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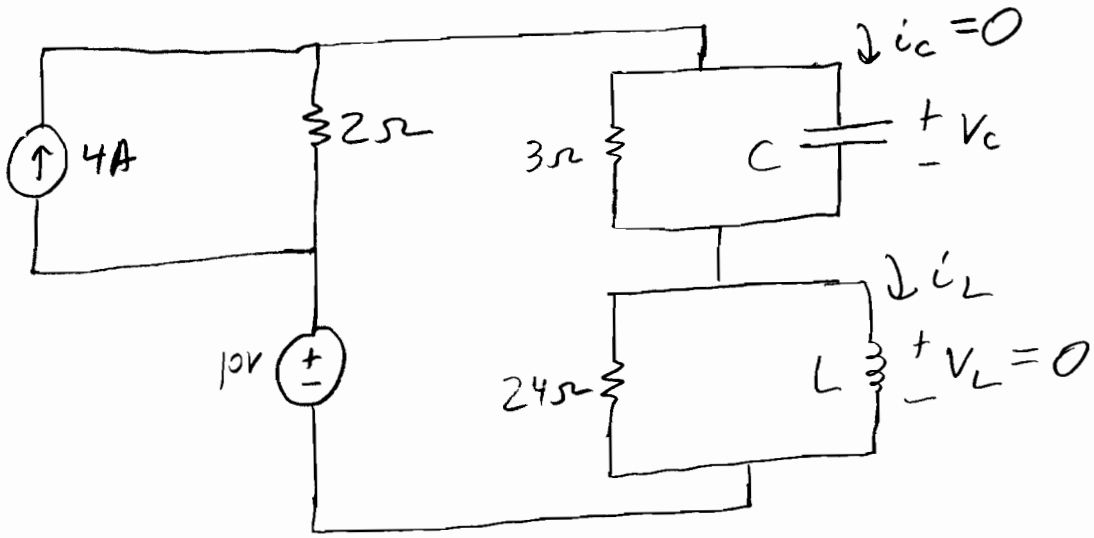
EECS 215 WN10
PS 8 Solutions

F. Terry

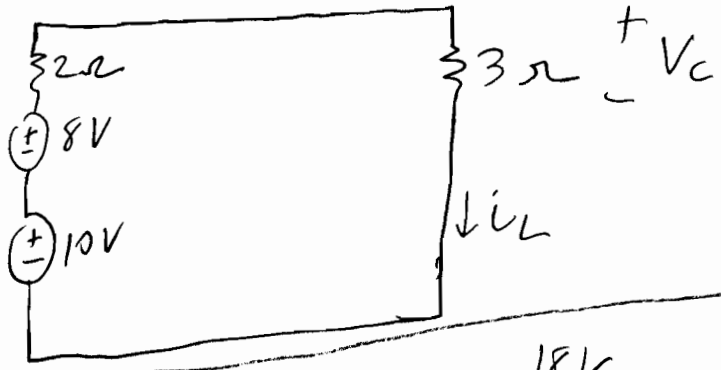
1) 6.6. $t<0$ $(t=0^-)$



$i_C(0^-) = 0$, $V_L(0^-) = 0$ (DC conditions)

Source transform 4A/2Ω combination

C → open, L → short



$$i_L(0^-) = i_L(0^+) = \frac{18V}{5\Omega} = +3.6A$$

$$V_C(0^-) = V_C(0^+) = (3.6A)3\Omega = +10.8V$$

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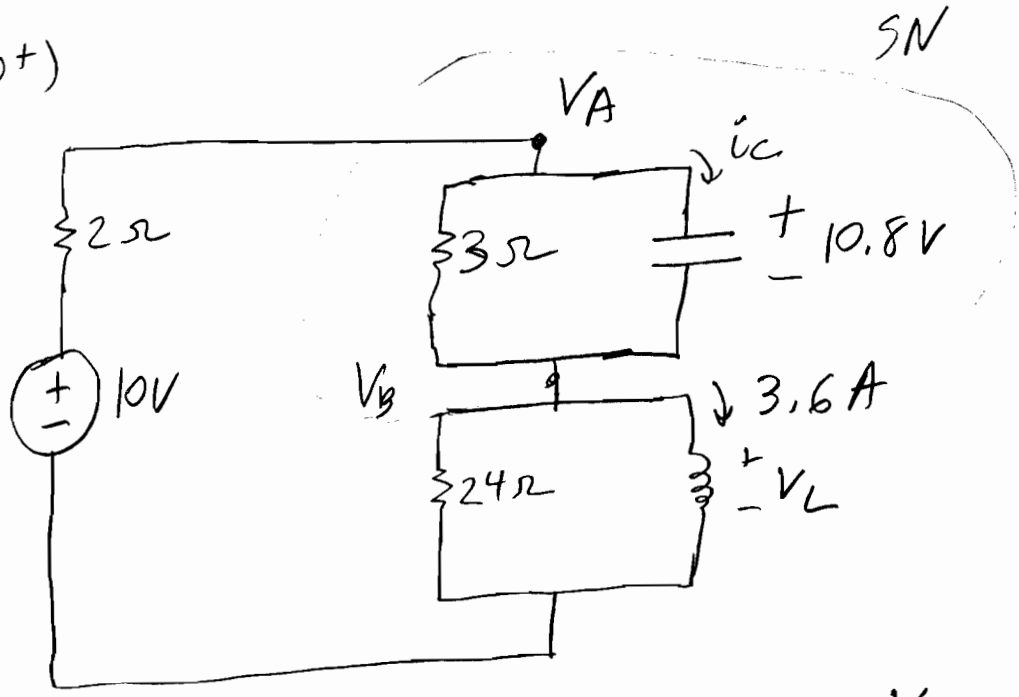
$t = 0 \quad (0^+)$



$V_A - V_B = +10.8V$ KVL, $V_B = V_L$

$$\frac{V_A - 10V}{2\Omega} + \frac{V_B}{24\Omega} + 3.6\,A = 0 \quad \text{SN KCL}$$

