EECS 215 HW#1 Solution

1. Electron charge : $1.6021 * 10^{-19}\ C$
   Doubly positively charged He nuclei charge : $2 * 1.6021 * 10^{-19}\ C$

$$\Delta Q_{electron} = 1.6021 * 10^{-19} * 4 * 10^{20}\ C$$
$$\Delta Q_{He\ nuclei} = 2 * 1.6021 * 10^{-19} * 2 * 10^{20}\ C$$

   Electron direction : -z direction → current in +z direction
   He nuclei direction : +z direction → current in +z direction
   The direction of current is opposite of the direction of electron. And the direction of current is same with the direction of positively charged nuclei flow.

$$i_{electron} = \frac{\Delta Q_{electron}}{\Delta t} = \frac{1.6021 * 10^{-19} * 4 * 10^{20}}{5} = 12.8168\ A$$
$$i_{He\ Nuclei} = \frac{\Delta Q_{He\ Nuclei}}{\Delta t} = \frac{2 * 1.6021 * 10^{-19} * 2 * 10^{20}}{5} = 12.8168\ A$$

   Summing up two currents, $i = i_{electron} + i_{He\ Nuclei}$ = 25.6336 A

2. U&M 1.8(d)

$$q(t) = 1.7 * t * [1 - e^{-1.2t}]\ nC$$

$$i(t) = \frac{dq(t)}{dt} = \frac{d}{dt}[1.7t * (1 - e^{-1.2t})] * 10^{-9}$$
$$= \{1.7 * (1 - e^{-1.2t}) + 1.7t * (1.2e^{-1.2t})\} * 10^{-9}$$
$$= 1.7 * [1 - e^{-1.2t} + 1.2 * e^{-1.2t}]\ nA$$

3. U&M 1.10(b)

$$\Delta Q = \int_{t_1}^{t_2} i\ dt = \int_0^{0.05} 4\sin(40\,\pi t)\cos(40\,\pi t) * 10^{-6}\ dt$$
$$= \int_0^{0.05} 2 * 2\sin(40\,\pi t)\cos(40\,\pi t) * 10^{-6}\ dt$$
$$= \int_0^{0.05} 2 * \sin(80\,\pi t) * 10^{-6}\ dt = 2 * 10^{-6} * \left[\frac{1}{80\pi t} * (-\cos(80\pi t))\right]_0^{0.05}$$
$$= 2 * 10^{-6} * \big(-1 - (-1)\big) = 0\ C$$

*Third equality comes from 2sin(ax)cos(ax) = sin(2ax)

4. U&M 1.1.5

$$q(t) = \begin{bmatrix} 2t & if \;\; 0 \le t \;\le 2 \\ 4 & if \; 2 < t \le 4 \\ -2t + 12 & if \; 4 < t \le 8 \end{bmatrix}$$

$$i(t) = \frac{dq(t)}{dt} = \begin{bmatrix} 2 & if \;\; 0 \le t \;\le 2 \\ 0 & if \; 2 < t \le 4 \\ -2 & if \; 4 < t \le 8 \end{bmatrix}$$

Accordingly,

(a) t = 1 → i(1) = 2A
(b) t = 3 → i(3) = 0A
(c) t = 6 → i(6) = -2A

5. Refer to the passive sign convention
When current flows into the (+) side of the device, p>0 and power is delivered to the device.
When current flows into the (-) side of the device, p<0 and power is supplied from the device.

(a) Device 1 : Current flows out from (+) side of the device = current flows into the (-) side of the device → Device 1 is supplier
P = VI = 16 x (-4) = -64 → 64W is supplied by Device 1
(b) Device 2 : Current flows into the (+) side of the device → Device 2 is recipient
P = VI = 6 x 4 = 24 → Device 2 received 24W
(c) Device 3 : Current flows into the (+) side of the device → Device 3 is recipient
P = VI = 10 x 1 = 10 → Device 3 received 10W
(d) Device 4 : Current flows into the (+) side of the device → Device 4 is recipient
P = VI = 8 x 1 = 8 → Device 4 received 8W
(e) Device 5 : Current flows into the (+) side of the device → Device 5 is recipient
P = VI = 4 x 1 = 4 → Device 5 received 4W
(f) Device 6 : Current flows into the (+) side of the device → Device 6 is recipient
P = VI = 12 x 2 = 24 → Device 6 received 24W
(g) Device 7 : current flows into the (-) side of the device → Device 7 is supplier
P = VI = 9 x (-3) = -27 → 27W is supplied by Device 7
(h) Device 8 : Current flows into the (+) side of the device → Device 8 is recipient
P = VI = 21 x 3 = 21 → Device 6 received 21W

