EECS 215: Introduction to Electronic Circuits WN 2010
Problem Set 7

Issued: 2/26/10

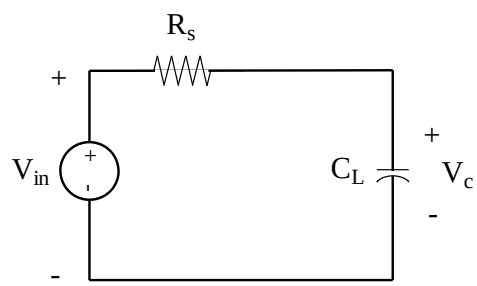
Due: 3/12/10, **9:30 AM** **Note: Problem sets are DUE at the start of lecture on Friday mornings. Do not skip class to work on homework further. Your paper may not be accepted.**

U&M refers to a problem from Ulaby and Maharbiz.

This assignment is not that bad; however, making a good start before Thursday, 2/25/10 is highly advised.

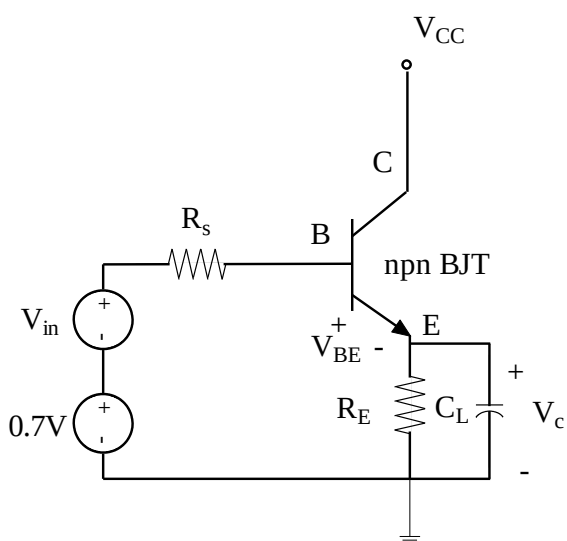
Problems:

1. Rise time, t_r, is usually defined from the 10% to 90% points of a transient response to a step. Assume we have a source $V_{in}(t)=1Vu(t)$, a source resistance $R_s=1K\Omega$, and load capacitance of $C_L=1nF$ (circuit below). Find $V_c(t)$ for $t\geq 0$ and find the rise time.

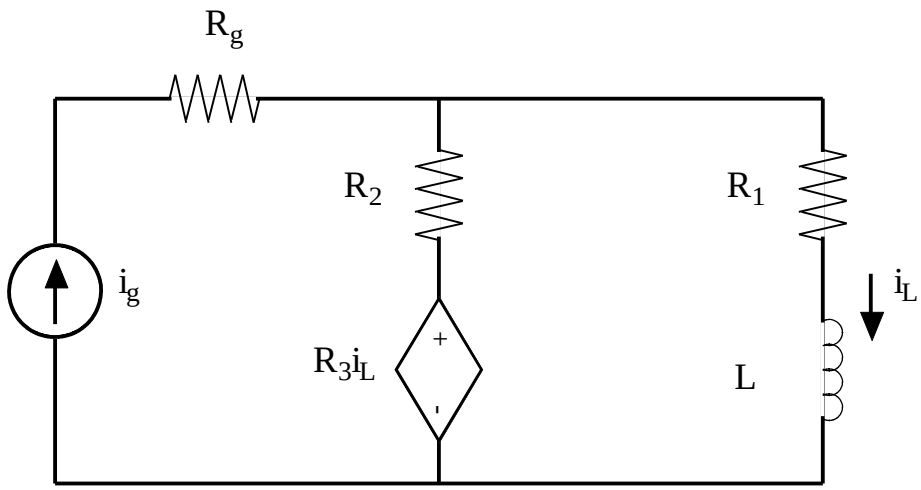


2. Now the source is buffered by a BJT common collector amp as shown below. Find $V_c(t)$ for $t\geq 0$ and find the rise time. Assume our standard linear model for the BJT in forward active mode operation.

