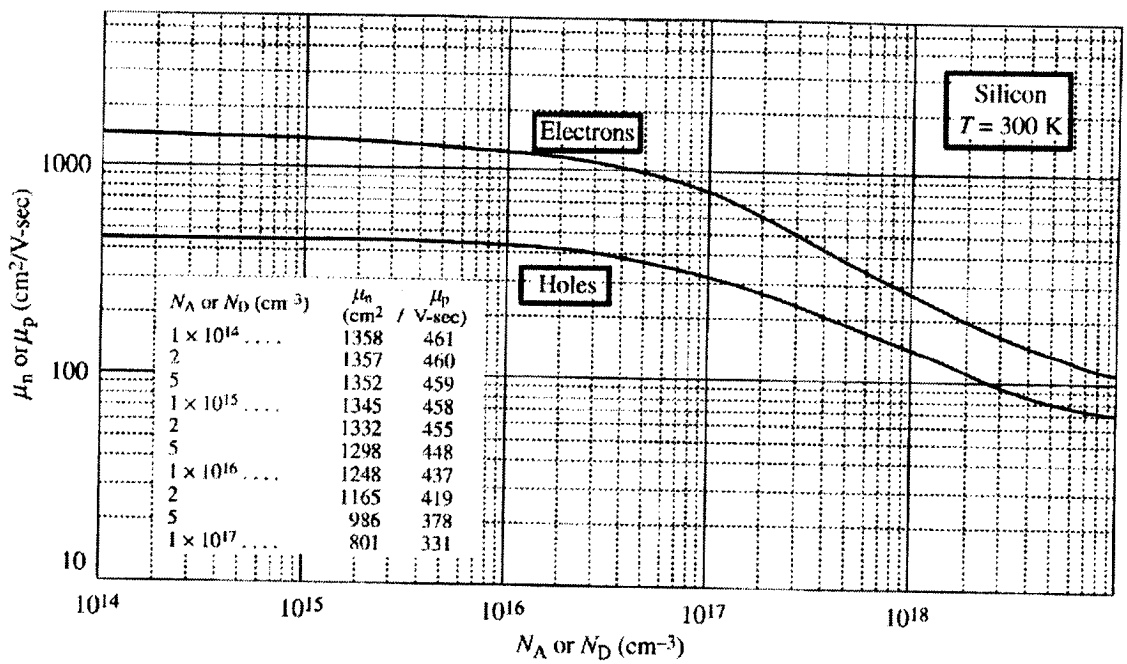
intercepts (2, 3, 1)

invert $(\frac{1}{2}, \frac{1}{3}, 1)$

LCM $6(\frac{1}{2}\ \frac{1}{3}\ 1) \doteq (3\ 2\ 6)$

Problem 2: Basic Conductivity (a 3pts ,b 4 pts)

a) Suppose we have a sample of pure Si at 300K doped only with shallow donors $N_D=1\times10^{13}cm^{-3}$. Estimate the resistivity of this sample using the data from Pierret figure 3.5a reproduced below.



N_A or N_D (cm⁻³)	μ_n (cm² / V-sec)	μ_p (cm² / V-sec)
1×10^{14}	1358	461
2	1357	460
5	1352	459
1×10^{15}	1345	458
2	1332	455
5	1298	448
1×10^{16}	1248	437
2	1165	419
5	986	378
1×10^{17}	801	331

(a)

Pierret figure 3.5 a

$n \approx 1\times10^{13} cm^{-3}$ $\quad p \approx n_i^2/N_D = 1.145\times10^{-3} \ll n$

$\sigma \approx q\mu_n n \cong q(1358)(1\times10^{13}) \approx 2.176\times10^{-3}\ \Omega^{-1}cm^{-1}$

$$\boxed{\rho = \frac{1}{\sigma} \cong 459.6\ \Omega\text{-cm}}$$

b) Now we have a sample of pure Si at 300K doped with shallow donors and shallow acceptors $N_D=5\times10^{16}cm^{-3}$ and $N_A=4.999\times10^{16}cm^{-3}$. Estimate the resistivity of this sample and compare your result in part b.