Summing up from (a) to (h) → 0

6. U&M2.3

From table 2-1 in page 34, resistivity ρ of pure Germanuim = 0.47.

And we have $R = \rho \frac{l}{A}$

(a) Top and bottom

$$A = (2 * 10^{-3}) * (1 * 10^{-3}) = 2 * 10^{-6}$$
$$l = 0.2 * 10^{-3} = 2 * 10^{-4}$$
$$R = 0.47 * \frac{(2 * 10^{-4})}{2 * 10^{-6}} = 47\ \Omega$$

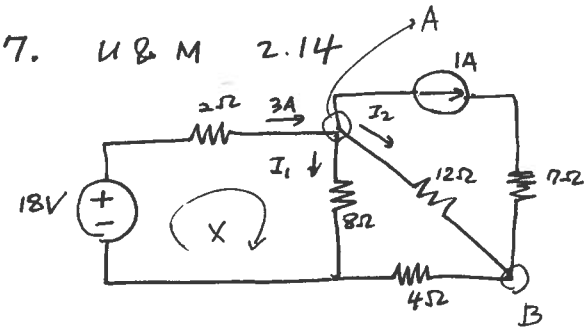
(b) Front and back

$$A = (0.2 * 10^{-3}) * (2 * 10^{-3}) = 4 * 10^{-7}$$
$$l = 1 * 10^{-3}$$
$$R = 0.47 * \frac{(1 * 10^{-3})}{4 * 10^{-7}} = 1175\ \Omega$$

(c) Right and left

$$A = (1 * 10^{-3}) * (0.2 * 10^{-3}) = 2 * 10^{-7}$$
$$l = 2 * 10^{-3}$$
$$R = 0.47 * \frac{(2 * 10^{-3})}{2 * 10^{-7}} = 4700\ \Omega$$

7. U & M 2.14



- USE KCL at Node A and B.

A → $3 = I_1 + I_2 + 1$ → $I_1 + I_2 = 2$ — ①

B → $1 + I_2 = I_3$ → $-I_2 + I_3 = 1$ — ②

- use KVL at loop X.

→ $18 = 2 \times 3 + I_1 \times 8$

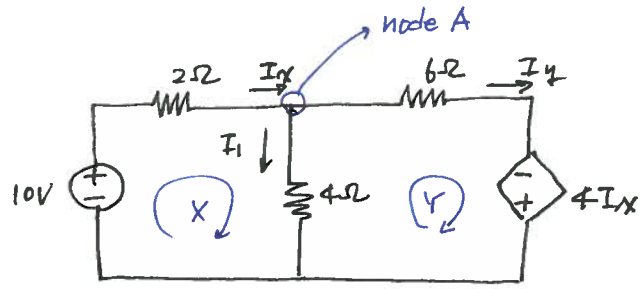
→ $8 I_1 = 12$

→ $\boxed{I_1 = 1.5 \text{ A}}$

- From eq. ① → $1.5 + I_2 = 2$ → $\boxed{I_2 = 0.5 \text{A}}$

From eq. ② → $-0.5 + I_3 = 1$ → $\boxed{I_3 = 1.5 \text{A}}$

8. U&M 2.19.



- At node A, use KCL

  $\rightarrow I_x = I_1 + I_y$ — ①

- At loop X, use KVL

  $10V = 2 \times I_x + 4 \times I_1$ — ②

- At loop Y, use KVL

  $-4 \times I_1 + 6 \times I_y - 4 \times I_x = 0.$ — ③

- From eq ① $\rightarrow$ $I_1 = I_x - I_y$

  Substitute to eq. ② and ③.

