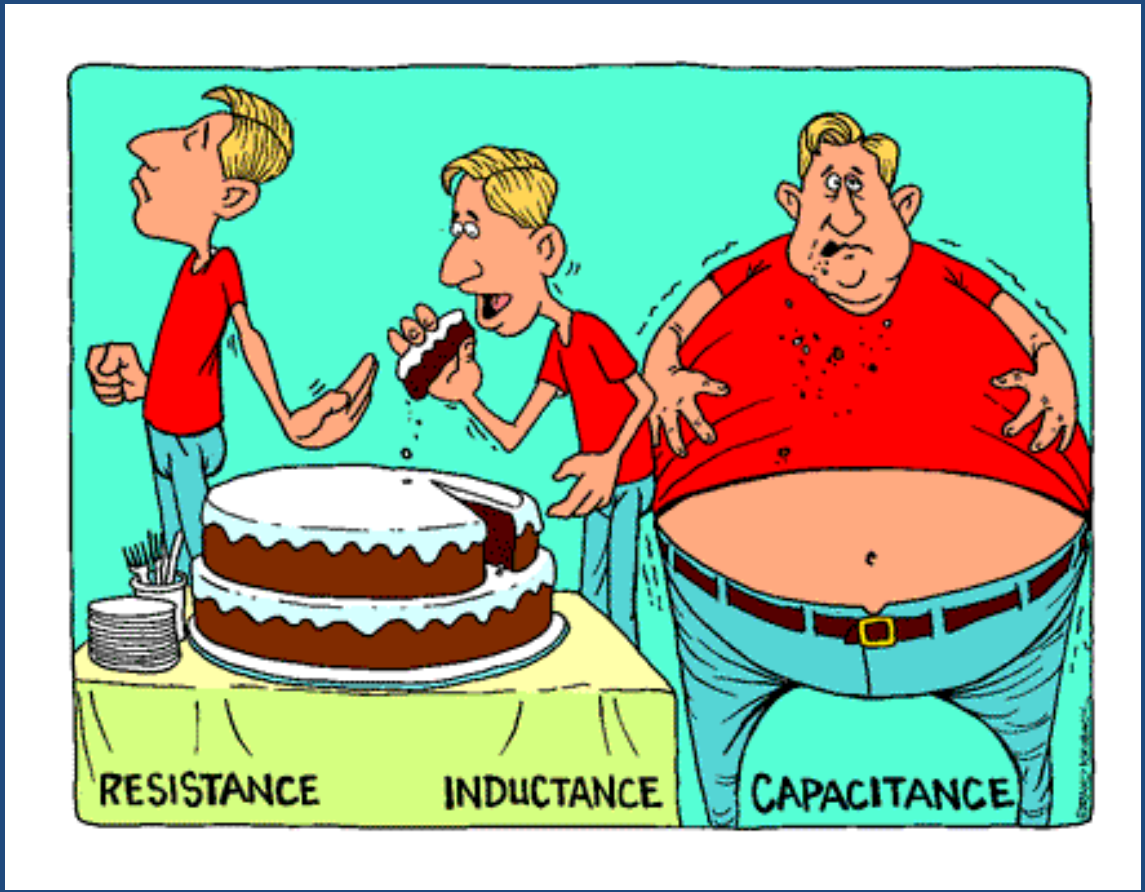


CAPACITORS AND INDUCTORS

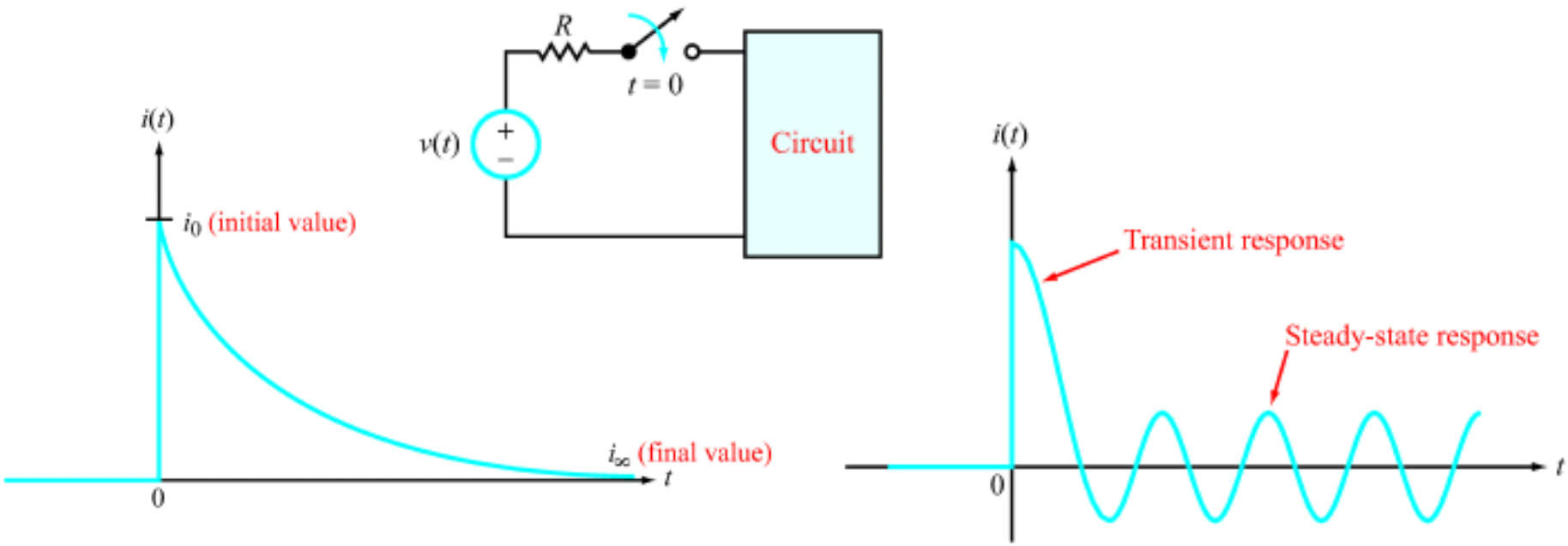
© Fred Terry
Winter, 2010

BIG IDEAS

- **Information Is Carried ONLY By Time Varying Signals**
- **Energy Storage & Finite Power <u>DIRECTLY</u> Imply Frequency Response Limitations / Non-Instant Time Responses**

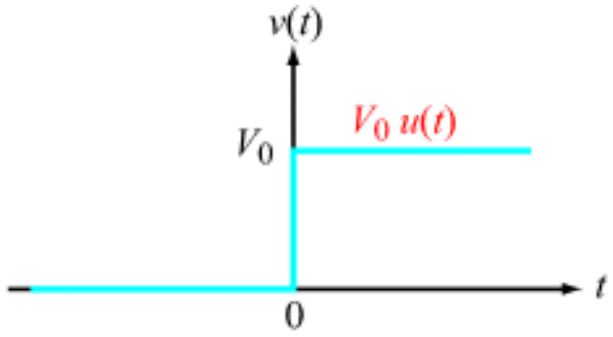
Time Response of Circuits

- Transient response due to change in circuit
- Steady state response can be AC or time varying, but is periodic in nature

