EECS 215 PS 11
Solutions

1) UM 9.20 Series RLC

$R = 5\,\Omega \quad L = 20\,mH \quad C = 0.5\,\mu F$

(a) $\omega_0 = \left[\frac{1}{LC}\right]^{1/2} = 10^4\ r/s$

$$Q = \frac{\omega_0 L}{R} = 40$$

$$B = \frac{\omega_0}{Q} = 250\ r/s$$

$$\omega_{c1,2} = \left[\mp\frac{1}{2Q} + \left[1 + \frac{1}{4Q^2}\right]^{1/2}\right]\omega_0$$

$$= \begin{cases} 9.8758 \times 10^3\ r/s & \omega_{c1} \\ 1.0126 \times 10^4\ r/s & \omega_{c2} \end{cases}$$

(b) Yes. If $Q_{new} = 2Q_{old}$ + R, ω_0 unchanged

then $\underline{L_{new} = 2L_{old} = 40\,mH}$

since $\omega_0 = \left[\frac{1}{L_{new}C_{new}}\right]^{1/2} = 10^4\ r/s$

$$\Rightarrow C_{new} = \frac{1}{L_{new}\omega_0^2} = 2.5 \times 10^{-7}\,F$$

$$\underline{= 0.25\,\mu F}$$

(2) UM 9.24

Parallel RLC with $f_0 = 4\,kHz$ $Q = 100$

$R_{in} = 25\,k\Omega$ at Resonance

At Resonance $Z_{in} = R$ so $R = 25\,k\Omega$

$\omega_0 = 2\pi f_0 = 8\pi \times 10^3\ r/s = \left(\frac{1}{LC}\right)^{1/2}$

$Q = \frac{R}{\omega_0 L} = 100 \Rightarrow L = \frac{R}{\omega_0 Q} \cong 9.9472\ mH$

$C = \frac{1}{L\omega_0^2} \cong 1.5915 \times 10^{-7} F = 0.15915\ \mu F$

(3) UM 9,3

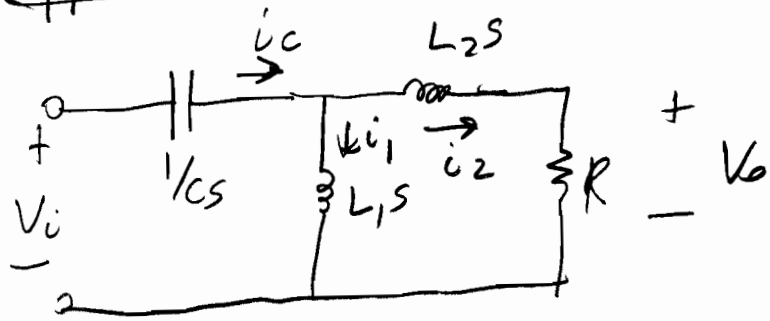
(a) Band pass filter

$\omega \to 0$ Cap $\to$ open, L's $\to$ short $\Rightarrow V_o = 0$

$\omega \to \infty$ $L_2 \to$ open $\Rightarrow V_o \Rightarrow 0$

(b) Transfer function, Multiple Ways to Do this

Approach 1



$$i_2 = i_c \left(\frac{L_1 s}{L_1 s + L_2 s + R} \right)$$ complex current Divider

$$i_c = V_i \frac{1}{\frac{1}{Cs} + [L_1 s \| (L_2 s + R)]}$$

$$= V_i \left[\frac{1}{\frac{1}{Cs} + \frac{L_1 s (L_2 s + R)}{L_1 s + L_2 s + R}} \right]$$

$$V_o = i_2 R$$

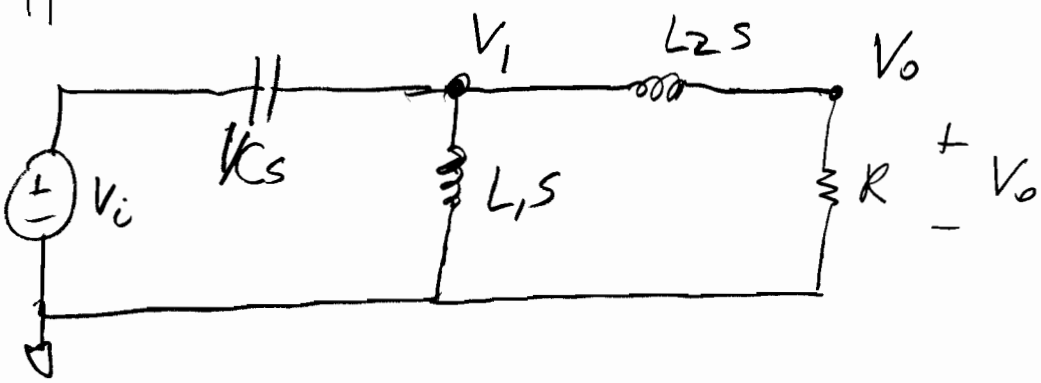
$$V_o = V_i R \left[\frac{L_1 s}{(L_1+L_2)s + R}\right]\left[\frac{1}{\frac{1}{Cs} + \frac{L_1 s(L_2 s + R)}{(L_1+L_2)s + R}}\right]$$

$$= V_i R \frac{L_1 C s^2}{(L_1+L_2)s + R + CL_1 s^2 (L_2 s + R)}$$

$$= V_i R \frac{L_1 C s^2}{L_1 L_2 C s^3 + RCL_1 s^2 + (L_1+L_2)s + R}$$

$$\boxed{H = \frac{V_o}{V_i} = \left[\frac{RCL_1 s^2}{L_1 L_2 C s^3 + RCL_1 s^2 + (L_1+L_2)s + R}\right]}$$

Approach 2 ; Nodal



KCL ①

$$V_1\left[Cs + \frac{1}{L_1 s} + \frac{1}{L_2 s}\right] - V_o\left[\frac{1}{L_2 s}\right] = V_i Cs$$

$$V_o\left[\frac{1}{R} + \frac{1}{L_2 s}\right] - V_1\left[\frac{1}{L_2 s}\right] = 0 \quad \text{KCL ②}$$

$$\Rightarrow V_1 = V_o\left[\frac{L_2 s}{R} + 1\right] = V_o\left[\frac{L_2 s + R}{R}\right]$$

$$V_o\left[\frac{L_2 s+R}{R}\right]\left[Cs+\frac{1}{L_1 s}+\frac{1}{L_2 s}\right]-V_o\left[\frac{1}{L_2 s}\right]=V_i Cs$$

multiply both sides by $L_1 L_2 R s$

$$V_o[L_2 s+R][L_1 L_2 C s^2+L_2+L_1]-V_o[RL_1]=V_i RCL_1L_2 s^2$$

$$V_o\{L_1 L_2^2 C s^3+L_2(L_1+L_2)s+RL_1L_2Cs^2+R(L_1+L_2)-RL_1\}=V_i RCL_1L_2 s^2$$

$$V_o\{L_1L_2^2Cs^3+RL_1L_2Cs^2+L_2(L_1+L_2)s+RL_2\}=RCL_1L_2s^2V_i$$

$$\frac{V_o}{V_i}=\left[\frac{RCL_1 s^2}{L_1L_2Cs^3+RCL_1s^2+(L_1+L_2)s+R}\right] \checkmark$$

Approach 3 ; Mesh
(you do it)

$$H(\omega)=\frac{-RCL_1\omega^2}{R(1-CL_1\omega^2)+j[(L_1+L_2)\omega-L_1L_2C\omega^3]}$$

$$=\frac{-RCL_1\omega^2}{R(1-CL_1\omega^2)+j\omega[(L_1+L_2)-L_1L_2C\omega^2]}$$

H goes pure real when

$$\omega_0^2 = \left(\frac{L_1 + L_2}{CL_1L_2}\right)$$

$$\Rightarrow \omega_0 = \left(\frac{L_1 + L_2}{CL_1L_2}\right)^{1/2}$$

(c) with $R = 500\,\Omega$ $C = \frac{1}{3}\,\mu F$ $L_1 = 1\,\mu H$ $L_2 = 3\,\mu H$

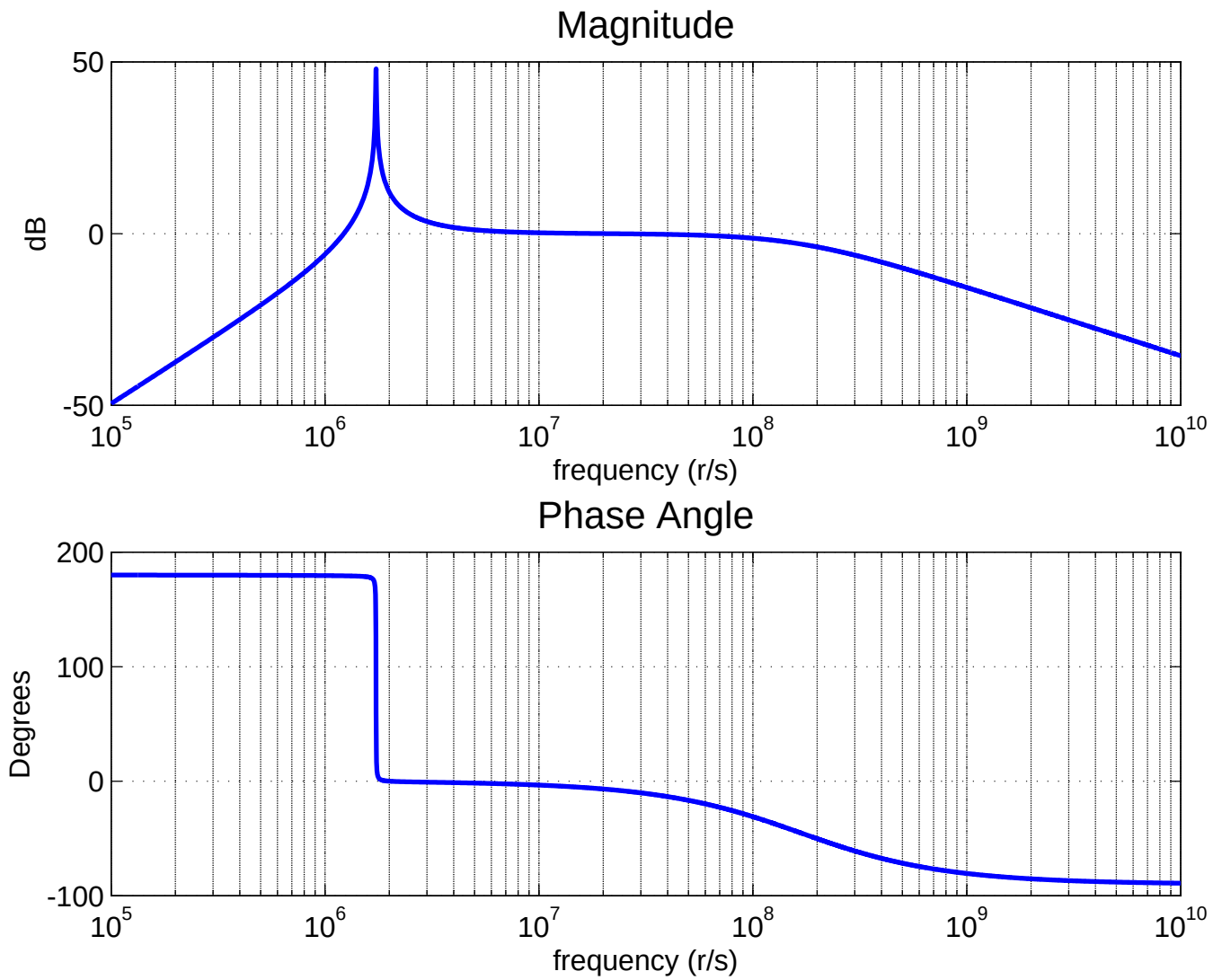
$$\omega_0 = \left(\frac{4\,\mu H}{\frac{1}{3}\,\mu F \; 3\times10^{-12}\,H^2}\right)^{1/2} = \left(2\times10^6\ r/s\right)$$

In the extra credit problem, I will illustrate getting the poles & zeroes.
For the Matlab plots, a little trial & error knowing it is a <u>bandpass</u> filter yields interesting plot range to be

$\sim 10^5$ to 10^{10} r/s

plot follows

The peak response (max of $|H|$) is $\sim \underline{1.73\times10^{\cancel{7}\,6}\ r/s}$ note this is lower than $\omega_0 = 2\times10^6$ r/s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dB
10^5
10^6
10^7
10^8
10^9
10^{10}
frequency (r/s)
Phase Angle
200
100
0
-100
Degrees
10^5
10^6
10^7
10^8
10^9
10^{10}
frequency (r/s)

4. UM 9.6

(a) As stated, it is a <u>high pass filter</u>

Why?

$\omega = 0$ All caps → open, ind → short } both take $V_o \to 0$

$\omega \to \infty$ All caps → shorts, ind → open } $V_o \to \frac{1}{2} V_i$

(b) See book section 9-2

We want: $\omega_c' = \omega_c \times 10^5$

$L' = L$

Hence we cannot do Frequency scaling only

Must do both magnitude + Frequency scaling

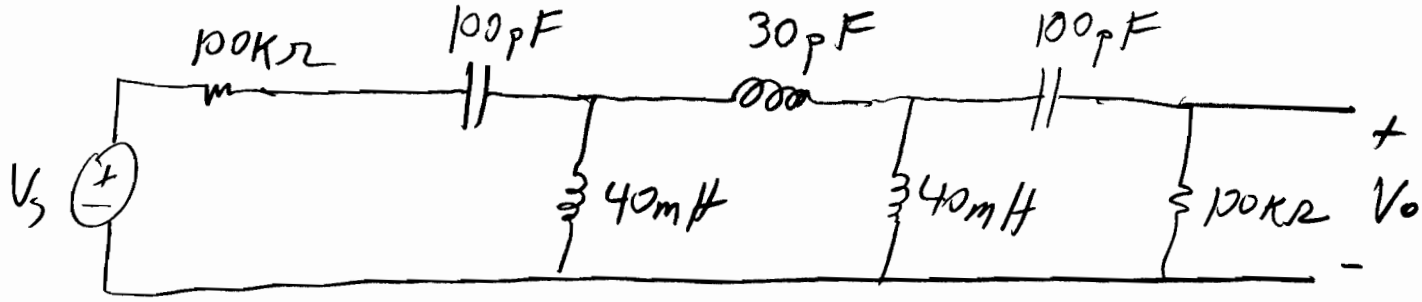
$\omega_c' = K_f \omega_c$ $\quad L' = \frac{K_m}{K_f} L = L$

$= 10^5 \omega_c$ $\quad \Rightarrow K_m = K_f = 10^5$

$C' = \frac{1}{K_m K_f} C = \frac{1}{10^{10}} C = 10^{-10} C$

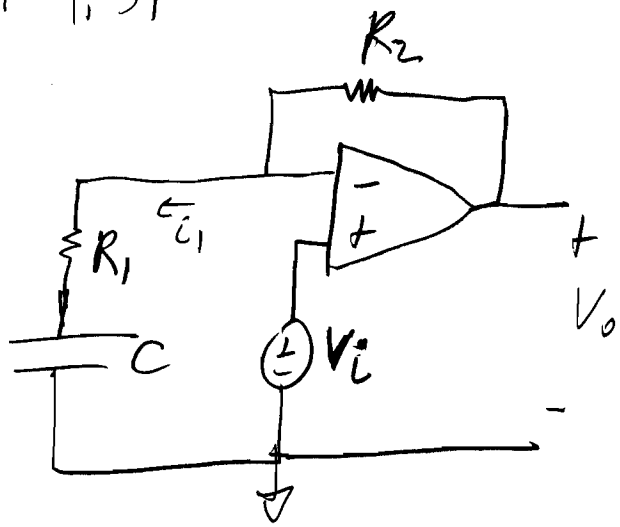
$R' = K_m R = 10^5 R$

New Scaled Circuit



5) UM 9.31

(a)



$$V_p = V_n = V_i$$

$$i_1 = \frac{V_n}{R_1 + 1/Cs} = \frac{V_i Cs}{1 + R_1 Cs}$$

$$V_o = V_n + i_1 R_2 = V_i + i_1 R_2$$

$$= V_i + \frac{V_i R_2 Cs}{1 + R_1 Cs}$$

$$\frac{V_o}{V_i} = 1 + \underbrace{\frac{R_2 Cs}{1 + R_1 Cs}}_{\text{1st order highpass}} = \frac{1 + (R_1 + R_2)Cs}{1 + R_1 Cs}$$

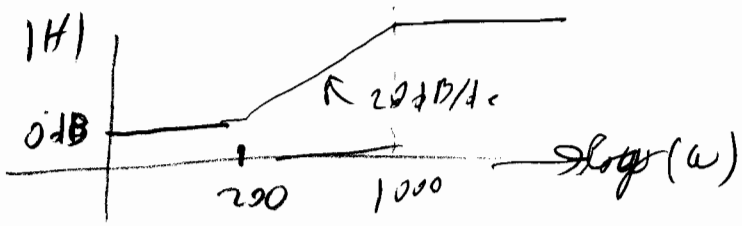
b) Bode Plots follow for

$R_1 = 1k\Omega \quad R_2 = 4k\Omega \quad C = 1\mu F$

$$H(s) = \frac{1 + (5k\Omega)(1\mu F)s}{1 + (1k\Omega)(1\mu F)s} = \frac{1 + (5\times10^{-3}\,s^{-1})s}{1 + (1\times10^{-3}\,s^{-1})s}$$

zero at $s = -200$ r/s

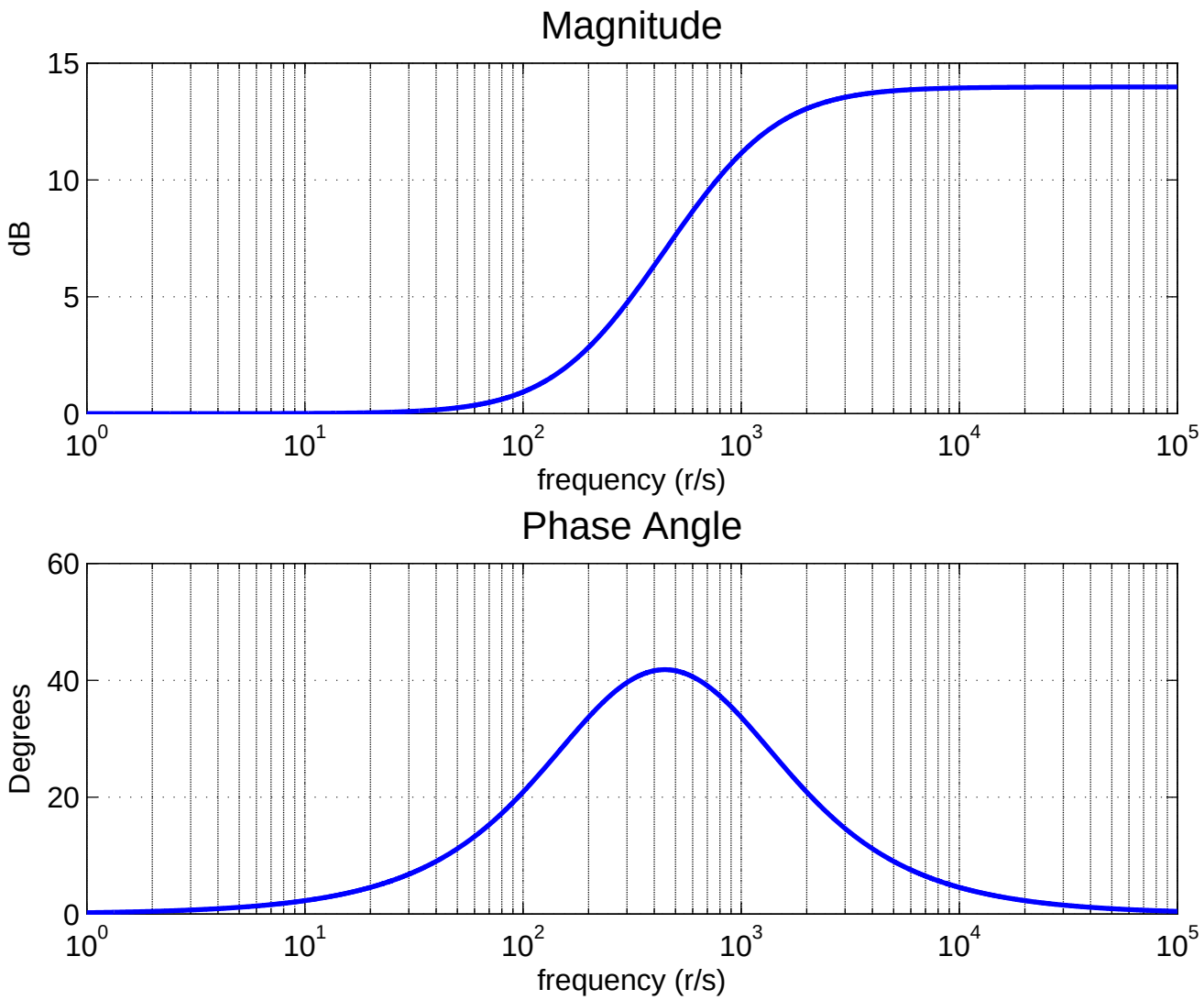
pole at $s = -1000$ r/s



c) It's basically a high pass filter but doesn't cut-off completely as $\omega \to \infty$

$$\frac{V_o}{V_i} \to \frac{R_1 + R_2}{R} = 1 + \frac{R_2}{R_1} = 5 \text{ (here)}$$

$$|H|_{max} = 5 \cong 13.98 \text{ dB}$$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dB
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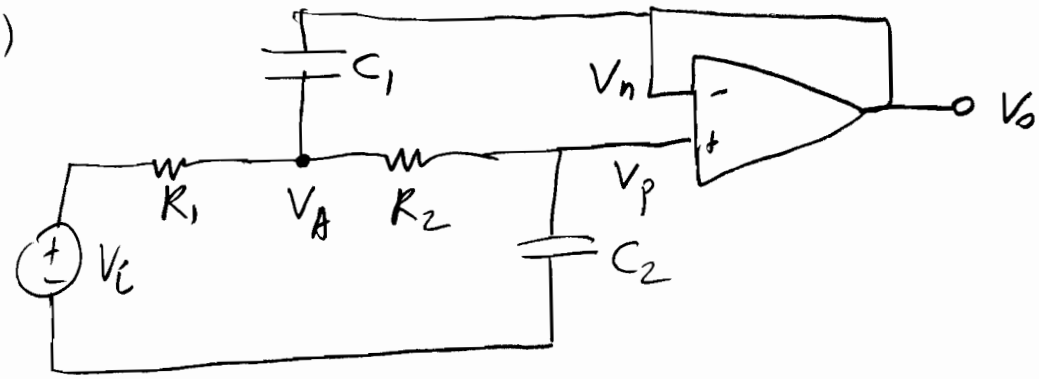
10^1
10^2
10^3
10^4
10^5
frequency (r/s)
Phase Angle
60
40
20
0
Degrees
10^0
10^1
10^2
10^3
10^4
10^5
frequency (r/s)