$$R_S = R_E = 1K\Omega \quad \beta_F = 100 \quad V_{cc} = +12V$$



3. For the circuit below:



a) Find the differential equation that relates $i_L(t)$ to $i_g(t)$

b) Now assume L=1 H, $R_1=R_2=R_g=10\Omega$, and $R_3=5\ \Omega$. Find $i_L(t)$ for $t\geq 0$ if

$$i_g(t) = (1A)u(t) + (-2A)u(t-0.2s)$$

Plot your result using Matlab or another good scientific plotting program for $-0.2s \leq t \leq 0.8s$

4. U&M 5.50
5. U&M 5.51
6. U&M 5.59
7. U&M 5.63

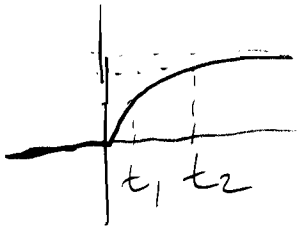
EECS 215 WN10
Problem Set # 7 Solutions

1. $V_c(t) = (1V)(1-e^{-t/\tau}) \quad t \geq 0$

where $\tau = RC = (1k\Omega)(1nF) = 1\mu s$

This is the basic RC circuit. If you did not get this part easily, please see your professor's help immediately,

t_r = 10% to 90% time, so here 0.1 V to 0.9V



$$0.1V = (1V)(1-e^{-t_1/\tau})$$

$$-0.9 = -e^{-t_1/\tau}$$

$$\Rightarrow t_1 = -\tau \ln(0.9)$$

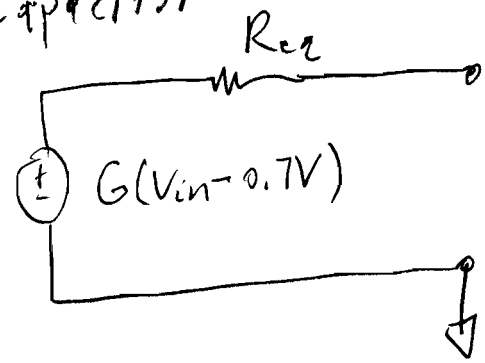
Similarly $t_2 = -\tau \ln(0.1)$

$$t_r = t_2 - t_1 = \tau[-\ln(0.1) + \ln(0.9)]$$

$$= \tau \ln\left(\frac{0.9}{0.1}\right) = \tau \ln 9 \approx (2.197)\tau$$

$\boxed{\text{Here } t_r \approx 2.197\mu s}$

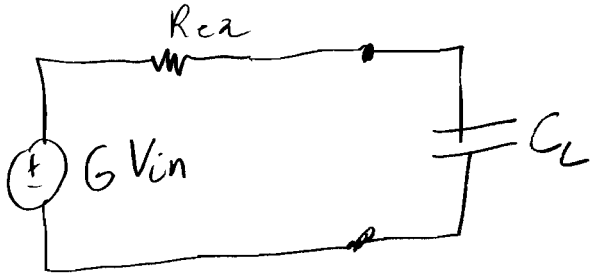
2. From Midterm 2 (see detail solutions) we had the follow the Thevenin Equivalent for the BJT circuit connected to the Capacitor



$$G = \frac{(\beta+1)R_E}{R_S + (\beta+1)R_E} = \frac{101}{102} \simeq 0.9902$$

$$R_{eq} = \frac{R_S R_E}{R_S + (\beta+1)R_E} = \frac{1k\Omega}{102} \simeq 9.804\,\Omega$$