② $\Rightarrow$ $10V = 2I_x + 4 \times (I_x - I_y) = 6I_x - 4I_y$ — ④

③ $\Rightarrow$ $-4 \times (I_x - I_y) + 6I_y - 4I_x = -8I_x + 10I_y = 0.$ — ⑤

$4 \times$ ④ $+$ $3 \times$ ⑤

$\Rightarrow$ $40 = 24I_x - 16I_y$

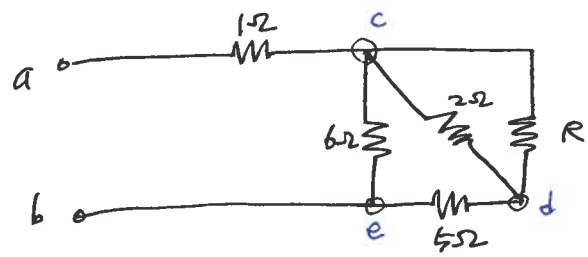
$+ \quad 0 = -24I_x + 30I_y$

$40 = 14I_y$

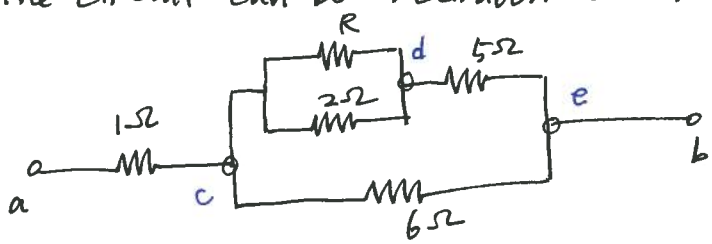
$$\boxed{I_y = \frac{20}{7} A}$$

From eq. ⑤ $\boxed{I_x = \frac{5}{4} I_y = \frac{5}{4} \times \frac{20}{7} = \frac{25}{7} A.}$

9. U & M 2.25



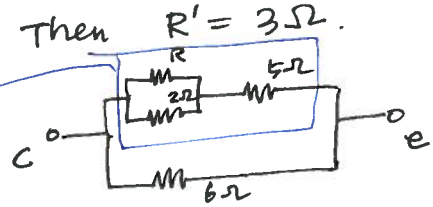
The circuit can be redrawn as follows.



And, we know $R_{eq} = 4\Omega$. We can replace resistances between c and e as R'

$\Rightarrow$

Then $R' = 3\Omega$.



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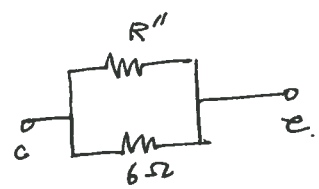
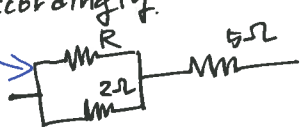


$$\Rightarrow \frac{1}{R'} = \frac{1}{R''} + \frac{1}{6} \Rightarrow \frac{1}{3} = \frac{1}{R''} + \frac{1}{6} \Rightarrow \frac{1}{R''} = \frac{1}{6} \Rightarrow R'' = 6\Omega.$$

Accordingly.



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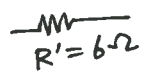

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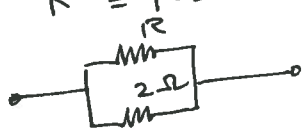
Then, $R''' = 1\Omega$

$\Rightarrow$ 

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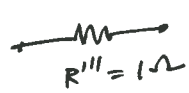
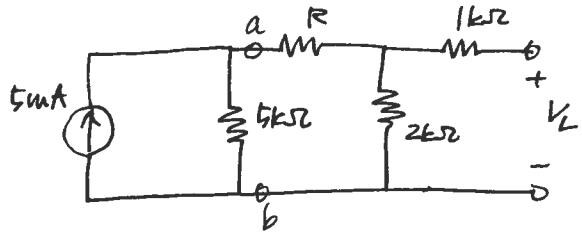


$$\frac{1}{R'''} = \frac{1}{R} + \frac{1}{2} = 1 \Rightarrow \boxed{R = 2\Omega.}$$

