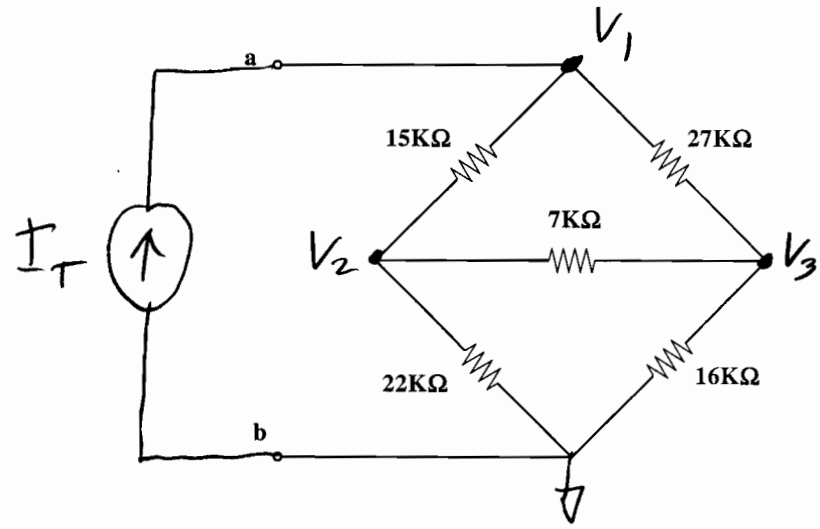
Problem Set 4 Solutions

1.a)



The point of (a) & (b) both is that we can solve any passive network problem like these with a test source approach. I will use a current source & node analysis.

$$I_T = \frac{V_1 - V_2}{15k\Omega} + \frac{V_1 - V_3}{27k\Omega}$$

$$0 = \frac{V_2 - V_1}{15k\Omega} + \frac{V_2 - V_3}{7k\Omega} + \frac{V_2}{22k\Omega}$$

$$0 = \frac{V_3 - V_1}{27k\Omega} + \frac{V_3 - V_2}{7k\Omega} + \frac{V_3}{16k\Omega}$$

Solving I get

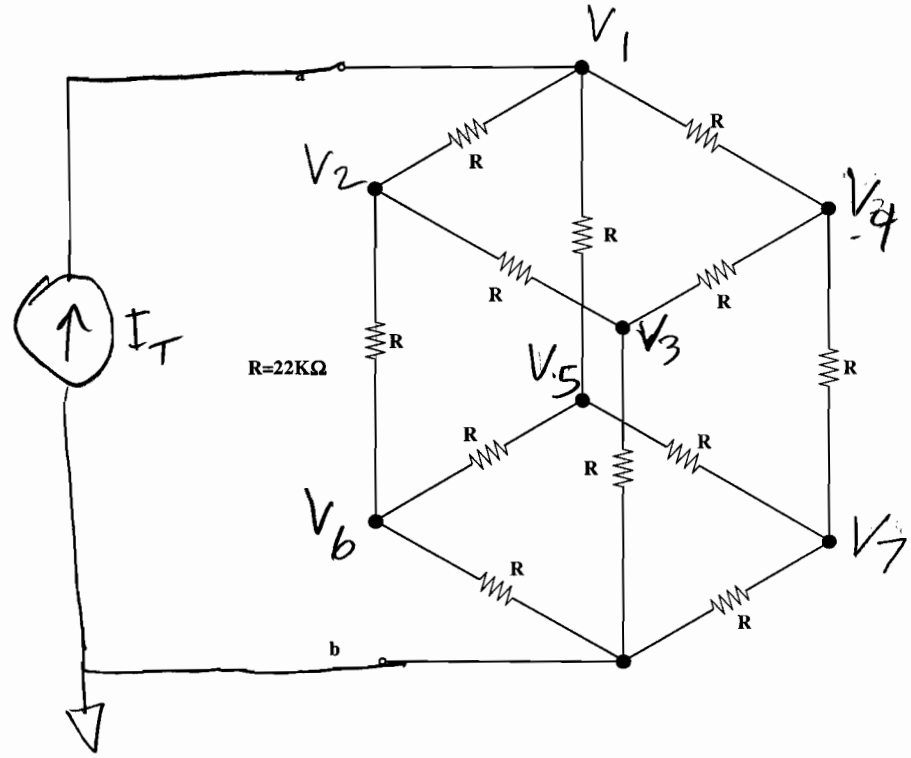
$$V_1 = \frac{10{,}327{,}750}{539} I_T$$

$$\Rightarrow R_{eq} = \frac{V_1}{I_T} = \frac{10{,}327{,}750}{539}\,\Omega \simeq 19.1609\,k\Omega$$

(Sang hoon's mesh solution follows)

1b)

T-2



KCL's

$$I_T = \frac{V_1 - V_2}{R} + \frac{V_1 - V_4}{R} + \frac{V_1 - V_5}{R} \quad (1)$$

$$0 = \frac{V_2 - V_1}{R} + \frac{V_2 - V_3}{R} + \frac{V_2 - V_6}{R} \quad (2)$$

$$0 = \frac{V_3 - V_2}{R} + \frac{V_3}{R} + \frac{V_3 - V_4}{R} \quad (3)$$

$$0 = \frac{V_4 - V_1}{R} + \frac{V_4 - V_3}{R} + \frac{V_4 - V_7}{R} \quad (4)$$

$$0 = \frac{V_5 - V_1}{R} + \frac{V_5 - V_6}{R} + \frac{V_5 - V_7}{R} \quad (5)$$

$$0 = \frac{V_6}{R} + \frac{V_6 - V_2}{R} + \frac{V_6 - V_5}{R} \quad (6)$$

$$0 = \frac{V_7}{R} + \frac{V_7 - V_4}{R} + \frac{V_7 - V_5}{R} \quad (7)$$

Solving yields

$$V_1 = \left(\frac{5}{6}R\right) I_T$$

$$V_2 = V_4 = V_5 = \left(\frac{1}{2}R\right) I_T$$

$$V_3 = V_6 = V_7 = \left(\frac{1}{3}R\right) I_T$$

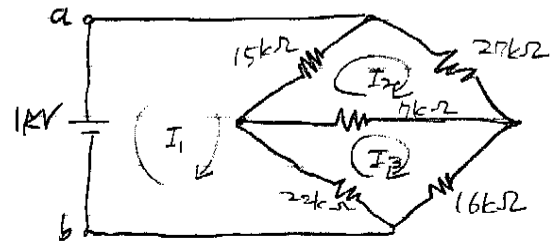
$$R_{eq} = \frac{V_1}{I_T} = \frac{5}{6}R = \frac{55}{3}\,k\Omega$$

$$\simeq 18\tfrac{1}{3}\,k\Omega$$

A clever symmetry argument solves this quickly (Sanghoon's soln follows)

Problem 1.

a. using external source method, applied 1kV voltage source



mesh1 : $-10^3 + 15\times10^3 \times (I_1 - I_2) + 22\times10^3 \times (I_1 - I_3) = 0$

mesh2 : $15\times10^3 \times (I_2 - I_1) + 27\times10^3 \times (I_2) + 7\times10^3 \times (I_2 - I_3) = 0$

mesh3 : $7\times10^3 \times (I_3 - I_2) + 22\times10^3 \times (I_3 - I_1) + 16\times10^3 \times I_3 = 0$

$$\Rightarrow \begin{cases} 15(I_1 - I_2) + 22(I_1 - I_3) = 1 \\ 15(I_2 - I_1) + 27I_2 + 7(I_2 - I_3) = 0 \\ 7(I_3 - I_2) + 22(I_3 - I_1) + 16I_3 = 0 \end{cases}$$