Now we replace V_{in} above by $(V_{in} + 0.7V)$ in this problem statement. So we now have



$$\tau_2 \equiv R_{eq} C_L \simeq 9.8\,ns$$

so now

$$V_c(t) = G(1V)\left(1 - e^{-t/\tau_2}\right) \qquad t \geq 0$$

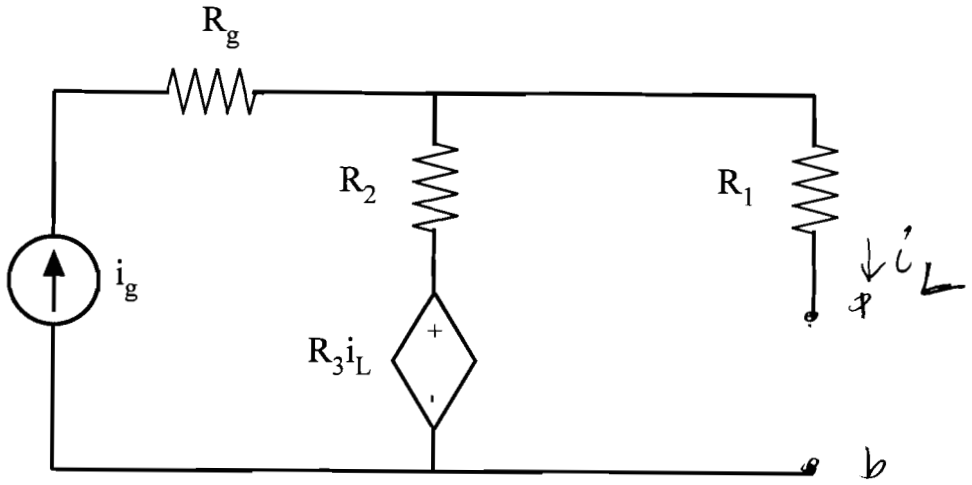
$$V_C \approx (0.9902V)(1 - e^{-t/\tau_2}) \qquad \tau_2 = 9.804\ ns$$
$$t \geq 0$$

$$\boxed{t_{r,2} = \tau_2 \ln 9 \approx 21.54\ ns}$$

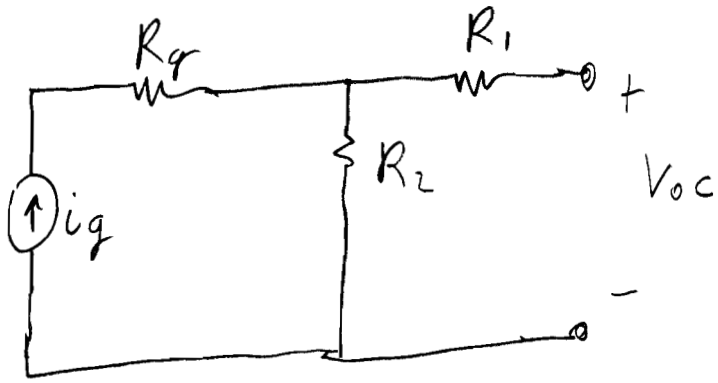
Note that this is ~ 102 X faster than problem 1

This illustrates directly why a unity gain buffer amplifier is useful, even in a digital circuit