$n \simeq N_D - N_A = 1\times10^{13} cm^{-3}$

Mobility determined by ionized impurity scattering related to $N_D + N_A \simeq 1\times10^{17}$

$\Rightarrow \mu_n \simeq 801$

$\sigma \cong q\mu_n n \approx 1.283\times10^{-3}\ \Omega^{-1} cm^{-1}$

$\boxed{\rho \simeq 779.2\ \Omega\text{-cm}}$

resistivity higher by factor of $\frac{1358}{801} \simeq 1.70$

Problem 3: Doping (7 pts)

Suppose we have Si at 300K doped with Phosphorus (E_D=E_C-44meV) and Boron (E_A=E_V+44meV). If N_A=1x10^{17}cm^{-3}, what N_D is needed so that n=1x10^{18} cm^{-3}?

$$-n + p + N_D^+ - N_A^- = 0$$

$$\Rightarrow n \simeq N_D^+ - N_A^-$$

$N_A^- \simeq N_A$ since n-type (all acceptors ionized

so $N_D^+ \simeq n + N_A = 1.1 \times 10^{18}$

$$E_f \simeq E_c + kT \ln\left(\frac{n}{N_c}\right)$$

$$\simeq E_c - 86.69\,meV$$

$$\frac{N_D^+}{N_D} = \frac{1}{1 + 2\exp\left(\frac{E_F - E_D}{kT}\right)}$$

$E_D = E_c - 44meV \Rightarrow E_F - E_D = 44meV - 86.69meV$

$= -42.69meV$

$= 1.65\,kT$

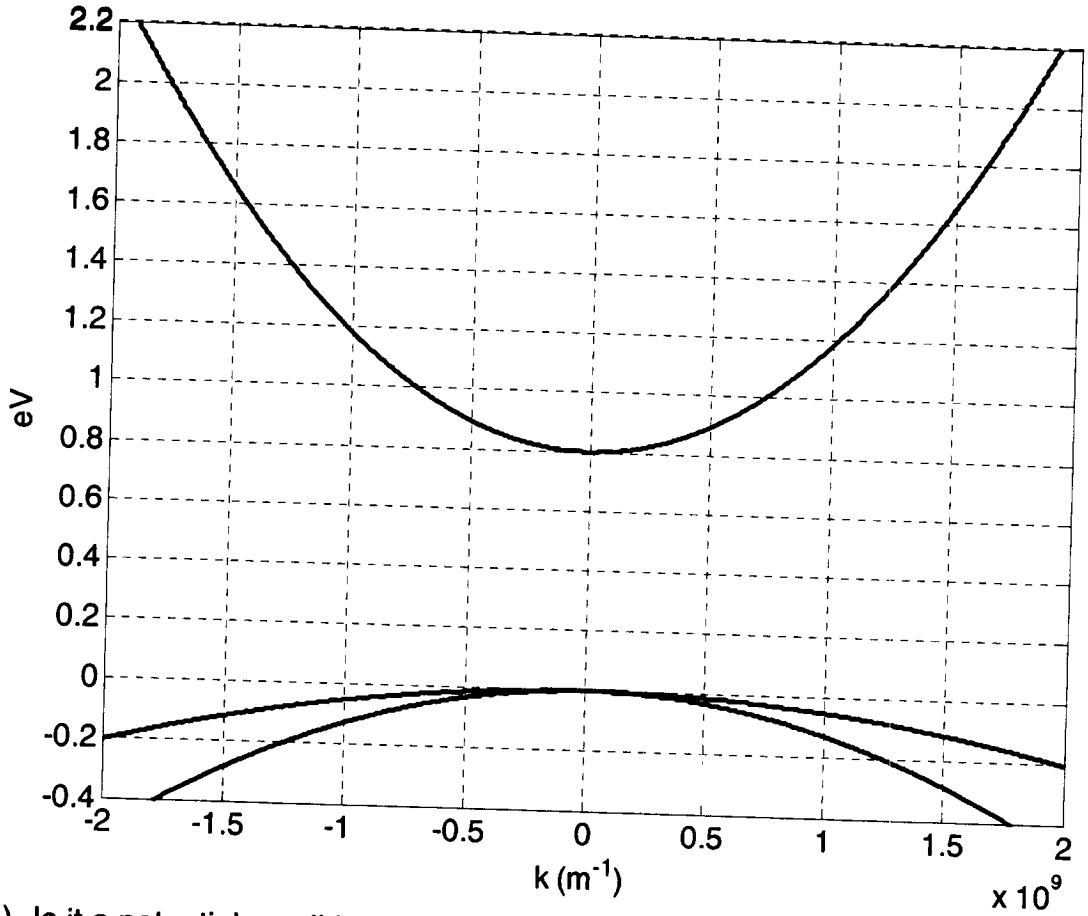
$$\frac{N_D^+}{N_D} = 0.723$$

$$\Rightarrow N_D = (1.3835) N_D^+ = \underline{1.522 \times 10^{18}\,cm^{-3}}$$

Problem 4: Band structure and statistics (a 3pts, b 3pts, c 15 pts)