$$\frac{V_A - 10V}{2\Omega} + \frac{V_A - 10.8V}{24\Omega} + 3.6\,A = 0$$

$$\Rightarrow V_A\left(\frac{1}{2\Omega} + \frac{1}{24\Omega}\right) = 1.85A$$

$$V_A = \frac{222}{65}V \approx 3.41538V$$

$$V_B = -\frac{480}{65}V = -\frac{96}{13}V \approx -7.38462V$$

$$\Rightarrow \boxed{V_L(0^+) = -\frac{96}{13}V \approx -7.38462V}$$

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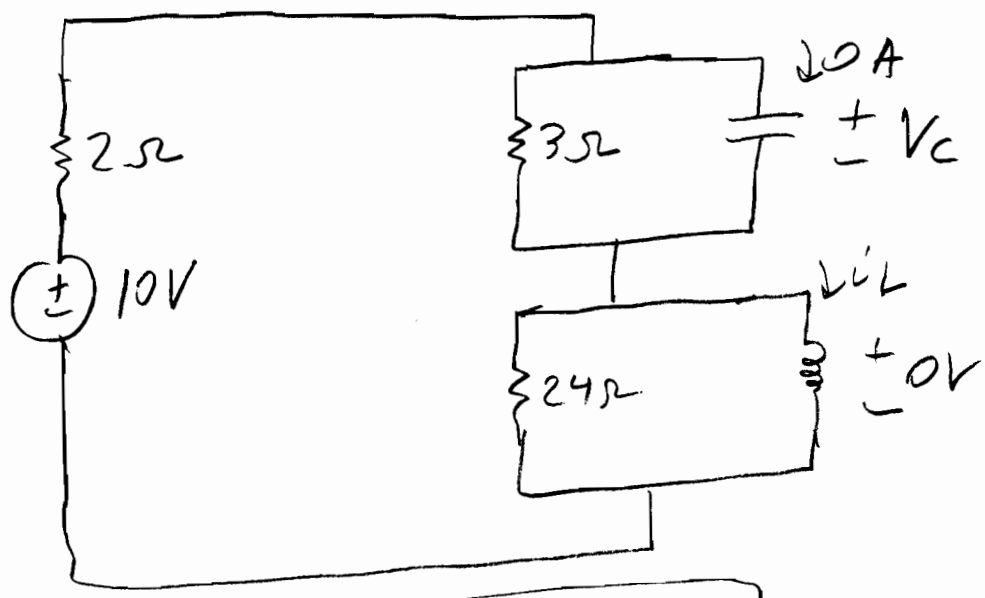
$$i_C(0^-) = i_C(0^+) = \frac{V_B}{24\Omega} + i_L(0) - \frac{V_C(0)}{3\Omega}$$

$$= -\frac{4}{13}A + 3.6A - 3.6A$$

$$\boxed{i_C(0^+) = -\frac{4}{13}A \approx -0.307692A}$$

$\underline{t \to \infty}$



$$\boxed{\begin{aligned} i_L(\infty) &= \frac{10V}{2\Omega + 3\Omega} = +2A \\ V_C(\infty) &= (2A)(3\Omega) = +6V \end{aligned}}$$

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2) 6.14 Series RLC

$$V_c(t) = (6\cos(4t) - 3\sin(4t))e^{-2t}\,u(t) \quad V$$

$$i_c(t) = -(0.24\cos(4t) + 0.18\sin(4t))e^{-2t}\,u(t) \quad A$$

so $\omega_d = 4s^{-1} = [\omega_0^2 - \alpha^2]^{1/2}$ } from form of underdamped response

& $\underline{\alpha = 2s^{-1}}$

$$\Rightarrow \omega_0^2 = \omega_d^2 + \alpha^2 = 20\,s^{-2}$$

$$\Rightarrow \underline{\omega_0 = \sqrt{20}\,s^{-1} \cong 4.472\,s^{-1}}$$

$$i_c = C\frac{dV_c}{dt} = C[-2(6\cos 4t - 3\sin 4t) + (-24\sin 4t - 12\cos 4t)]e^{-2t}$$

$$= C[-24\cos 4t - 18\sin 4t]e^{-2t}$$

so
$$\boxed{\begin{aligned} -24C &= -0.24 \Rightarrow C = 10^{-2}F \\ -18C &= -0.18 \Rightarrow C = 10^{-2}F \checkmark \end{aligned}}$$

$$\omega_0^2 = \frac{1}{LC} = 20s^{-1} \Rightarrow \underline{L = \frac{1}{\omega_0^2 C} = 5H}$$

$$\alpha = \frac{R}{2L} = 2s^{-1} \Rightarrow \underline{R = 2L\alpha = 20\Omega}$$

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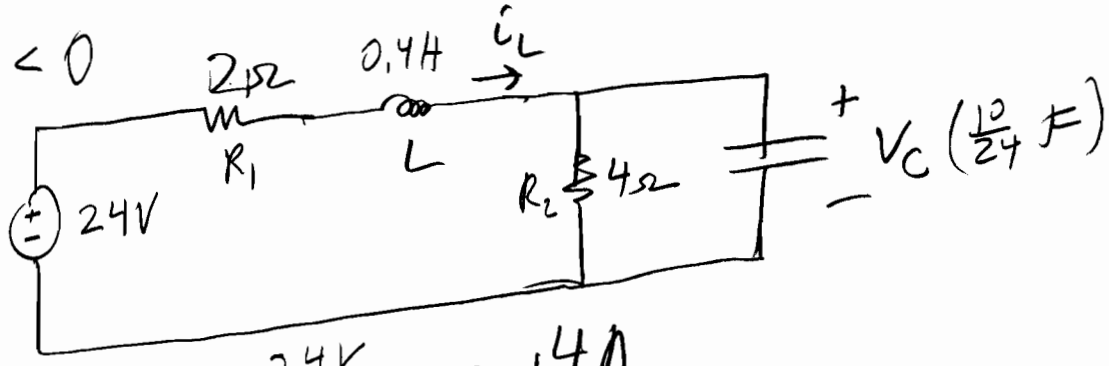
3) 6.23

$t < 0$



$$i_L(0) = \frac{24V}{2\Omega + 4\Omega} = +4A$$

$$V_C(0) = 4\Omega(4A) = +16\,V$$

$t \geq 0$

$$\alpha = \frac{R_1}{2L} = \frac{2\Omega}{2(0.4H)} = 2.5\ s^{-1}$$

$$\omega_0 = \left(\frac{1}{LC}\right)^{1/2} = \sqrt{6}\ s^{-1} \simeq 2.44949\ s^{-1}$$

