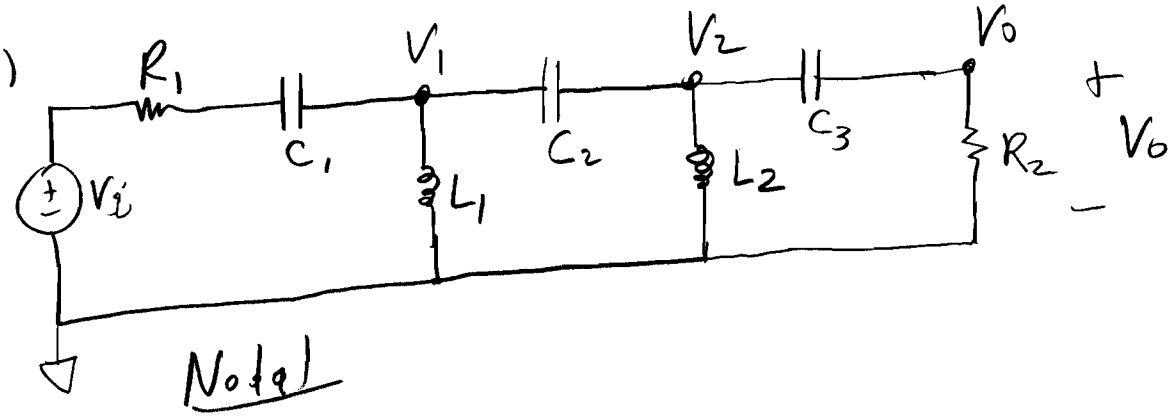
PS11 Extra Credit

(1) a)



Nodal

$$\frac{V_1 - V_s}{R_1 + 1/C_1 s} + \frac{V_1}{L_1 s} + \frac{V_1 - V_2}{1/C_2 s} = 0 \quad \text{KCL}$$

$$\frac{V_2 - V_1}{1/C_2 s} + \frac{V_2}{L_2 s} + \frac{V_2 - V_o}{1/C_3 s} = 0 \quad \text{KCL}$$

$$\frac{V_o - V_2}{1/C_3 s} + \frac{V_o}{R_2} = 0 \quad \text{KCL}$$

Using $C_3 = C_1$, $L_1 = L_2 = L$, $R_1 = R_2 = R$ and solving using Mathematica yields

$$\boxed{H(s) = \frac{V_o}{V_i} = \frac{C_1^2 C_2 L^2 R s^5}{[(1 + C_1 R s + C_1 L s^2)(1 + C_1 L s^2 + 2 C_2 L s^2 + 2 C_1 C_2 L R s^3)]}}$$

with the scaled component values this becomes

$$H(s) = \frac{3 s^5}{\left[(6.25 \times 10^{28})\left(1 + \frac{s}{10^5} + \frac{s^2}{2.5 \times 10^{11}}\right) \times \left(1 + \frac{s}{10^5} + \frac{s^2}{1.5625 \times 10^{11}} + \frac{3 s^3}{1.25 \times 10^{17}}\right)\right]}$$

b) There are 5 zeros at DC $(s=0)$
+ 5 poles

Quadratic term yields 2 simple, real poles

$$p = (2.5 \times 10^5)(-5 \pm \sqrt{21}) \text{ r/s}$$

$$f_p \approx \begin{cases} -16.609 \text{ kHz} & f_{p2} \\ -381.279 \text{ kHz} & f_{p5} \end{cases}$$