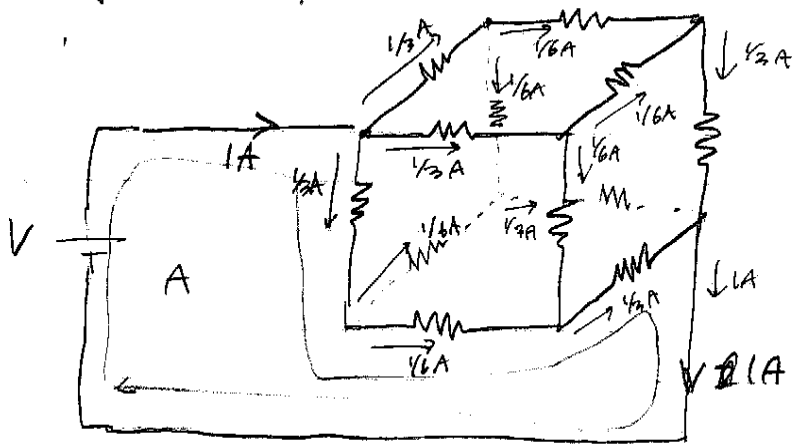
$$\Rightarrow \begin{cases} 37I_1 - 15I_2 - 22I_3 = 1 \\ -15I_1 + 49I_2 - 7I_3 = 0 \\ -22I_1 - 7I_2 + 45I_3 = 0 \end{cases} \Rightarrow \begin{bmatrix} 37 & -15 & -22 \\ -15 & 49 & -7 \\ -22 & -7 & 45 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 37 & -15 & -22 \\ -15 & 49 & -7 \\ -22 & -7 & 45 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.0522 \\ 0.0201 \\ 0.0286 \end{bmatrix}$$

$$R_{eq} = \frac{1\times10^3}{I_1} = 1.9161\times10^4\ \Omega$$

p. 1. (b)

using symmetry of the circuit



Let's assume we applied V voltage source to the circuit so that 1A current flows into the circuit.

The cube is symmetric in every node
So, the current is divided equally in each node

Then KVL for mesh A

$$\Rightarrow V = \tfrac{1}{3} \times R + \tfrac{1}{6} \times R + \tfrac{1}{3} \times R$$

$$= \tfrac{5}{6} R \quad \Rightarrow \quad V = \tfrac{5}{6} R$$

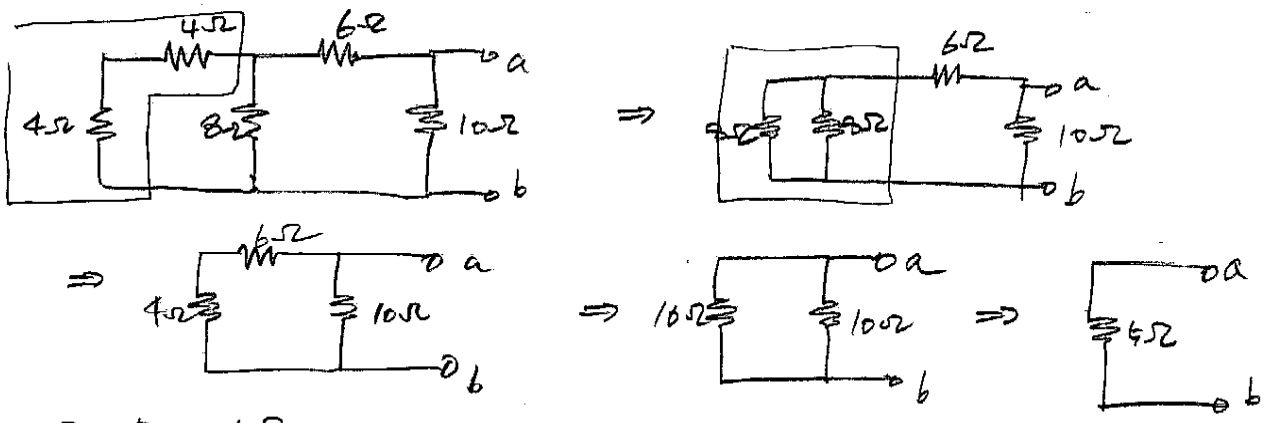
$$\Rightarrow R_{eq} = \frac{V}{I} = \frac{5/6\, R}{1} = \frac{5}{6} R = \boxed{\frac{5}{6} \times 22k\Omega}$$

<prob. 2> H&M 3.53.

(1) R_{TH} = equivalent resistance between the same terminals when all independent sources in the circuit have been deactivated

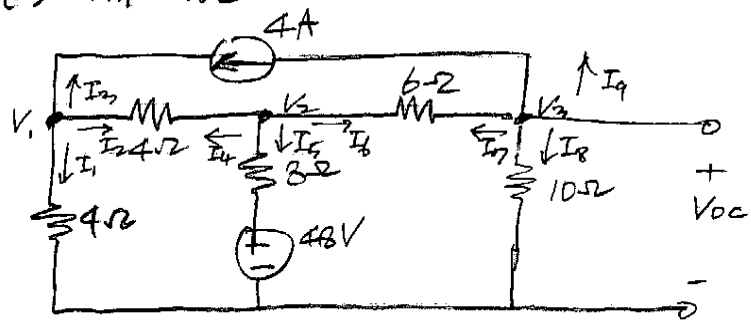
ⓐ current source → open circuit

ⓑ voltage source → short "



$\Rightarrow R_{TH} = 6\Omega$

(2) $V_{TH} = V_{oc}$



Nodal Analysis.

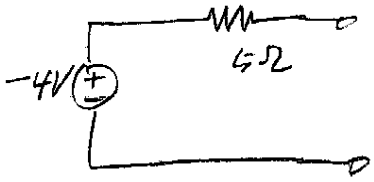
$$\begin{cases} I_1 + I_2 + I_3 = 0 \\ I_4 + I_5 + I_6 = 0 \\ I_7 + I_8 + I_9 = 0 \end{cases} \Rightarrow \begin{cases} \frac{V_1}{4} + \frac{V_1 - V_2}{4} - 4 = 0 \\ \frac{V_2 - V_1}{4} + \frac{V_2 - V_3}{6} + \frac{V_2 - 48}{8} = 0 \\ \frac{V_3 - V_2}{6} + \frac{V_3}{10} + 4 = 0 \end{cases}$$

$$\Rightarrow \begin{cases} 2V_1 - V_2 = 16 \\ -6V_1 + 13V_2 - 4V_3 = 144 \\ -5V_2 + 8V_3 = -120 \end{cases} \Rightarrow \begin{bmatrix} 2 & -1 & 0 \\ -6 & 13 & -4 \\ 0 & -5 & 8 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 16 \\ 144 \\ -120 \end{bmatrix}$$

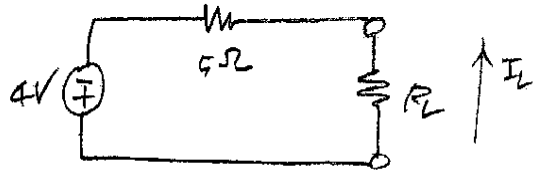
$$\Rightarrow \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 2 & -1 & 0 \\ -6 & 13 & -4 \\ 0 & -5 & 8 \end{bmatrix}^{-1} \begin{bmatrix} 16 \\ 144 \\ -120 \end{bmatrix} = \begin{bmatrix} 16.8 \\ 19.6 \\ -4 \end{bmatrix}$$

$\Rightarrow V_{oc} = V_3 = -4V$

Thévenin equivalent circuit



(b)