10. U&M 2.29



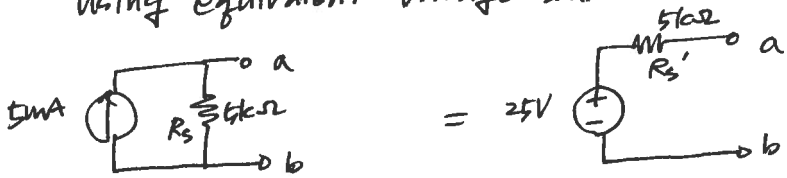
Because 1kΩ resistance is not connected, it is open circuit.

No current flows through 1kΩ resistance

→ No effect on V_L.

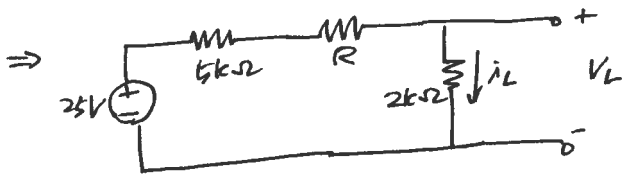
We can remove 1kΩ resistance without any effect.

Using equivalent voltage source



$V_s = I_s R_s$

$R_s = R_s'$



We know $V_L = 5V$ → $i_L = \frac{V_L}{2k\Omega} = 2.5\,mA$

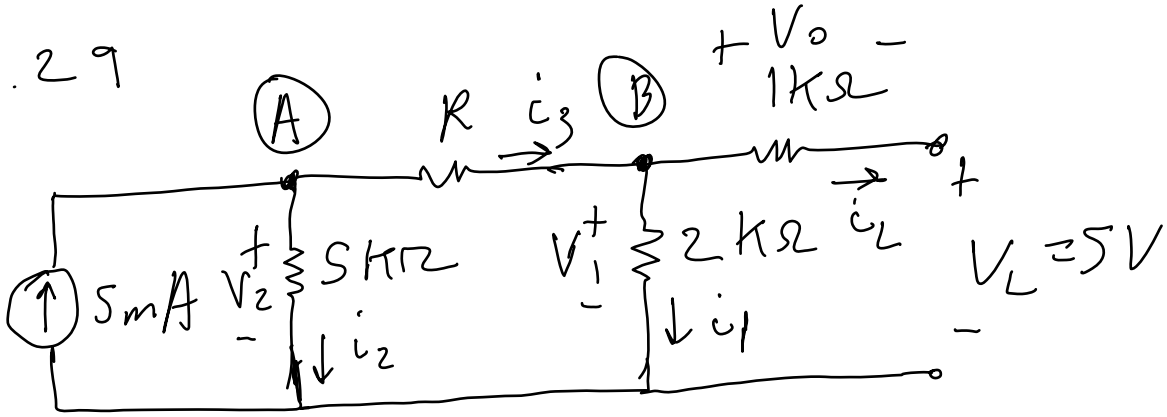
Using KVL

$$25 = (5\times10^3)\times(2.5\times10^{-3}) + R\times(2.5\times10^{-3}) + 5$$

$$= 12.5 + 5 + R\times(2.5\times10^{-3})$$

$$7.5 = R\times 2.5\times10^{-3} \Rightarrow R = 3\times10^3 = 3k\Omega.$$

Alternative (even easier) approach to 2.29



$i_L = 0$ since we have an open circuit

