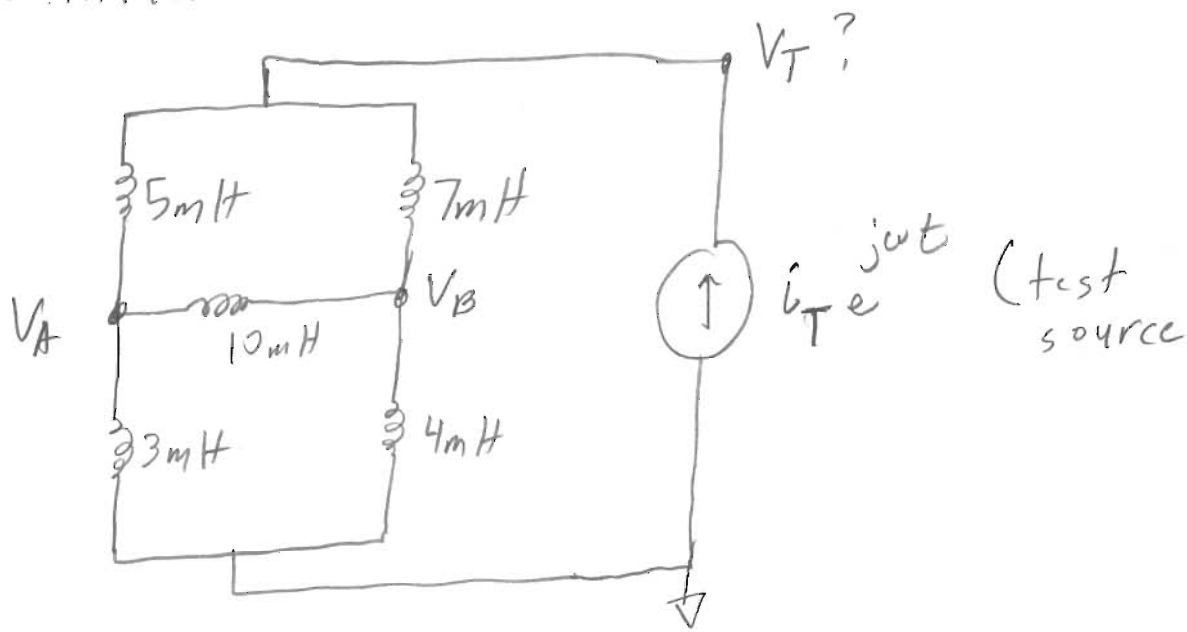
EECS 215 PS 10
Solutions

1) Norton Equivalent. This can be done multiple ways, but the key is the equivalent impedance of the inductor network



<u>complex Nodal</u>, $s = j\omega$ KCL's:

$$\frac{V_T - V_A}{(5mH)s} + \frac{V_T - V_A}{(7mH)s} = i_T$$

$$\frac{V_A - V_T}{(5mH)s} + \frac{V_A - V_B}{(10mH)s} + \frac{V_A}{(3mH)s} = 0$$

$$\frac{V_B - V_T}{(7mH)s} + \frac{V_B - V_A}{(10mH)s} + \frac{V_B}{(4mH)s} = 0$$

solving yields

$$V_T = \frac{987}{212}\, mH\, i_T\, s$$

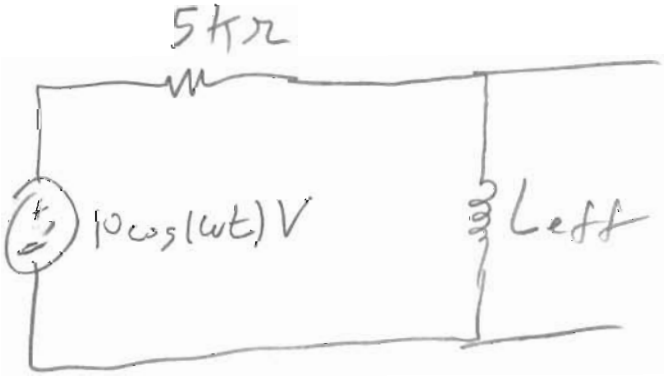
also $V_a = \frac{553}{318}\, mH\, i_T\, s$ $\qquad V_b = \frac{812}{477}\, mH\, i_T\, s$

so

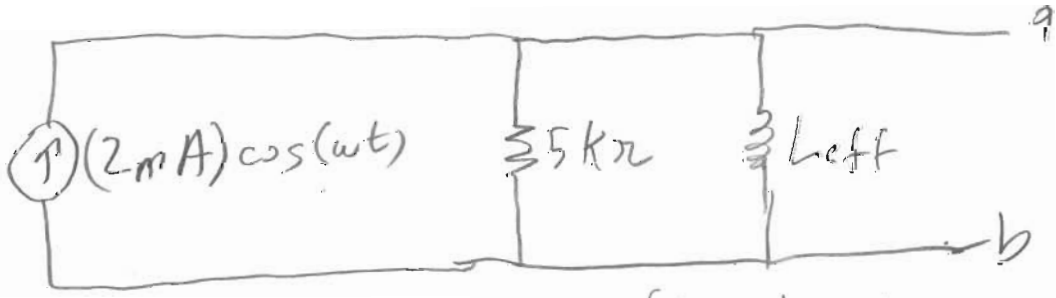
$$V_T(\omega) = \left(\left(\frac{987}{212}\right) mH\right] j\omega\, i_T = j\omega L_{eff}\, i_T$$

hence $L_{eff} = \frac{987}{212}\, mH \approx 4.656\, mH$

So Circuit becomes

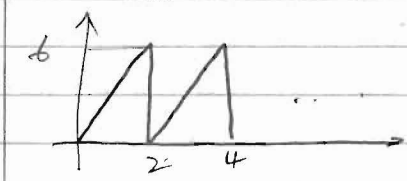


+ then by source transform



$$L_{eff} = \left(\frac{987}{212}\right) mH$$

2. UBM 8.4.



(a) average value.

$T=2$. Let's consider one period. $0 \le t \le 2$

$i(t) = 3t \qquad 0 \le t \le 2$

$$I_{av} = \frac{1}{T}\int_0^T i(t)\,dt = \frac{1}{2}\int_0^2 3t\,dt = \frac{1}{2}\cdot\left[\frac{3}{2}t^2\right]_0^2$$

$$= \frac{1}{2}\cdot\left[\frac{3}{2}\times 4 - 0\right] = \frac{1}{2}\times 6 = 3A.$$

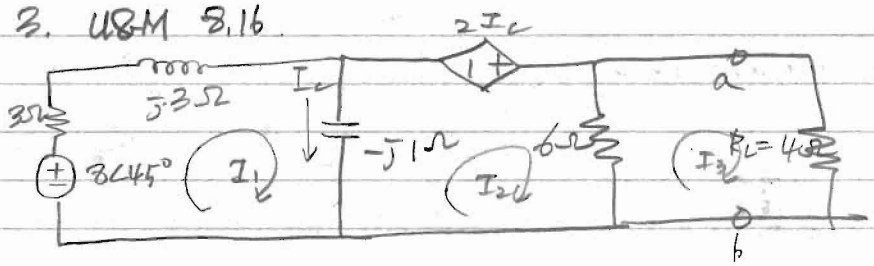
(b). Root mean square value

$$I_{rms} = \left[\frac{1}{T}\int_0^T i(t)^2\,dt\right]^{1/2}$$

$$= \left[\frac{1}{2}\int_0^2 (3t)^2\,dt\right]^{1/2} = \left[\frac{1}{2}\int_0^2 9t^2\,dt\right]^{1/2}$$

$$= \left[\frac{1}{2}\cdot\left[3t^3\right]_0^2\right]^{1/2} = \left[\frac{1}{2}\cdot(24)\right]^{1/2} = \sqrt{12} = \underline{\underline{2\sqrt{3}}}\ A.$$

3. USM 8.16



KVL.