Cubic term yield 1 real + 2 complex poles

$$f_p \approx -16.5896 \text{ kHz} \qquad f_{p1}$$

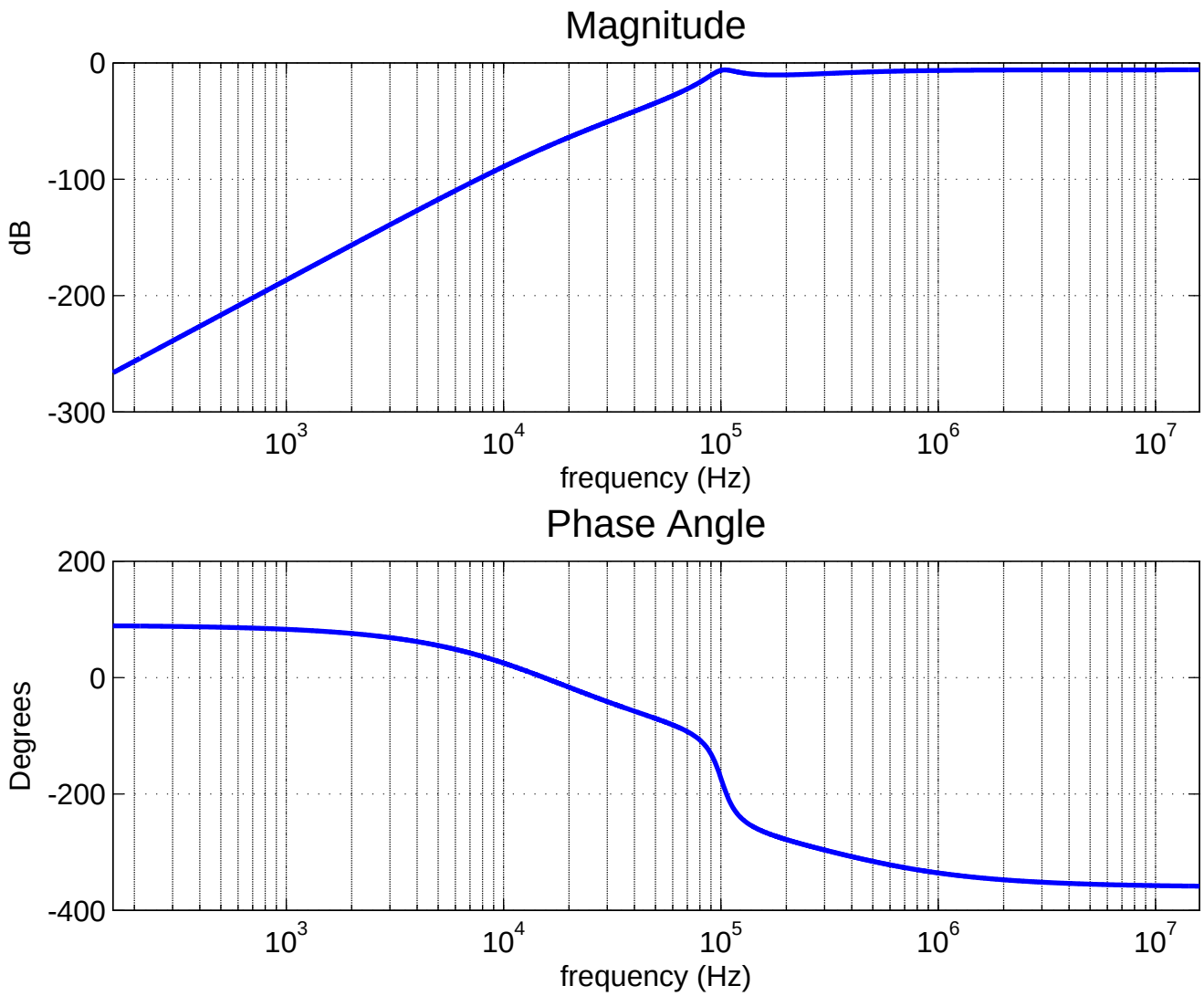
$$f_p \approx \begin{cases} 100.625 \angle 97.3804^\circ & f_{p3} \\ 100.625 \angle -97.3804^\circ & f_{p4} \end{cases}$$

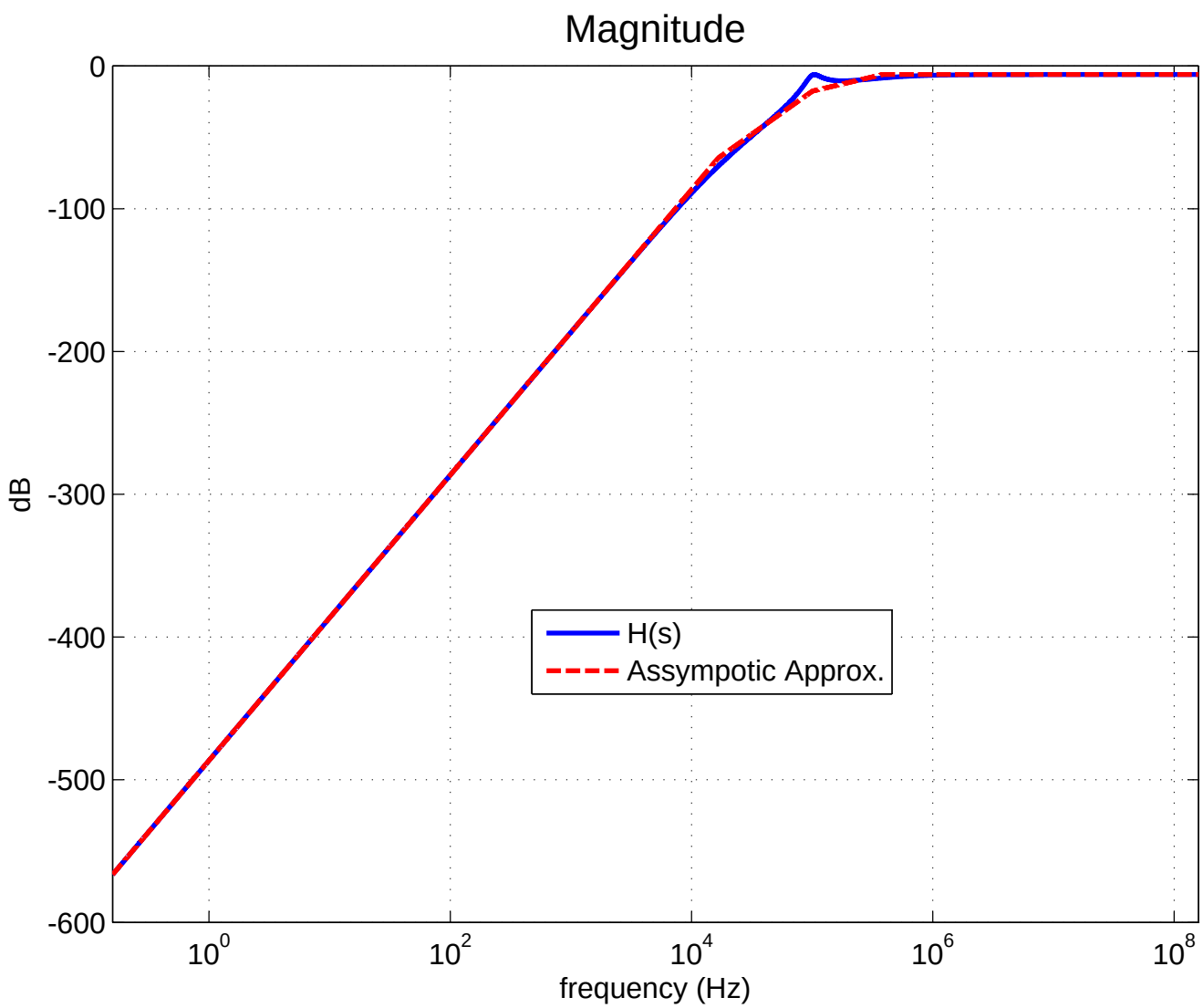
c) Bode Plots follow
Putting in $H(s)$ in standard form