$\alpha > \omega_0$ barely, but it is

$\Rightarrow$ Overdamped

$$s_{1,2} = -\alpha \pm [\alpha^2 - \omega_0^2]^{1/2}$$

$$= -2.5 \pm [6.25 - 6]^{1/2}$$

$$= -2.5 \pm 0.5$$

$$= -2, -3\ s^{-1}$$

$$V_{Cf} = +24V$$

so $V_C(t) = 24V + [A_1 e^{s_1 t} + A_2 e^{s_2 t}]$

$V_c(0) = 16\,V = 24V + A_1 + A_2 \Rightarrow A_1 + A_2 = -8V$

$i_L(0) = i_c(0) = 4A = \frac{10}{24}(s_1 A_1 + s_2 A_2)$

$(-2A_1 - 3A_2) = 9.6$

$A_2 = -8V - A_1$

$\Rightarrow -2A_1 - 3(-8V - A_1) = 9.6$

$A_1 = 9.6 - 24 = -14.4\,V$

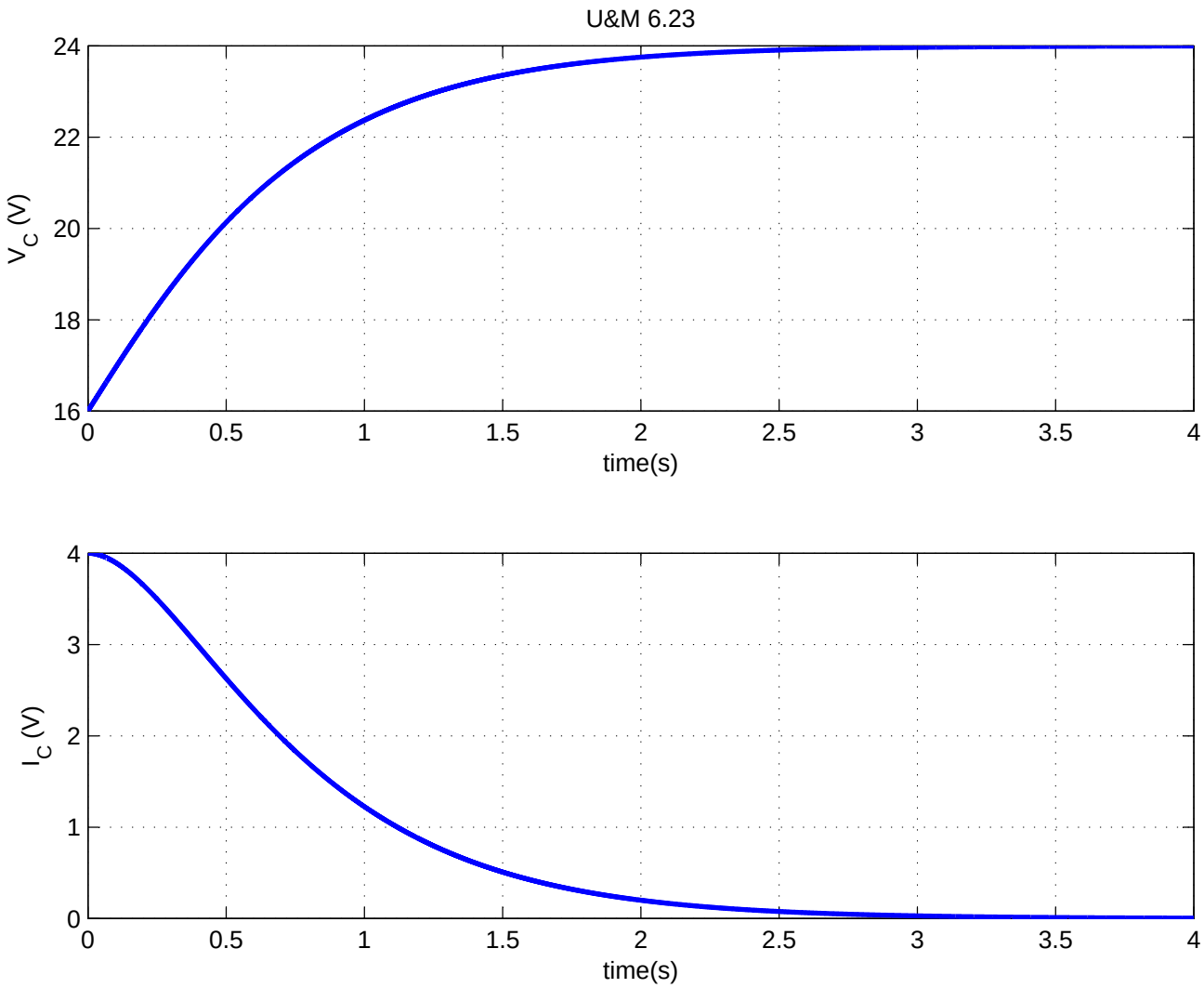
$A_2 = 6.4\,V$

$$\boxed{V_c(t) = \left[24 + (-14.4e^{-2t} + 6.4e^{-3t})\right] V}$$

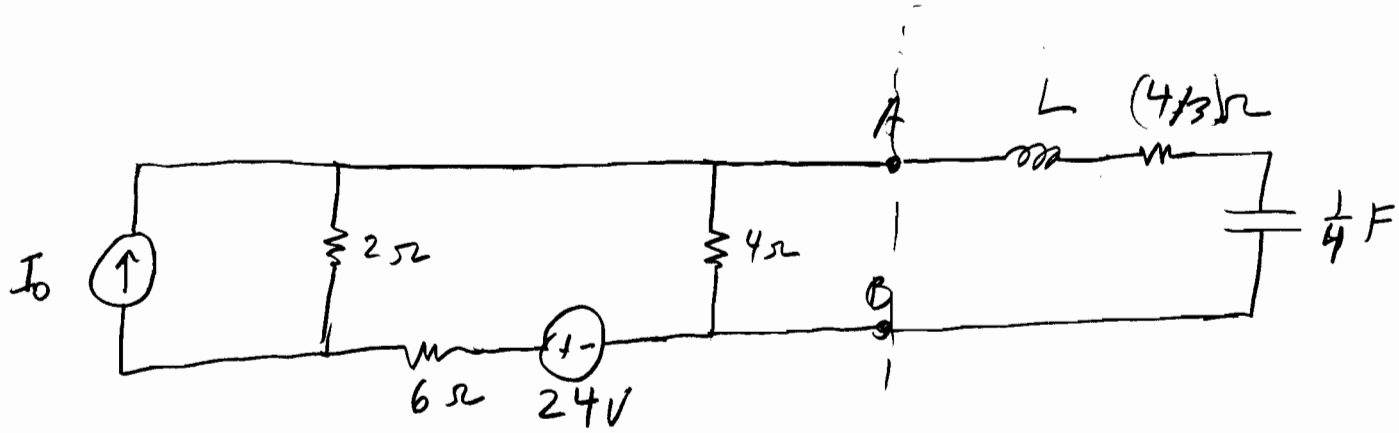
$i_c(t) = \frac{10}{24}(s_1 A_1 e^{-2t} + s_2 A_2 e^{-3t})\,A$

$$\boxed{i_c(t) = (12e^{-2t} - 8e^{-3t})\,A}$$

Plot follows


U&M 6.23
24
22
20
18
16
V_C (V)
0
0.5
1
1.5
2
2.5
3
3.5
4
time(s)
4
3
2
1
0
I_C (V)
0
0.5
1
1.5
2
2.5
3
3.5
4
time(s)

4) 6.28 1st Redraw



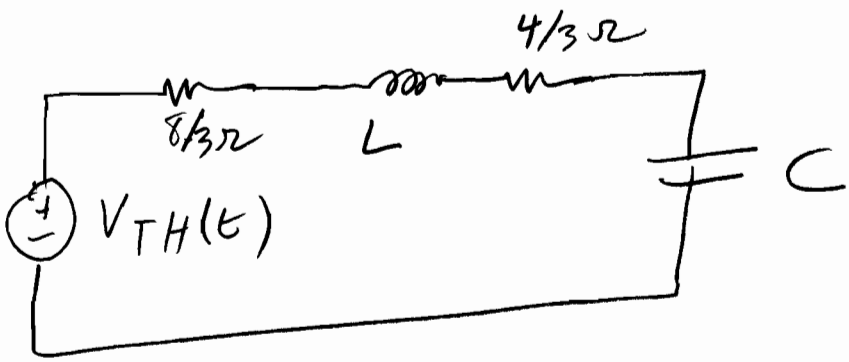
$I_0 = 3A\,[u(t) - u(t-0.4s)]$

Find Thevenin Equivalent For circuit to left of A, B

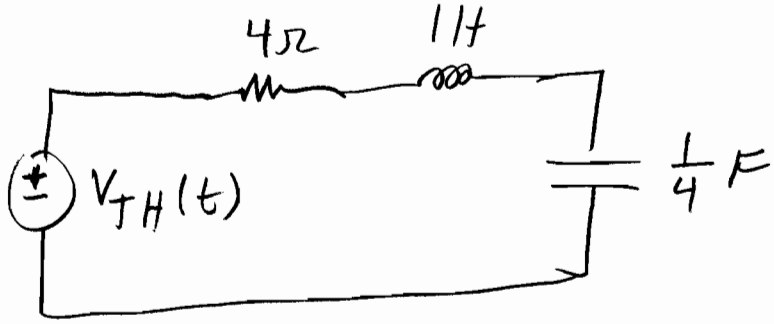
$R_{eq} = 4\Omega \parallel (2\Omega + 6\Omega) = \frac{32}{12} = \frac{8}{3}\Omega$