① : $8\angle 45^\circ \Rightarrow 4\sqrt{2} + j4\sqrt{2}$

$\Rightarrow (4\sqrt{2} + j4\sqrt{2}) = 3 \times I_1 + j3 \times I_1 + (-j) \times (I_1 - I_2)$

② $(-j)(I_2 - I_1) - 2I_c + 6(I_2 - I_3) = 0.$

$\Rightarrow (-j)(I_2 - I_1) - 2 \times (I_1 - I_2) + 6(I_2 - I_3)$

$I_c = I_1 - I_2$

③ $6(I_3 - I_2) + 4I_3 = 0.$

① $\Rightarrow I_1(3 + j3 - j) + I_2(j) = 4\sqrt{2} + j4\sqrt{2}$

② $\Rightarrow I_1(j - 2) + I_2(-j + 2 + 6) + I_3(-6) = 0.$

③ $\Rightarrow I_2(-6) + I_3(10) = 0.$

$I_1 = 1.9948 + j0.2268$

$I_2 = 0.9869 - j0.1260$

$I_3 = 0.5921 - j0.0756$

$V_L = I_3 R_L = I_3 \times 4 = 2.3686 - j0.3024$

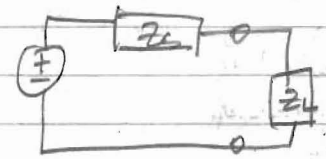
$I_L = 0.5921 - j0.0756$

$S = V_L I_L^* \times \frac{1}{2} = 0.7127$

$P_{av} = Re[S] = \frac{1}{2}|I_L|^2 R = 0.7127$

4. UFM 8.27

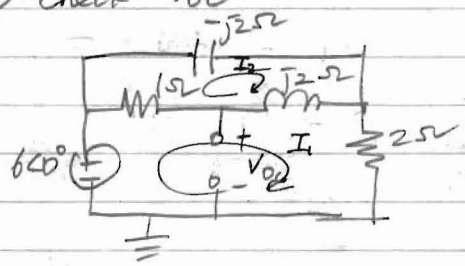
Maximum power transfered when



$Z_L = Z_s^*$

We can represent given circuit to Thevenin equivalent circuit

① check V_{oc}



KVL

① $1\times(I_1-I_2) + (j2)\times(I_1-I_2) + 2\times I_1 = 6$

$\rightarrow I_1(3+j2) + I_2(-1-j2) = 6.$

② $(-j2)I_2 + 1\times(I_2-I_1) + j2(I_2-I_1) = 0$

$I_1(-1-j2) + I_2(1-j2+j2) = 0$

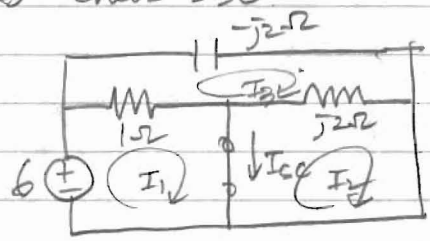
$\rightarrow I_1(-1-j2) + I_2 = 0$

$\Rightarrow I_1 = 0.9 + j0.3$

$I_2 = 0.3 + j2.1$

$V_{oc} = 6 - 1\times(I_1-I_2) = 6-(0.6-j1.8) = \boxed{5.4+j1.8}$

② Check I_{sc}



KVL

① $6 = 1\times(I_1-I_3)$

② $j2\Omega\times(I_2-I_3) = 0$

③ $-j2\times(I_3) + 1\times(I_3-I_1) + j2(I_3-I_2) = 0$

$\Rightarrow I_1(-1) + I_2(-j2) + I_3(1) = 0.$

$\Rightarrow$

$I_1(1) + I_3(-1) = 6$

$I_2(j2) + I_3(-j2) = 0.$

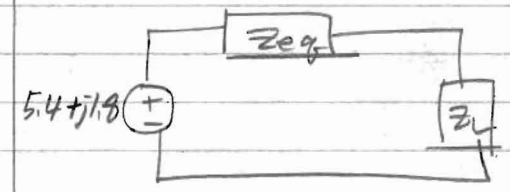
$I_1(-1) + I_2(-j2) + I_3(1) = 0.$

$\Rightarrow$ $I_1 = 6+j3$, $I_2 = j3$, $I_3 = j3$ $\Rightarrow$ $I_{sc} = I_1 - I_2 = \underline{\underline{6}}.$

$V_{oc} = 5.4 + j1.8$

$I_{sc} = 6$

$$\boxed{Z_{eq} = \frac{V_{oc}}{I_{sc}} = 0.9 + j0.3}$$



$5.4 + j1.8 \Rightarrow 5.6921 \angle 18.4349^\circ$

maximum power transfer

if $Z_L = Z_{eq}^* = 0.9 - j0.3$

$$P_{av}(max) = \frac{1}{8}\frac{|V_s|^2}{R_L} = \frac{(5.6921)^2}{8 \times 0.9} = \underline{\underline{4.5W}}$$

5. U&M 9.14

(a) $$H(\omega) = \frac{4\times10^4\,(60+j6\omega)}{(4+j2\omega)(100+j2\omega)(400+j4\omega)}$$

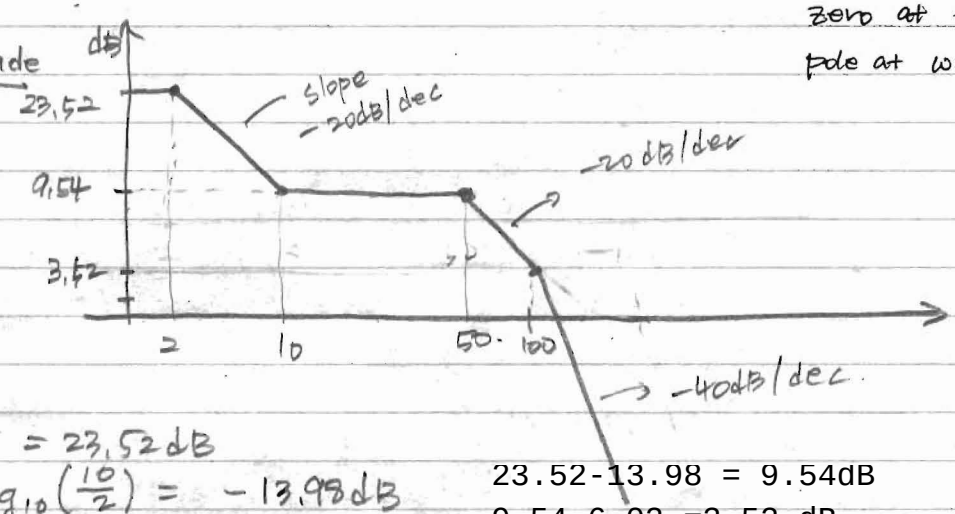
$$= \frac{\frac{1}{4}\times\frac{1}{100}\times\frac{1}{400}}{\frac{1}{4}\times\frac{1}{100}\times\frac{1}{400}} \times \frac{4\times10^4\,(60+j6\omega)}{(4+j2\omega)(100+j2\omega)(400+j4\omega)}$$

$$= \frac{15\left(1+j\frac{\omega}{10}\right)}{\left(1+j\frac{\omega}{2}\right)\left(1+j\frac{\omega}{50}\right)\times\left(1+j\frac{\omega}{100}\right)}$$

zero at $\omega=10$

pole at $\omega=2, 50, 100$

Magnitude



$20\log_{10}15 = 23.52\,dB$

$-20\times\log_{10}\left(\frac{10}{2}\right) = -13.98\,dB$

$-20\times\log_{10}\left(\frac{100}{50}\right) = -6.02\,dB$

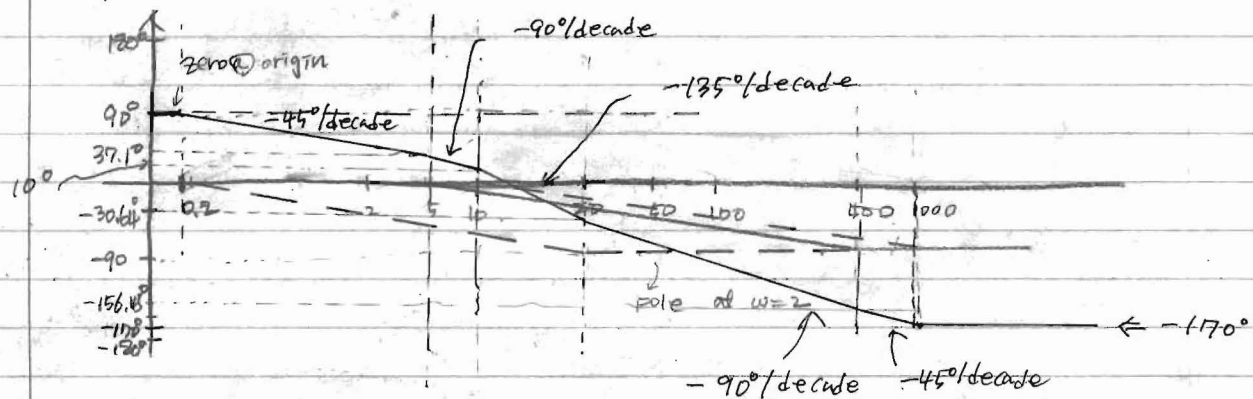
23.52-13.98 = 9.54dB

9.54-6.02 =3.52 dB

Phase

zero at $\omega=0$.

pole at $\omega=2, 50, 100$ (all first order)



$20(1+j\omega)$

9.14 (e)

$$H(\omega) = \frac{j5\times10^3\,\omega\,(20+j2\omega)}{(2500-\omega^2+j20\omega)} = \frac{\frac{1}{2500}}{\frac{1}{2500}} \times \frac{j5\times10^3\,\omega\,(20+j20\omega)}{2500-\omega^2+j20\omega}$$

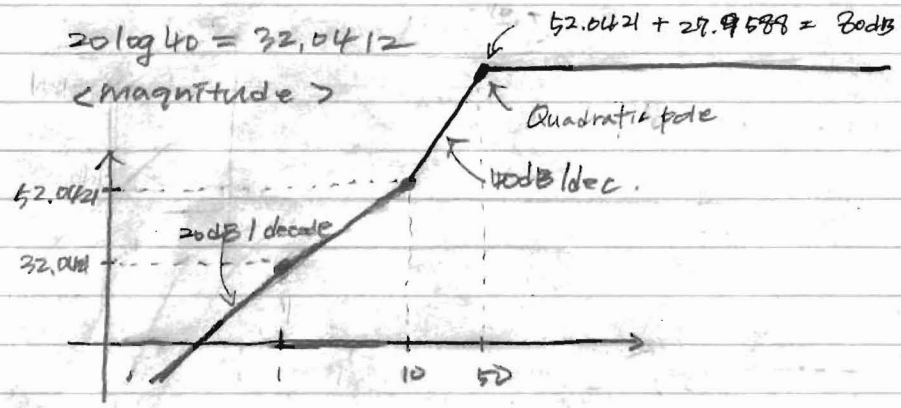
$$= \frac{40j\omega\left(1+j\frac{\omega}{10}\right)}{\left(1+j\frac{\omega}{125}+\left(\frac{\omega}{50}\right)^2\right)}$$

→ Zeros at $\omega=0$, $\omega=10$.

→ Quadratic pole at $\omega_c=50$

$20\log 40 = 32.0412$

<magnitude>



~~40log10 = 27.9588~~

20log(10/1) = 20dB

32.0421 + 20 = 52.0421 dB

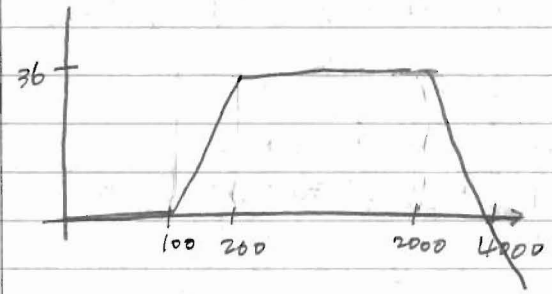
52.0421 + 40log(50/10) = 52.0421+27.9588 = 80 dB

<phase>

$j\omega = s$. $\quad (j\omega)^2 = -\omega^2 = s^2$

$$H(s) = \frac{5000\,s\,(20+2s)}{2500+20s+s^2} = \frac{10000\,s^2+10^5 s}{s^2+20s+2500}$$

6. U&M 9.18



We have zero at $\omega = 100$.

pole at $\omega = 200$ and $\omega = 2000$

At the region of $100 \le W \le 200$

$$M_2 - M_1 = 36 - 0 = (20N)\log(200) - (20N)\log(100)$$
$$= 20N(\log 2) = 20N \times 0.3$$

$\Rightarrow$ $120 = 20N$ $\boxed{N=6}$ $\Rightarrow$ <u>6th order zero at $\omega = 100$</u>

Slope = 120 dB / decades

At the range of $200 \le W \le 2000$

slope = 0 $\Rightarrow$ 6th order zero at $\omega = 200$

At the range of $2000 \le W$

$$\text{Slope} = -36 = (-20N)\log\left(\frac{4000}{2000}\right)$$
$$= -20N \times 0.3$$