3. (a) Find the Norton Equivalent Seen by the inductor

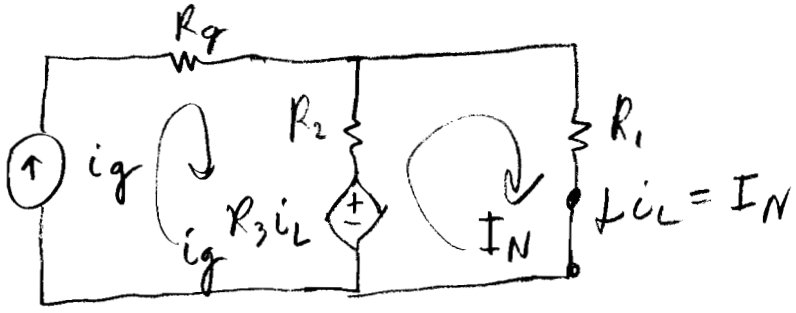


V_{oc}? $i_L = 0$ so



$$V_{oc} = i_g R_2 = V_{TH}$$

$I_{sc} = I_N$? One case where mesh analysis is easy



1 loop for KVL

$$-R_3 i_L + (I_N - i_g) R_2 + I_N R_1 = 0$$

$$-R_3 I_N + (I_N - i_g) R_2 + I_N R_1 = 0$$

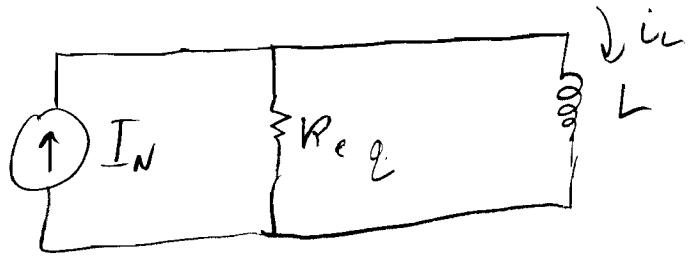
$$I_N (R_1 + R_2 - R_3) = R_2 i_g$$

$$I_N = \left[\frac{R_2}{R_1 + R_2 - R_3}\right] i_g$$

$$R_{eq} = V_{TH} / I_N = (R_1 + R_2 - R_3)$$

(also easy to get by turning of i_g + applying a test source)

so



will let us solve for i_L

$$I_N = \frac{v_L}{R_{eq}} + i_L = \frac{L}{R_{eq}} \frac{di_L}{dt} + i_L$$

or

$$\frac{di_L}{dt} + \left(\frac{R_{eq}}{L}\right) i_L = \left(\frac{R_{eq}}{L}\right) I_N$$

$$\frac{di_L}{dt} + \frac{(R_1 + R_2 - R_3)}{L} i_L = \frac{(R_1 + R_2 - R_3)}{L} \left(\frac{R_2}{R_1 + R_2 - R_3}\right) i_g$$

$$\boxed{\frac{di_L}{dt} + \left(\frac{R_1+R_2-R_3}{L}\right) i_L = \frac{R_2}{L} i_g}$$

b) With $L = 1H$ $R_1 = R_2 = R_g = 10\Omega$ & $R_3 = 5\Omega$

we have

$$\frac{di_L}{dt} + (15 s^{-1}) i_L = (10 s^{-1}) i_g$$

$$i_g = [u(t) - 2u(t-0.2s)] (1A)$$

$t < 0$ $i_g = 0 \Rightarrow i_L = 0$

$0 \le t < 0.2s$

$i_L(0) = 0$ Boundary condition

$i_{Lf} = \frac{10}{15} A = (\frac{2}{3} A)$ both from the differential eqn & from $\underline{\underline{I_N}}$ (circuit)

So

$$i_L(t) = (\tfrac{2}{3} A)[1 - e^{-(15 s^{-1})t}]$$

$t \ge 0.2s$

$$i_L(0.2s) = (\tfrac{2}{3} A)[1 - e^{-(15 s^{-1})(0.2s)}]$$

$$= (\tfrac{2}{3} A)(1 - e^{-3}) \approx 0.633475A$$

$i_{Lf} = I_N = \left(-\frac{4}{3}A\right) + i_{Ln} = Be^{-(15s^{-1})(t-0.2s)}$

so B.C. matching

$i_L(0.2s) = -\frac{4}{3}A + B$

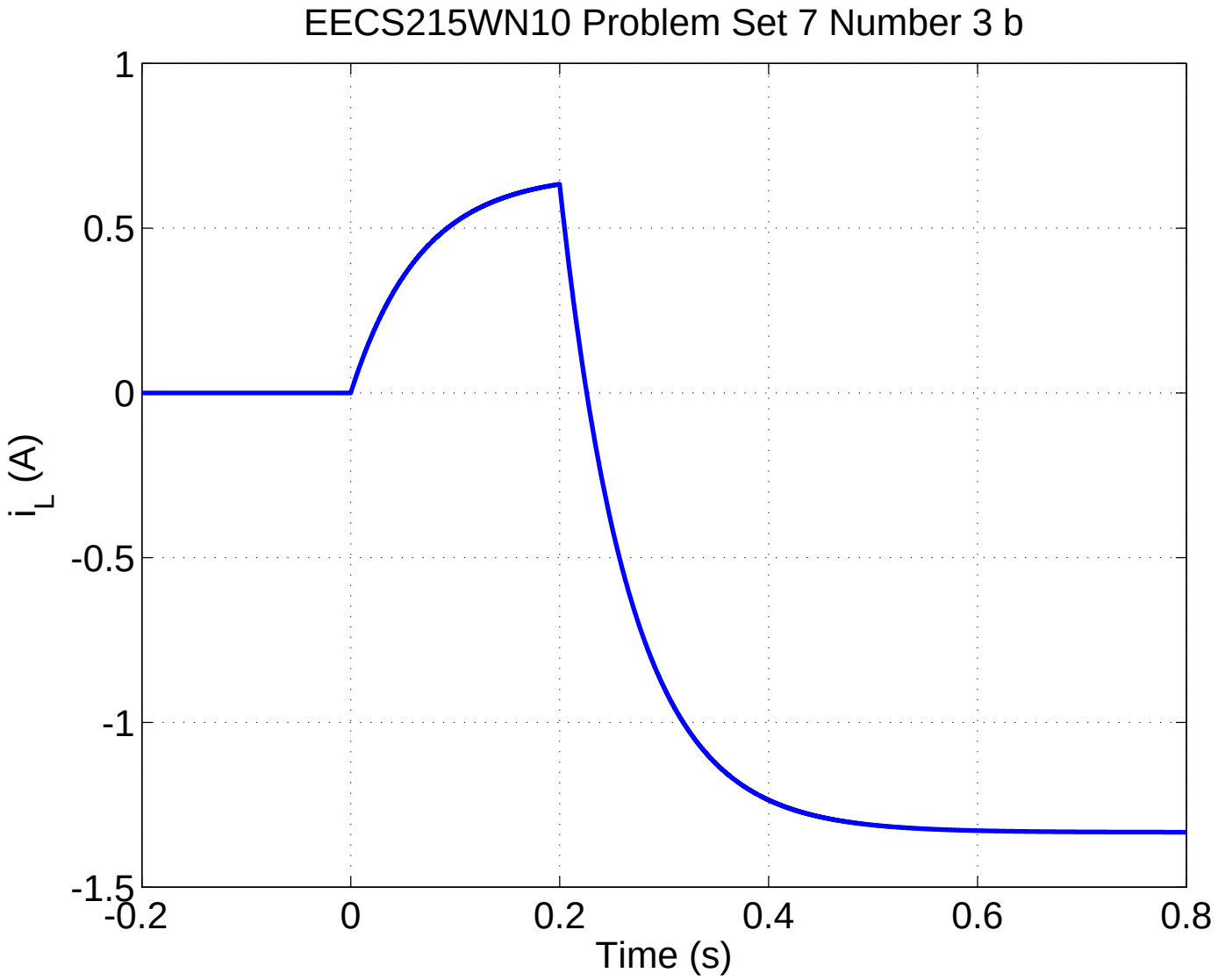
$\Rightarrow B = \frac{4}{3}A + i_L(0.2s) \approx 1.96681\ A$

& $i_L(t) = \left(-\frac{4}{3}A\right) + Be^{-(15s^{-1})(t-0.2s)}$

for $t \geq 0.2s$

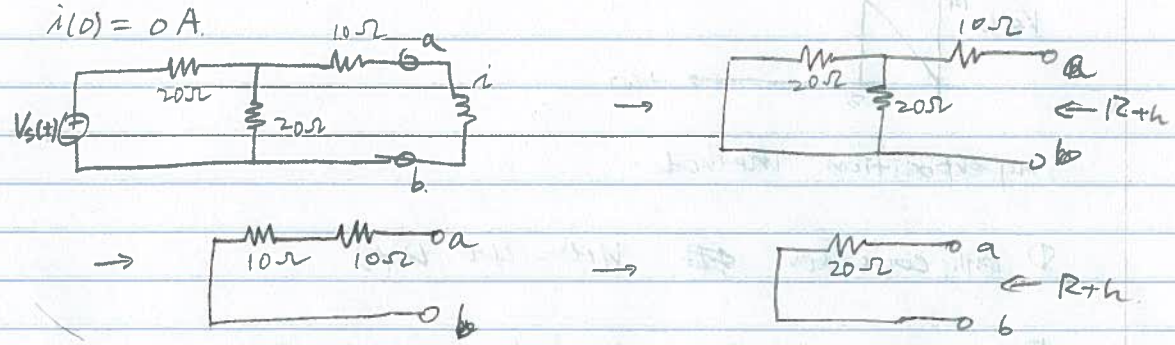
matlab code + plot follows

```
%plotting solution of problem set 7 question 3 (b)
clear all;
close all;
t=-0.2:1e-4:0.8;
iL02=2/3*(1-exp(-3));
B=4/3+iL02;
for jj=1:length(t),
    t0=t(jj);
    if t(jj)<0,
        iL(jj)=0;
    else if t(jj)<0.2,
            iL(jj)=(2/3)*(1-exp(-15*t0));
        else
            iL(jj)=-4/3+B*exp(-15*(t0-0.2));
        end;
    end;
end;
plot(t,iL,'linewidth',2);
set(gca,'fontsize',16);
xlabel('Time (s)','fontsize',16);
ylabel('i_L (A)', 'fontsize',16);
title('EECS215WN10 Problem Set 7 Number 3 b','fontsize',16);
grid;
```


EECS215WN10 Problem Set 7 Number 3 b
1
0.5
0
-0.5
-1
-1.5
-0.2
0
0.2
0.4
0.6
0.8
Time (s)
i_L (A)

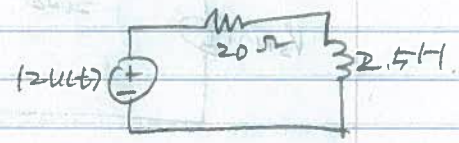
4. U&M 5.50

$i(0) = 0\,A.$



$$i(\infty) = \frac{V_s(\infty)}{20} = \frac{12}{20} = 0.6\,A$$

$$i(0) = i(0_-) = 0\,A$$



<SIMPLE RL CIRCUIT>

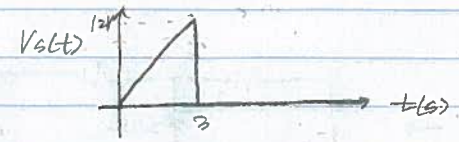
$$i(t) = i(\infty) + [i(0) - i(\infty)]\,e^{-t/\tau}$$

$$\tau = \frac{L}{R} = \frac{2.5}{20} = \frac{12.5}{100} = 0.125\ s.$$

$$i(t) = 0.6 + [-0.6]\,e^{-t/0.125}$$
$$= 0.6\,(1 - e^{-t/\tau}) \qquad \tau = 0.125$$

5 RGM 5.51

$V_s(t) = 4t \cdot u(t) - 4t \cdot u(t-3)$



Superposition method

① only consider $V_1(t) = 4t \cdot u(t)$

From previous problem Equivalent circuit is

$\tau = \frac{L}{R} = \frac{2.5}{20} = 0.125$

Using KVL $\quad -V_s + i_1(t) \cdot R + L\frac{di_1(t)}{dt} = 0$

$$\frac{di_1(t)}{dt} + \frac{R}{L} i_1(t) = \frac{V_s}{L} \Rightarrow \frac{di_1(t)}{dt} + a\, i_1(t) = bt$$

where $a = \frac{R}{L}$, $b = \frac{4}{L}$ $\rightarrow$ $a = 8$. $b = 1.6$

$$\Rightarrow \int_0^t \left(e^{at}\frac{di_1(t)}{dt} + a i_1(t) e^{at} \right) dt = \int_0^t bt e^{at}\, dt$$

$$\text{LHS} := \int_0^t \left[\frac{d}{dt} i_1(t) e^{at} \right] dt = i_1(t) e^{at} - i_1(0).$$

$$\text{RHS} := \int_0^t bt e^{at}\, dt = \left[bt \cdot \frac{1}{a} e^{at} - b \cdot \frac{1}{a^2} e^{at} \right]_0^t = \frac{b}{a^2}\left[e^{at}(at-1) + 1 \right]$$

we have $i_1(0) = 0$ $\Rightarrow$ $i_1(t) e^{at} = \frac{b}{a^2}\left[e^{at}(at-1) + 1 \right]$

$i_1(t) = \frac{b}{a^2}\left[(at-1) + e^{-at} \right]$ $\quad a = 8.\ b = 1.6.$

for $t \geq 0$

② only consider $V_2(t) = -4t \cdot u(t-3)$

$t \geq 3$ sec.

$\frac{di_2(t)}{dt} + a i_2(t) = -bt$ $\quad b = \frac{4}{L}$ $\quad \rightarrow \int_3^t \text{LHS}\, dt = \int_3^t \text{RHS}\, dt$

$-bt \cdot \frac{1}{a} e^{at}$

$-(-b) \cdot \frac{1}{a^2} e^{at}$

$$\Rightarrow i_2(t) e^{at} - i_2(3) = -\frac{b}{a^2}\left[e^{at}(at-1) - e^{3a}(3a-1) \right]$$

$i_2(3) = 0$

$$\Rightarrow i_2(t) e^{at} = -\frac{b}{a^2}\left[e^{at}(at-1) - e^{3a}(3a-1) \right]$$

$$\Rightarrow i_2(t) = -\frac{b}{a^2}\left[(at-1) - (3a-1) e^{-a(t-3)} \right]$$ for $t \geq 3$

We have $a = 8$, $b = 1.6$

$$i_1(t) = \frac{1.6}{64}\left[(8t-1) + e^{-8t}\right] \quad \text{for } t \geq 0$$

$$i_2(t) = -\frac{1.6}{64}\left[(8t-1) - 23\,e^{-8(t-3)}\right] \quad \text{for } t \geq 3$$

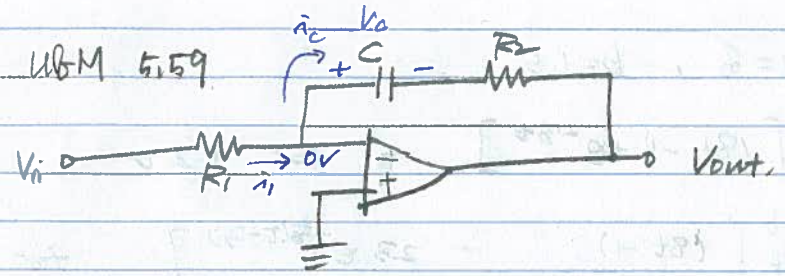
$$\Rightarrow \quad i(t) = \begin{cases} i_1(t) & \text{for } 0 \leq t \leq 3 \\ i_1(t) + i_2(t) & \text{for } t \geq 3 \end{cases}$$

$$= \begin{cases} 0.025\left[(8t-1)\,e^{-8t}\right] & \text{for } 0 \leq t \leq 3 \\ 0.025 \times \left(23\,e^{24} \cdot e^{-8t} + e^{-8t}\right) & t \geq 3 \end{cases}$$

$$= \begin{cases} 0.025\left[(8t-1)\,e^{-8t}\right] & 0 \leq t \leq 3 \\ 0.025\left[23\,e^{24} + 1\right] e^{-8t} & t \geq 3 \end{cases}$$

this part looks extreme, but $t \geq 3$ e^{-8t} term cancels out this part exponentially.

6 UBM 5.59



$i_1 = \frac{V_i}{R_1}$ $\quad i_c = C\frac{dV_c}{dt}$ $\quad$ and $\quad i_c = i_1$.

$$V_{out} = -V_c - i_c R_2 = -V_c - R_2 C \frac{dV_c}{dt}$$

$$\frac{V_i}{R_1} = C\frac{dV_c}{dt} \rightarrow \frac{dV_c}{dt} = \frac{1}{R_1 C} V_i \rightarrow \int_0^t \frac{dV_c}{dt}\,dt = \int_0^t \frac{1}{R_1 C} V_i(t)\,dt$$

$$\Rightarrow V_c(t) = \frac{1}{R_1 C}\int_0^t V_i(t)\,dt + V_c(0)$$

we assumed $V_c(0) = 0$

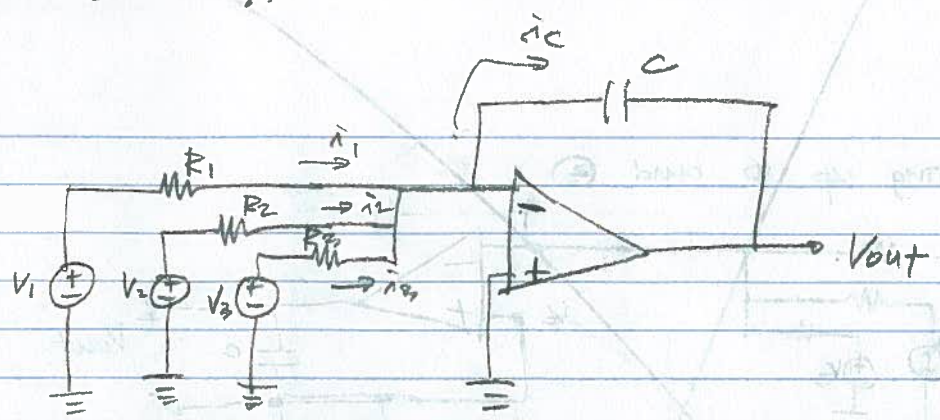
$$\Rightarrow V_c(t) = \frac{1}{R_1 C}\int_0^t V_i(t)\,dt$$

$$V_{out} = -V_c(t) - R_2 C \frac{dV_c(t)}{dt}$$

$$= -\frac{1}{R_1 C}\int_0^t V_i(t)\,dt - \frac{R_2}{R_1} V_i(t)$$

$$\Rightarrow V_{out}(t) = -\frac{1}{R_1 C}\int_0^t V_i(t)\,dt - \frac{R_2}{R_1} V_i(t)$$

7. U&M 5.59.



KCL: $i_C = i_1 + i_2 + i_3$

$i_1 = V_1/R_1$

$i_2 = V_2/R_2$

$i_3 = V_3/R_3$

$V_{out} = -V_C$

$$i_C = C\frac{dV_C}{dt} = \frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3}$$

$$\frac{dV_C}{dt} = \frac{V_1}{R_1C} + \frac{V_2}{R_2C} + \frac{V_3}{R_3C}$$

Assume $V_C(0) = 0$. $\quad \int_0^t \frac{dV_C}{dt}\,dt = \int\left(\frac{V_1}{R_1C} + \frac{V_2}{R_2C} + \frac{V_3}{R_3C}\right)dt$

$$\Rightarrow V_C(t) = \int\left(\frac{V_1}{R_1C} + \frac{V_2}{R_2C} + \frac{V_3}{R_3C}\right)dt$$

$$\Rightarrow V_{out} = -V_C(t) = -\int\left(\frac{V_1}{R_1C} + \frac{V_2}{R_2C} + \frac{V_3}{R_3C}\right)dt$$

If we set $R_1C = \frac{1}{5}$, $R_2C = \frac{1}{2}$, $R_3C = 1$

then $V_{out} = -\int(5V_1 + 2V_2 + V_3)\,dt$

Let $C = 1\,\mu F$

and $R_1 = \frac{1}{5}M\Omega = 200k\Omega$

$R_2 = \frac{1}{2}M\Omega = 500k\Omega$

$R_3 = 1M\Omega$

$\Rightarrow$ you can set different values that satisfy relations

Then $V_{out} = -\int(5V_1 + 2V_2 + V_3)\,dt$