6) 9,33

(a)



$V_p = V_n = V_o$

$$V_A\left[\frac{1}{R_1} + \frac{1}{R_2} + C_1 s\right] + V_p\left[-\frac{1}{R_2}\right] + V_o[-C_1 s] = V_i/R_1$$

KCL at (A)

$$\Rightarrow V_A\left[\frac{1}{R_1} + \frac{1}{R_2} + C_1 s\right] - V_o\left[C_1 s + \frac{1}{R_2}\right] = V_i/R_1$$

$$V_p\left[\frac{1}{R_2} + C_2 s\right] - V_A\left[\frac{1}{R_2}\right] = 0$$ KCL at + input node

$$\Rightarrow V_o\left[\frac{1}{R_2} + C_2 s\right] - V_A\left[\frac{1}{R_2}\right] = 0$$

$$\Rightarrow V_A = V_o[1 + R_2 C_2 s]$$

So

$$V_o\left\{[1 + R_2 C_2 s]\left[\frac{1}{R_1} + \frac{1}{R_2} + C_1 s\right] - \left[C_1 s + \frac{1}{R_2}\right]\right\} = V_i/R_1$$

$$V_o\left\{[1+R_2C_2s][R_1+R_2+R_1R_2C_1s]-R_1R_2\left[C_1s+\frac{1}{R_2}\right]\right\} = V_iR_2$$

$$V_o\left\{R_1+R_2+R_1R_2C_1s+(R_1+R_2)R_2C_2s+R_1R_2^2C_1C_2s^2 - R_1R_2C_1s - R_1\right\} = V_iR_2$$

$$V_o\left\{R_2+R_2(R_1+R_2)C_2s+R_1R_2^2C_1C_2s^2\right\} = V_iR_2$$

$$V_o\left\{1+(R_1+R_2)C_2s+R_1R_2C_1C_2s^2\right\} = V_i$$

$$\boxed{H=\frac{V_o}{V_i}=\frac{1}{R_1R_2C_1C_2s^2+(R_1+R_2)C_2s+1}}$$

(b) $R_1=R_2=100\ \Omega$ $C_1=10\mu F$ $C_2=0.4\mu F$

$$H(\omega)=\frac{1}{[1-(4\times10^{-8}\omega^2)]+j(8\times10^{-5})\omega}$$

poles:

$$(4\times10^{-8})s^2+8\times10^{-5}s+1=0$$

$$s^2+4\times10^{+3}s+\frac{1}{4}\times10^{+8}=0$$

$$\alpha=2\times10^3 \quad \omega_0=\frac{1}{2}\times10^4=5\times10^3$$

$$p_{1,2} = -\alpha \pm j[\omega_0^2 - \alpha^2]^{1/2}$$

$$= -2\times10^3 \pm j4.583\times10^3$$

$$= (5\times10^3)\exp(\pm j\, 1.9823 \text{ rads})$$

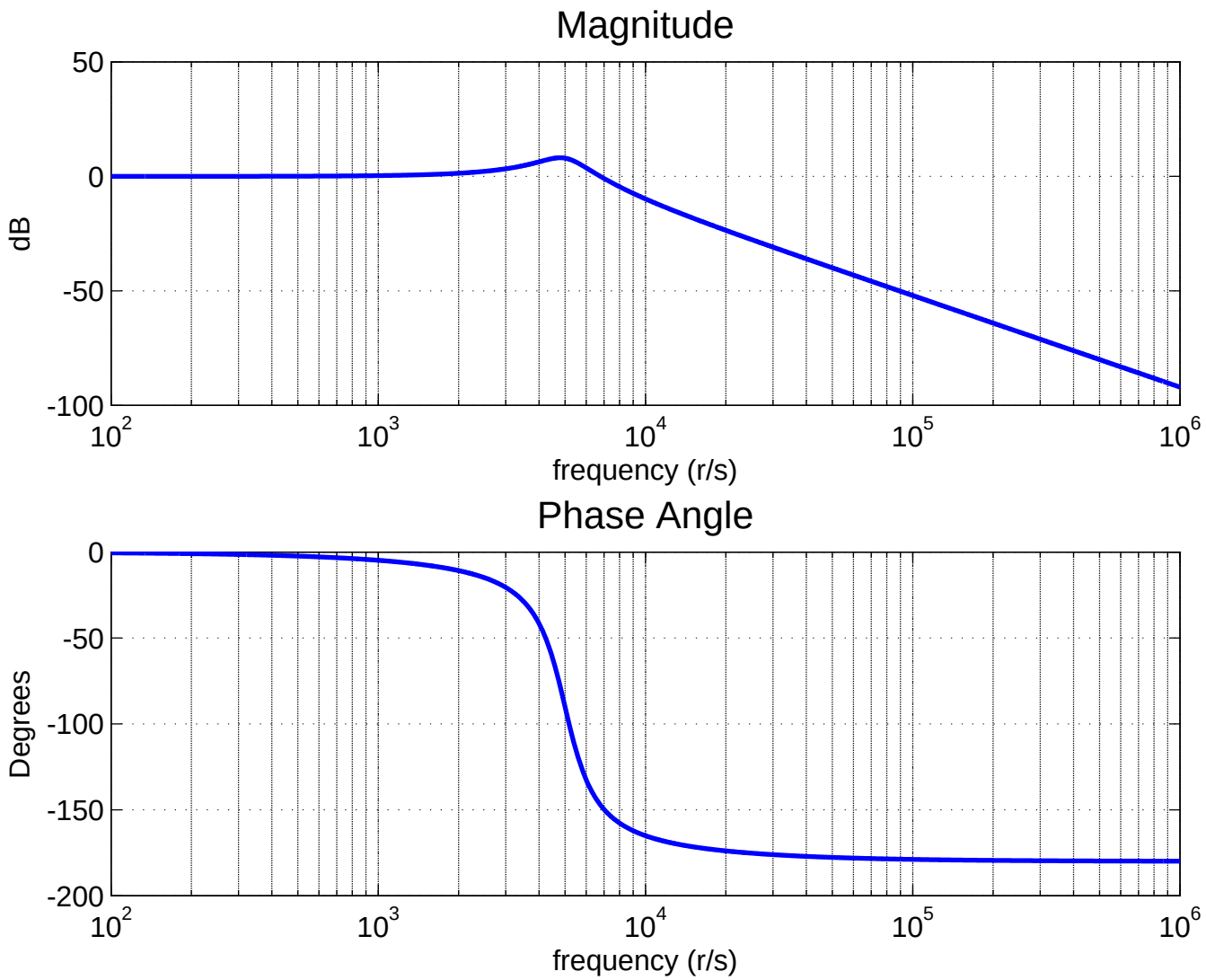
$$= (5\times10^3)\exp(\pm j\, 113.578^\circ)$$

c) Low pass filter with max gain of ~2.55 (~8.136 dB) @ ~4800 r/s

$\omega \to 0$ caps open & unity gain

$\omega \to \infty$ $V_p \to 0$ $C_2 \to$ short

$\Rightarrow V_n \to 0$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dB
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10^3
10^4
10^5
10^6
frequency (r/s)
Phase Angle
0
-50
-100
-150
-200
Degrees
10^2
10^3
10^4
10^5
10^6
frequency (r/s)