$\Rightarrow N = 6$. Slope = -120 dB / decade

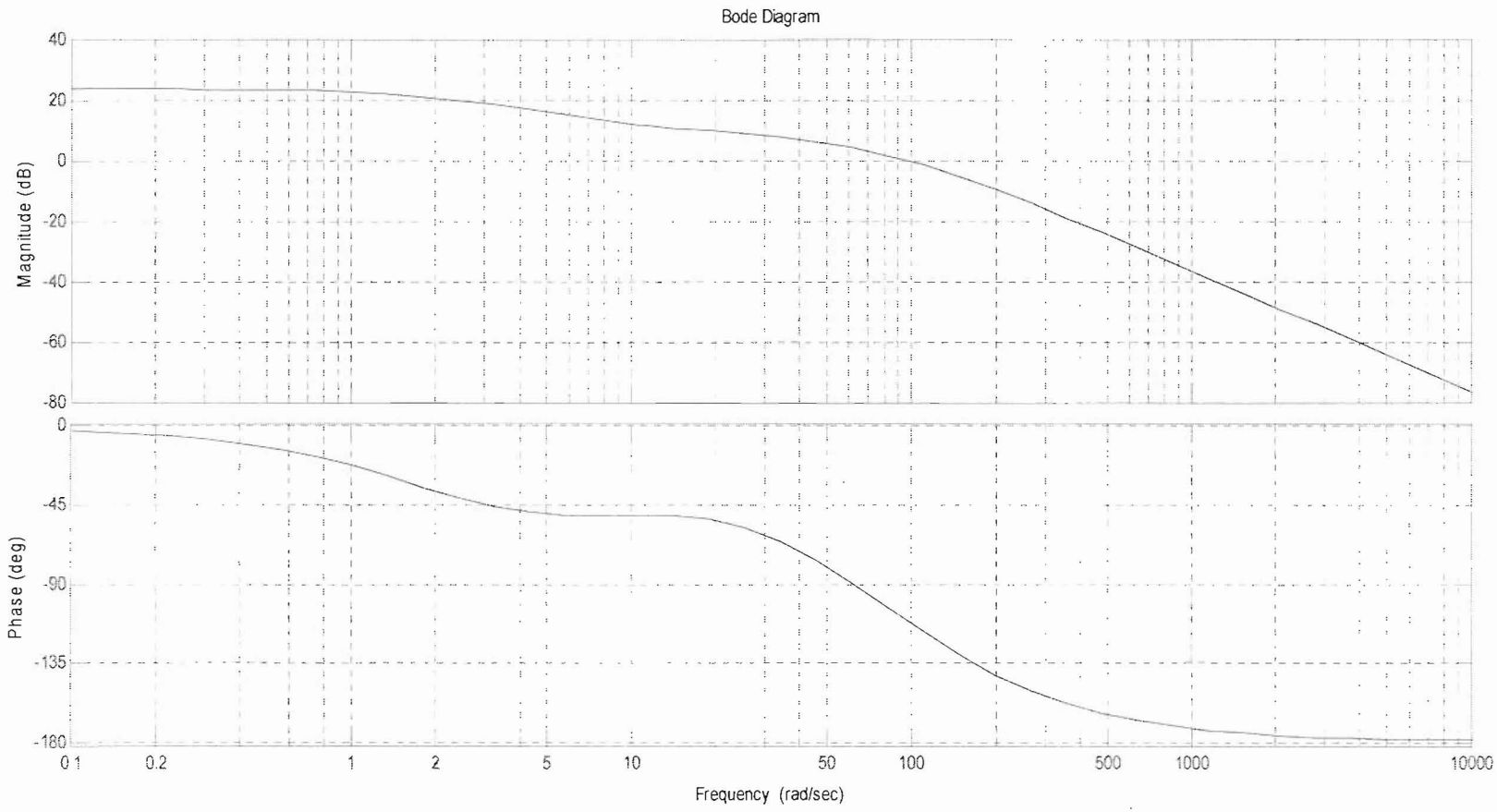
$\rightarrow$ 6th order zero at $\omega = 2000$.

$$\angle H(0) = \angle -90^\circ = \angle -\frac{\pi}{2}(\text{rad}) = -j$$

$$\Rightarrow \boxed{H(\omega) = \frac{-j\left(1 + \frac{j\omega}{100}\right)^6}{\left(1 + \frac{j\omega}{200}\right)^6\left(1 + \frac{j\omega}{2000}\right)^6}}$$

U&M 9.14 (a)

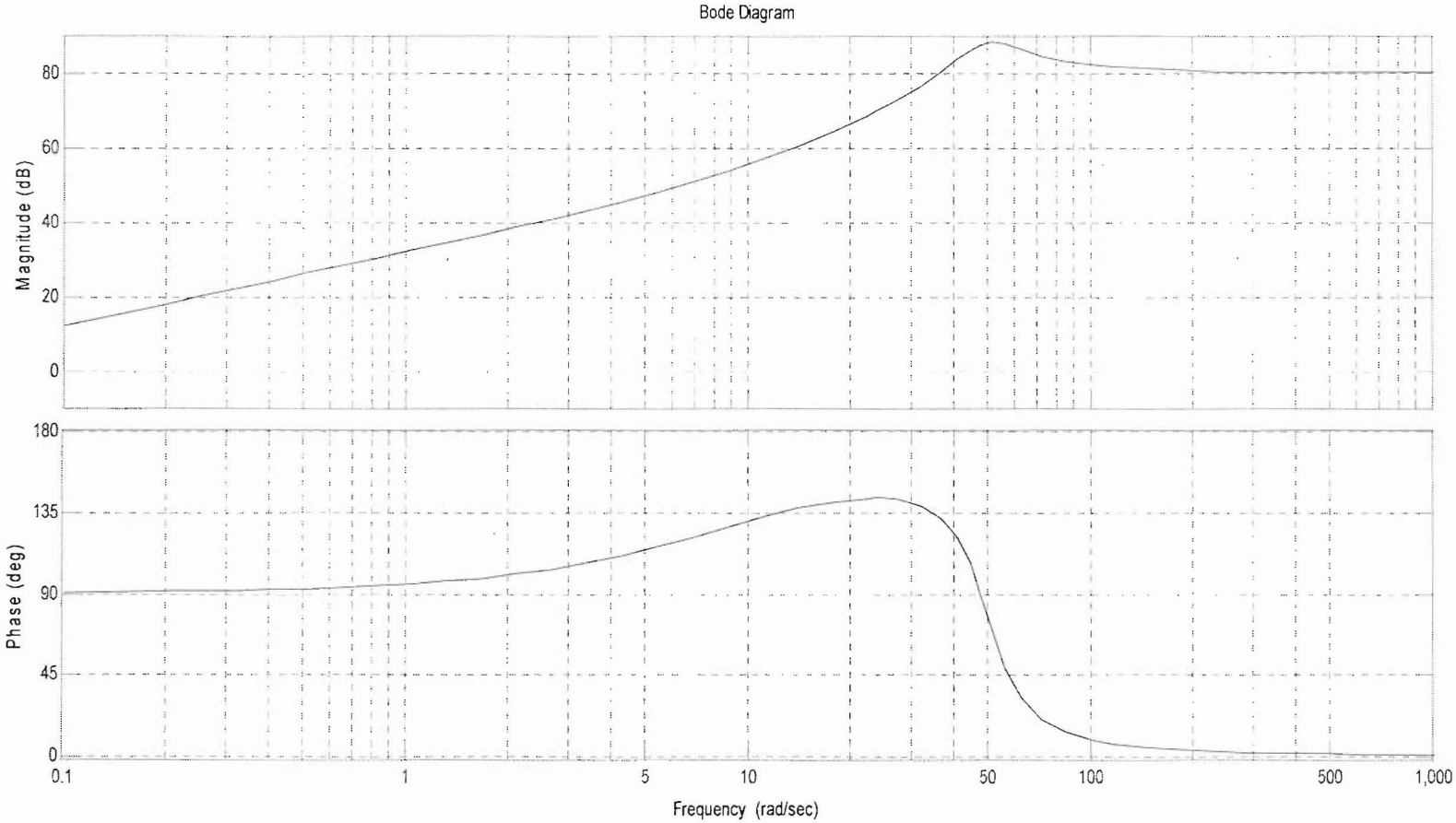
```
>> g=tf([240000 2400000],[16 2432 84800 160000])

>> bode(g)
```



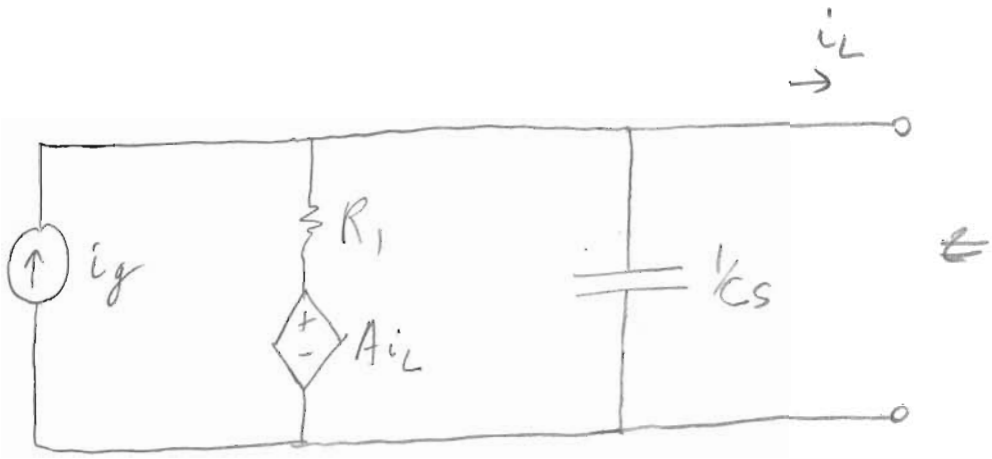
U&M 9.14 (e)

```
>> g=tf([10000 100000 0],[1 20 2500])     >> bode(g)
```

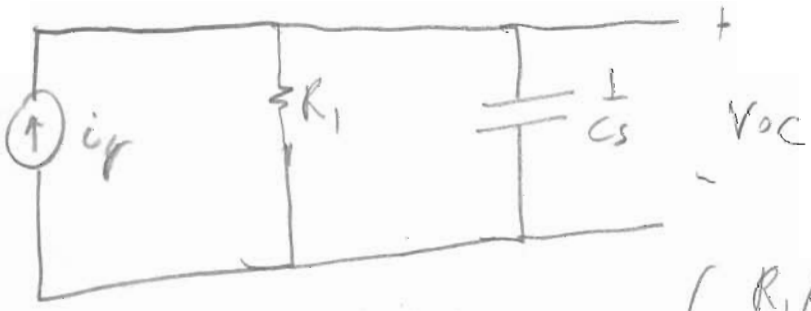
Transfer function : (10000 s^2 + 100000 s) / (s^2 + 20 s + 2500)



7. a) Find Thevenin Equivalent of circuit seen by the load



V_{oc} : $i_L = 0$ so circuit becomes



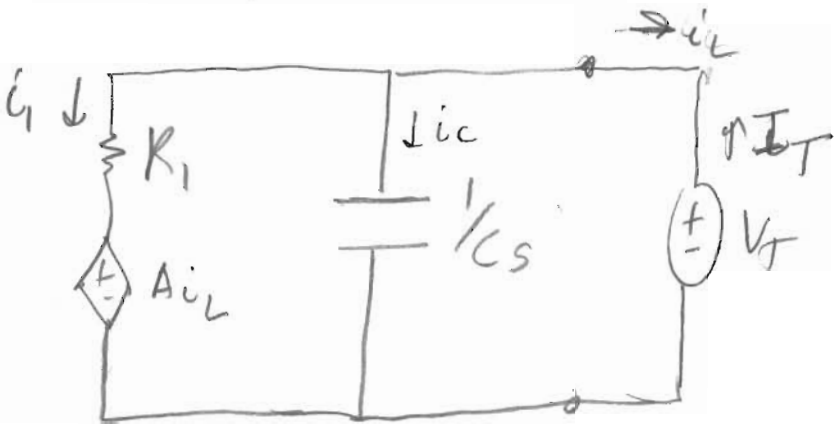
$$V_{oc} = \left(R_1 \| \frac{1}{Cs}\right) i_g = \left(\frac{R_1/Cs}{R_1 + 1/Cs}\right) i_g$$

$$= \left(\frac{R_1}{1 + R_1 Cs}\right) i_g \quad \Big| \quad V_{oc}(\omega) = \left(\frac{R_1}{1 + j\omega R_1 C}\right) i_g(\omega)$$

Now we need either Z_{eq} directly or I_{sc}

I'll do both for completeness / check

Z_{eq} $i_g \to$ off (open)



$$i_L = -I_T$$

$$i_c = V_T Cs$$

$$i_1 R_1 + A i_L = V_T = i_1 R_1 - A I_T$$

$$i_1 + i_c = I_T \Rightarrow i_1 = I_T - i_c$$

$$= I_T - V_T Cs$$

so $$V_T = R_1(I_T - V_T Cs) - A I_T$$

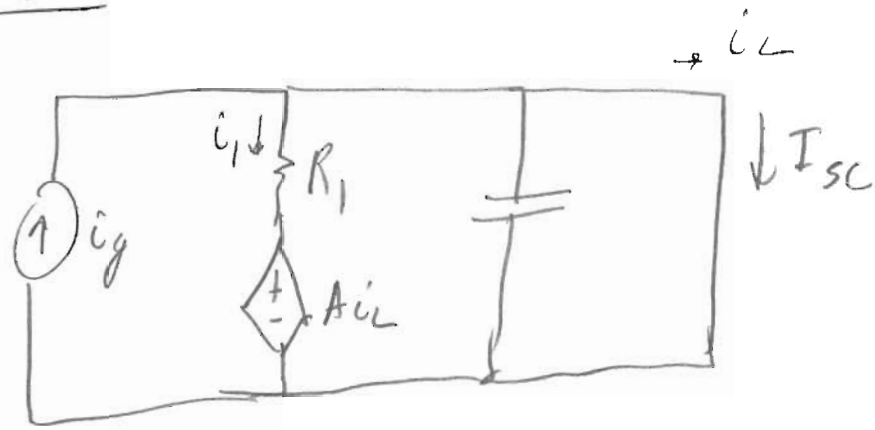
$$V_T(1 + R_1 Cs) = (R_1 - A) I_T$$

$$Z_{eq} = \frac{V_T}{I_T} = \left[\frac{(R_1 - A)}{1 + R_1 Cs}\right]$$

$$Z_{eq}(\omega) = \left[\frac{R_1 - A}{1 + j\omega R_1 C}\right]$$

$$= \left[\frac{R_1 - A}{1 + (\omega R_1 C)^2}\right](1 - j\omega R_1 C)$$

I_{sc}



$i_L = I_{sc}$

$i_g = i_1 + I_{sc}$ KCL $\Rightarrow i_1 = (i_g - I_{sc})$

$R_1 i_1 + A i_L = 0$ KVL

$\Rightarrow R_1 (i_g - I_{sc}) + A I_{sc} = 0$

$$R_1 i_g = (R_1 - A) I_{sc}$$

$$I_{sc} = \left(\frac{R_1}{R_1 - A}\right) i_g$$

$$Z_{eq} = \frac{V_{oc}}{I_{sc}} = \left(\frac{R_1}{1+R_1 Cs}\right)\left(\frac{R_1 - A}{R_1}\right)$$

$$= \left(\frac{R_1 - A}{1+R_1 Cs}\right)$$ (checks with prior result)

so with $i_g = 1mA \cos(\omega t)$ $\omega = 2000 s^{-1}$

$R_1 = 1k\Omega$ $A = 996\,\Omega$ $C = 1\mu F$

we have

$V_{TH} \cong 0.4472\ V \measuredangle -63.435°$

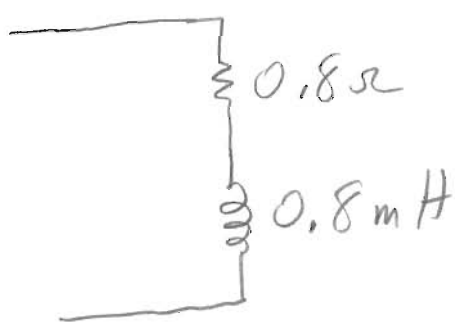
$Z_{eq} = \underbrace{0.8}_{R_{eq}} \underbrace{-1.6j}_{\text{capacitative reactance}}\ \Omega$