A semiconductor has band structure show below. We have

$m_n^* = 0.1m_0$ $m_{lh}^* = 0.3m_0$ $m_{hh}^* = 0.75m_0$ at 300K



a) Is it a potential candidate for building light emitting diodes or lasers? Briefly justify your answer.

yes, it is a direct gap semiconductor

b) At what wavelength would expect to begin seeing significant optical absorption?

$$h\nu = 0.8\,eV = E_g$$

$$\nu\lambda = c \approx 3\times10^{8}\ m/s$$

$$\frac{hc}{\lambda} = 0.8\,eV = 1.282\times10^{-19}\,J$$

$$\lambda = \frac{(6.626\times10^{-34}\,J\cdot s)(3\times10^{8}\,m/s)}{1.282\times10^{-19}\,J} = 1.551\times10^{-6}\,m$$

$$= 1.55\,\mu m$$

c) Find the effective densities of states N_c and N_v, the intrinsic carrier concentration n_i, and E_i relative to E_v.

N_c=	$7.934\times10^{17}\ cm^{-3}$
N_v=	$2.042\times10^{19}\ cm^{-3}$
n_i=	$7.670\times10^{11}\ cm^{-3}$
E_i=E_v+	0.442 eV

$$N_c = \left(\frac{m_n^*}{m_0}\right)^{3/2}(2.509\times10^{19}\,cm^{-3}) = 7.934\times10^{17}\,cm^{-3}$$

$$N_v = \left[\left(\frac{m_{ep}}{m_0}\right)^{3/2} + \left(\frac{m_{hp}}{m_0}\right)^{3/2}\right](2.509\times10^{19}) = 2.041\times10^{19}\,cm^{-3}$$

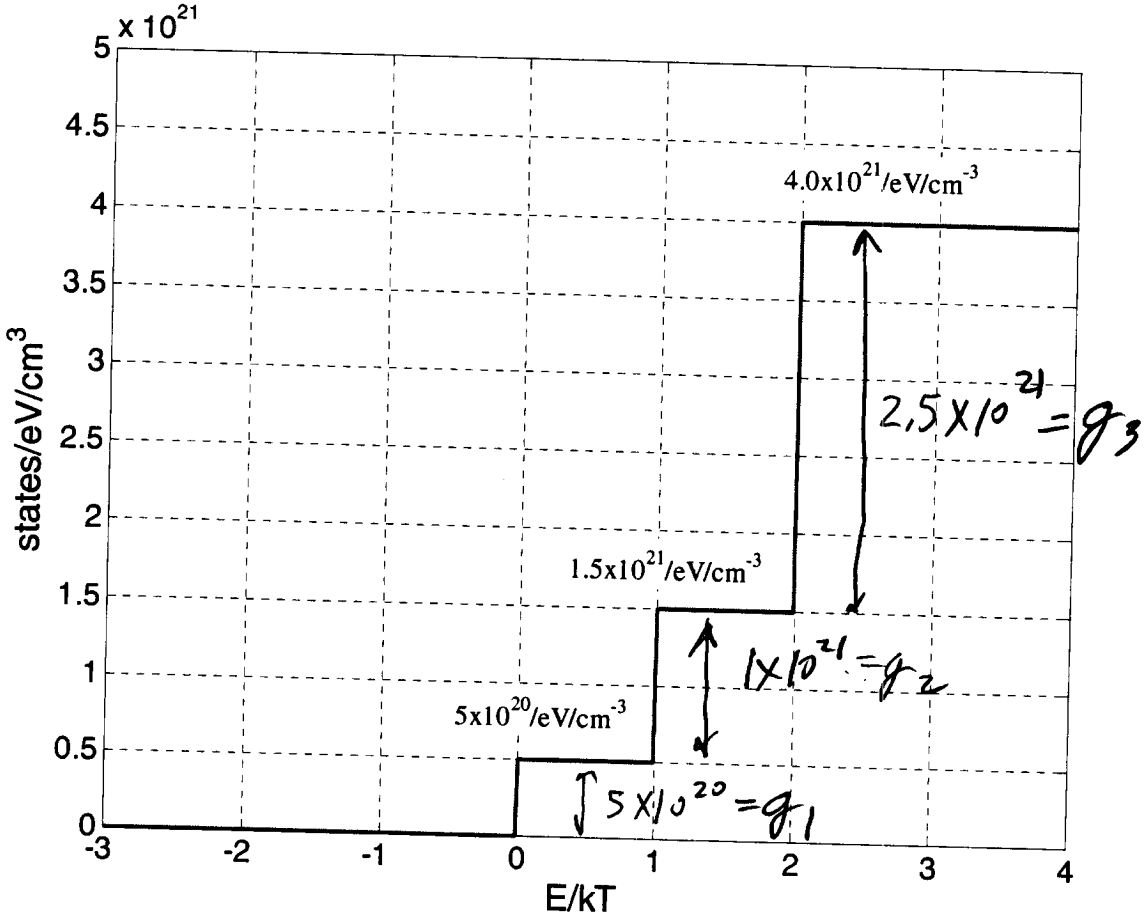
$$n_i = \left[N_c N_v \exp\left(-\frac{E_g}{kT}\right)\right]^{1/2} = 7.670\times10^{11}$$

$$E_g = 0.8\,eV$$

$$E_i = E_v + kT\ln\left(\frac{N_v}{n_i}\right) = E_v + 0.442\,eV$$

Problem 5: Density of States and Bands (15 pts) ($E_c = 0$ for picture)

Suppose we have the following density of states $g_c(E)$ for a conduction band. Find the electron concentration in the band if $E_F = E_C - 3kT$



Easy way is linearity

$$n \cong \int_{E_c}^{\infty} g_c(E)\, e^{(E_F - E)/kT}\, dE \qquad \text{(Boltzmann)}$$

$$n_1 = e^{E_F/kT} \int_{E_c}^{\infty} g_1 e^{-E/kT}\, dE$$

$$= g_1 e^{E_F/kT} \left[-kT e^{-E/kT} \right]_{E_c}^{\infty}$$

$$= g_1 kT\, e^{-E_c/kT} e^{+E_F/kT}$$

Additional workspace for problem 5

$$n_2 = g_2 kT\, e^{-(E_c + 1kT)/kT}\, e^{E_F/kT}$$

$$n_3 = g_3 kT\, e^{-(E_c + 2kT)/kT}\, e^{E_F/kT}$$

$$n = n_1 + n_2 + n_3$$

$$= kT\left[g_1 e^{(E_F - E_c)/kT} + g_2 e^{(E_F - E_c - 1kT)/kT} + g_3 e^{(E_F - E_c - 2kT)/kT} \right]$$

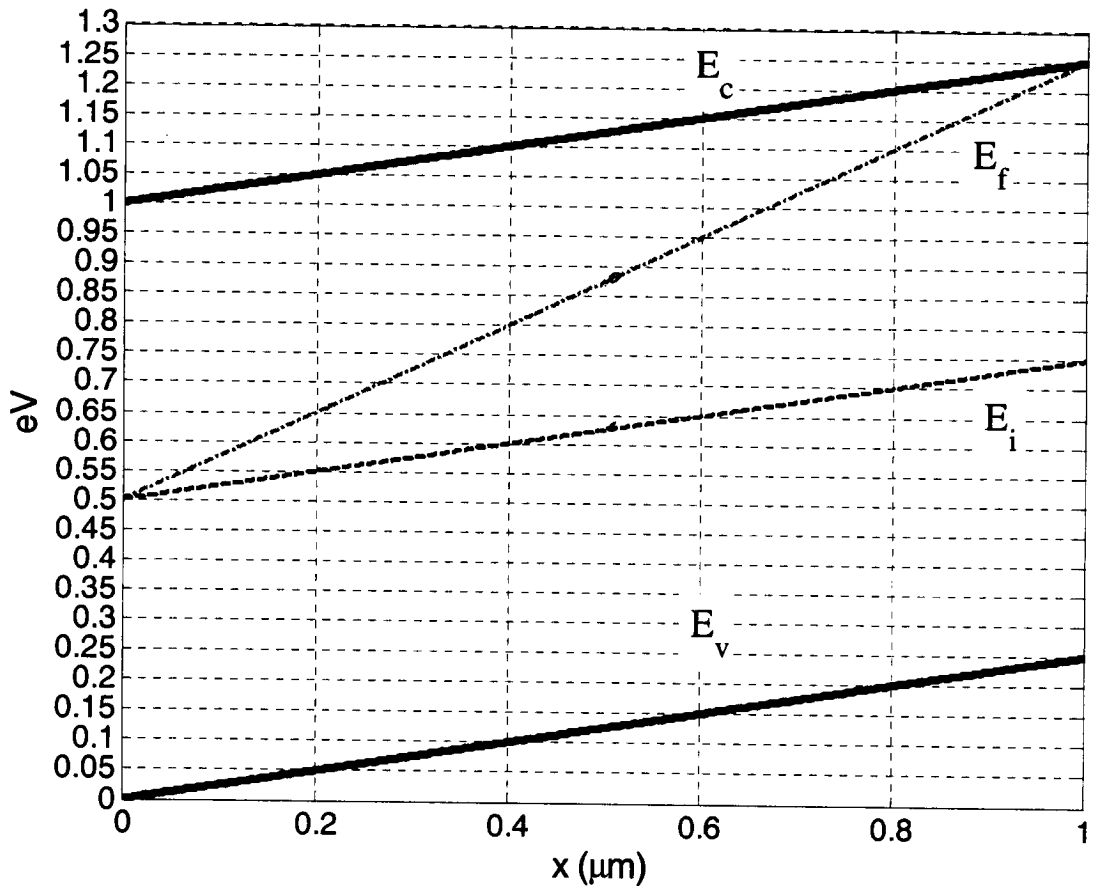
$$= kT\left[g_1 e^{-3} + g_2 e^{-4} + g_3 e^{-5} \right]$$

$$\simeq kT\,(6.0054 \times 10^{19})$$

$$\boxed{n \cong 1.552 \times 10^{18}\ cm^{-3}}$$

Problem 6: Drift Diffusion Transport (20 pts)

We have a semiconductor with $N_v = N_c$ $n_i = 1x10^{10}$ $E_g = 1eV$ $\mu_n = 1000cm^2v^{-1}s^{-1}$ at 300K. A sample of this semiconductor has the band picture shown below. Find $J_{n,total}$, $J_{n,drift}$, and $J_{n,diffusion}$ at x=0.5μm.



$J_{n,total}$=	+ 190.44 A/cm²
$J_{n,drift}$=	+ 63.48 A/cm²
$J_{n,diffusion}$=	+ 126.96 A/cm²

$J_n = \mu_n n \frac{\partial E_F}{\partial x}$ total D-D current

$\frac{\partial E_F}{\partial x} = \frac{(1.25 - 0.5)eV}{1\mu m} = \frac{+0.75eV}{1\mu m} = 7.5 \times 10^3 \frac{eV}{cm}$

Additional workspace for problem 6

$$E_F = 0.5\,eV + \left(\frac{x}{1\mu m}\right) 0.75\,eV$$

$$E_i = 0.5\,eV + \left(\frac{x}{1\mu m}\right) 0.25$$

$$\left.\begin{array}{l} E_F(0.5\mu m) = 0.875\,eV \\ E_i(0.5\mu m) = 0.625\,eV \end{array}\right\} \begin{array}{l} E_F - E_i = +0.25\,eV \\ @\ 0.5\mu m \end{array}$$

$$n = n_i \exp\left(\frac{E_F - E_i}{kT}\right) = 1.585 \times 10^{14}\,cm^{-3}$$

so $$J_{n,tot} = (1000\,cm^2/V{\cdot}s)\, n \left(q\,\frac{7500\,eV}{cm}\right)$$

$$= +190.44\,A/cm^2$$

$$\mathcal{E} = \frac{1}{q}\frac{\partial E_c}{\partial x} = 2500\,V/cm$$

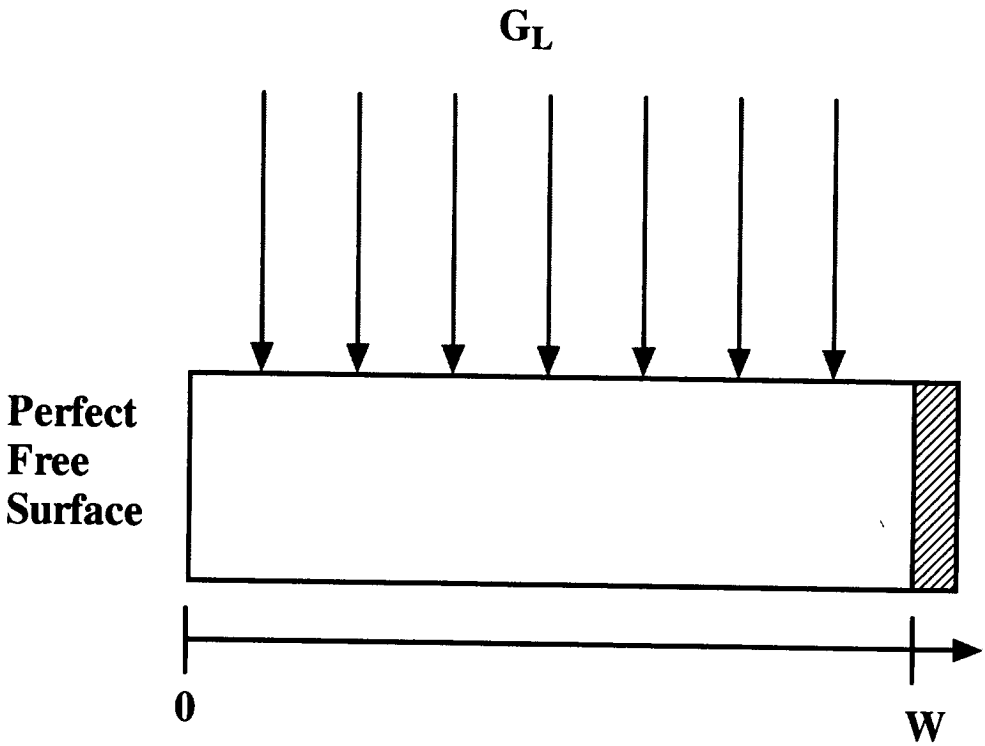
$$J_{n,drift} = q\mu_n n \mathcal{E} = +63.48\,A/cm^2$$

$$J_{n,diff} = J_{n,tot} - J_{n,drift}$$

$$\simeq 126.96\,A/cm^2$$

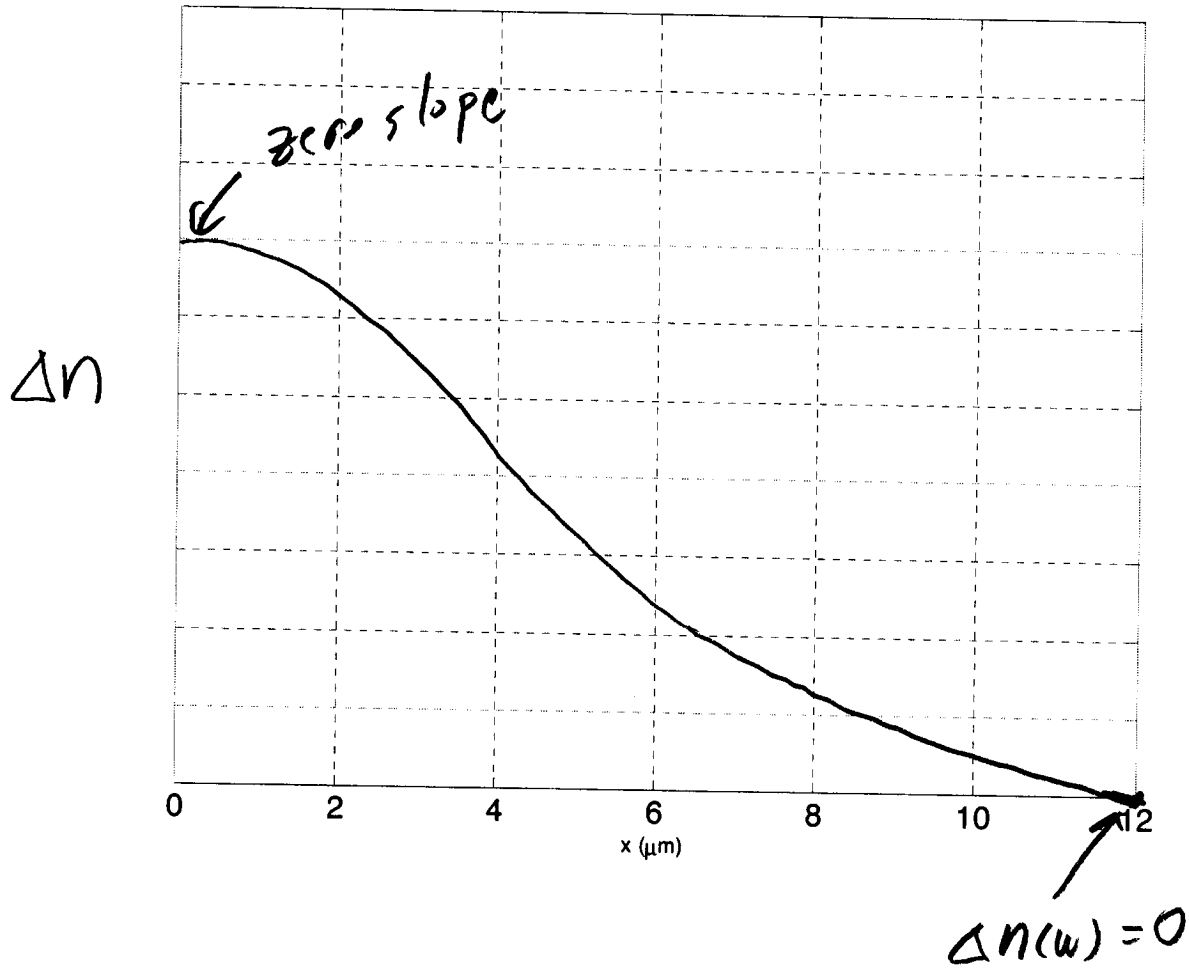
Problem 7 Generation-Recombination (a 5pts, b 20 pts) Steady State Assumed

A sample of Si is uniformly illuminated as illustrated below. The left side is a perfect free surface (x=0), the right side has a perfect ohmic contact at x=W=12μm. The sample is uniformly p-type doped with shallow acceptors $N_A=1x10^{17}cm^{-3}$, $D_n=18cm^2/s$, and $\tau_n=20ns$. Also, $G_L=6.8x10^{23}pairs/cm^3/s$.



$N_A = 1x10^{17} cm^{-3}$ $D_n = 18cm^2 s^{-1}$ $\tau_n = 20ns$ $G_L = 6.8x10^{23}\ pairs/cm^3/s$ $W = 12\mu m$

a) Sketch the expected excess minority carrier lifetime $\Delta n(x)$ vs. x



b) Find $\Delta n(x)$ assuming LLI conditions apply. Are the assumptions of LLI statisfied?

$N_A = 1x10^{17} cm^{-3}$ $D_n = 18 cm^2 s^{-1}$ $\tau_n = 20ns$ $G_L = 6.8x10^{23}\ pairs/cm^3/s$ $W = 12\mu m$

$$\Delta n = A_1 e^{x/L_n} + A_2 e^{-x/L_n} + G_L \tau_n \qquad L_n = [D_n \tau_n]^{1/2}$$

$$\Delta n(W) = 0 = A_1 e^{W/L_n} + A_2 e^{-W/L_n} + G_L \tau_n$$

$$\left.\frac{\Delta n}{\partial x}\right|_0 = 0 = \frac{A_1}{L_n} + \frac{-A_2}{L_n} = 0$$

Perfect Free surface nothing leaves/recombines

$$\Rightarrow A_2 = A_1$$

$$A_1 (e^{W/L_n} + e^{-W/L_n}) + G_L \tau_n = 0$$

$$2A_1 \cosh(W/L_n) = -G_L \tau_n$$

$$A_1 = \frac{-G_L \tau_n}{2\cosh(W/L_n)}$$

$$\Delta n(x) = A_1 (e^{x/L_n} + e^{-x/L_n}) + G_L \tau_n$$

$$= 2A_1 \cosh(x/L_n) + G_L \tau_n$$

$$= -G_L \tau_n \frac{\cosh(x/L_n)}{\cosh(W/L_n)} + G_L \tau_n$$

or

$$\boxed{\Delta n(x) = G_L \tau_n \left[1 - \frac{\cosh(x/L_n)}{\cosh(W/L_n)}\right]}$$

Additional space for 9b

$N_A = 1x10^{17} cm^{-3}$ $D_n = 18 cm^2 s^{-1}$ $\tau_n = 20 ns$ $G_L = 6.8x10^{23}$ $pairs/cm^3/s$ $W = 12 \mu m$

$$\Delta n(0) = G_L \tau_n \left[1 - \frac{2}{\cosh(W/L_n)} \right]$$

$$G_L \tau_n = 1.36 \times 10^{16} \, cm^{-3}$$

$$L_n = [18 \times 20 \times 10^{-9}]^{1/2} = 6 \times 10^{-4} \, cm$$
$$= 6 \, \mu m$$

$$\Delta n(0) = (1.36 \times 10^{16} \, cm^{-3}) \left(1 - \frac{2}{\cosh(2)} \right)$$

$$\simeq 6.37 \times 10^{15} \, cm^{-3}$$

Since $\Delta n(0) < N_A/10$ we are safely in LLI

EECS 320 Equation sheet

Physical Constants

$h = 6.626\times10^{-34}$ J - s

$\hbar = 1.054\times10^{-34}$ J-s

$q = 1.60217\times10^{-19}$ C

$eV = 1.602\times10^{-19}$ J

$m_0 = 9.11\times10^{-31}$ kg

$k = 8.617\times10^{-5}$ eV/K

$kT = 0.02585eV$ at T=300K

Electrostatics

$$\frac{d^2V}{dx^2} = -\frac{\rho}{\varepsilon}$$

$$F = -\frac{dV}{dx} = \frac{1}{q}\frac{dE}{dx}$$, F is electric field

Parabolic Band Density of States

$$g_C(E) = \frac{\sqrt{2}m_n^{*3/2}(E-E_C)^{1/2}}{\pi^2\hbar^3}$$

$$g_V(E) = \frac{\sqrt{2}m_p^{*3/2}(E_V-E)^{1/2}}{\pi^2\hbar^3}$$

Fermi Distribution Function

$$f(E) = \frac{1}{1+\exp\left(\frac{E-E_f}{kT}\right)}$$

Boltzmann approx:

$$f(E) = \exp\left(-\frac{E-E_f}{kT}\right)$$

Donor and Acceptor Statistics

$$\frac{N_D^+}{N_D} = \frac{1}{1+2\exp\left(\frac{E_f-E_D}{kT}\right)}$$

$$\frac{N_A^-}{N_D} = \frac{1}{1+4\exp\left(\frac{E_A-E_f}{kT}\right)}$$

Parabolic Band Model

$$N_C = 2\left(\frac{m_{n,dos}^* kT}{2\pi\hbar^2}\right)^{3/2}$$

$$= 2.509\times10^{19}\left(\frac{m_{n,dos}^*}{m_0}\right)^{3/2}\left(\frac{T}{300}\right)^{3/2} \text{ cm}^{-3}$$

$$N_V = 2\left(\frac{m_{p,dos}^* kT}{2\pi\hbar^2}\right)^{3/2}$$

$$= 2.509\times10^{19}\left(\frac{m_{p,dos}^*}{m_0}\right)^{3/2}\left(\frac{T}{300}\right)^{3/2} \text{ cm}^{-3}$$

$$n_i = \sqrt{N_C N_V}\exp\left(-\frac{E_G}{2kT}\right)$$

Carrier Statistics

$$n = \int g_C(E)f(E)dE$$

$$p = \int g_V(E)(1-f(E))dE$$

Calculate relative to E_C E_V

$$n = N_C\exp\left(\frac{E_f-E_C}{kT}\right)$$

$$p = N_V\exp\left(\frac{E_V-E_f}{kT}\right)$$

$$E_f = E_C + kT\ln\left(\frac{n}{N_C}\right)$$

$$E_f = E_V - kT\ln\left(\frac{p}{N_V}\right)$$

Calculate relative to E_i

$$n = n_i\exp\left(\frac{E_f-E_i}{kT}\right)$$

$$p = n_i\exp\left(-\frac{E_f-E_i}{kT}\right)$$

$$E_f - E_i = kT \ln\left(\frac{n}{n_i}\right)$$

$$E_i - E_f = kT \ln\left(\frac{p}{n_i}\right)$$

$$E_i = \frac{E_C + E_V}{2} + \frac{1}{2} kT \ln\left(\frac{N_V}{N_c}\right)$$

Doping and Charge Neutrality

$$n - N_d^+ - p + N_a^- = 0$$

$$np = n_i^2$$

$$n = \frac{N_D - N_A}{2} + \sqrt{\left(\frac{N_D - N_A}{2}\right)^2 + n_i^2}$$

$$p = \frac{N_A - N_D}{2} + \sqrt{\left(\frac{N_A - N_D}{2}\right)^2 + n_i^2}$$

Drift-Diffusion

$$\sigma = nq\mu_n + pq\mu_p$$

$$\mu = \frac{q\tau_{sc}}{m^*}$$

$$\frac{D}{\mu} = \frac{kT}{q}$$

$$J_n = q\mu_n nF + qD_n \frac{dn}{dx}$$

$$J_p = q\mu_p pF - qD_p \frac{dp}{dx},$$

F is electric field

Continuity Equations

$$\frac{\partial n}{\partial t} = \frac{1}{q}\frac{\partial J_n}{\partial x} + (G - R)$$

$$\frac{\partial \Delta n_p}{\partial t} = D_n \frac{\partial^2 \Delta n_p}{\partial x^2} - \frac{\Delta n_p}{\tau_n} + G$$

$$\frac{\partial \Delta p_n}{\partial t} = D_n \frac{\partial^2 \Delta p_n}{\partial x^2} - \frac{\Delta p_n}{\tau_p} + G$$

$$L_N = \sqrt{D_N \tau_n}$$

$$L_P = \sqrt{D_P \tau_p}$$

Table of properties of selected semiconductors (at 300K)

Property	Si	Ge	GaAs
N_C (cm^{-3})	2.86×10^{19}	1.04×10^{19}	4.45×10^{17}
N_V (cm^{-3})	3.10×10^{19}	6.0×10^{18}	7.72×10^{18}
n_i (cm^{-3})	1.07×10^{10}	2.3×10^{13}	1.8×10^{6}
E_G (eV)	1.1242	0.66	1.42
m_n^*/m_0	1.18	0.55	0.067
m_p^*/m_0	0.81	0.36	0.52
ε_r	11.8	16	13.1
χ (eV)	4.05	4.0	4.07