$\Rightarrow V_o = (1K\Omega)\, i_L = 0$

KVL $\Rightarrow V_1 = V_L = 5V$

Ohm's Law $\Rightarrow i_1 = V_1/2K\Omega = 2.5mA$

KCL node B

$\Rightarrow i_3 = i_1 = 2.5mA$

KCL node A

$5mA = i_2 + i_3$

$\Rightarrow i_2 = 2.5\,mA$

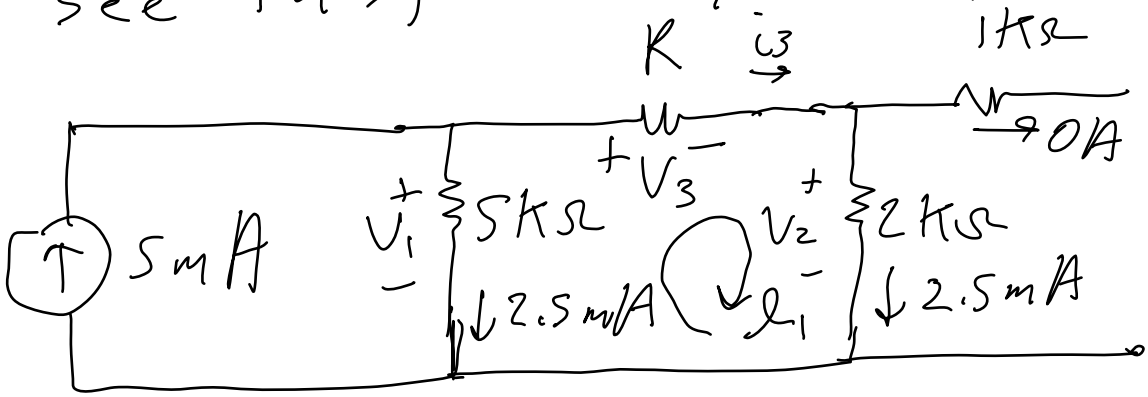
easiest way:

Effectively we have a current divider (since $i_L = 0$) with equal currents

Thus $R + 2k\Omega = 5k\Omega$

$$\Rightarrow \underline{R = 3k\Omega}$$

Slightly more work if you don't see this, but only slightly:



KVL e_1: $-V_1 + V_3 + V_2 = 0$

Ohm's Law: $-(5k\Omega)2.5mA + (2.5mA)R + (2k\Omega)2.5mA = 0$

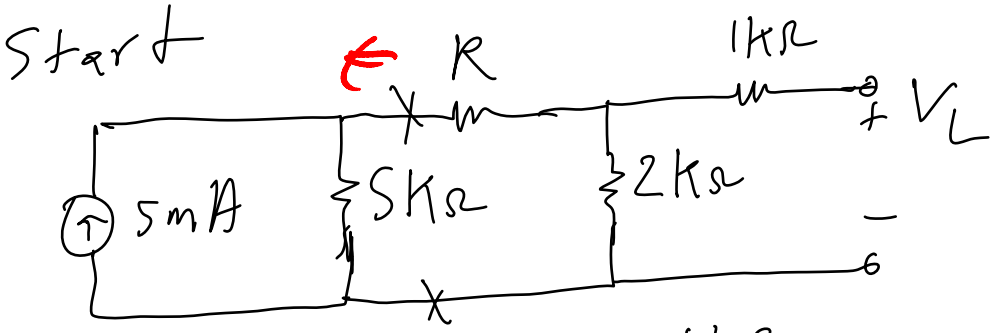
$$\Rightarrow -5k\Omega + R + 2k\Omega = 0$$

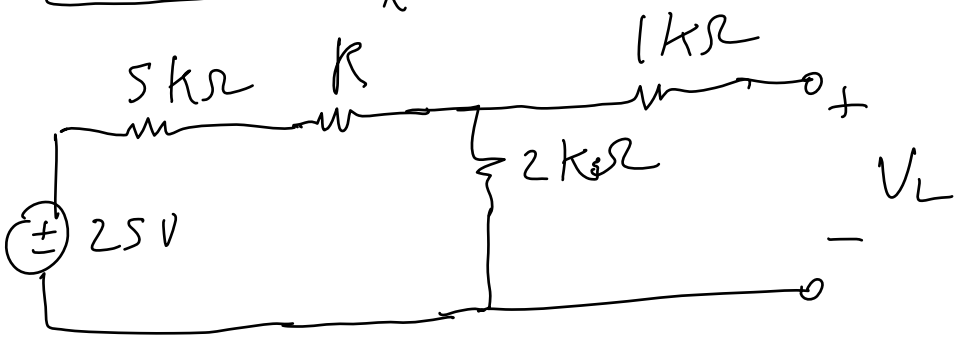
$$\Rightarrow \underline{R = 3k\Omega}$$