$Z_L = Z_{eq}^* = 0.8 + 1.6j\ \Omega$

$\omega L = 1.6\,\Omega \Rightarrow L = \frac{1.6\,\Omega}{\omega}$

$L \cong 0.8\,mH$

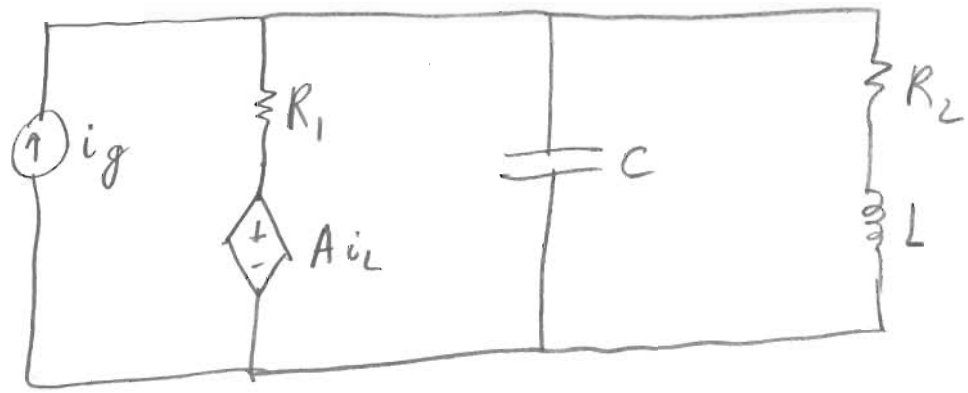
Load



Not requested, but

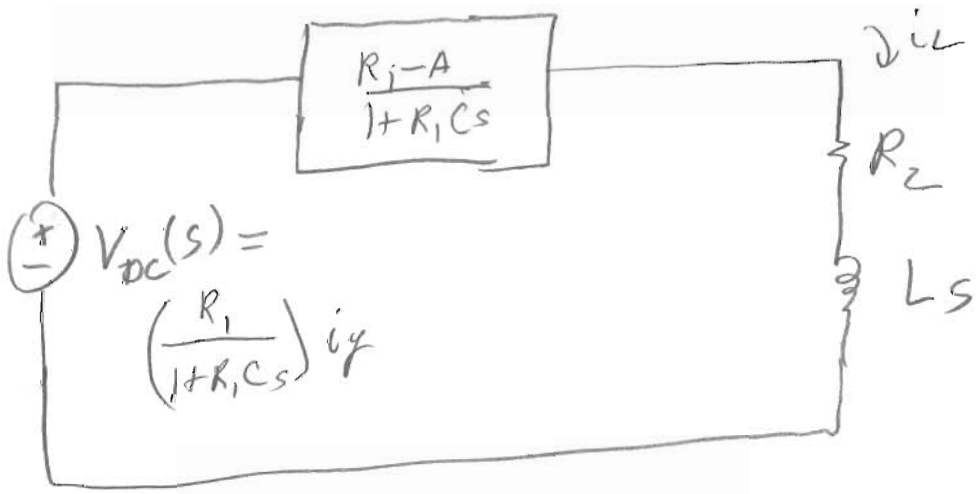
$P_L = \frac{1}{8} \frac{V_{TH}^2}{R_{eq}} = 31.25\ mW$

b) Circuit is now



$R_2 = 0.5\,\Omega$
$L = 0.8\,mH$

We found the diff eqn on Midterm 2, but here's another way, using our work from (a)



$$i_L(s) = \frac{V_{oc}}{\left(\frac{R_1 - A}{1+R_1 C s}\right) + R_2 + Ls}$$

$$= \left(\frac{R_1 i_g}{1+R_1 C s}\right) \frac{1}{\frac{R_1 - A}{1+R_1 C s} + R_2 + Ls}$$

$$= \frac{R_1 i_g}{(R_1 - A) + R_2(1+R_1 C s) + Ls(1+R_1 C s)}$$

$$i_L = \frac{R_1 \, i_g}{R_1 L C s^2 + (L + R_1 R_2 C)s + (R_1 + R_2 - A)}$$

$$\Rightarrow \left[R_1 L C s^2 + (L + R_1 R_2 C)s + (R_1 + R_2 - A)\right] i_L = R_1 \, i_g$$

$$\left[s^2 + \left(\frac{1}{R_1 C} + \frac{R_2}{L}\right)s + \left(\frac{R_1 + R_2 - A}{R_1}\right)\frac{1}{LC}\right] i_L = \left(\frac{R_1}{R_1 + R_2 - A}\right)\frac{1}{LC} i_g$$

$$\Rightarrow \frac{d^2 i_L}{dt^2} + \underbrace{\left(\frac{1}{R_1 C} + \frac{R_2}{L}\right)}_{2\alpha} \frac{di_L}{dt} + \underbrace{\left[\left(\frac{R_1 + R_2 - A}{R_1}\right)\frac{1}{LC}\right]}_{\omega_0^2} i_L = \left(\frac{R_1}{R_1 + R_2 - A}\right)\frac{1}{LC} i_g$$

Plugging in

$\alpha = 10^3 \, s^{-1}$ $\quad \omega_0 \cong 2.4495 \times 10^3 \, s^{-1}$

$\Rightarrow$ under damped

$\omega_d = [\omega_0^2 - \alpha^2]^{1/2} \cong 2.236 \times 10^3 \, s^{-1}$

So $i_{Ln}(t) = \left[B_1 \cos(\omega_d t) + B_2 \sin(\omega_d t)\right] e^{-\alpha t}$

Forced Response ?

Define $H(s) = \dfrac{i_L}{i_g} = \dfrac{R_1}{\{R_1 L C s^2 + (L + R_1 R_2 C)s + (R_1 + R_2 - A)\}}$

Let $s = j\omega$, then

$$i_{Lf}(t) = |i_g|\,|H(\omega)|\cos(\omega t + \measuredangle H(\omega))$$

$$\cong (0.2795A)\cos(\omega t - 63.4395^\circ) \qquad \omega = 2000\,s^{-1}$$

B.C.?

$$\left.\begin{array}{l} i_{L1}(0) = 0 \\ V_C(0) = 0 \end{array}\right\} \Rightarrow V_L(0) = 0$$

since $V_C = i_L R_2 + V_L$

so

$$i_L(0) = B_1 + i_{Lf}(0)$$

$$V_L(0) = L\left[(-\alpha B_1 + B_2\omega_d) - \omega|i_g|\,|H(\omega)|\sin(\measuredangle H(\omega))\right] = 0$$

so $B_1 = -i_{Lf}(0) \simeq -0.125\ A$

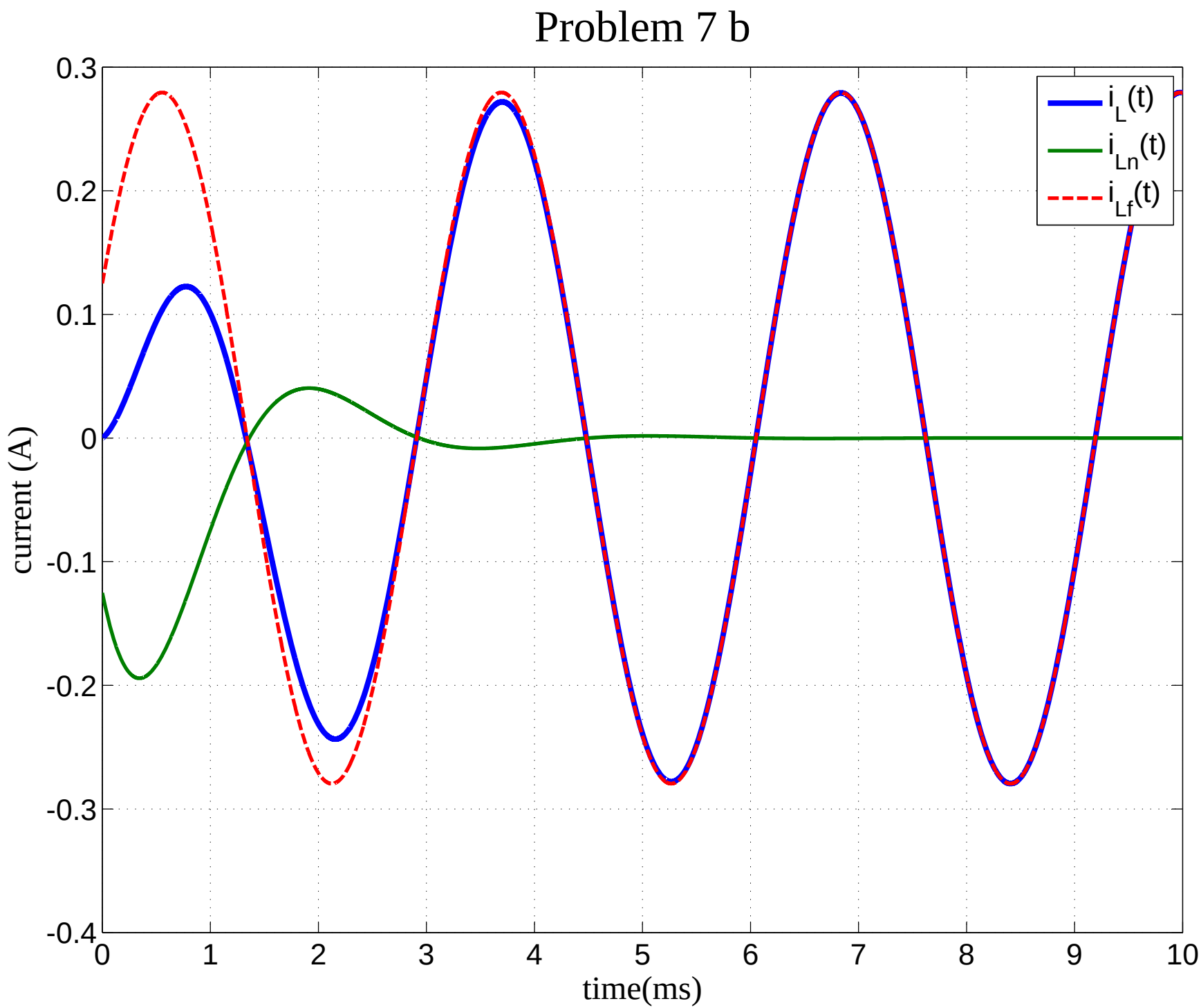
$$B_2 = \frac{1}{\omega_d}\left[\alpha B_1 + \omega|i_g|\,|H(\omega)|\sin(\measuredangle H(\omega))\right]$$

$$\simeq -0.2795\ A$$

$\therefore$ $i_L(t) = i_{Ln}(t) + i_{Lf}(t)$

plot + matlab code follow.

plot is from 0 to $\frac{10}{\alpha} = 10\,ms$


Problem 7 b
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
current (A)
0
1
2
3
4
5
6
7
8
9
10
time(ms)
i_L(t)
i_Ln(t)
i_Lf(t)

```
%problem set 10 number 7
clear all;
close all;
format long;
r1=1e3;
a=996;
c=1e-6;
ig=1e-3;
w=2000;
% Part a
zeq=(r1-a)/(1+(j*w*r1*c))
zl=conj(zeq)
req=real(zl);
L=imag(zl)/w
voc=r1/(1+(j*w*r1*c))*ig
abs(voc)
angle(voc)*180/pi
pl=abs(voc)^2/8/req
r2=req;
% Part b
alpha=0.5*(1/(r1*c)+r2/L)
w02=(r1+r2-a)/r1/L/c
w0=sqrt(w02)
wd=sqrt(w02-alpha^2)
s=j*w;
h=r1/(r1*L*c*s^2+(L+r1*r2*c)*s+r1+r2-a);
t=0:1e-3:10;
t=t/alpha;
%magnitude of ilf
ig*abs(h)
%phase in degrees of ilf
angle(h)*180/pi
ilf=ig*abs(h)*cos(w*t+angle(h));
b1=-ilf(1)
b2=1/wd*(alpha*b1+w*ig*abs(h)*sin(angle(h)))
iln=(b1*cos(w*t)+b2*sin(w*t)).*exp(-alpha*t);
il=iln+ilf;
h=plot(t*1000,il,t*1000,iln,t*1000,ilf,'--');
set(h,'linewidth',2);
legend('i_L(t)','i_{Ln}(t)','i_{Lf}(t)');
xlabel('time(ms)');
ylabel('current (A)');
title('Problem 7 b');
grid;
```

c) H(s) result given in (b)

$$H(s) = \frac{R_1}{R_1 L C s^2 + (L + R_1 R_2 C)s + (R_1 + R_2 - A)}$$

Plugging in #'s

$$H(s) = \frac{(1000\Omega)}{(8\times10^{-7}\Omega\, s^2)s^2 + (1.6\times10^{-3}\Omega s)s + 4.8\Omega}$$

$$= \frac{(1.25\times10^{9}\, s^{-1})}{s^2 + (2000 s^{-1})s + (6\times10^{6} s^{-2})}$$

2 poles

$$s_{1,2} = -1000 s^{-1} \pm [10^6 s^{-1} - 6\times10^6 s^{-2}]^{1/2}$$

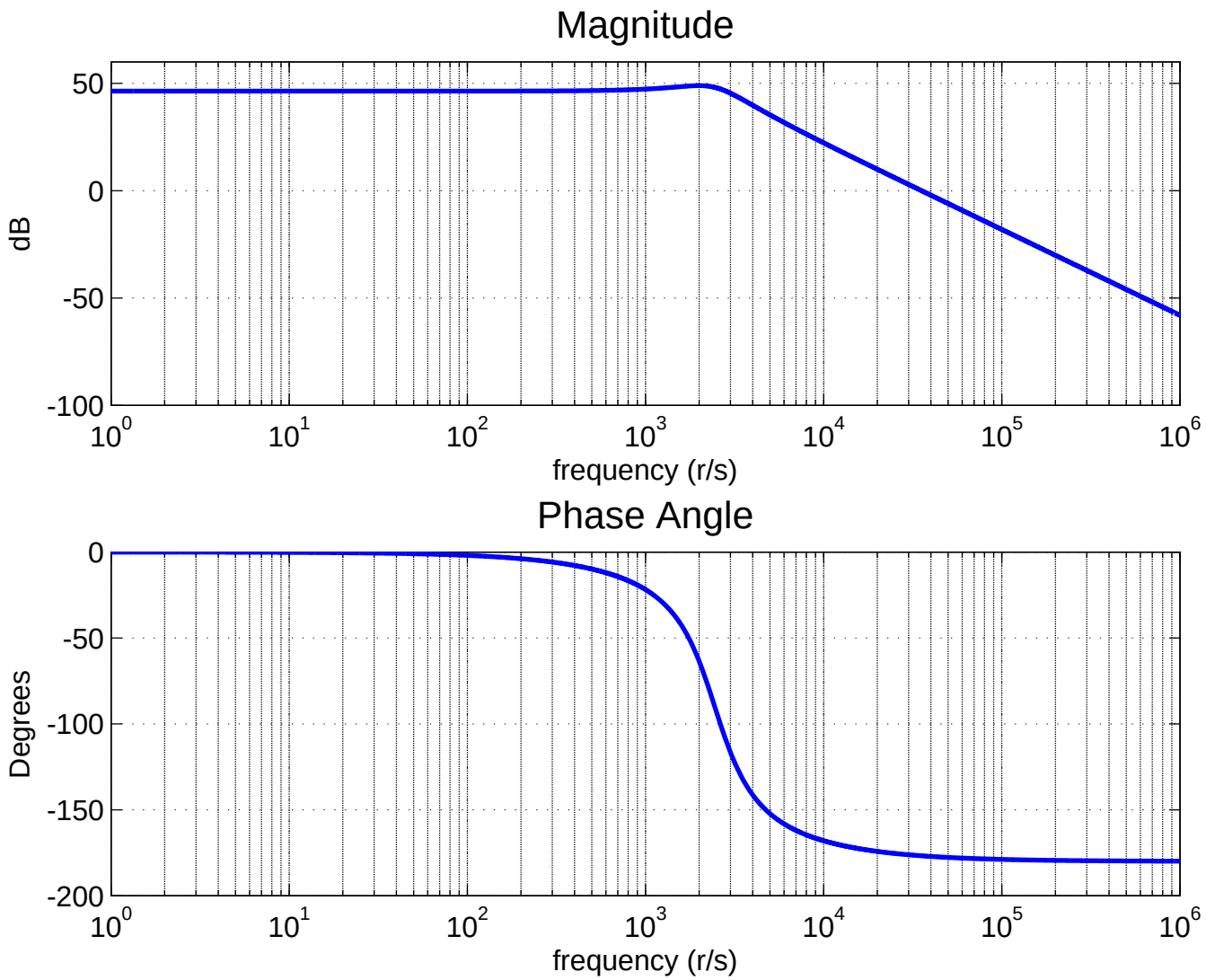
$$= -10^3 \pm j10^3[5]^{1/2} s^{-1} = 10^3(1 \pm j\sqrt{5}) s^{-1}$$