$$H(s) = \frac{A\, s^5}{\prod_{i=1}^{5} \left(1 - \frac{s}{p_i}\right)}$$

yields $A = 4.8 \times 10^{-29}$

The phase plot goes through a 150 total span (5 poles/zeros, 5*90° = 450°) (Dropping 450° from DC to end). Matlab considers the phase to be +90° at the start & drops toward -360°. You could argue that it should start at +450° and drop to 0°. This is just a modulo 360° issue.


Magnitude
0
-100
-200
-300
dB
10^3
10^4
10^5
10^6
10^7
frequency (Hz)
Phase Angle
200
0
-200
-400
Degrees
10^3
10^4
10^5
10^6
10^7
frequency (Hz)


Magnitude
0
-100
-200
-300
-400
-500
-600
dB
10^0
10^2
10^4
10^6
10^8
frequency (Hz)
H(s)
Assympotic Approx.

(2) We have the transfer function for the circuit from the main PS 3b result

$$H(s) = \left[\frac{RCL_1 s^2}{L_1 L_2 C s^3 + RCL_1 s^2 + (L_1 + L_2)s + R}\right]$$

Using Fourier's Theorem and superposition, our output will be

$$V_{out} = \sum_{n=-\infty}^{+\infty} C_n H(n\omega_0) e^{jn\omega_0 t}$$

where $\omega_0 = \frac{2\pi}{T}$ T = period of V_{in} signal

&

$$C_n = \frac{1}{T}\int_0^T V_{in}(t) e^{-jn\omega_0 t} dt$$

For our square wave integrate $[-T/2, T/2]$

$$C_n = \frac{1}{T}\left[\int_{-T/2}^{-T/4} (-1) e^{-jn\omega_0 t} + \int_{-T/4}^{T/4} (+1) e^{-jn\omega_0 t} dt + \int_{+T/4}^{+T/2} (-1) e^{-jn\omega_0 t} dt\right]$$

$$\int e^{-jn\omega_0 t} dt = \frac{1}{-jn\omega_0} e^{-jn\omega_0 t} \qquad n \neq 0$$

If $n = 0$ we have $\int 1 \, dt = t$

and ($n=0$ case) $c_0 = \frac{1}{T}\int_{-T/2}^{T/2} 1\,dt = 0$

for $n \neq 0$, noting that $\omega_0 T = 2\pi$

$$c_n = \frac{1}{T}\left(\frac{j}{n\omega_0}\right)\left\{ -e^{-jn\omega_0 t}\Big|_{-T/2}^{-T/4} + e^{-jn\omega_0 t}\Big|_{-T/4}^{+T/4} - e^{-jn\omega_0 t}\Big|_{+T/4}^{+T/2} \right\}$$

$$= \left(\frac{j}{2\pi n}\right)\left\{ e^{+jn\pi} - e^{+jn\pi/2} + e^{-jn\pi/2} - e^{+jn\pi/2} + e^{-jn\pi/2} - e^{-jn\pi} \right\}$$

$$= \left(\frac{j}{2\pi n}\right)\left\{ 2j\underbrace{\sin(n\pi)}_{0} - 4j\sin(n\pi/2) \right\}$$

$$= \left(\frac{+2}{\pi n}\right)\sin(n\pi/2)$$

There are tricks to reduce this further, but this will do.