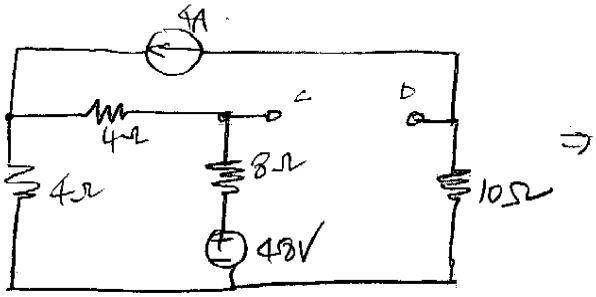


$$I_L = \frac{4V}{5 + R_L} = 0.5 \Rightarrow \underline{R_L = 3\Omega}$$

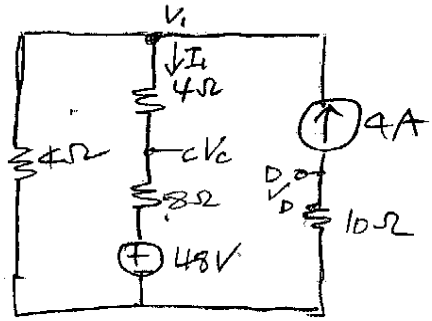
<problem 3> 3.54

6Ω resister is a load register

⇒ It should be teared off from the circuit to get V_{TH} and R_{TH}



⇒



Nodal Analysis $\quad \frac{V_1}{4} + \frac{V_1 - 48}{12} + (-4) = 0$

$\Rightarrow 3V_1 + V_1 - 48 = 48 \Rightarrow 4V_1 = 96 \Rightarrow V_2 = 24$

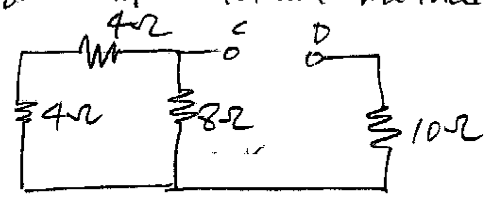
$\Rightarrow I_1 = (V_1 - 48)/12 = -2A$

$\Rightarrow V_c = 48 - 2\times 8 = 32V$

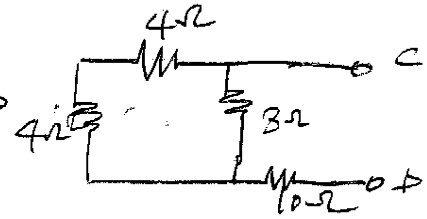
$\Rightarrow V_D = 0 - 4\times 10 = -40V$

$\Rightarrow V_{cD} = V_c - V_D = 32 - (-40) = 72V.$

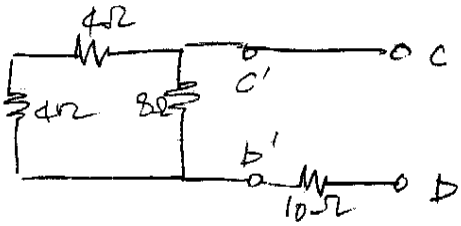
Equivalent resistance method



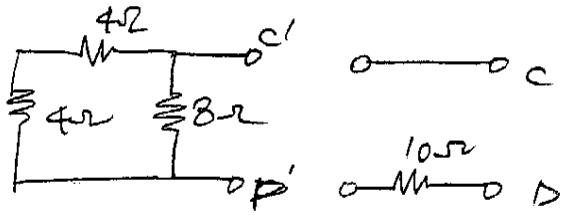
$\Rightarrow$



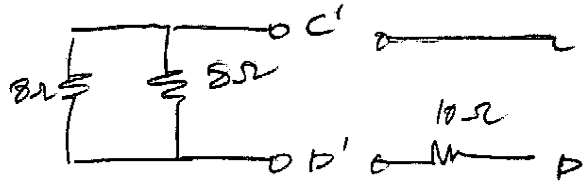
$\Rightarrow$



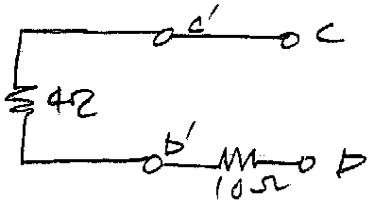
$\Rightarrow$



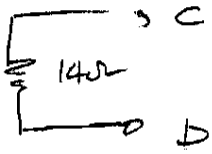
$\Rightarrow$



$\Rightarrow$

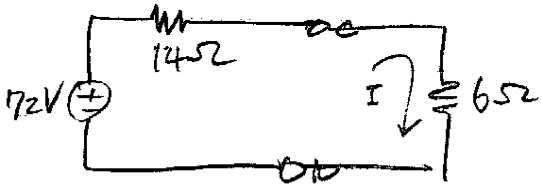


$\Rightarrow$



$\Rightarrow R_{TH} = 10\Omega$

$\Rightarrow$ Thevenin equivalent circuit.



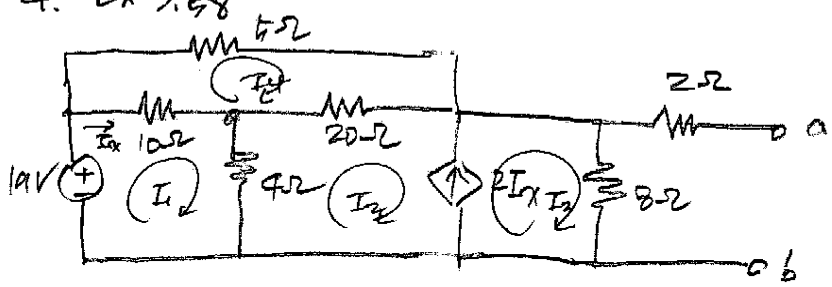
$$I = \frac{72}{14+6} = \frac{72}{20} = \underline{3.6A}$$

$\Rightarrow$ It's matching with the result from previous problem.

$$\Rightarrow \frac{V_2 - V_3}{6\Omega} = \frac{13.6-(-4)}{6}$$

$$= \underline{3.6A}$$

4. Ex 3.58



As we know 2Ω resistor is not effective unless node a and b are connected.

- Mesh 1: $-19V + 10\times(I_1 - I_4) + 4\times(I_1 - I_2) = 0$
- Supermesh: mesh 2 and mesh 3

  $4\times(I_2 - I_1) + 20\times(I_2 - I_4) + 8\times I_3 = 0$

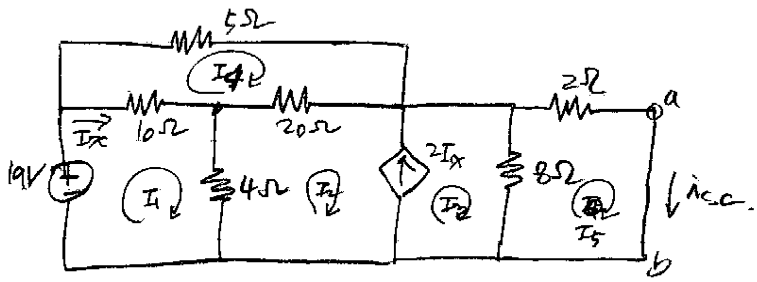
  $I_3 - I_2 = 2I_x$
- Mesh 4: $5\times I_4 + 10\times(I_4 - I_1) + 20\times(I_4 - I_2) = 0$
- $I_x = I_1 - I_4$

$$\Rightarrow 14I_1 - 4I_2 - 10I_4 = 19$$
$$-4I_1 + 24I_2 + 8I_3 - 20I_4 = 0$$
$$-10I_1 - 20I_2 + 35I_4 = 0$$
$$I_3 - I_2 = 2I_x = 2(I_1 - I_4) \Rightarrow 2I_1 + I_2 - I_3 - 2I_4 = 0$$

$$\Rightarrow \begin{bmatrix} 14 & -4 & 0 & -10 \\ -4 & 24 & 8 & -20 \\ -10 & -20 & 0 & 35 \\ 2 & 1 & -1 & -2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} 19 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} 14 & -4 & 0 & -10 \\ -4 & 24 & 8 & -20 \\ -10 & -20 & 0 & 35 \\ 2 & 1 & -1 & -2 \end{bmatrix}^{-1} \begin{bmatrix} 19 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1.5079 \\ -0.2262 \\ 2.1865 \\ 0.3016 \end{bmatrix}$$