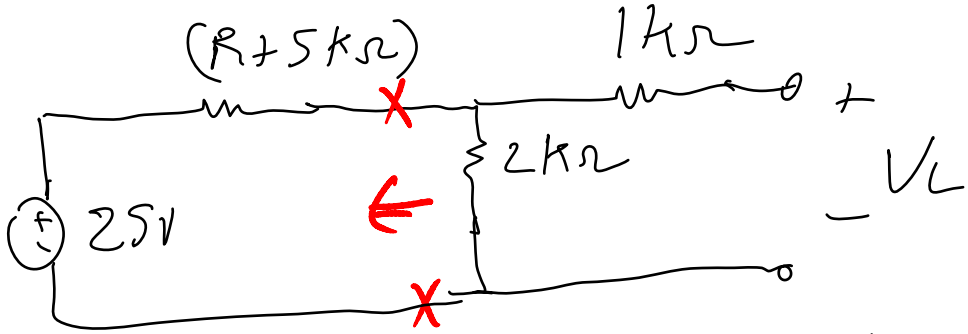
Hardest way:

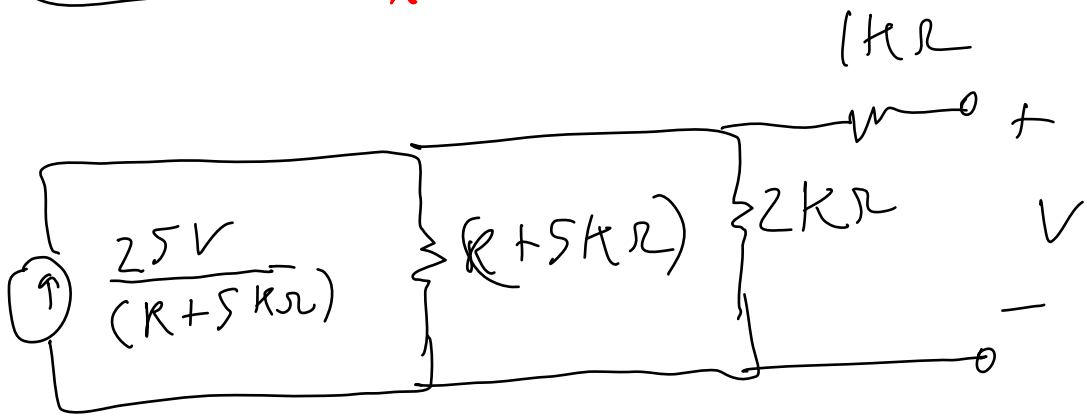
Still need $i_L = 0$, then do source transformations like crazy