So $$c_n = \begin{cases} \left(\frac{+2}{\pi n}\right)\sin\left(\frac{n\pi}{2}\right) & n \neq 0 \\ 0 & n = 0 \end{cases}$$

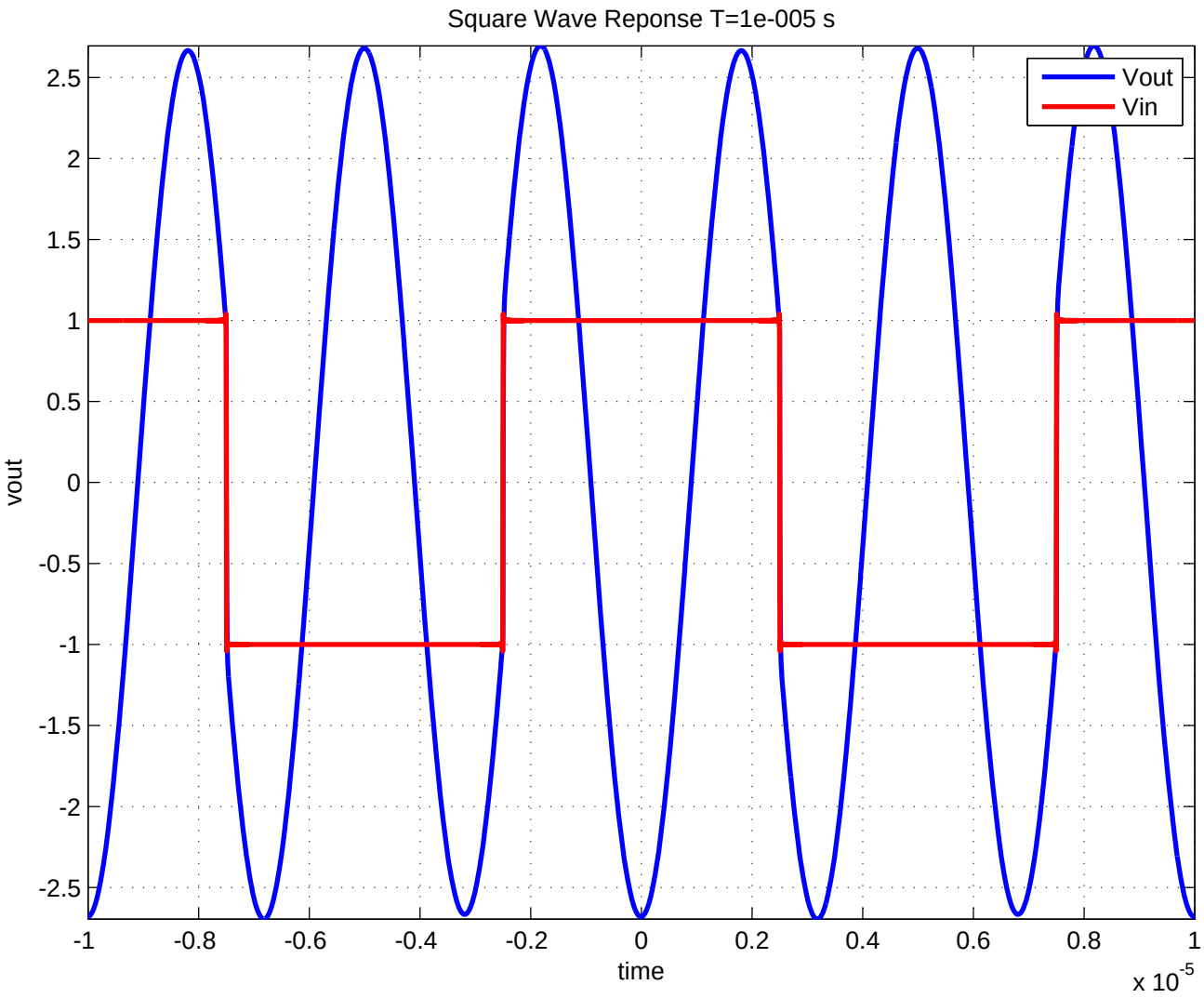
$$V_{out} = \sum_{n=-\infty}^{+\infty} c_n H(n\omega_0) e^{jn\omega_0 t}$$