$V_{oc} = V_{ab} = 8\times I_3 = 17.4921\,V$



M1: $-19 + 10\times(I_1 - I_4) + 4\times(I_1 - I_2) = 0$

M2-M3. $4\times(I_2 - I_1) + 20\times(I_2 - I_4) + 8\times(I_3 - I_5) = 0$. I_3

M4 $5\times I_4 + 10\times(I_4 - I_1) + 20\times(I_4 - I_2)$

M5 $8\times(I_5 - I_4) + 2\times I_5 = 0$.

etc: $2I_x = I_3 - I_2$, $I_x = I_1 - I_4$ $\Rightarrow$ $I_3 - I_2 = 2I_1 - 2I_4$

$\Rightarrow 2I_1 + I_2 - I_3 - 2I_4 = 0$

I_3

$$\Rightarrow \begin{aligned} 14I_1 - 4I_2 \qquad - 10I_4 &= 19 \\ -4I_1 + 24I_2 + 8I_3 - 20I_4 - 8I_5 &= 0 \\ -10I_1 - 20I_2 + \qquad 35I_4 \qquad &= 0 \\ -8I_4 + 10I_5 &= 0 \\ 2I_1 + I_2 - I_3 - 2I_4 \qquad &= 0 \end{aligned}$$

$$\Rightarrow \begin{bmatrix} 14 & -4 & 0 & -10 & 0 \\ -4 & 24 & 8 & -20 & -8 \\ -10 & -20 & 0 & 35 & 0 \\ 0 & 0 & \cancel{0}\,-8 & \cancel{-8}\,0 & 10 \\ 2 & 1 & -1 & -2 & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \end{bmatrix} = \begin{bmatrix} 19 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

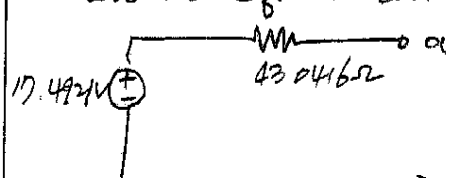
3.5874
2.1591
4.8164
2.2587
3.8531

$$\Rightarrow \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \end{bmatrix} = \begin{bmatrix} 14 & -4 & 0 & -10 & 0 \\ -4 & 24 & 8 & -20 & -8 \\ -10 & -20 & 0 & 35 & 0 \\ 0 & 0 & 0 & -8 & 10 \\ 2 & 1 & -1 & -2 & 0 \end{bmatrix}^{-1} \begin{bmatrix} 19 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \cancel{1.7273} \\ \cancel{0.0254} \\ \cancel{2.4039} \\ \cancel{0.5080} \\ \cancel{0.4064} \end{bmatrix}$$

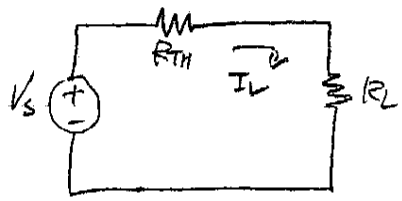
$I_{sc} = I_5 =$ ~~0.4064~~ 3.8531A $\Rightarrow R_{TH} = \frac{V_{oc}}{i_{sc}} = \frac{\cancel{17.4921}}{\cancel{0.4064}} = \cancel{43.0416}$

17.4921V/3.8531A=4.93568 ohms

Thevenin eq. circuit



pg. 68M 3.68



$$i_L = \frac{V_s}{R_{TH} + R_L}$$

① $3mA = \dfrac{V_s}{R_{TH} + 1.2k\Omega} \Rightarrow (3\times10^{-3}) \times (R_{TH} + 1.2\times10^{3}) = V_s$

② $2.5mA = \dfrac{V_s}{R_{TH} + 2k\Omega} \Rightarrow (2.5\times10^{-3}) \times (R_{TH} + 2\times10^{3}) = V_s$

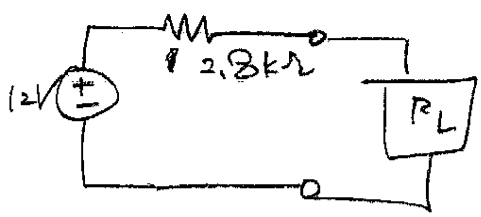
$\Rightarrow 3\times R_{TH}\times10^{-3} + 3.6 = V_s = 2.5\times R_{TH}\times10^{-3} + 5$