$V_{TH} = \frac{4\Omega}{(4+6+2)\Omega} 24V + 4\Omega\left(\frac{2\Omega}{(2+4+6)\Omega}\right) I_0$

$= 8V + 2V\,[u(t) - u(t-0.4s)]$



$\Rightarrow$



equivalent from L, C perspective

$$V_{TH} = \begin{cases} 8V & t < 0 \\ 10V & 0 \le t \le 0.4s \\ 8V & t \ge 0.4s \end{cases}$$

$i_L(0) = 0$ A

$V_C(0) = 8$ V

KVL

$-V_{TH}(0^+) + V_L(0^+) + V_C(0^+) = 0$

since $i_L(0^+) = 0$

$\Rightarrow V_L(0^+) = -V_C(0^+) + V_{TH}(0^+)$

$= -8V + 10V = \underline{+2V}$

$\alpha = \frac{R}{2L} = 2s^{-1}$ $\quad \omega_0 = \left(\frac{1}{LC}\right)^{1/2} = 2s^{-1}$

$\Rightarrow$ critically damped

$\underline{0 \le t < 0.4s}$

$i_{Lf} = 0$

$i_L(t) = (A_1 + A_2 t)e^{-\alpha t}$

$i_L(0) = 0 = A_1$

$V_L(0) = 2V = L(-\alpha A_1 + A_2)$

$\Rightarrow A_2 = \frac{2V}{L} = 2 A/s$

$$\boxed{\begin{array}{l} 0 \le t \le 0.4s \\ i_L(t) = \left(2\frac{A}{s}\right) t\, e^{-2t} \end{array}}$$

$\underline{t \ge 0.4s}$

$$i_L(0.4s) = \left(0.8\, e^{-0.8}\right) A$$

$$\simeq 0.359463\ A$$

need $V_C(0.4s)$ to get $V_L(0.4s^+)$

$$V_C(t) = V_{TH} - i_L(4\Omega) - V_L$$

$$= 10V - \left(8\tfrac{V}{s}\right) t\, e^{-2t} + \left(4\tfrac{V}{s} t - 2V\right) e^{-2t}$$

$$V_C(0.4s) = 10V - \left(3.2\, e^{-0.8}\right) + \left(1.6 - 2\right) e^{-0.8}$$

$$= 10 - (3.6)\, e^{-0.8}\ V \simeq 8.38242V$$

$$V_L(0.4s^+) = -V_C(0.4s) + V_{TH}(0.4s^+) - \left[i_L(0.4s)\right] 4\Omega$$

$$= -10V + 3.6\, e^{-0.8V} + 8V - 3.2\, e^{-0.8}$$

$$= -2V + 0.4\, e^{-0.8V}$$

$$\simeq -1.82027V$$

$$i_{Lf} = 0$$

$$i_L(t) = \left[B_1 + B_2(t - 0.4s)\right] e^{-2(t-0.4s)}$$

$i_L(0.4s) = B_1 = 0.8e^{-0.8}\ A \simeq 0.359463\ A$

$v_L(0.4s) = L[-2B_1 + B_2] \Rightarrow B_2 = \frac{v_L(0.4s)}{L} + 2B_1$

$= (2e^{-0.8} - 2)\ A/s$

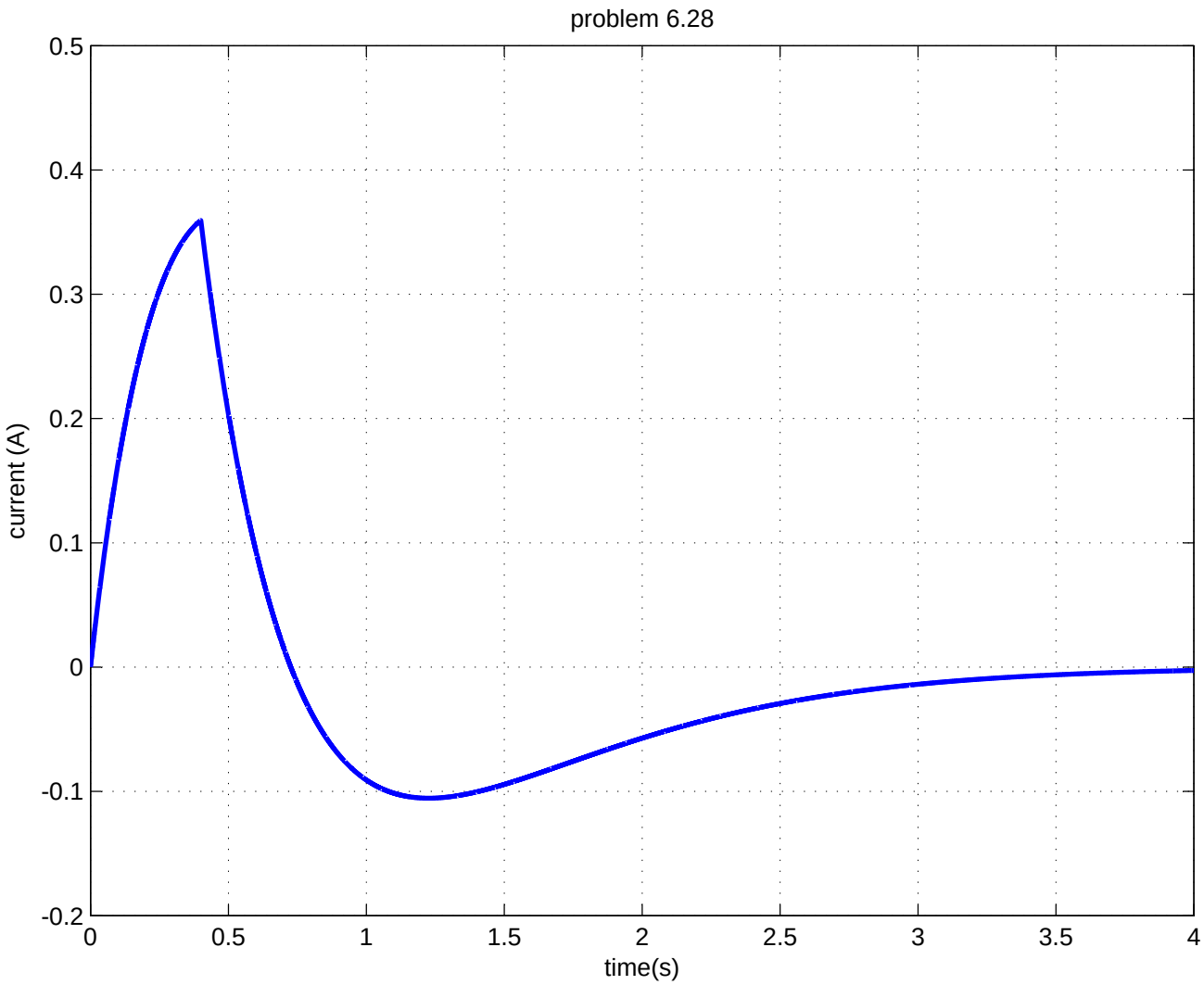
$\simeq -1.10134\ A/s$

$t \geq 0.4s$

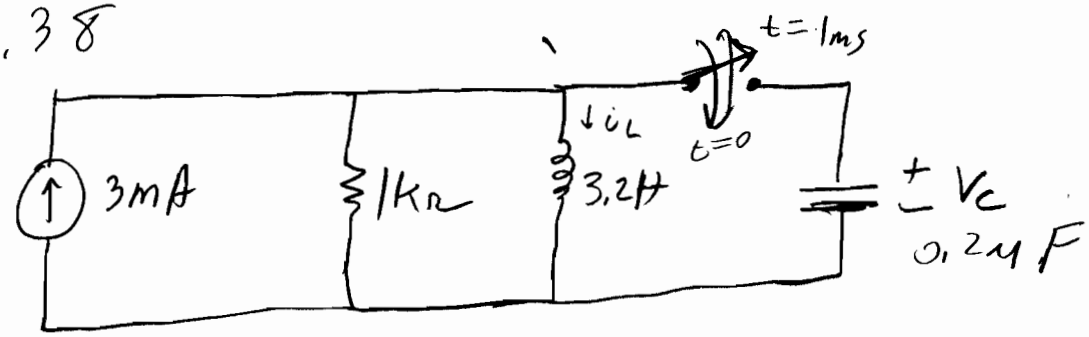
$i_L(t) = [B_1 + B_2(t - 0.4s)]e^{-2(t-0.4s)}$

with above B_1, B_2

Plot follows


problem 6.28
current (A)
time(s)
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
0
0.5
1
1.5
2
2.5
3
3.5
4

5) 6.38



$t < 0 \quad i_L = 3mA \quad V_c = 0$

$0 \le t < 1ms$ <u>Parallel RLC</u>

$\alpha = \frac{1}{2RC} = 2.5 \times 10^3 \, s^{-1}$

$\omega_0 = \left(\frac{1}{LC}\right)^{1/2} = 1.25 \times 10^3 \, s^{-1} = \frac{1}{2}\alpha$

$\alpha > \omega_0 \Rightarrow$ overdamped

$s_{1,2} = -\alpha \pm [\alpha^2 - \omega_0^2]^{1/2}$

$= \alpha\left[-1 \pm \frac{\sqrt{3}}{2}\right]$

$\simeq -334{,}936 \, s^{-1}, \; -4665.06 \, s^{-1}$

$i_{Lf} = 3mA$

$i_L(t) = 3mA + A_1 e^{s_1 t} + A_2 e^{s_2 t}$

$i_L(0) = 3mA = 3mA + A_1 + A_2$

$V_L(0) = V_c(0) = 0 = L[A_1 s_1 + A_2 s_2]$

only solution is $\underline{A_1 = A_2 = 0}$

Hence $0 \le t < 1\,ms$

$i_L(t) = 3mA$

$V_L(t) = V_c(t) = 0V$

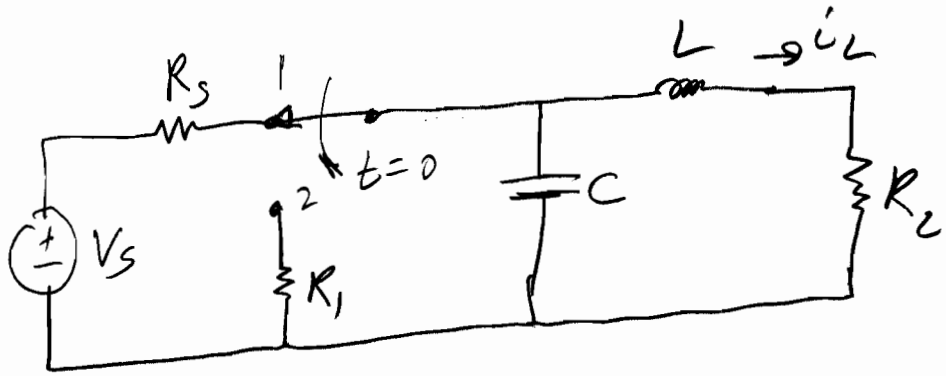
$t \ge 1ms$?

$V_c(t) = 0V$

$i_L(t) = 3mA$

This circuit was started in its stable final state, so nothing happened

6) 6.43



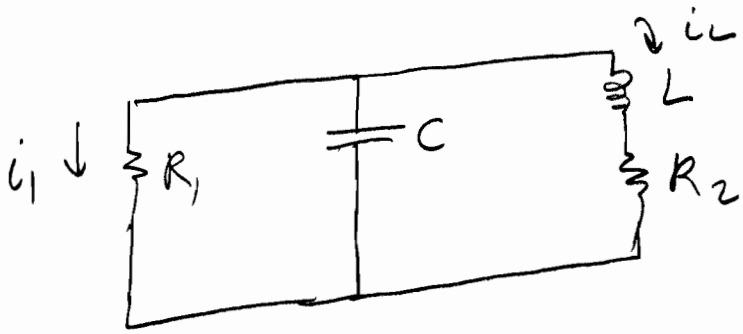
a) $i_L(0) = \dfrac{V_s}{R_s+R_2}$

$V_c(0) = i_L(0)R_2 = \dfrac{R_2}{R_s+R_2}V_s$

$V_c(0^+) = V_L(0^+) + i_L(0^+)R_2$

Hence $V_L(0^+) = V_c(0) = \dfrac{R_2}{R_s+R_2}V_s$

b)



$i_1 = \dfrac{V_c}{R_1}$

$i_1 + i_c + i_L = 0 \Rightarrow \dfrac{V_c}{R_1} + C\dfrac{dV_c}{dt} + i_L = 0$

$V_c = V_L + i_L R_2$

$= L\dfrac{di_L}{dt} + i_L R_2$ plug in

$$\frac{L}{R_1}\frac{di_L}{dt} + \frac{R_2}{R_1}i_L + LC\frac{d^2i_L}{dt^2} + R_2C\frac{di_L}{dt} + i_L = 0$$

$$\boxed{\frac{d^2 i_L}{dt^2} + \left(\frac{1}{R_1 C} + \frac{R_2}{L}\right)\frac{di_L}{dt} + \left[\left(1+\frac{R_2}{R_1}\right)\frac{1}{LC}\right] i_L = 0}$$

c) $\alpha = \frac{1}{2}\left(\frac{1}{R_1 C} + \frac{R_2}{L}\right) = 5.002 \times 10^3 \, s^{-1}$

$\omega_0 = \left[\left(1+\frac{R_2}{R_1}\right)\frac{1}{LC}\right]^{1/2} = \left[\frac{3}{50 \times 10^{-6}}\right]^{1/2} = (6 \times 10^4)^{1/2}$

$\approx 2.4494 \times 10^2 \, s^{-1}$

$\alpha > \omega_0$ overdamped

$s_{1,2} = -\alpha \pm [\alpha^2 - \omega_0^2]^{1/2}$

$= -5.002 \times 10^3 \pm 4.996 \times 10^3$

$\underline{s_{1,2} = -6.0012 \, s^{-1}, -9.998 \times 10^3 \, s^{-1}}$

$i_{Lf} = 0$ (natural decay)

$\underline{i_L(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}}$

$i_L(0) = \frac{12V}{400\Omega} = 30mA = A_1 + A_2$

$V_L(0) = \frac{R_2}{R_s + R_2} 12V = \frac{100}{400} 12V = +3V$

$= L(A_1 s_1 + A_2 s_2)$

$\Rightarrow A_1 s_1 + A_2 s_2 = 300 \, A/s$

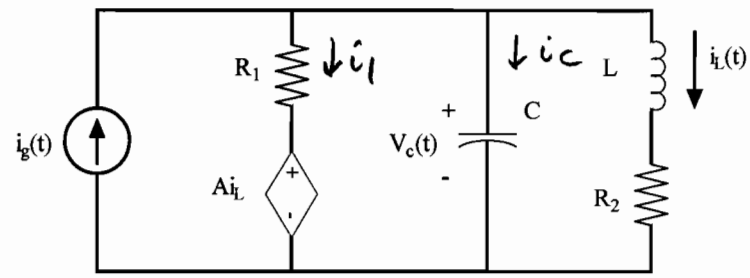
solving

$\underline{A_1 = +60.0420 mA}$

$\underline{A_2 = -30.0420 mA}$

7. a)

KVL/KCL Approach



① $i_g = i_1 + i_c + i_L$ KCL

② $i_1 = \dfrac{V_c - A i_L}{R_1}$ KVL + Ohm's Law

$[i_1 R_1 + A i_L = V_c$ by KVL$]$

③ $V_c = V_L + i_L R_2$ KVL

$= L\dfrac{di_L}{dt} + i_L R_2$

② → ① to get rid of i_1

④ $i_g = \dfrac{V_c - A i_L}{R_1} + C\dfrac{dV_c}{dt} + i_L$

③ → ④ to eliminate V_c

$$i_g = \Big\{\underbrace{\left(\frac{L}{R_1}\frac{di_L}{dt} + \frac{R_2}{R_1} i_L - \frac{A}{R_1} i_L\right)}_{i_1} + \underbrace{\left(CL\frac{d^2 i_L}{dt^2} + R_2 C\frac{di_L}{dt}\right)}_{i_c} + i_L\Big\}$$

$$i_g = LC\frac{d^2 i_L}{dt^2} + \left(\frac{L}{R_1} + R_2 C\right)\frac{di_L}{dt} + \left(1 + \frac{R_2 - A}{R_1}\right) i_L$$

$$\left(\frac{1}{LC}\right) i_g = \left\{ \frac{d^2 i_L}{dt^2} + \left(\frac{1}{R_1 C} + \frac{R_2}{L}\right) \frac{di_L}{dt} + \left[\left(1 + \frac{R_2 - A}{R_1}\right)\frac{1}{LC}\right] i_L \right\}$$

b) $R_1 = 1K\Omega$ $R_2 = 5\Omega$ $A = 996\Omega$ $L = 1mH$
$C = 1\mu F$

$$\alpha = \frac{1}{2}\left(\frac{1}{R_1 C} + \frac{R_2}{L}\right) = 3 \times 10^3 s^{-1}$$

$2\alpha = 6 \times 10^3 s^{-1}$ (matches problem statement) ✓

$$\omega_0^2 = \left[\left(1 + \frac{R_2 - A}{R_1}\right)\frac{1}{LC}\right] = 9 \times 10^6 s^{-2} \quad ✓$$

$$\omega_0 = 3 \times 10^3 s^{-1}$$

$$\frac{1}{LC} = 10^9 s^{-2} \quad ✓$$

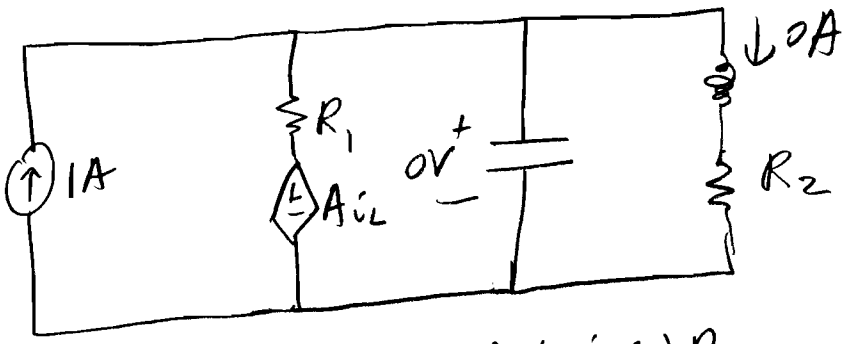
$\alpha = \omega_0 \Rightarrow$ critical damping

$$i_{Ln}(t) = (A_1 + A_2 t) e^{-\alpha t}$$

$\underline{t < 0}$

$i_g = 0$ so we will get $i_L(0) = 0$
$V_C(0) = 0$

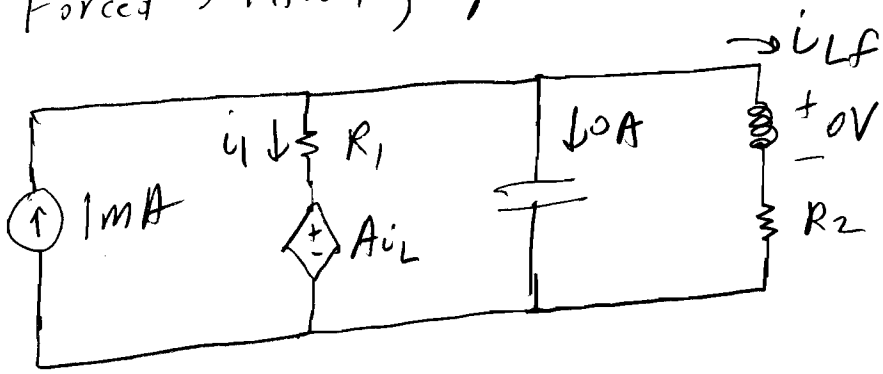
$t = 0^+$



KVL $\underbrace{V_c(0)}_{0} = V_L(0) + \underbrace{i_L(0)}_{0} R_2$

$\Rightarrow V_L(0) = 0$

Forced solution, $i_g = 1mA$ $(t \to \infty)$



$1mA = i_1 + i_{Lf}$

$V_{cf} = i_{Lf} R_2$

$i_1 = \dfrac{V_{cf} - A i_{Lf}}{R_1} = \left(\dfrac{R_2 - A}{R_1}\right) i_{Lf}$

$1mA = \left[\left(\dfrac{R_2 - A}{R_1}\right) + 1\right] i_{Lf}$

$i_{Lf} = (1/9\ A)$ note that this agrees with the diff. eqn.

$0 \le t < 0.4\,ms$

$$i_L(t) = \tfrac{1}{9}A + (A_1 + A_2 t)e^{-\alpha t}$$

$$i_L(0) = 0 = \tfrac{1}{9} + A_1 \Rightarrow A_1 = -1/9\,A$$

$$V_L(0) = 0 = -\alpha A_1 + A_2 \Rightarrow A_2 = +\alpha A_1$$
$$= -\tfrac{1}{3}\times 10^3\,A/s$$

so

$$i_L(t) = \tfrac{1}{9} - \left(\tfrac{1}{9} + \tfrac{1}{3}\times 10^3 t\right)e^{-\alpha t}$$
$$= \tfrac{1}{9}\left[1 - (1 + 3\times 10^3 t)e^{-\alpha t}\right]$$

$t = 0.4\,ms$

$$i_L(t) = \tfrac{1}{9}\left[1 - (1 + 1.2)e^{-1.2}\right]$$
$$\approx 37.4859\ mA$$

Now note that

$$V_C = V_L + i_L R_2 = L\frac{di_L}{dt} + i_L R_2 \quad \text{(here)}$$

since (V_C, i_L) both must be continuous, the circuit then constrains $V_L = L\frac{di_L}{dt}$ to also be continuous.

Thus $V_L(0.4ms^-) = V_L(0.4ms^+)$

$0 \le t \le 0.4ms$

$$V_L(t) = \frac{1}{9}L\left[+\alpha(1+3\times10^3 t)e^{-\alpha t} - (3\times10^3)e^{-\alpha t}\right]$$

$$= \frac{1}{9}L\left[(3\times10^3)^2 t\, e^{-\alpha t}\right] \quad \text{since } \alpha = 3\times10^3$$

$$= \left(10^3\tfrac{V}{s}\right)t\, e^{-\alpha t} \quad \text{note } V_L(0)=0 \text{ as required}$$

$$V_L(0.4ms) = (0.4V)e^{-1.2} \simeq 0.120478V$$

<u>$t > 0.4ms$</u>

$$i_{Lf} = 0$$

$$i_L(t) = (B_1 + B_2 t)e^{-\alpha t}$$

$$= \left[D_1 + D_2(t - 0.4ms)\right]e^{-\alpha(t-0.4ms)}$$

$$i_L(t_1) = \left[D_1 + D_2 t_1\right]e^{-\alpha t_1} \qquad t_1 \equiv t - 0.4ms$$

$$t = 0.4ms \rightarrow t_1 = 0$$

$$i_L(t_1 = 0) = i_L(0.4ms) = \frac{1}{9}\left[1 - 2.2e^{-1.2}\right]$$

$$= D_1$$

$$D_1 \simeq 37.4859\ mA$$

$$V_L(0.4ms) = \frac{1}{9}(1.2\times10^{-3})\, e^{-1.2} \ V$$

$$= L\left[-\alpha D_1 + D_2\right]$$

$$D_2 = \frac{1}{L} V_L(0.4ms) + \alpha D_1$$

$$= 120.478 \ A/s + \alpha \cdot D_1$$

$$\approx 232.935 \ A/s$$

So

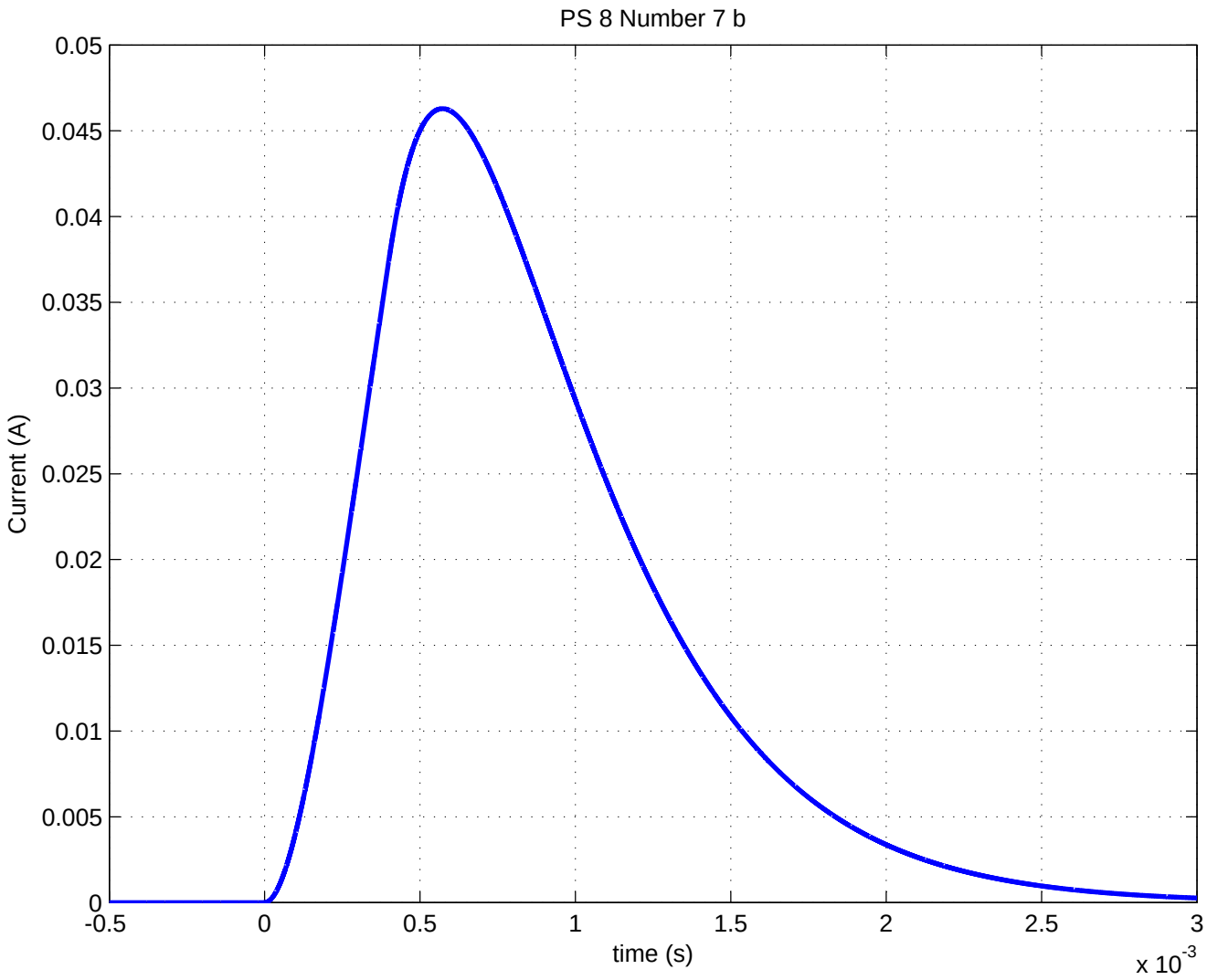
$$i_L(t) = \begin{cases} 0 & t<0 \\ \frac{1}{9}\left[1-(1+3\times10^3 t)\, e^{-\alpha t}\right] & 0 \le t < 0.4ms \\ \left[D_1 + D_2 t_1\right] e^{-\alpha t_1} & t \ge 0.4ms \\ D_1 \approx 37.4859 \ mA & \alpha = 3\times10^3 s^{-1} \\ D_2 \approx 232.935 \ A/s & t_1 \equiv t - 0.4ms \end{cases}$$

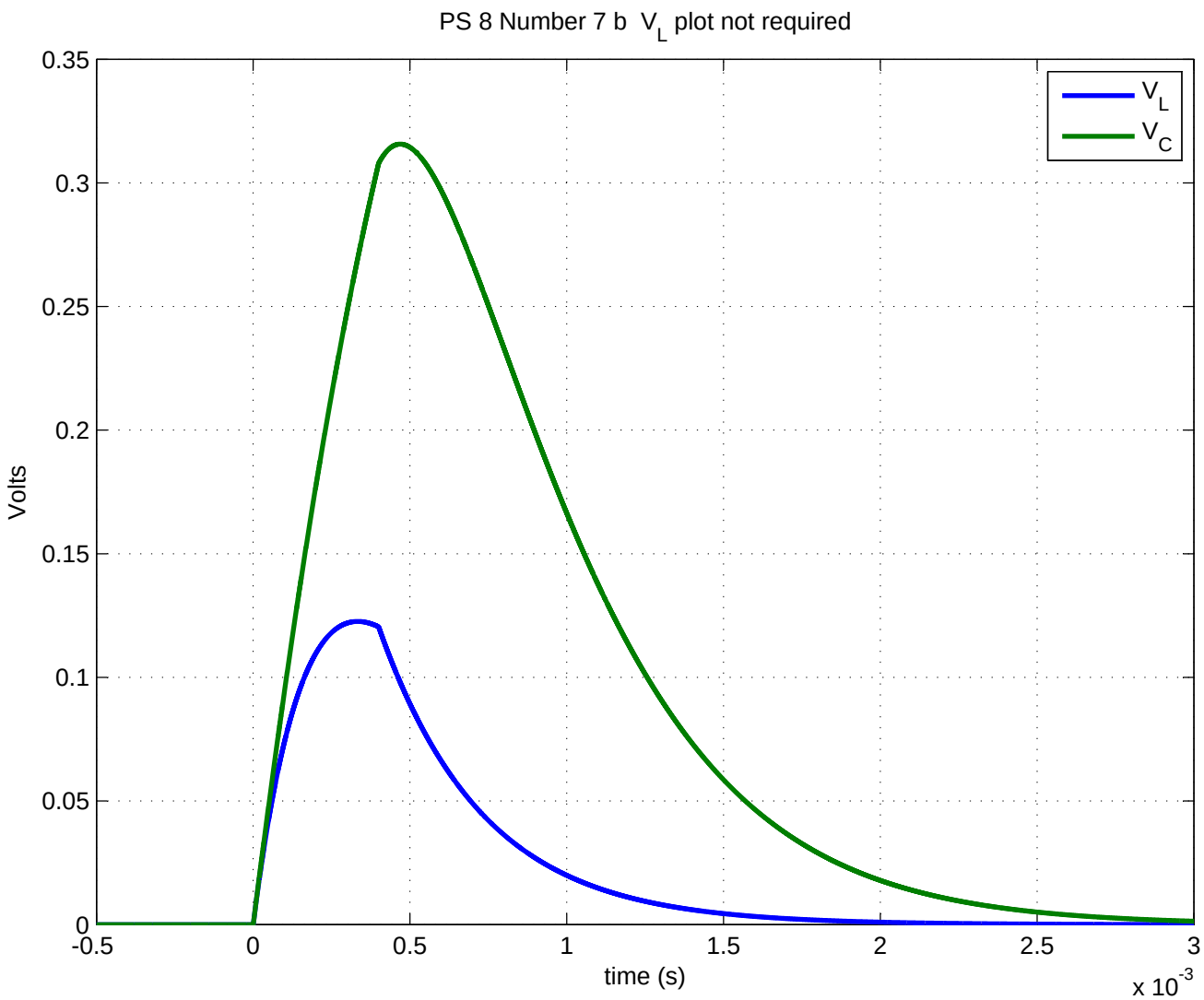
$$V_L(t) = \begin{cases} 0 & t \le 0 \\ 10^3 (V/s)\, t\, e^{-\alpha t} & 0 \le t < 0.4ms \\ L\left[D_2 - \alpha D_1 + D_2 t_1\right] e^{-\alpha t_1} & t \ge 0.4ms \\ & t_1 \equiv t - 0.4ms \end{cases}$$

$$V_C(t) = V_L(t) \overset{+}{\ominus} i_L R_2 \quad \text{always}$$

(silly error) code is correct I think

Matlab code + plots follow


PS 8 Number 7 b
Current (A)
0.05
0.045
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
-0.5
0
0.5
1
1.5
2
2.5
3
time (s)
x 10^-3


PS 8 Number 7 b V_L plot not required
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
Volts
-0.5
0
0.5
1
1.5
2
2.5
3
time (s)
x 10^-3
V_L
V_C

```
%problem set 8 number 7
clear all;
close all;
t=-0.5:1e-4:3;
t=t*1e-3;
il0=1/9*(1-2.2*exp(-1.2));
vl0=0.4*exp(-1.2);
d1=il0;
d2=vl0/1e-3+3e3*d1;
alpha=3e3;
for jj=1:length(t),
    if t(jj)<0,
        il(jj)=0;
        vl(jj)=0;
    else if t(jj)<0.4e-3,
            il(jj)=1/9*(1-(1+3e3*t(jj))*exp(-alpha*t(jj)));
            vl(jj)=1e3*t(jj)*exp(-alpha*t(jj));
        else
            il(jj)=(d1+d2*(t(jj)-0.4e-3))*exp(-alpha*(t(jj)-0.4e-3));
            vl(jj)=1e-3*(d2-alpha*d1+d2*(t(jj)-0.4e-3))*exp(-alpha*(t(jj)-0.4e-3));
        end;
    end;
end;
vc=vl+il*5;
plot(t,il,'linewidth',2);
title('PS 8 Number 7 b');
xlabel('time (s)');
ylabel('Current (A)');
grid;
figure;
plot(t,vl,t,vc,'linewidth',2);
grid;
title('PS 8 Number 7 b  V_L plot not required');
legend('V_L','V_C');
xlabel('time (s)');
ylabel('Volts');
```