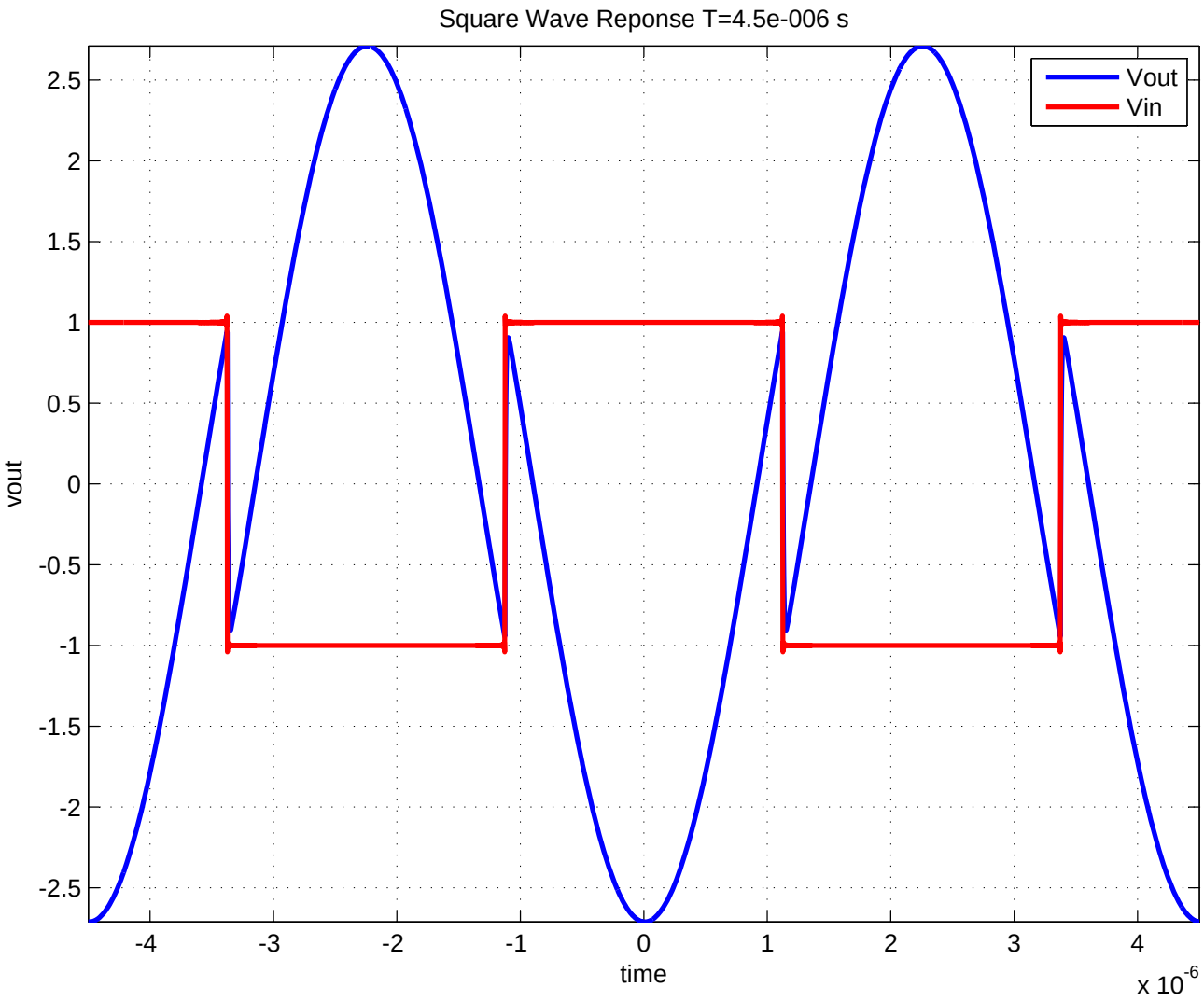
This alone will work & we can plot (a) – (e)

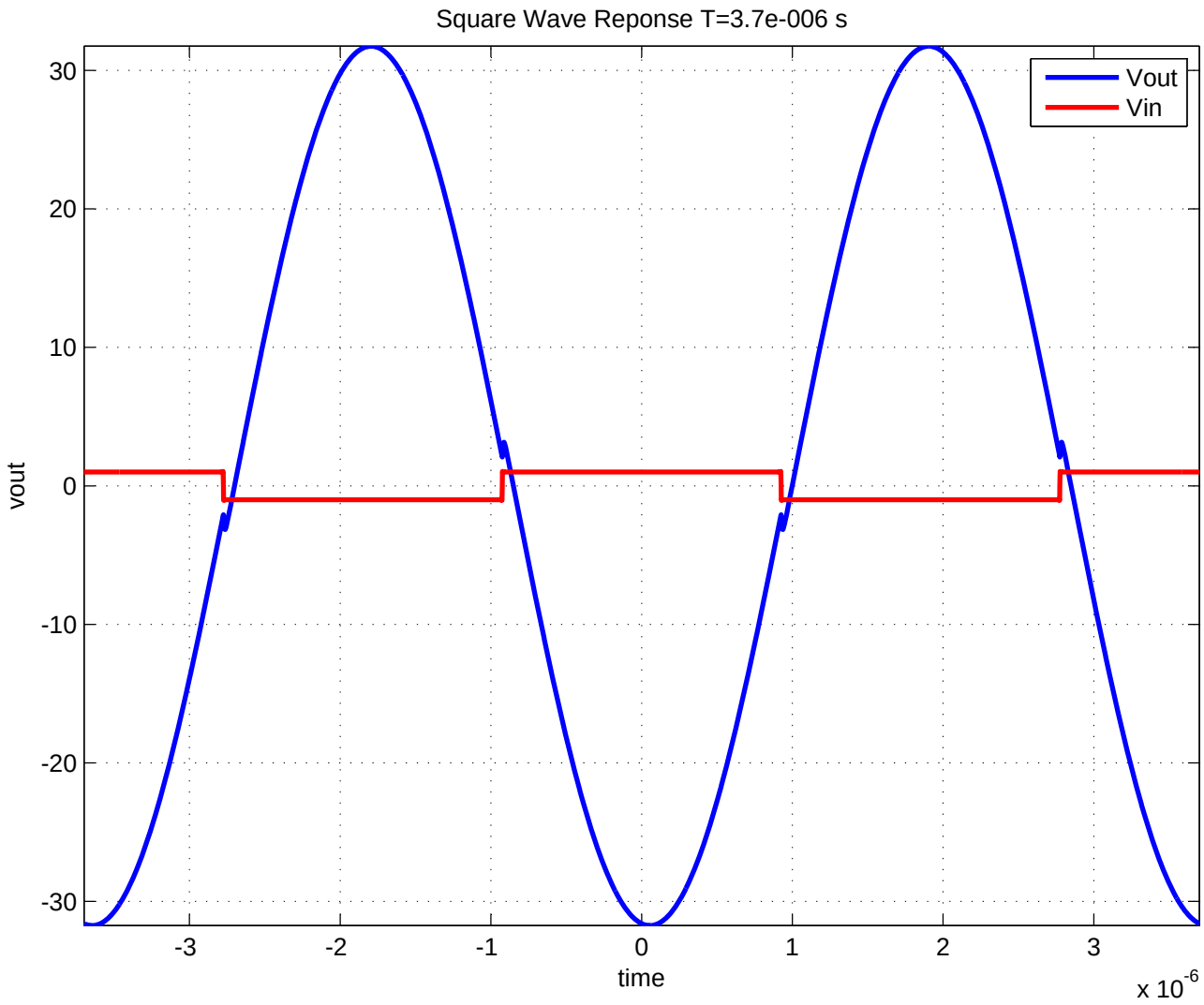
[illegible] due to the very sharp resonance

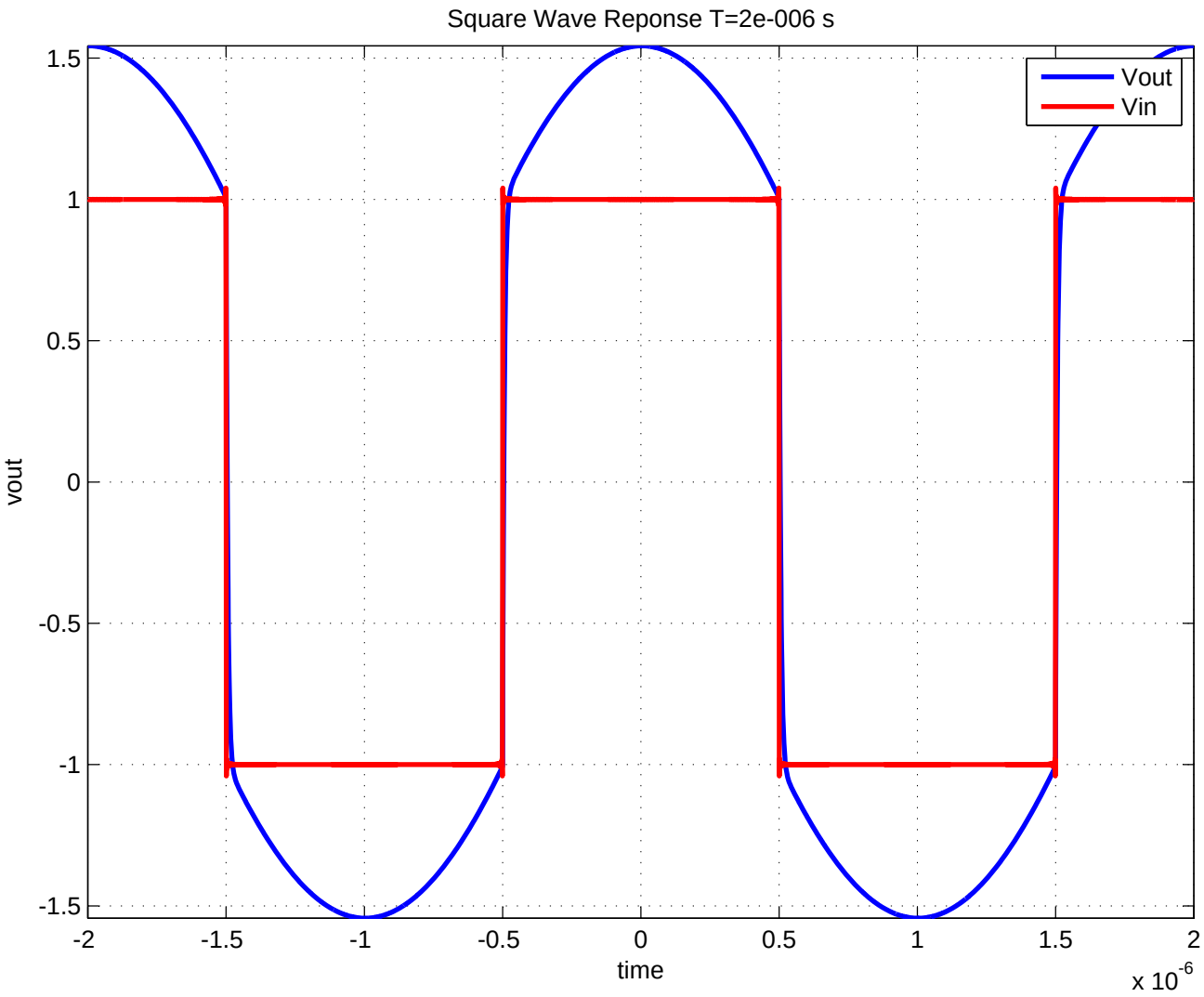
at $\sim 1.73 \times 10^{6}$ r/s $\rightarrow f \simeq 2.753 \times 10^{5}$ Hz

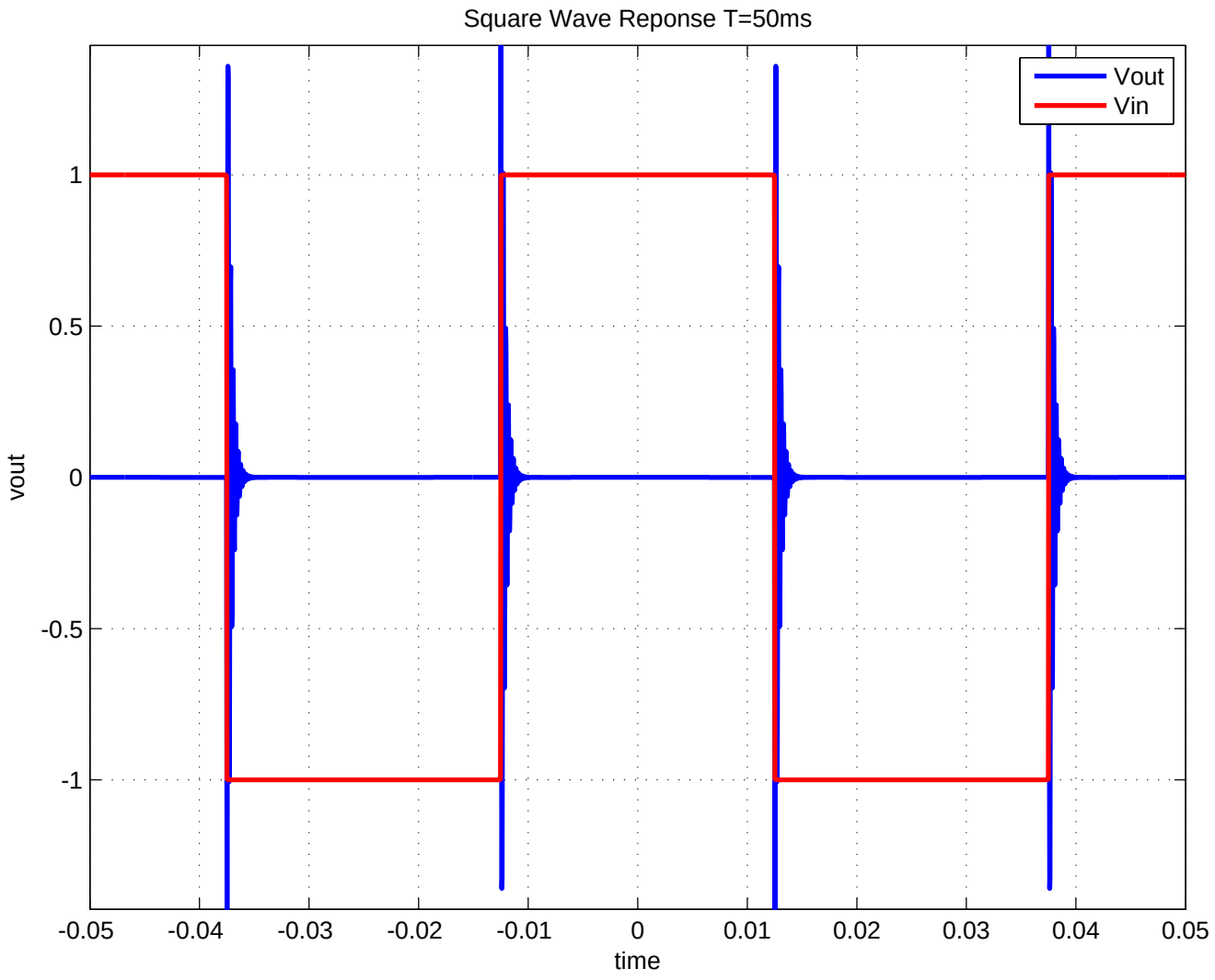
$T = 1/f \simeq 3.63\,\mu s$

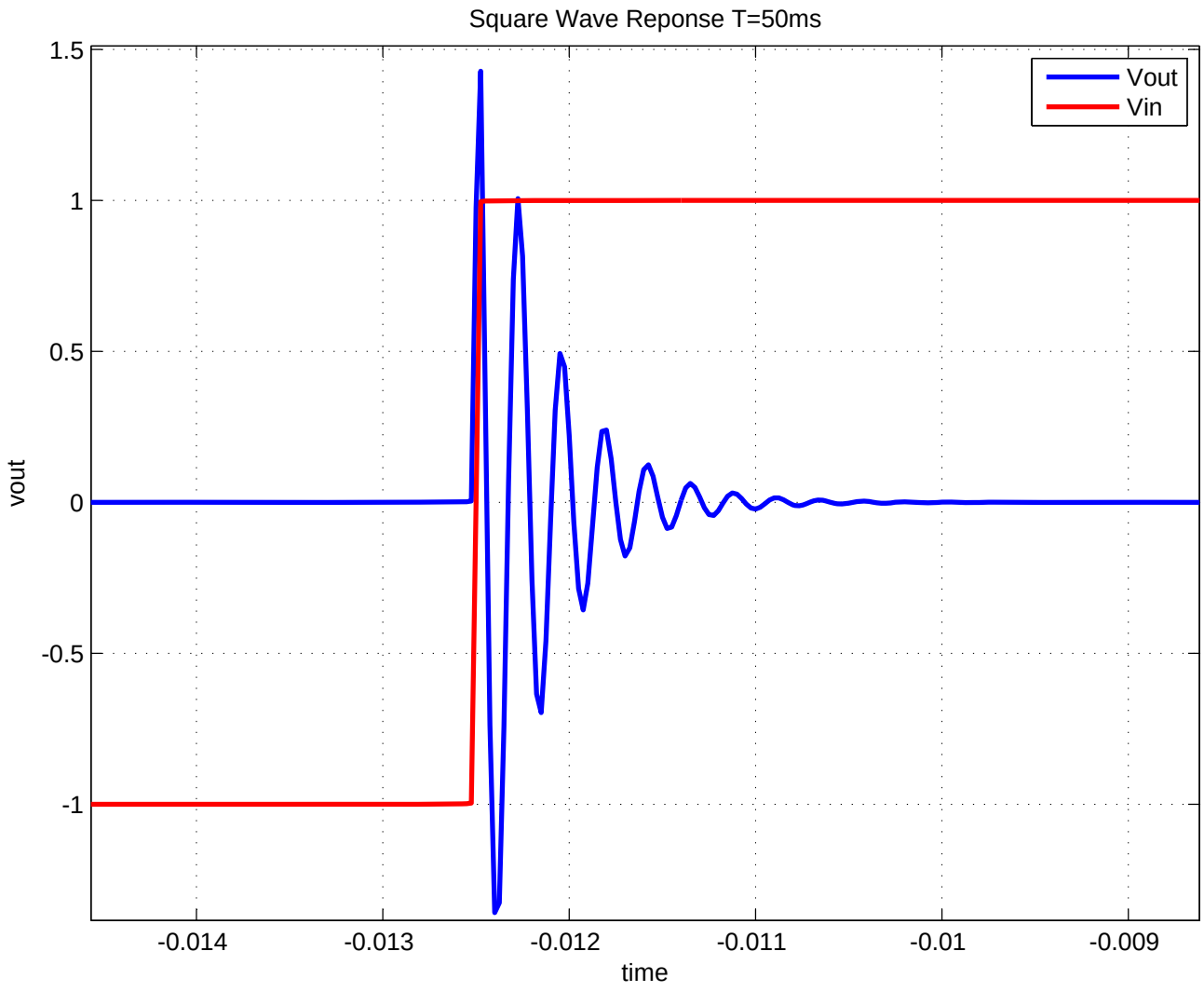

Square Wave Reponse T=1e-005 s
Vout
Vin
vout
time
x 10^-5
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1


Square Wave Reponse T=4.5e-006 s
Vout
Vin
vout
time
x 10^-6
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
-4
-3
-2
-1
0
1
2
3
4


Square Wave Reponse T=3.7e-006 s
Vout
Vin
30
20
10
0
-10
-20
-30
vout
-3
-2
-1
0
1
2
3
time
x 10^-6


Square Wave Reponse T=2e-006 s
Vout
Vin
vout
time
x 10^-6
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2


Square Wave Reponse T=50ms
Vout
Vin
1
0.5
0
-0.5
-1
vout
-0.05
-0.04
-0.03
-0.02
-0.01
0
0.01
0.02
0.03
0.04
0.05
time


Square Wave Reponse T=50ms
Vout
Vin
1.5
1
0.5
0
-0.5
-1
vout
-0.014
-0.013
-0.012
-0.011
-0.01
-0.009
time