$$= (\sqrt{6}\times10^3) e^{\pm j\phi} \qquad \phi = \tan^{-1}\sqrt{5} = 1.1503\ r$$
$$\simeq 65.905^\circ$$

$$\simeq (2.4495\times10^3 s^{-1}) e^{\pm j\phi}$$

I plotted the transfer function using the Bode plot code provided on Ctools (example plotting m-file)

The action in the plot is around $\omega_n = \sqrt{6}\times10^3 r/s$ and the -40 dB/dec slope at high ω make sense from the 2 poles


Magnitude
dB
50
0
-50
-100
10^0
10^1
10^2
10^3
10^4
10^5
10^6
frequency (r/s)
Phase Angle
Degrees
0
-50
-100
-150
-200
10^0
10^1
10^2
10^3
10^4
10^5
10^6
frequency (r/s)

d) My Multisim file will be included as an upload on the Ctools site

The Bode plot result follows agrees well with the Matlab result.

The ~49dB peak response occurs at $f \approx 316$ Hz

$\rightarrow \omega \approx 1.985 \times 10^3$ r/s

which matches well with Matlab

It is not required, but pasting the Multisim results into Matlab yields virtually exact agreement

PS10numb7d

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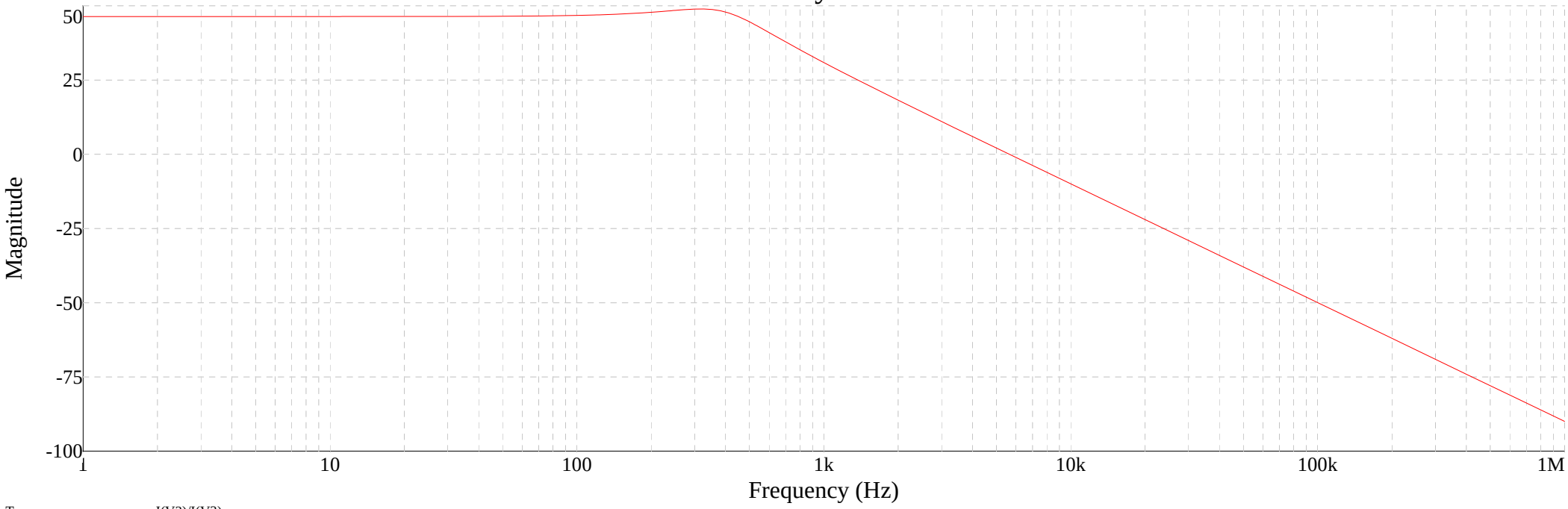
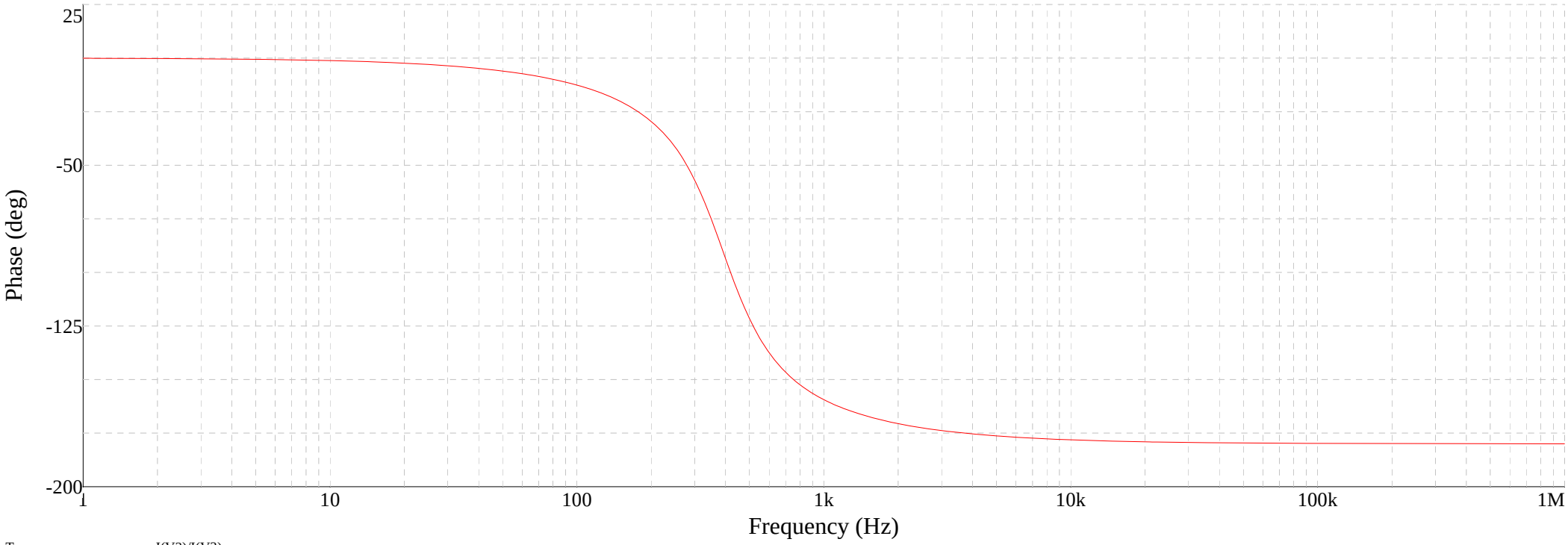
AC Analysis



Traces: I(V2)/I(V3),



Traces: I(V2)/I(V3),