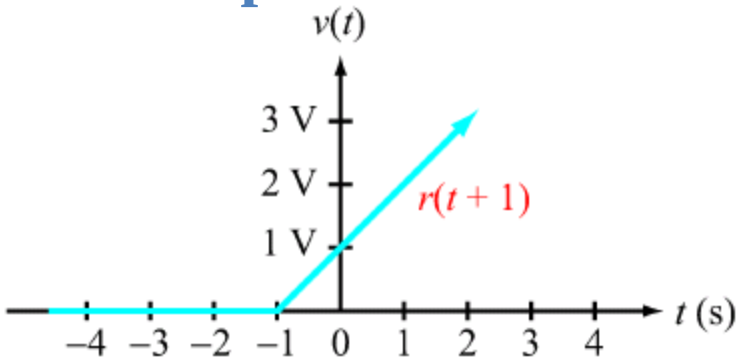


Non-Periodic Waveforms

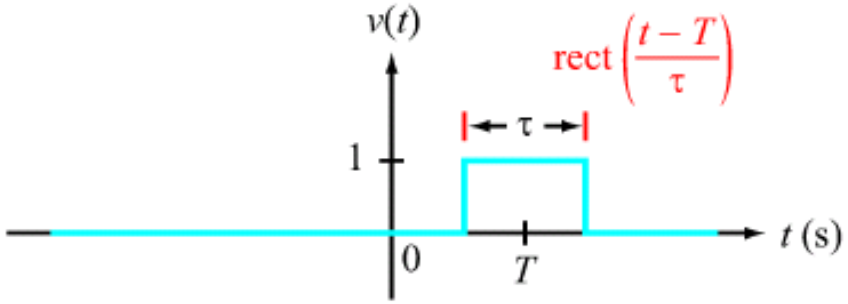
Step Function



Ramp Function



Square Pulse



Exponential

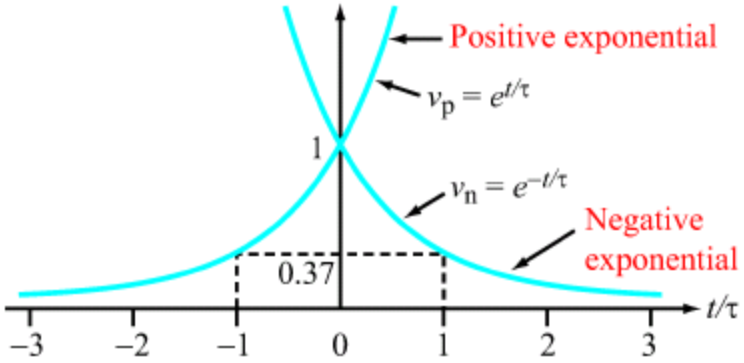
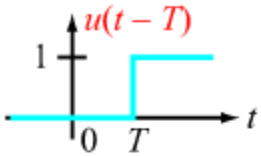
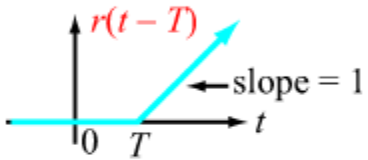
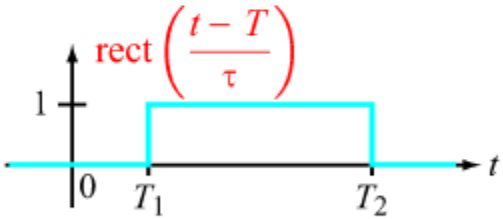
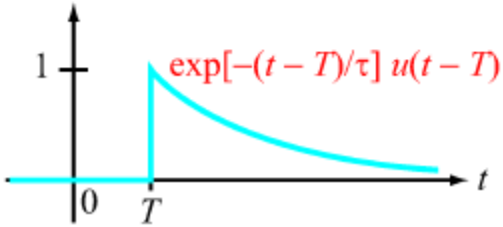


Table 5-1 Common Nonperiodic Functions

Table 5-1: Common nonperiodic waveforms.

Waveform	Expression	General Shape
Step	$u(t-T) = \begin{cases} 0 & \text{for } t < T \\ 1 & \text{for } t > T \end{cases}$	
Ramp	$r(t-T) = (t-T)\,u(t-T)$	
Rectangle	$\text{rect}\left(\frac{t-T}{\tau}\right) = u(t-T_1) - u(t-T_2)$ $T_1 = T - \frac{\tau}{2}; \quad T_2 = T + \frac{\tau}{2}$	
Exponential	$\exp[-(t-T)/\tau]\,u(t-T)$	

Energy Storage Elements

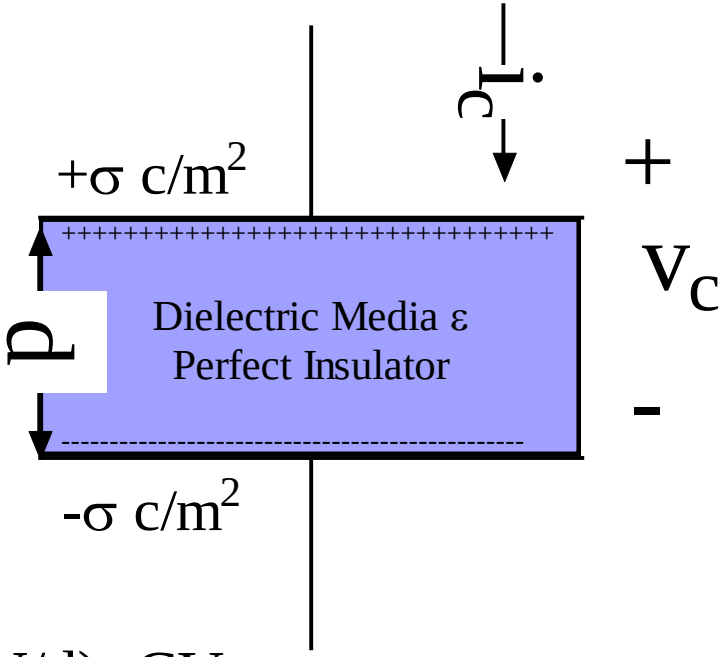
- Capacitors and Inductors Store Electromagnetic Energy
  - Useful in Power Conditioning (A.C. to D.C. Conversion)
  - Useful in Filtering Circuits
- Takes Time To Change Energy Storage (unless you have infinite *power* available).
- The Capacitors and Inductors in a Circuit Are Strong Factors in Determining Frequency Response and Speed of All Real Circuits
- Parasitic (unavoidable but present) Elements in Transistor Circuits (a little in EECS 215, more in EECS 311, 312, model building in EECS 320)
- Delays on Real Wires (Transmission Lines, EECS 230, 330).

Physics

- Detailed E&M from Maxwell's Equations
  - Phys 240
  - EECS 230
- Here We Will Use the Resulting Circuit Laws from This Physics

Ideal Capacitor Basics

$$i_c = C\frac{\partial V_c}{\partial t}$$



Charge on each plate Q=Aσ= Aε*E*=(AεV/d)=CV

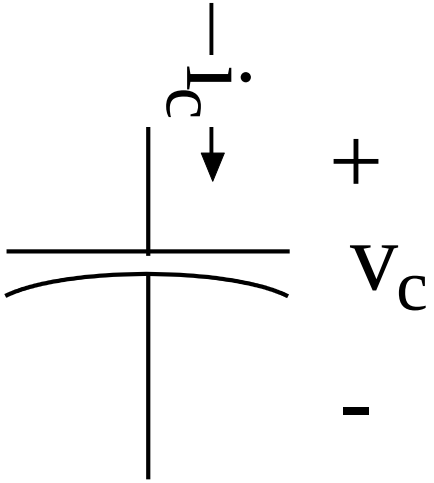
Equal and opposite total charges on each plate

Total Device is Neutral

Device storage energy in E-field

Ideal Capacitor Basics

$$i_c = C\frac{\partial V_c}{\partial t}$$



C in Farads 1F=1 C/V

V_c must be continuous function of time t. Otherwise i_c is infinite (& power is infinite).

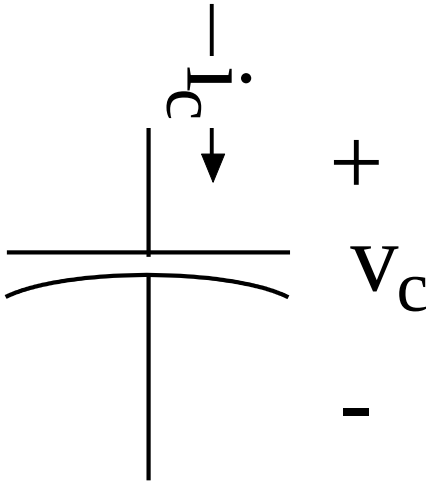
Voltage across a capacitor cannot change instantly !

ONLY exception: Idealized problem with a PERFECT V-source connected directly across a capacitor

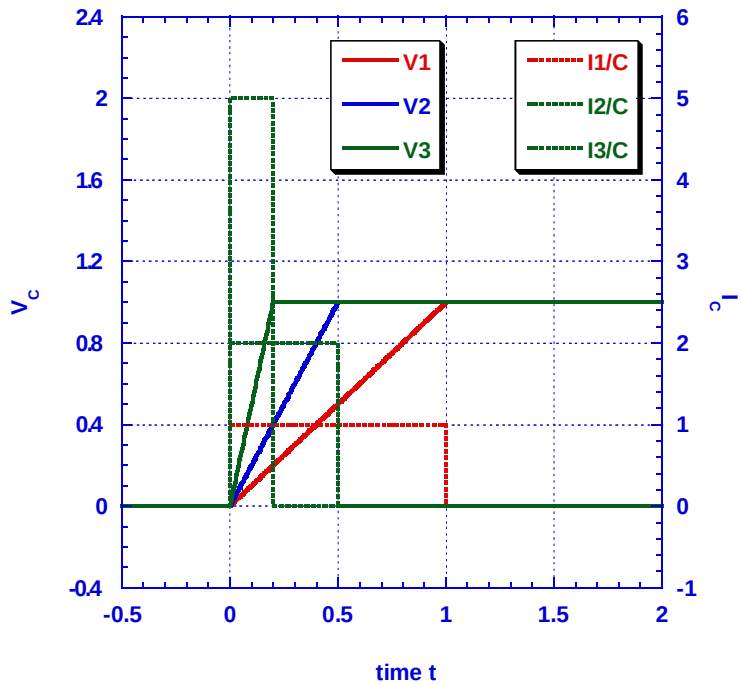
At DC, i_c=0→Capacitor looks like OPEN Circuit

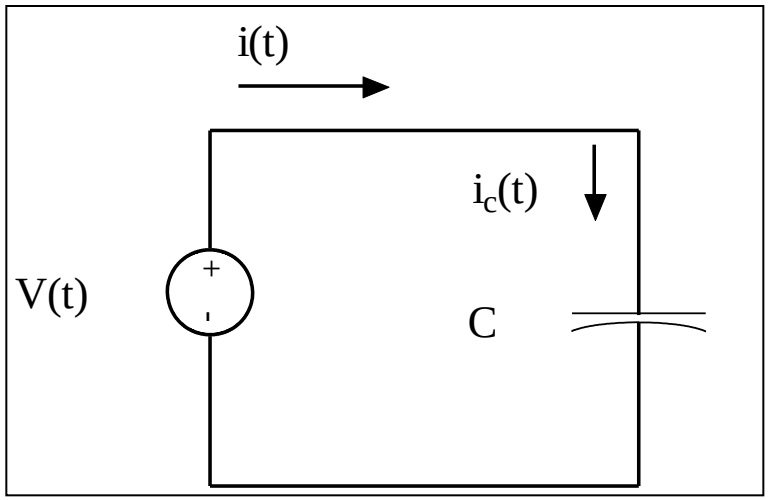
Ideal Capacitor Basics

$$i_c = C\frac{\partial V_c}{\partial t}$$



Voltage Ramp Into Capacitor





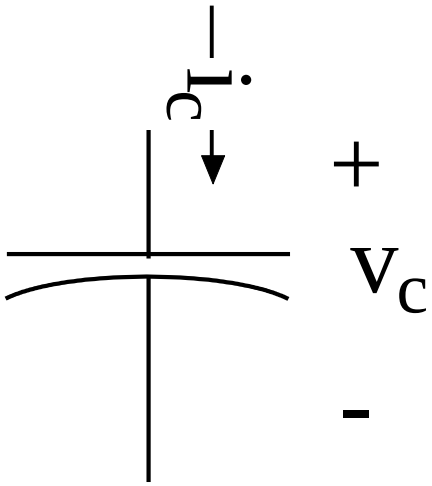
Limit: As Ramp Rate $\rightarrow\infty$ $i_c\rightarrow\infty$ at t=0 only (unit impulse of charge)

Infinite Instantaneous Power, but Finite Total Energy

Stored Energy in Capacitor

Work w to charge a capacitor

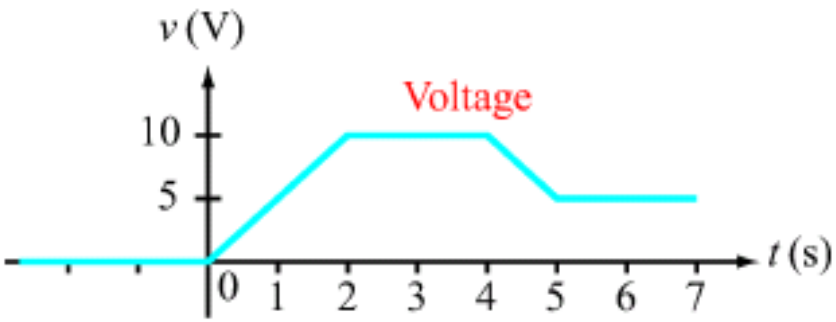
$$
\begin{aligned}
p(t) &= i_c(t)V_c(t) \\
&= C\left(\frac{\partial V_c}{\partial t}\right)V_c \\
w &= \int_0^t C\left(\frac{\partial V_c}{\partial t}\right)V_c \partial t \\
&= \int_{V_c(0)}^{V_c(t)} CV_c \partial V_c \\
&= \frac{1}{2}C\left[\left(V_c(t)\right)^2 - \left(V_c(0)\right)^2\right]
\end{aligned}
$$

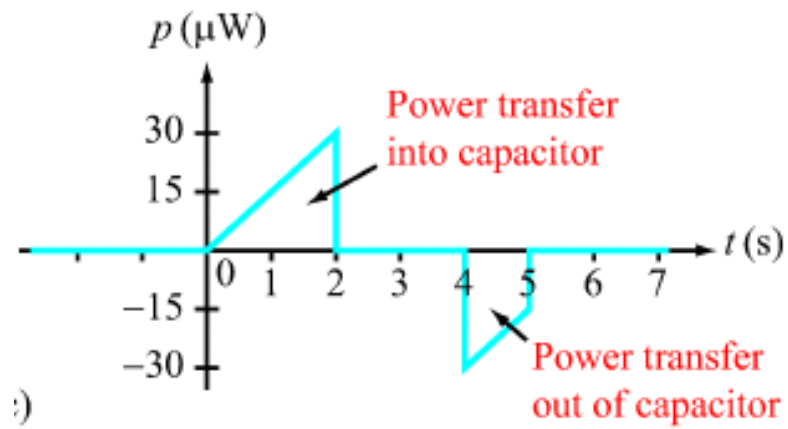


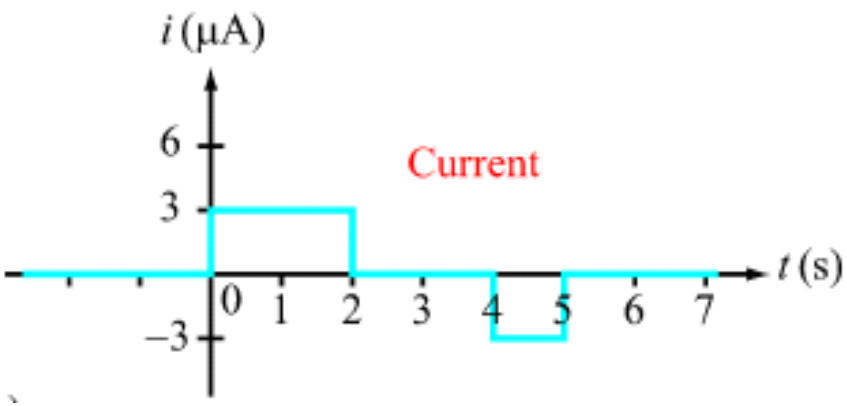
Total Stored Energy in Capacitor

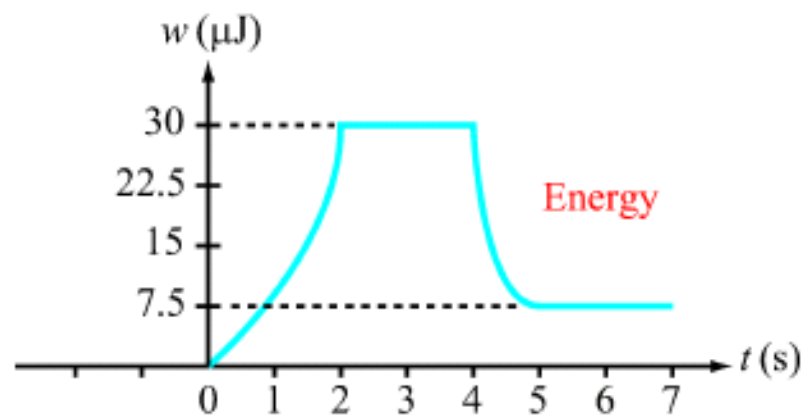
$$E_c = \frac{1}{2}CV_c^2$$

Capacitor Response



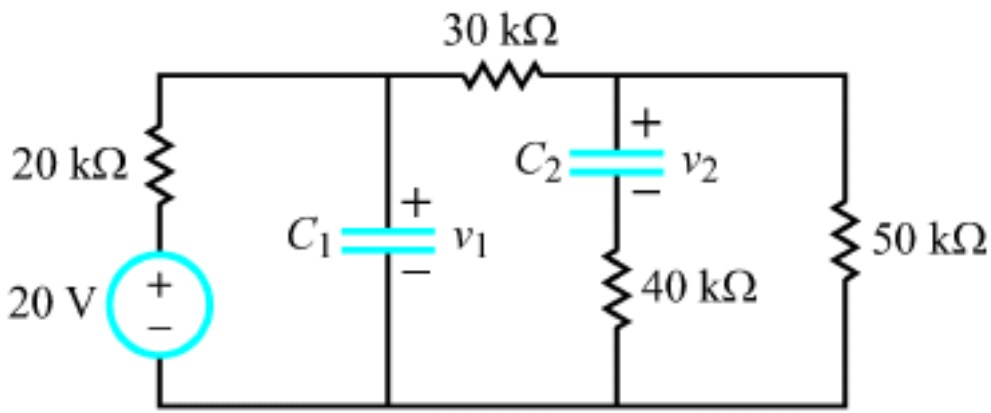


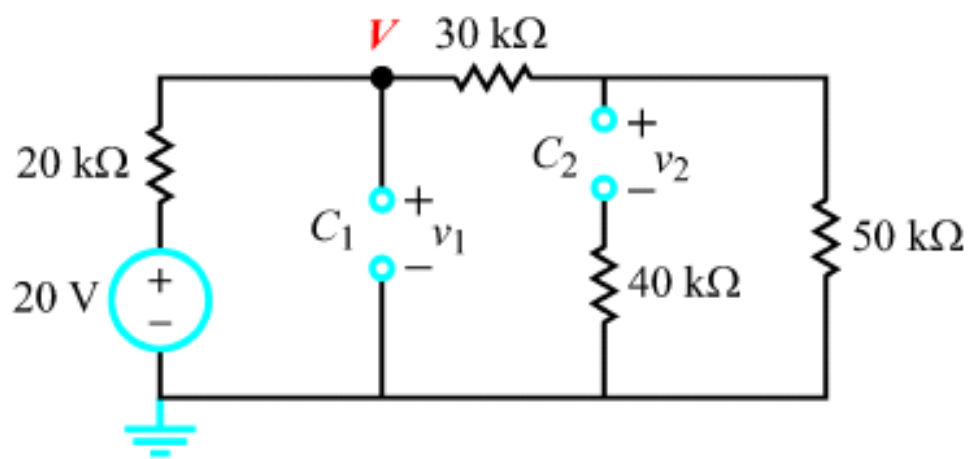




RC Circuits at DC

- No current flow through capacitors: open circuit





Parallel Capacitors

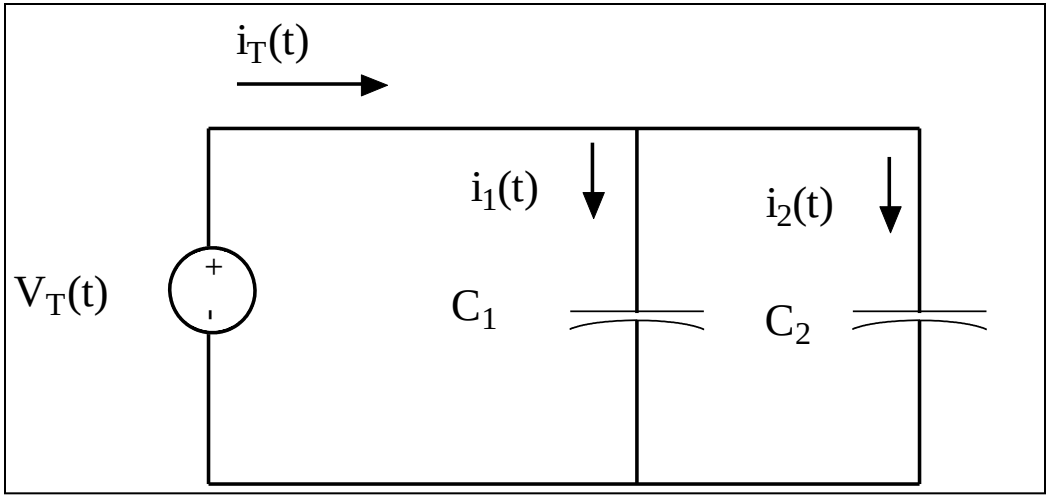
Parallel Capacitors Add

$$i_T = i_1 + i_2$$

$$= C_1 \frac{\partial V_{c1}}{\partial t} + C_2 \frac{\partial V_{c2}}{\partial t}$$

$$= \left(C_1 + C_2\right) \frac{\partial V_T}{\partial t}$$

$$= C_{eff} \frac{\partial V_T}{\partial t}$$



$$C_{eff} = C_1 + C_2$$

$$N - Caps - in - parallel:$$

$$C_{eff} = \sum_{i=1}^{N} C_i$$

Series Capacitors

Series Capacitors Add Reciprocally

$$V_T = V_{c1} + V_{c2}$$

$$i_T(t) = C_1 \frac{\partial V_{c1}}{\partial t} = C_2 \frac{\partial V_{c2}}{\partial t}$$

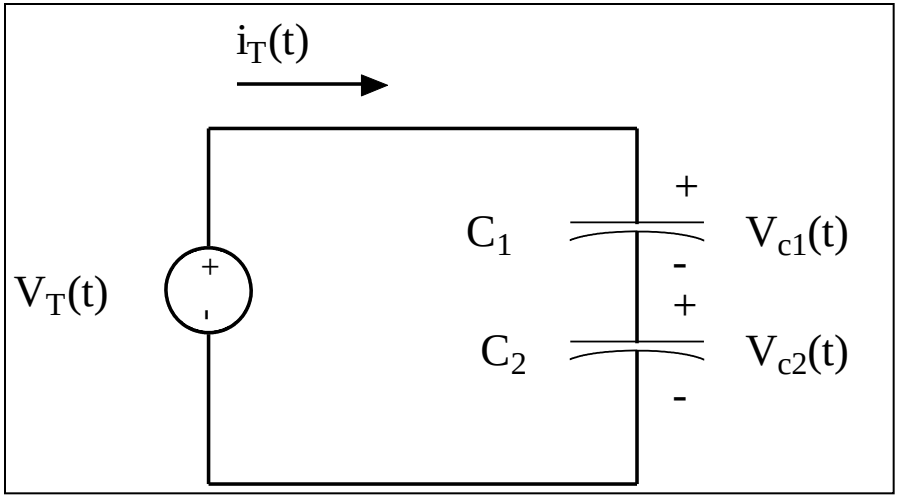
$$\frac{\partial V_T}{\partial t} = \frac{\partial V_{c1}}{\partial t} + \frac{\partial V_{c2}}{\partial t}$$

$$= \frac{i_T}{C_1} + \frac{i_T}{C_2}$$

$$= \left[\frac{1}{C_1} + \frac{1}{C_2} \right] i_T$$

$$i_T(t) = C_{eff} \frac{\partial V_T}{\partial t}$$

$$= \left[\frac{1}{C_1} + \frac{1}{C_2} \right]^{-1} \left(\frac{\partial V_T}{\partial t} \right)$$



$$C_{eff} = \left[C_1^{-1} + C_2^{-1} \right]^{-1}$$

$$N - Caps - in - series:$$

$$C_{eff} = \left[\sum_{i=1}^{N} C_i^{-1} \right]^{-1}$$

Series Capacitors: Capacitive V-Divider

$$V_T = V_{c1} + V_{c2}$$

$$Q_1 = Q_2$$

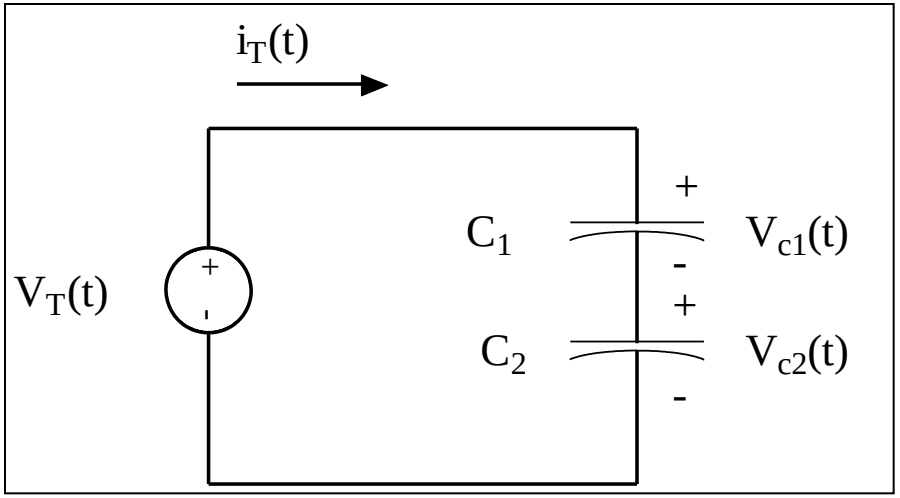
$$C_1 V_{c1} = C_2 V_{c2}$$

$$V_{c2} = \left(\frac{C_1}{C_2}\right) V_{c1}$$

$$V_T = V_{c1}\left(1 + \frac{C_1}{C_2}\right)$$

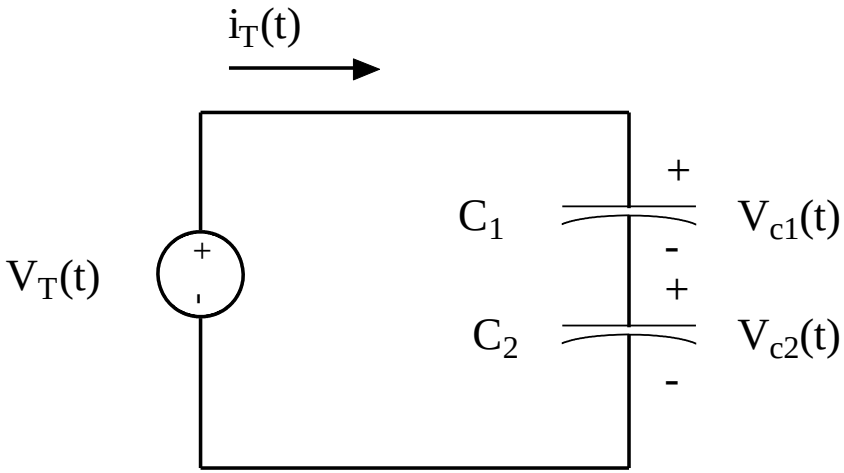
$$V_{c1} = \frac{V_T}{1 + \frac{C_1}{C_2}} = \left[\frac{C_2}{C_1 + C_2}\right] V_T$$

$$V_{c2} = \left[\frac{C_1}{C_1 + C_2}\right] V_T$$

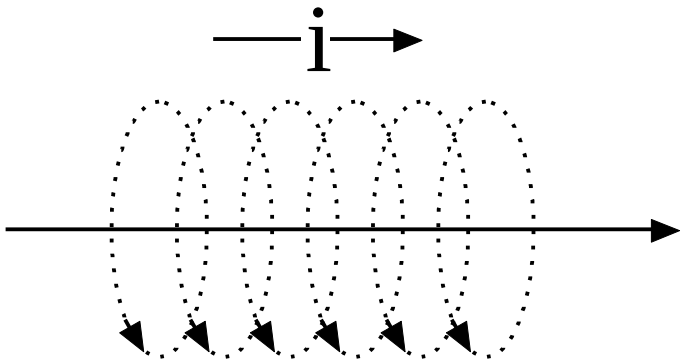


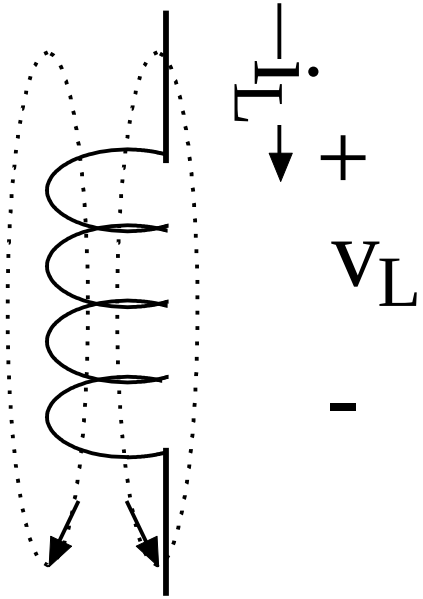
What would you get for 3 Cap's in Series?

Series Capacitors: Capacitive V-Divider



Ideal Inductor Basics





$$i = \oint \vec{H} \bullet \partial \vec{l}$$

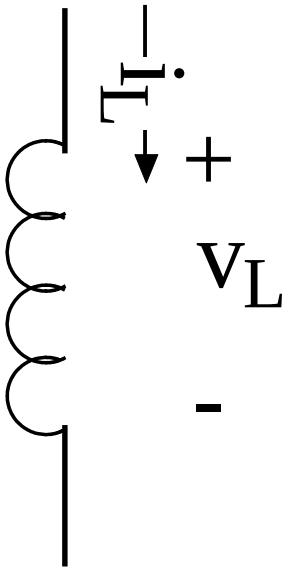
$$= \int_0^{2\pi} \vec{H}(r) r \partial \theta$$

Magnetic Energy Storage tied to CURRENT

Changing H-field requires Changing I

Ideal Inductor Basics

$$V_L = L\frac{\partial i_L}{\partial t}$$



L in Henries 1H= 1V-s/A

i_L must be continuous function of time t. Otherwise V_L is infinite (& power is infinite).

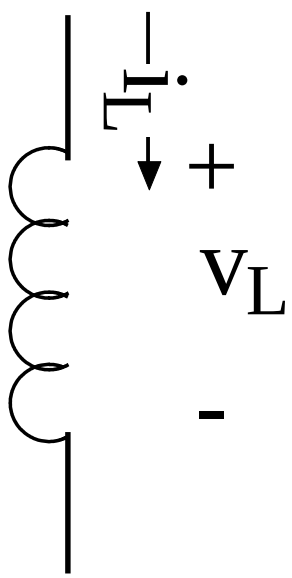
Current through an inductor cannot change instantly!

ONLY exception: Idealized problem with a PERFECT I-source connected directly to one node of an inductor

At DC, V_L=0→Inductor looks like Short Circuit

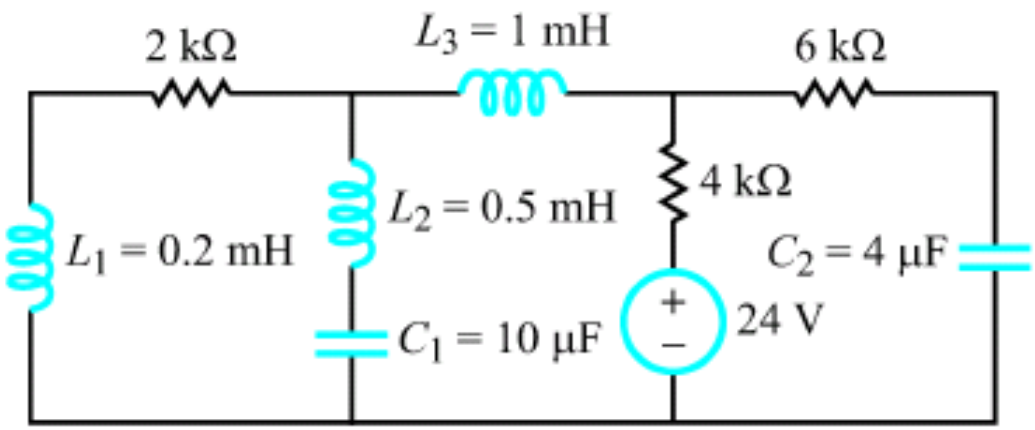
Ideal Inductor Basics

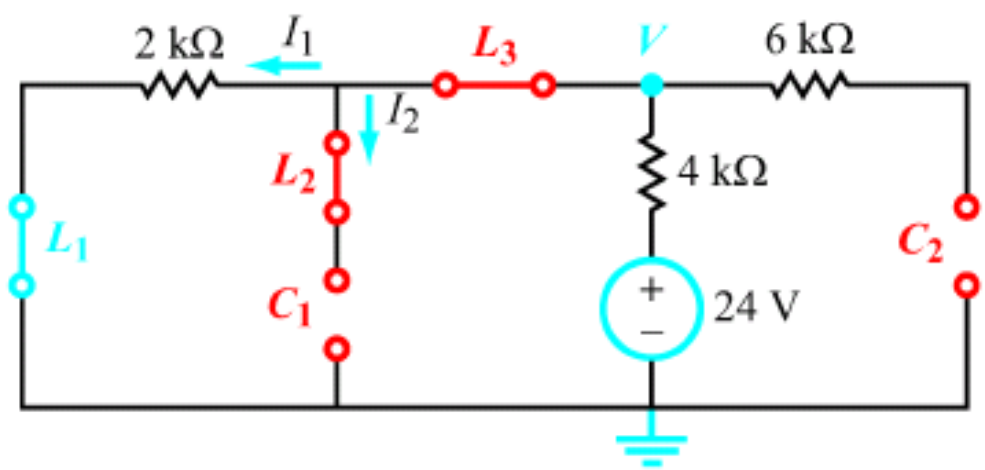
$$V_L = L\frac{\partial i_L}{\partial t}$$



RL Circuits at DC

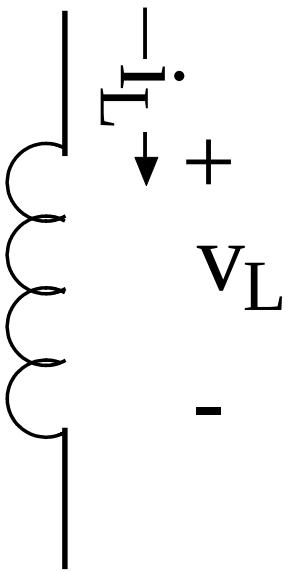
- No voltage across inductors: short circuit





Stored Energy in Inductor

Work w to energize an inductor



$$
\begin{aligned}
p(t) &= i_L(t)V_L(t) \\
&= L\left(\frac{\partial i_L}{\partial t}\right)i_L \\
w &= \int_0^t L\left(\frac{\partial i_L}{\partial t}\right)i_L \partial t \\
&= \int_{i_L(0)}^{i_L(t)} L i_L \partial i_L \\
&= \frac{1}{2}L\left[\left(i_L(t)\right)^2 - \left(i_L(0)\right)^2\right]
\end{aligned}
$$

Total Stored Energy in Inductor

$$E_L = \frac{1}{2}Li_L^2$$

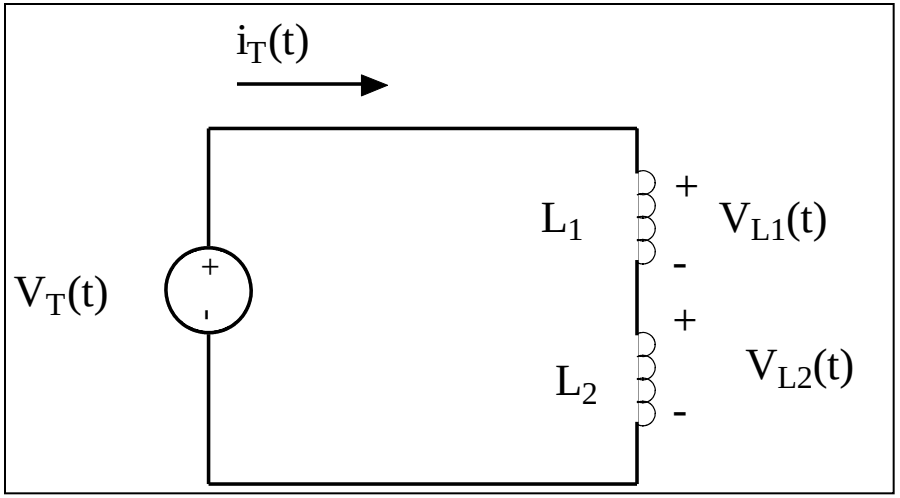
Series Inductors

Series Inductors Add

$$V_T = V_{L1} + V_{L2}$$

$$= L_1 \frac{\partial i_{L1}}{\partial t} + L_2 \frac{\partial i_{L2}}{\partial t}$$

$$= \left(L_1 + L_2\right) \frac{\partial i_T}{\partial t}$$



$$Leff = \left(L_1 + L_2\right)$$

$$N - Inductors - in - series:$$

$$L_{eff} = \left[\sum_{i=1}^{N} L_i\right]$$

Parallel Inductors

Parallel Inductors Add Reciprocally

$$i_T = i_1 + i_2$$

$$\frac{\partial i_T}{\partial t} = \frac{\partial i_1}{\partial t} + \frac{\partial i_2}{\partial t}$$

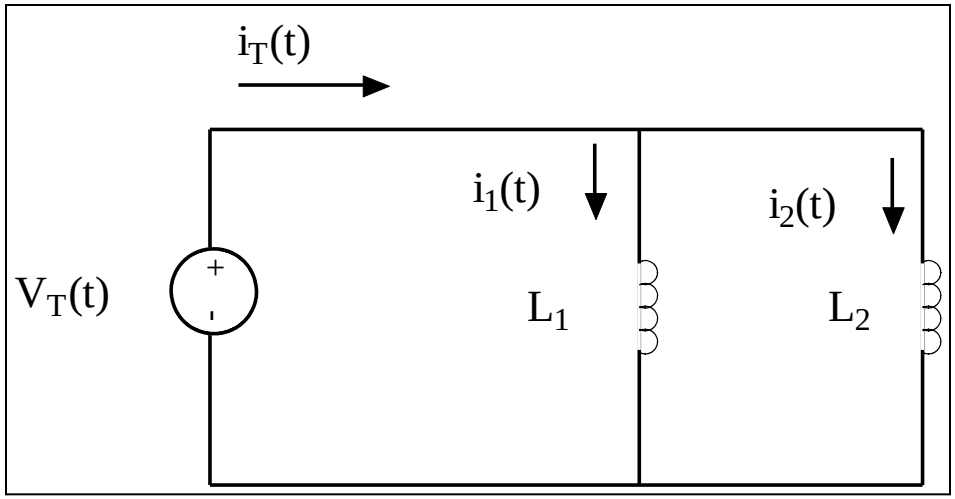
$$V_T = L_1 \frac{\partial i_1}{\partial t} = L_2 \frac{\partial i_2}{\partial t}$$

$$\frac{\partial i_1}{\partial t} = \frac{V_T}{L_1}$$

$$\frac{\partial i_2}{\partial t} = \frac{V_T}{L_2}$$

$$\frac{\partial i_T}{\partial t} = \frac{\partial i_1}{\partial t} + \frac{\partial i_2}{\partial t} = \left[\frac{1}{L_1} + \frac{1}{L_2}\right] V_T$$

$$V_T = \left[\frac{1}{L_1} + \frac{1}{L_2}\right]^{-1} \left(\frac{\partial i_T}{\partial t}\right)$$



$$L_{eff} = \left[\frac{1}{L_1} + \frac{1}{L_2}\right]^{-1}$$

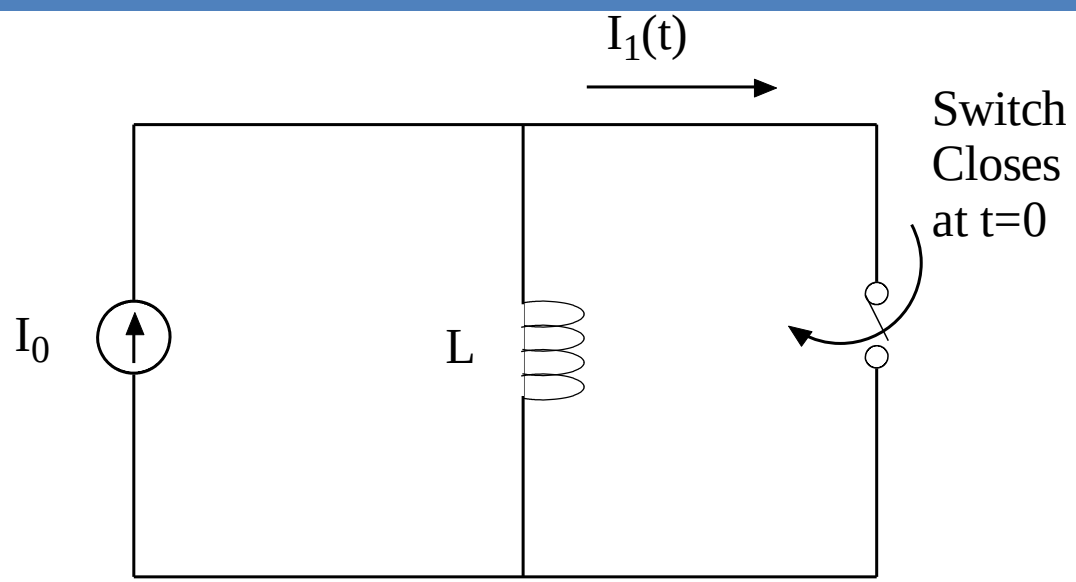
$$N - Inductors - in - parallel:$$

$$L_{eff} = \left[\sum_{i=1}^{N} L_i^{-1}\right]^{-1}$$

Example

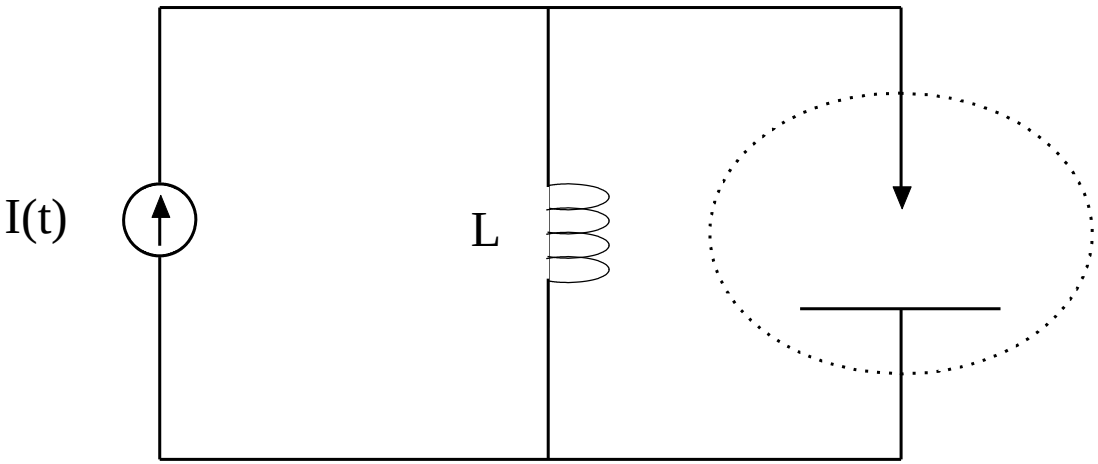
Suppose I_0 is constant and non-zero.

What is $I_1(t)$ for t>0?

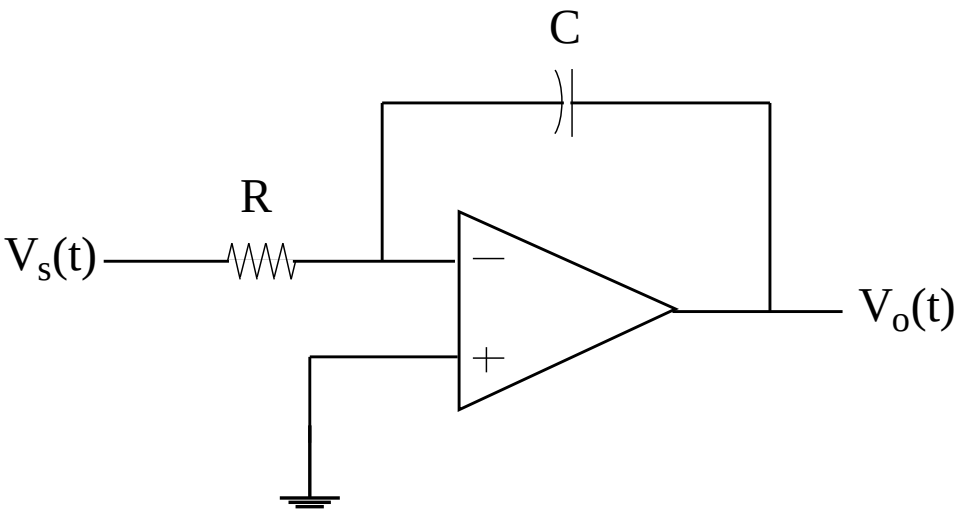
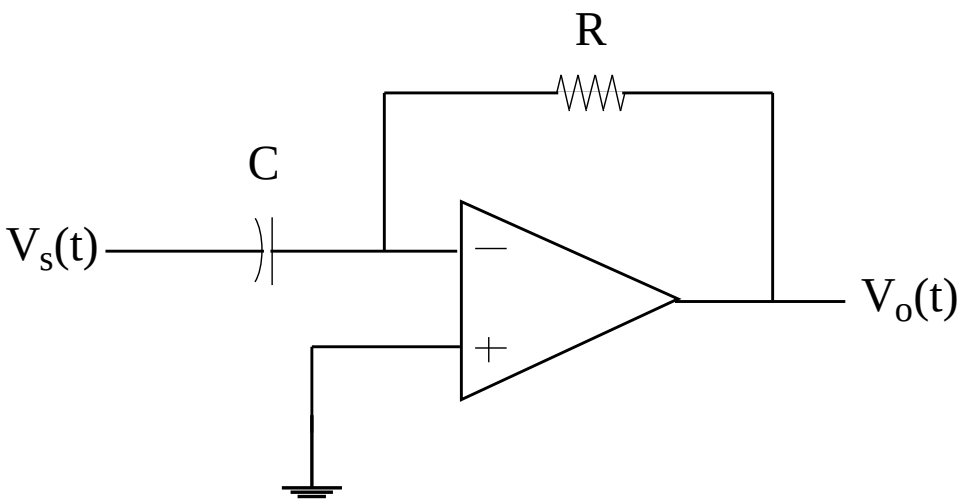


Example: Spark Gap

Suppose I(t) = constant t<0 but ramps to 0 for t>0 at rate B A/s



Example: Integrator



Example: Differentiator