$\Rightarrow 0.5\times R_{TH}\times10^{-3} = 1.4$

$\Rightarrow R_{TH} = 2.8\times10^{3} = 2.8k\Omega$

$\Rightarrow V_s = 3mA \times (2.8k\Omega + 1.2k\Omega) = 3\times4 = 12V$

Thevenin equivalent circuit



$P_{RL} = V_{RL} \cdot I_{RL}$ is maximized when $R_{TH} = R_L = 2.8k\Omega$

$\Rightarrow R_L = 2.8k\Omega$ maximizes power consumption at R_L

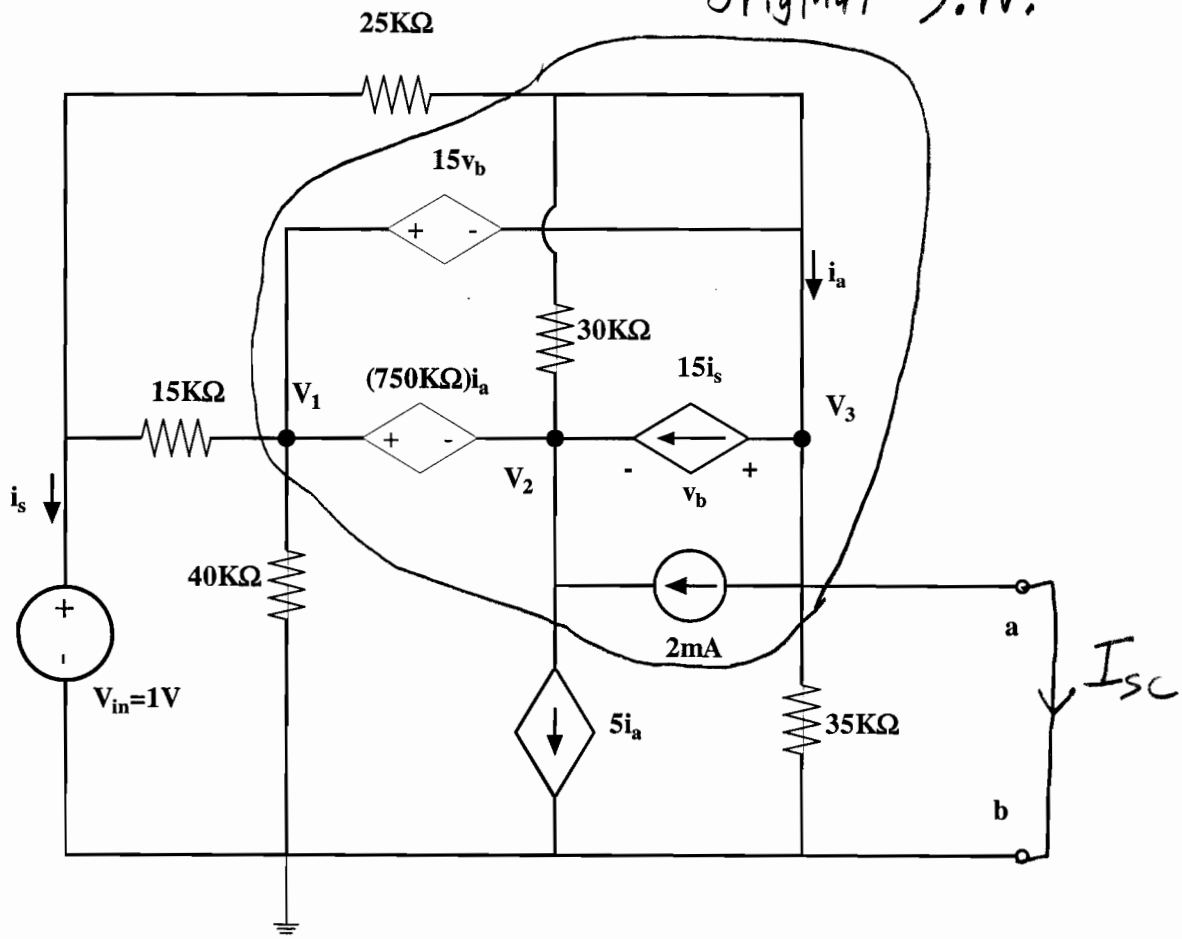
6. Solutions

a. Part a is the same as problem set 3 number 6. Please refer to this solution for details

$$V_{TH} = V_{OC} = -\frac{1{,}724{,}933}{10{,}117} \approx -170.498468$$

b. Short Circuit Current Calculation

"original" S.N.



There are many possible ways to do this, but the most straight-forward is to reuse & appropriately modify the work of P.S. 3 #6(a).

KVL 1 $(V_1 - V_2) = (750\,k\Omega)\,i_a$

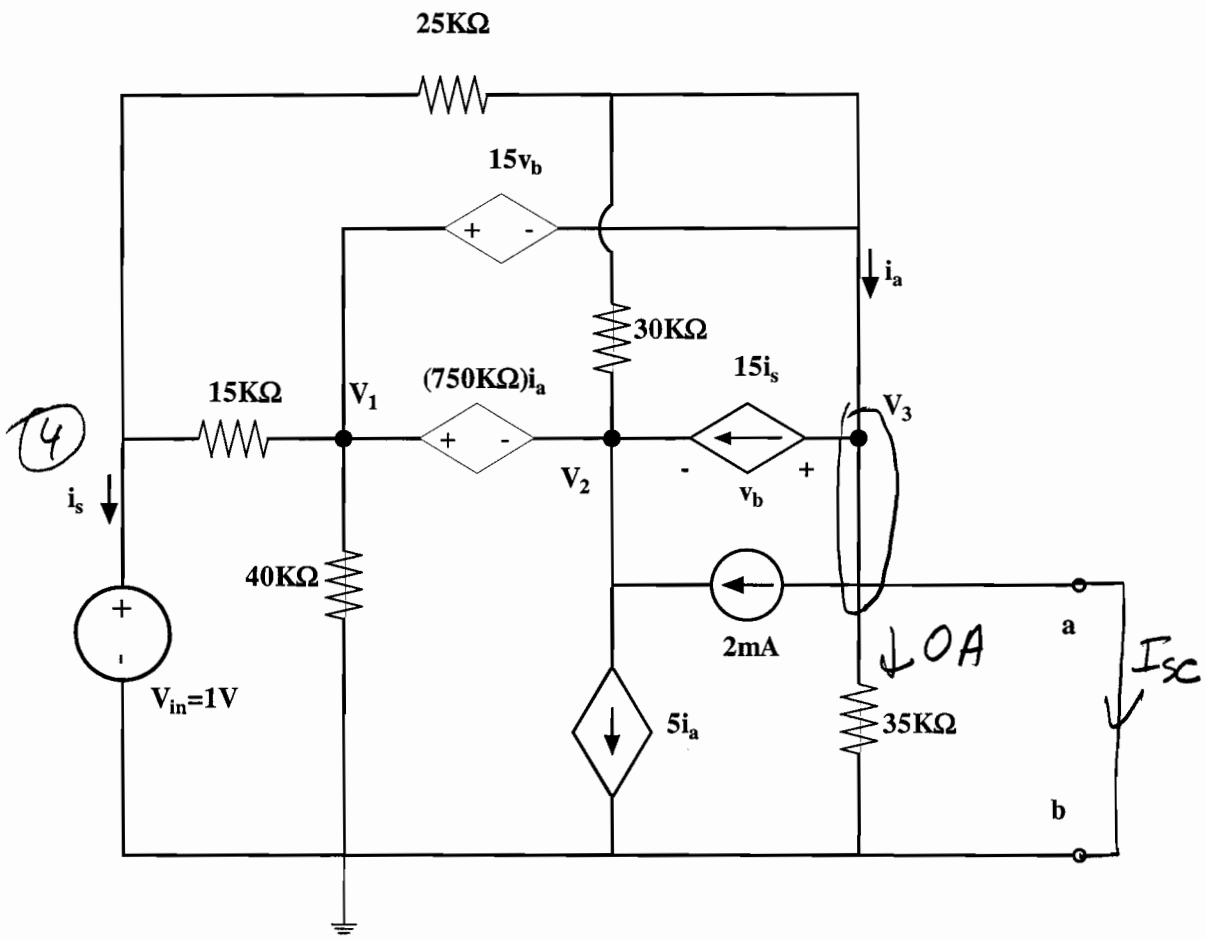
KVL 2 $(V_1 - V_3) = 15\,v_b$

Dep Source $v_b = V_3 - V_2$

All still apply

But now $V_3 = 0$ so these become

6-2



$v_b = -V_2$

$V_1 = 15 v_b = -15V_2$

$V_1 - V_2 = -16 V_2 = (750\,K\Omega) i_a$ (E1)

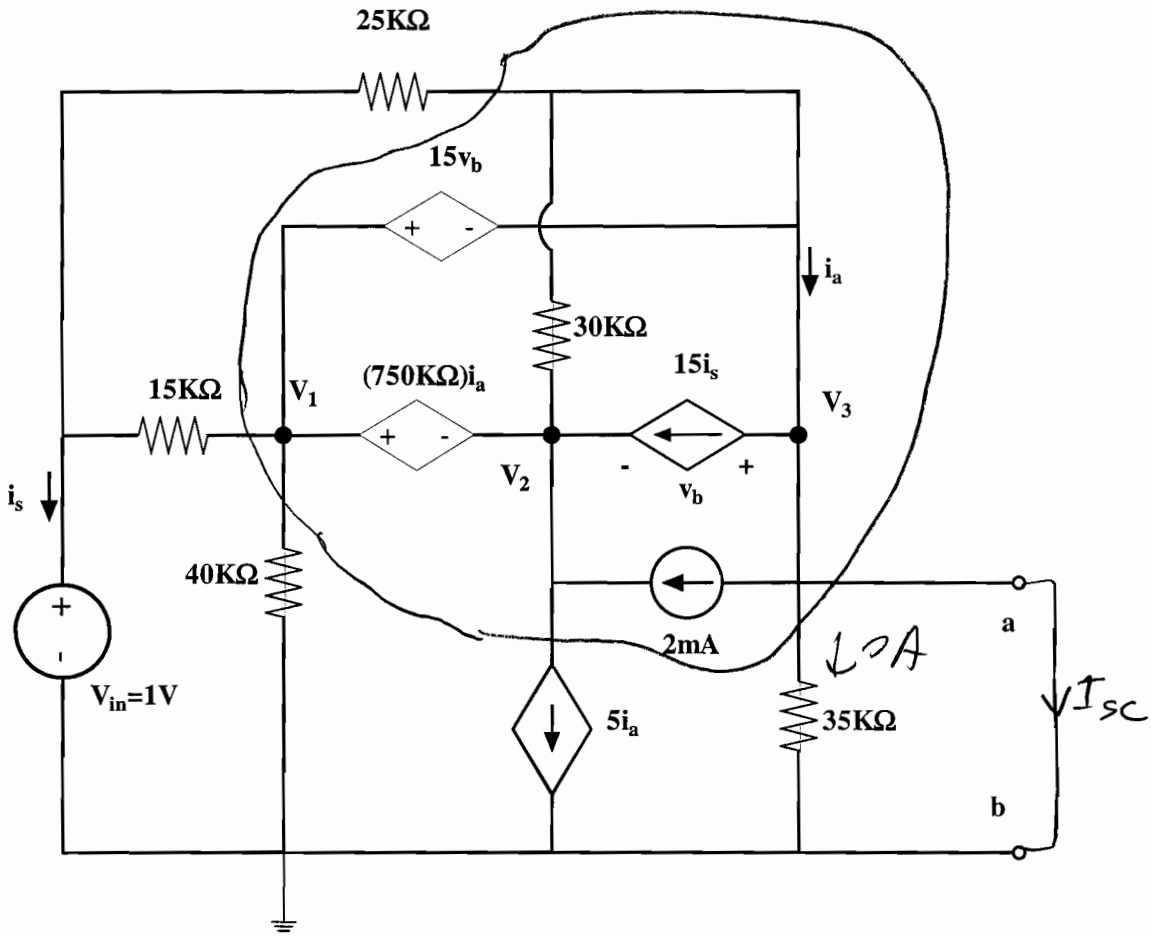
KCL for i_a on surface above

$i_a = 15 i_s + 2mA + I_{sc}$ ($i_{35\Omega} = 0$ due to short) (E2)

KCL for i_s at node 4

$$i_s = \frac{V_3 - V_{in}}{25K\Omega} + \frac{V_1 - V_{in}}{15\,K\Omega}$$

$$= -V_{in}\left[\frac{1}{25K\Omega} + \frac{1}{15K\Omega}\right] + (-V_2)\left(\frac{1}{1K\Omega}\right)$$ (E3)



Finally, KCL for the original S.N. Surface is now

$$\left(\frac{V_3 - V_{in}}{25K\Omega}\right) + \left(\frac{V_1 - V_{in}}{15K\Omega}\right) + \left(\frac{V_1}{40K\Omega}\right) + 5i_a + I_{sc} = 0$$

$$\Rightarrow -V_{in}\left[\frac{1}{25K\Omega} + \frac{1}{15K\Omega}\right] + (-15V_2)\left(\frac{1}{15K\Omega} + \frac{1}{40K\Omega}\right) + 5i_a + I_{sc} \quad \text{(E4)}$$

E1-E4 are 4 eqns in 4 unknowns (V_2, i_a, i_s, I_{sc})

Solving yields

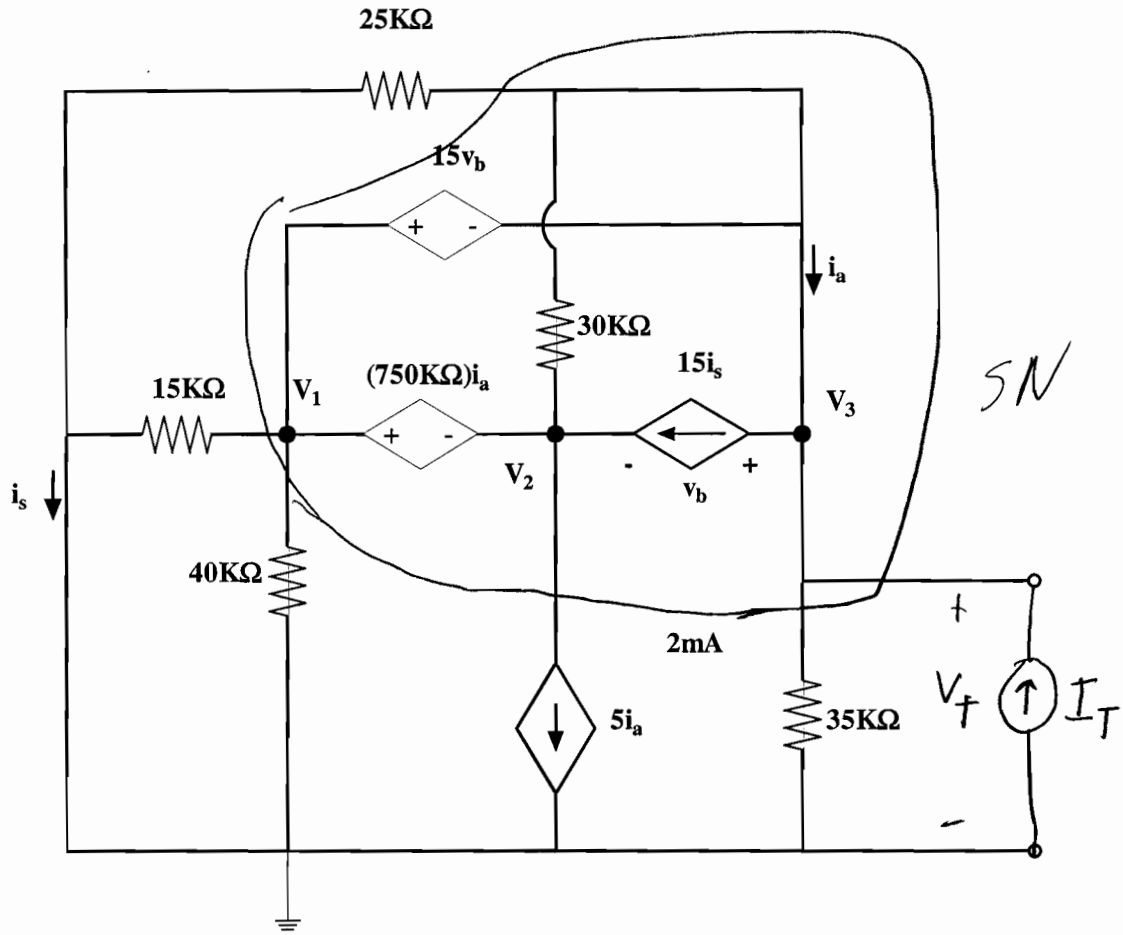
$$I_{sc} = I_N = \frac{333{,}375 - 86{,}956\, v_{in}}{1{,}518{,}412{,}500} \text{ A}$$

$$= +\frac{246{,}419}{1{,}518{,}412{,}500} \text{ A} \quad \text{when } v_{in} = 1$$

$$\approx +162.28725 \,\mu\text{A}$$

This matches the Multisim result shown later

c) Turn-off Ind Sources



Attach test source, find $V_3 = V_T$

$R_{eq} = V_3 / I_T$

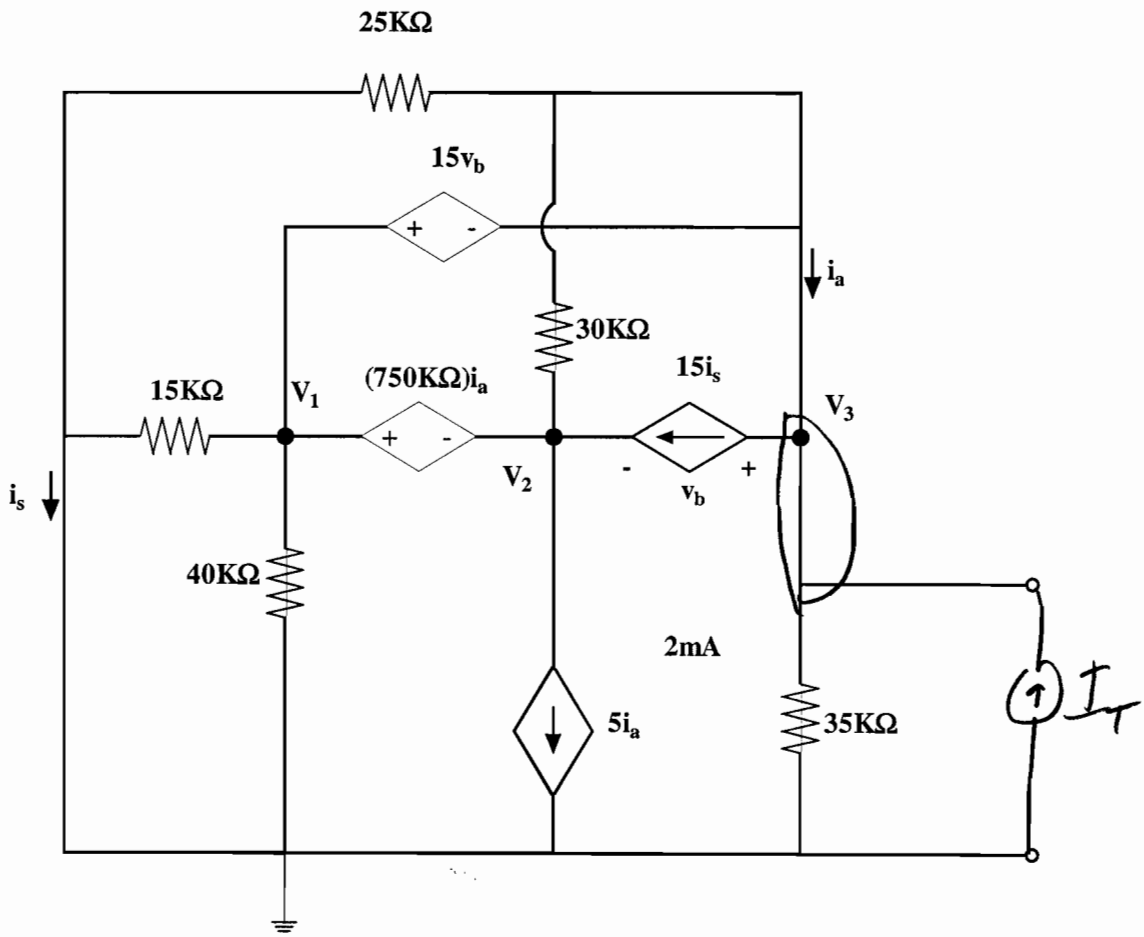
KVL 1: $V_1 - V_2 = (750\,k\Omega)\, i_a$ (E1)

KVL 2: $V_1 - V_3 = 15\, v_b$

Dep Sources: $v_b = V_3 - V_2$

$\Rightarrow V_1 - V_3 = 15(V_3 - V_2)$ (E2)

$$i_s = \frac{V_3}{25\,k\Omega} + \frac{V_1}{15\,k\Omega} \quad \text{(E3)}$$



$$i_a = 15 i_s + \frac{V_3}{35k\Omega} - I_T \quad (E4)$$

SN KCL

$$V_1\left[\frac{1}{15k\Omega} + \frac{1}{40k\Omega}\right] + 5 i_a + \frac{V_3}{35k\Omega} - I_T + \frac{V_3}{25k\Omega} = 0 \quad (E5)$$

5 equations, 5 unknowns (V_1, V_2, V_3, i_a, i_s)
Though it quickly reduces to 3/3 as (E3) gives i_s in terms of V_1, V_3 & plugging this into (E4) yields i_a in terms of V_1, V_2, V_3

Solving yields

$$V_3 = -\frac{10,628,887,500}{10,117} I_T$$

Thus $R_{eq} = -\frac{10,628,887,500}{10,117}\ \Omega$

$$\approx -1.0506\ M\Omega$$

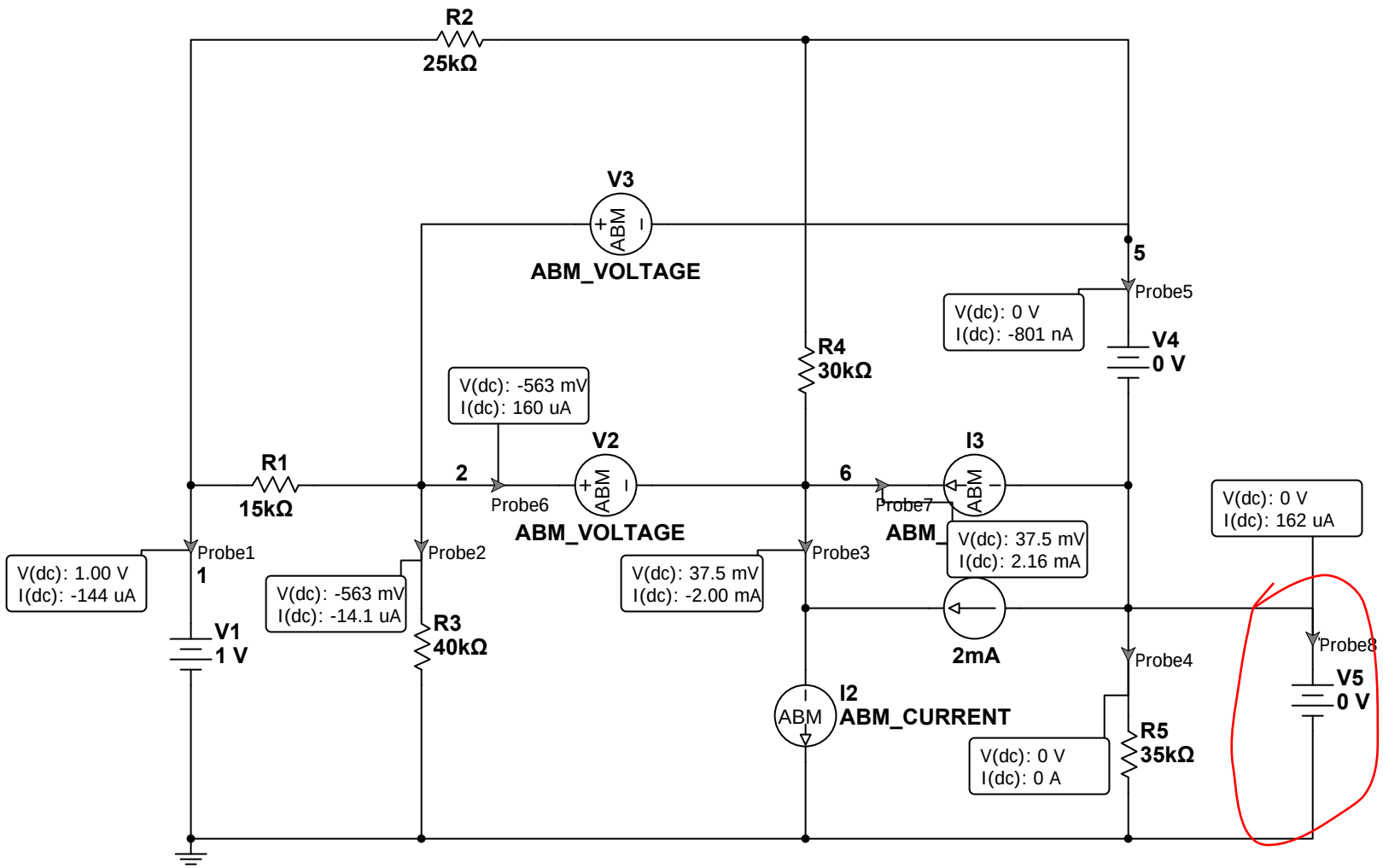
yes, it is <u>negative</u>

Taking the results of (a) + (b) we see that

$$R_{eq} = \frac{V_{TH}}{I_N}$$

The Multisim Results also match.

These circuits are straightforward modifications of the simulation file I used for PS3#6b and provided on the course website. I have circled the key modifications in red.



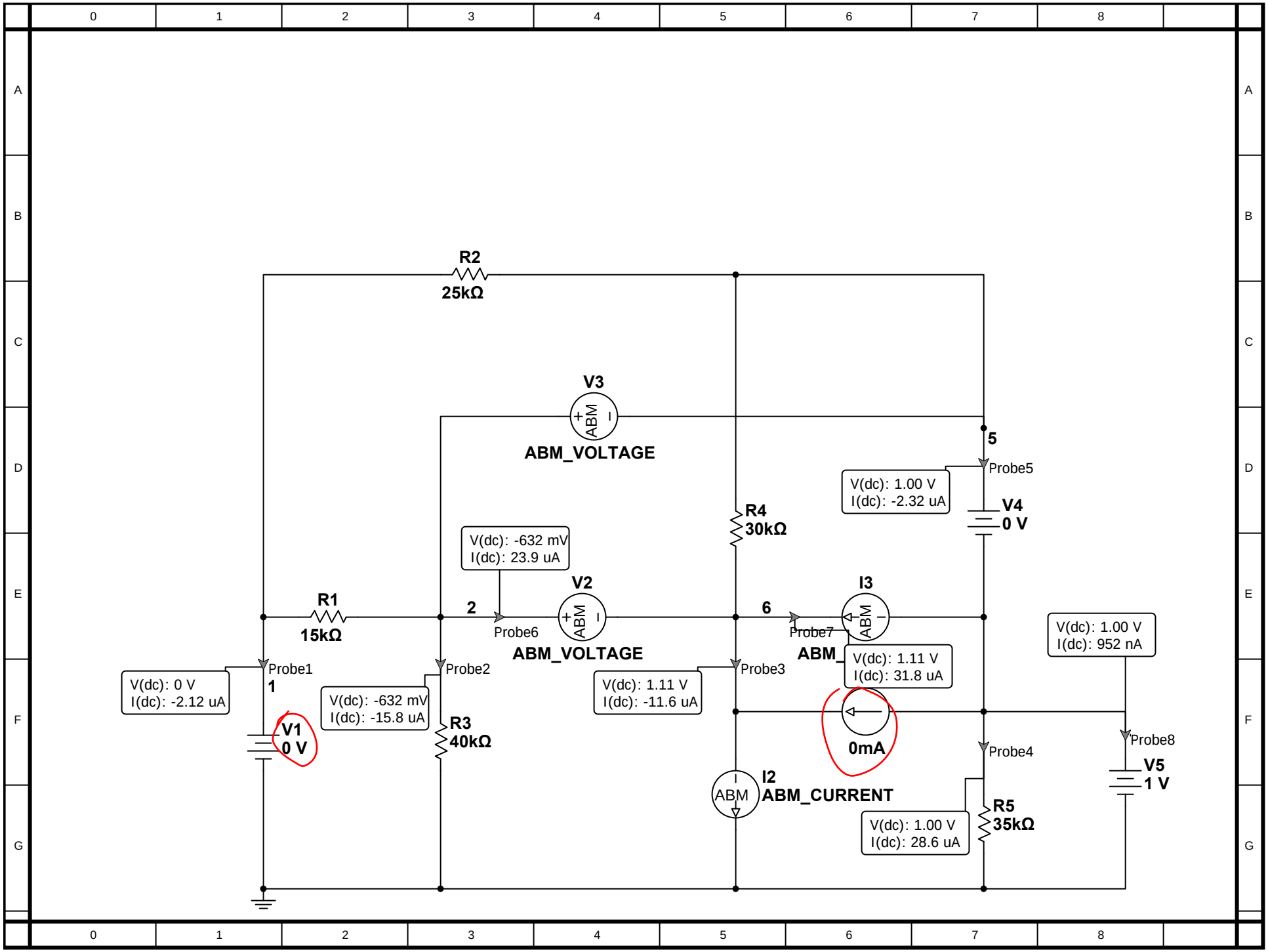
Problem set 4 number 6 b

Printing Time:February 01, 2010 15:47:43

DC Operating Point

	DC Operating Point	
1	V(Probe6)	-563.08806 m
2	V(Probe5)	0.00000
3	V(Probe4)	0.00000
4	V(Probe3)	37.53920 m
5	V(Probe2)	-563.08806 m
6	V(Probe1)	1.00000
7	I(Probe1)	-144.20587 u
8	I(Probe2)	-14.07720 u
9	I(Probe3)	-2.00400 m
10	I(Probe4)	0.00000
11	I(Probe5)	-800.83635 n
12	I(Probe6)	160.33522 u
13	V(Probe7)	37.53920 m
14	I(Probe7)	2.16309 m
15	V(Probe8)	0.00000
16	I(Probe8)	162.28726 u

$I_{sc} = I_N$


R2
25kΩ
V3
ABM_VOLTAGE
5
Probe5
V(dc): 1.00 V
I(dc): -2.32 uA
V4
0 V
R4
30kΩ
V(dc): -632 mV
I(dc): 23.9 uA
V2
ABM_VOLTAGE
I3
R1
15kΩ
2
Probe6
6
Probe7
ABM_
V(dc): 1.11 V
I(dc): 31.8 uA
V(dc): 1.00 V
I(dc): 952 nA
Probe1
1
V(dc): 0 V
I(dc): -2.12 uA
Probe2
V(dc): -632 mV
I(dc): -15.8 uA
V(dc): 1.11 V
I(dc): -11.6 uA
Probe3
V1
0 V
R3
40kΩ
0mA
Probe4
Probe8
V5
1 V
I2
ABM_CURRENT
R5
35kΩ
V(dc): 1.00 V
I(dc): 28.6 uA

Problem set 4 number 6 c

Printing Time:February 01, 2010 15:50:41

DC Operating Point

	DC Operating Point	
1	V(Probe6)	-631.84411 m
2	V(Probe5)	1.00000
3	V(Probe4)	1.00000
4	V(Probe3)	1.10879
5	V(Probe2)	-631.84411 m
6	V(Probe1)	0.00000
7	I(Probe1)	-2.12294 u
8	I(Probe2)	-15.79610 u
9	I(Probe3)	-11.60422 u
10	I(Probe4)	28.57143 u
11	I(Probe5)	-2.32084 u
12	I(Probe6)	23.86621 u
13	V(Probe7)	1.10879
14	I(Probe7)	31.84411 u
15	V(Probe8)	1.00000
16	I(Probe8)	951.83417 n

$$R_{eq} = -\frac{1V}{I(P8)} \simeq -1.0506\,M\Omega$$