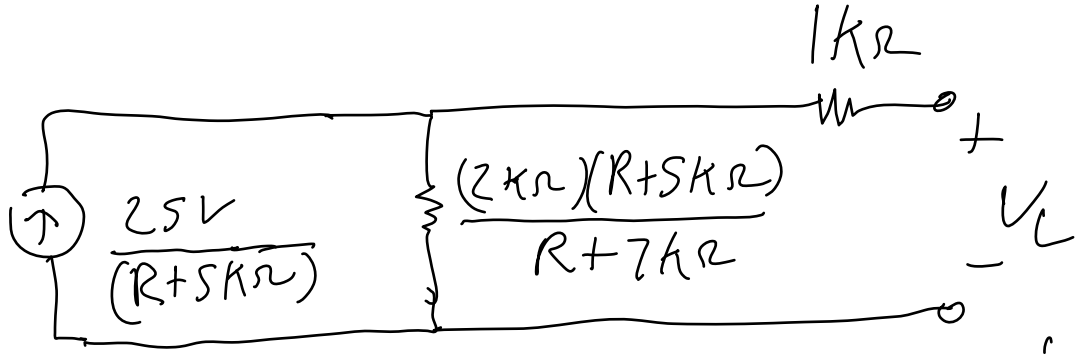
Start

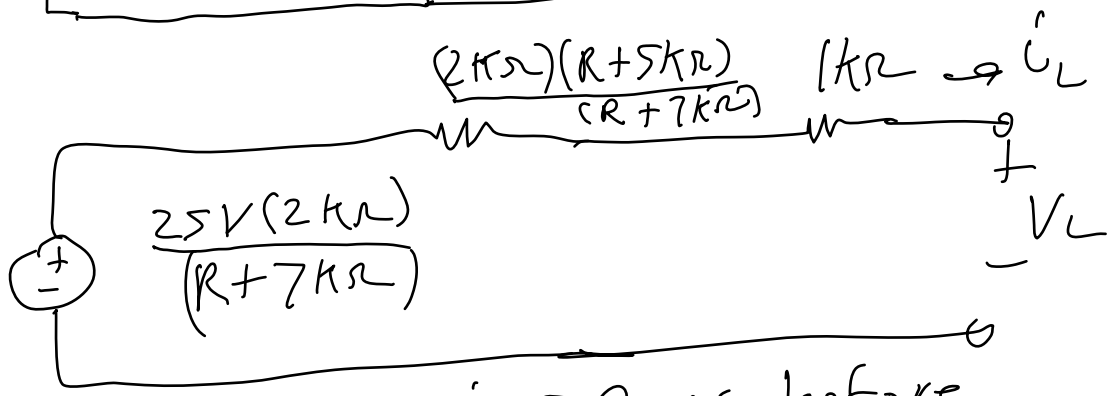












now with $i_L = 0$ as before

$$\frac{(25V)(2K\Omega)}{(R+7K\Omega)} = V_L = 5V$$

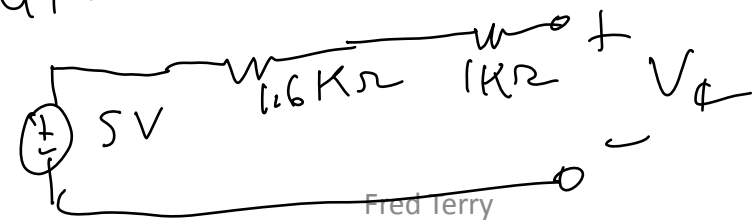
$$(25V)(2K\Omega) = (5V)(R+7K\Omega)$$

$$R+7K\Omega = 10K\Omega$$

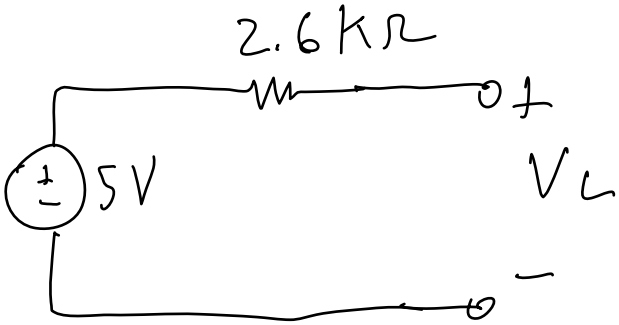
$$\Rightarrow \underline{R = 3K\Omega}$$

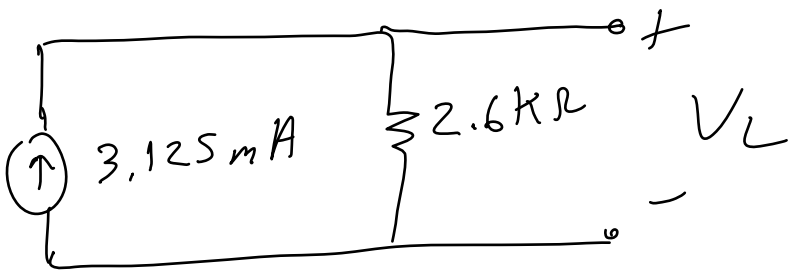
Lots more algebra, but it works

also yields full model for equivalent circuit at V_L output



or finally:



This is known as the Thevénin Equivalent Circuit

A source transform yields the Norton Equivalent: