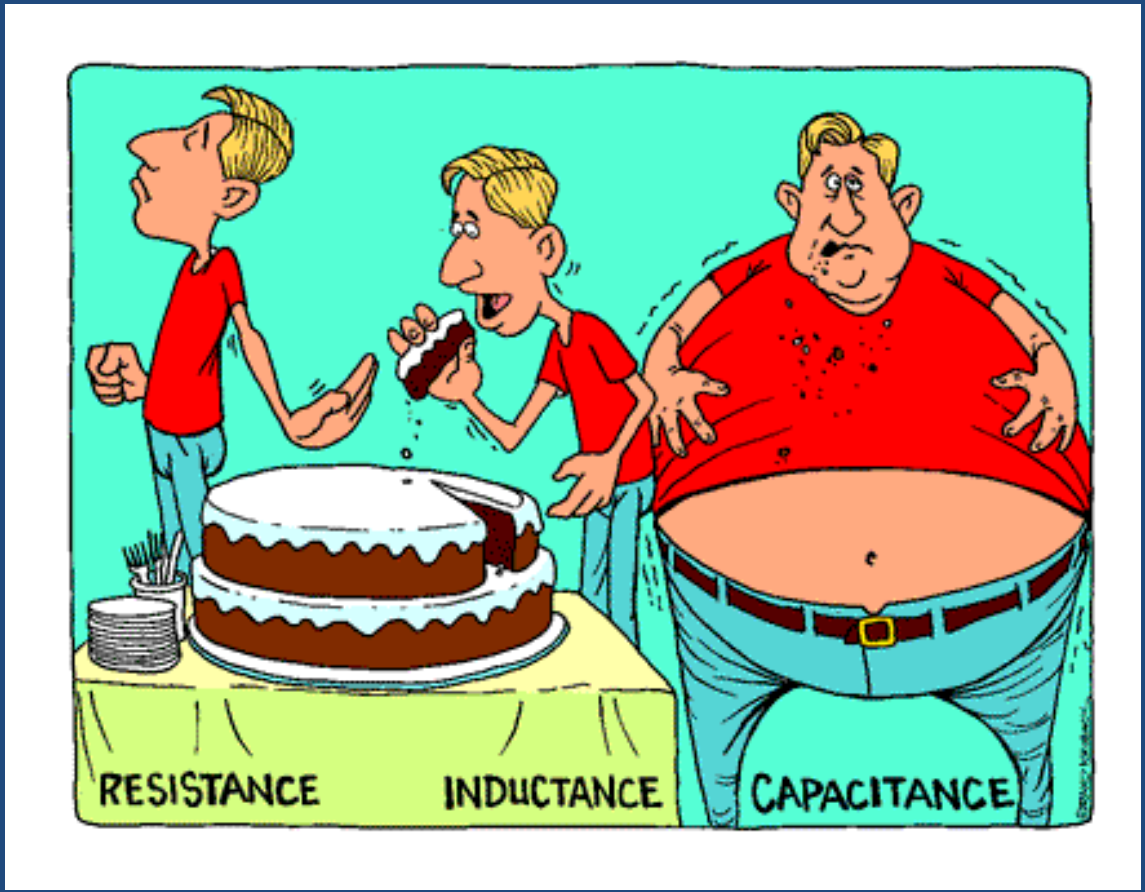


CAPACITORS AND INDUCTORS

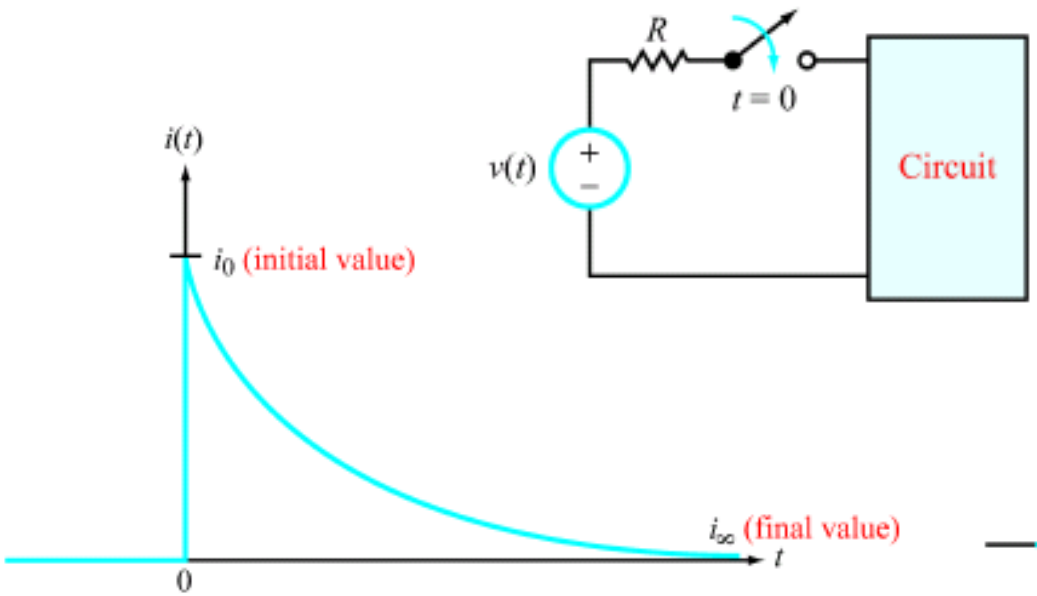
© Fred Terry
Fall, 2013

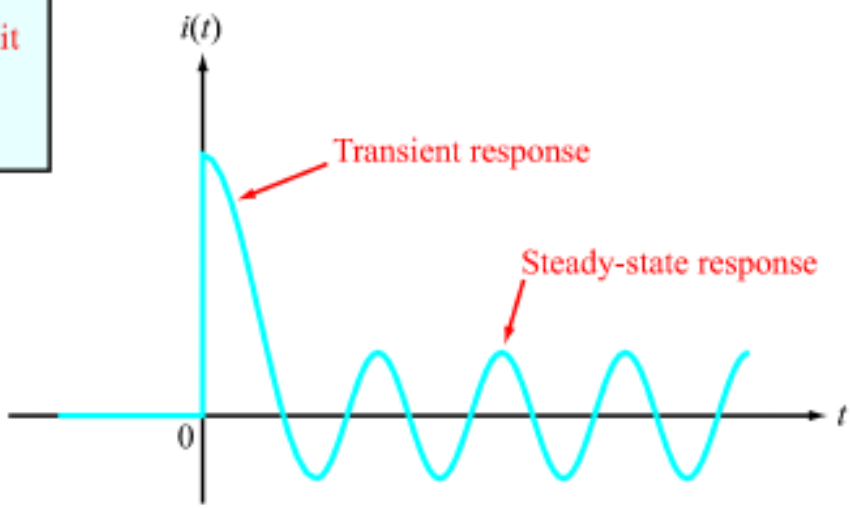
BIG IDEAS

- **Information Is Carried ONLY By Time Varying Signals**
- **All Real Circuit Have Energy Storage Elements Either As Intentional Circuit Elements and/or As Parasitics**
- **Energy Storage & Finite Power <u>DIRECTLY</u> Imply Frequency Response Limitations / Non-Instant Time Responses**

Time Response of Circuits

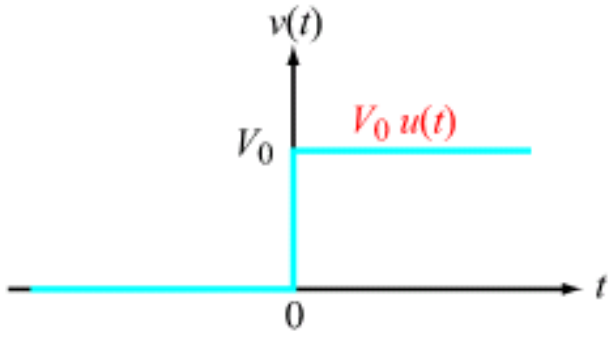
- Transient response due to change in circuit
- Steady state response can be AC or time varying, but is periodic in nature



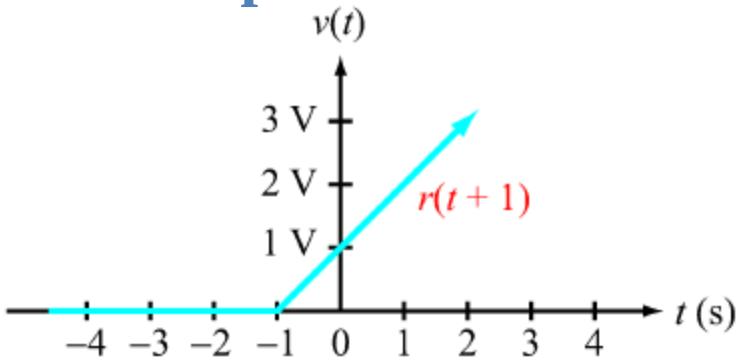


Non-Periodic Waveforms

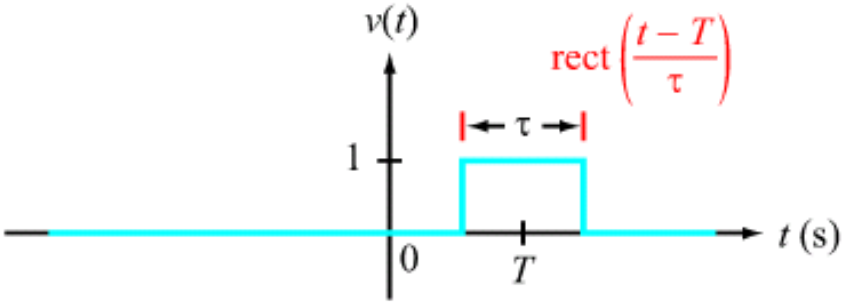
Step Function



Ramp Function



Square Pulse



Exponential

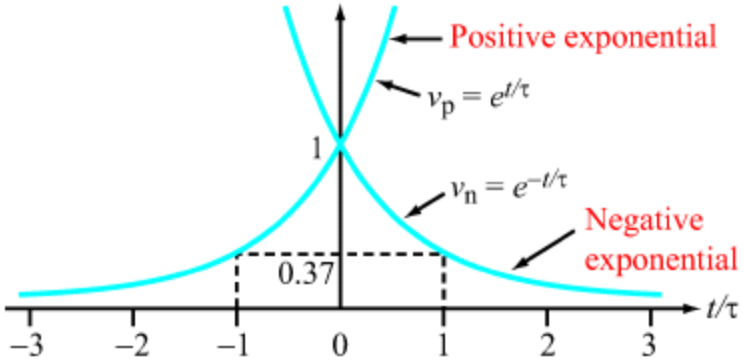
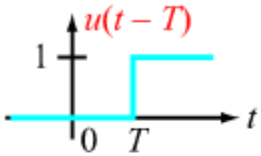
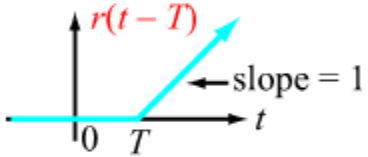
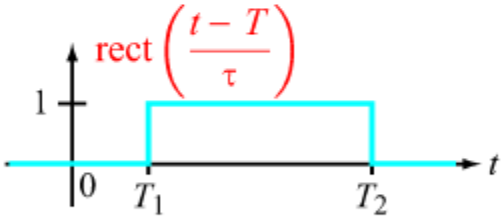
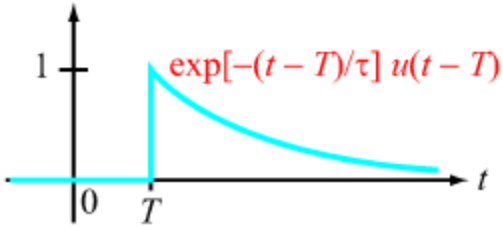
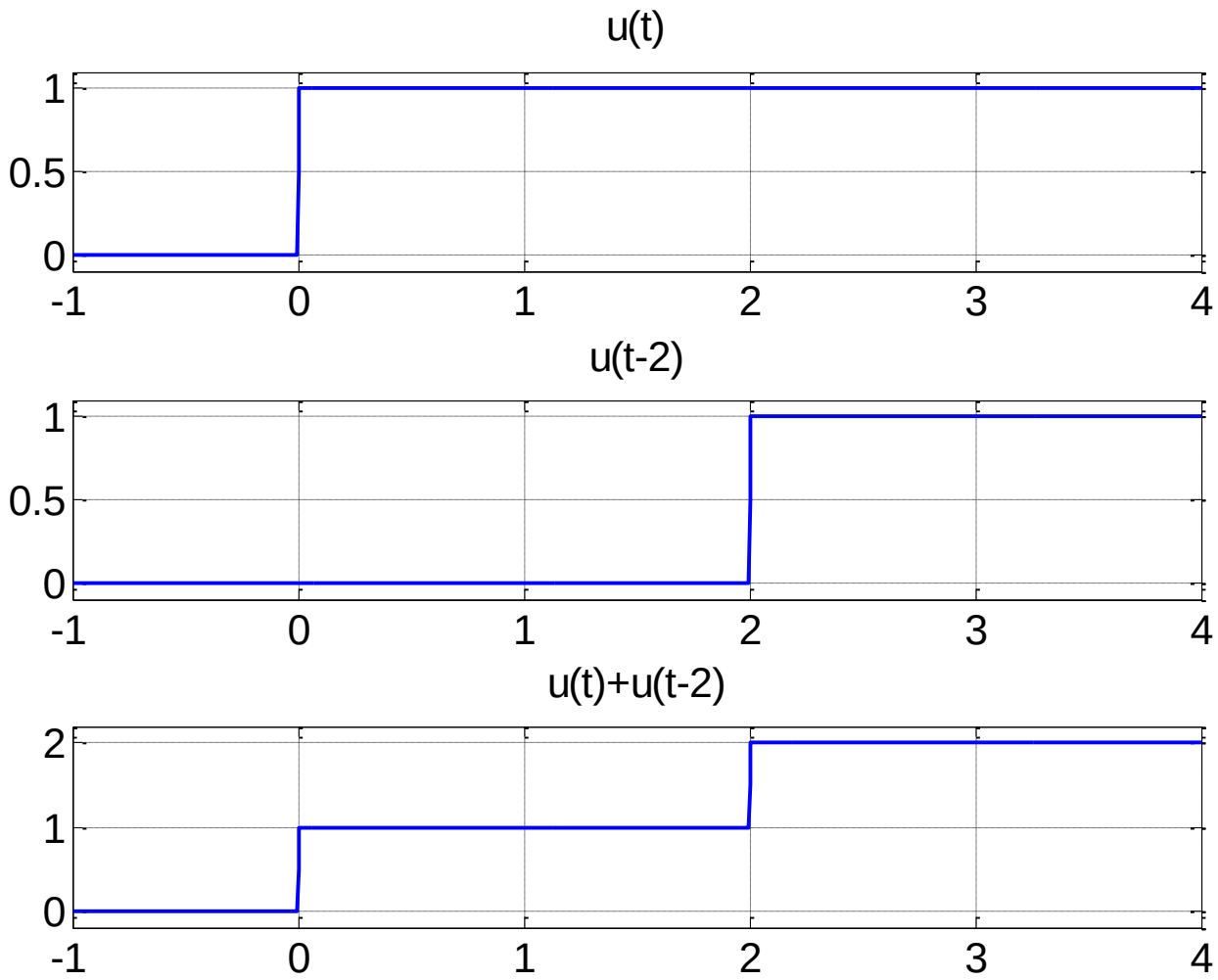


Table 5-1 Common Nonperiodic Functions

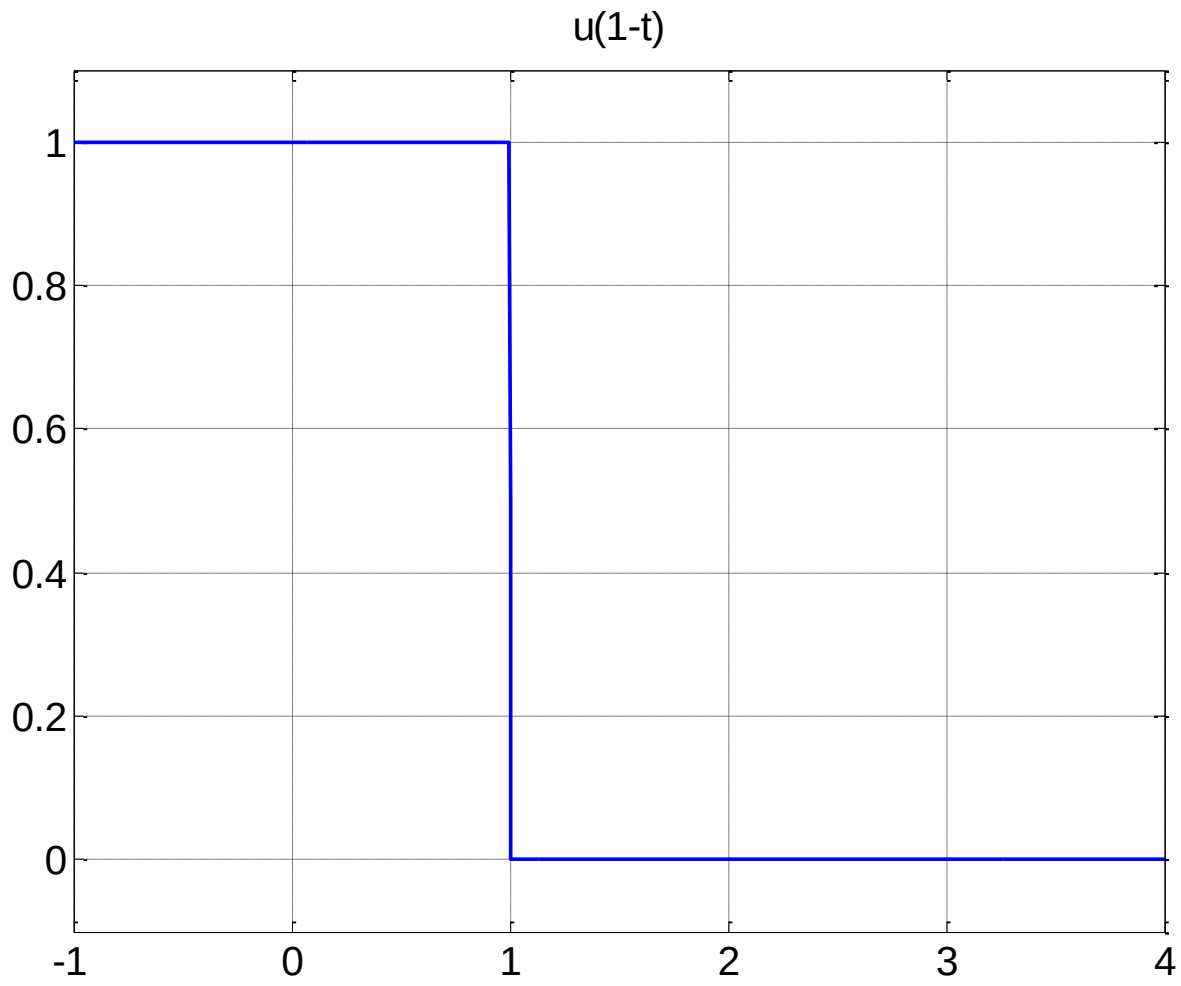
Table 5-1: Common nonperiodic waveforms.

Waveform	Expression	General Shape
Step	$u(t-T) = \begin{cases} 0 & \text{for } t < T \\ 1 & \text{for } t > T \end{cases}$	
Ramp	$r(t-T) = (t-T)\,u(t-T)$	
Rectangle	$\text{rect}\left(\frac{t-T}{\tau}\right) = u(t-T_1) - u(t-T_2)$ $T_1 = T - \frac{\tau}{2}; \quad T_2 = T + \frac{\tau}{2}$	
Exponential	$\exp[-(t-T)/\tau]\,u(t-T)$	

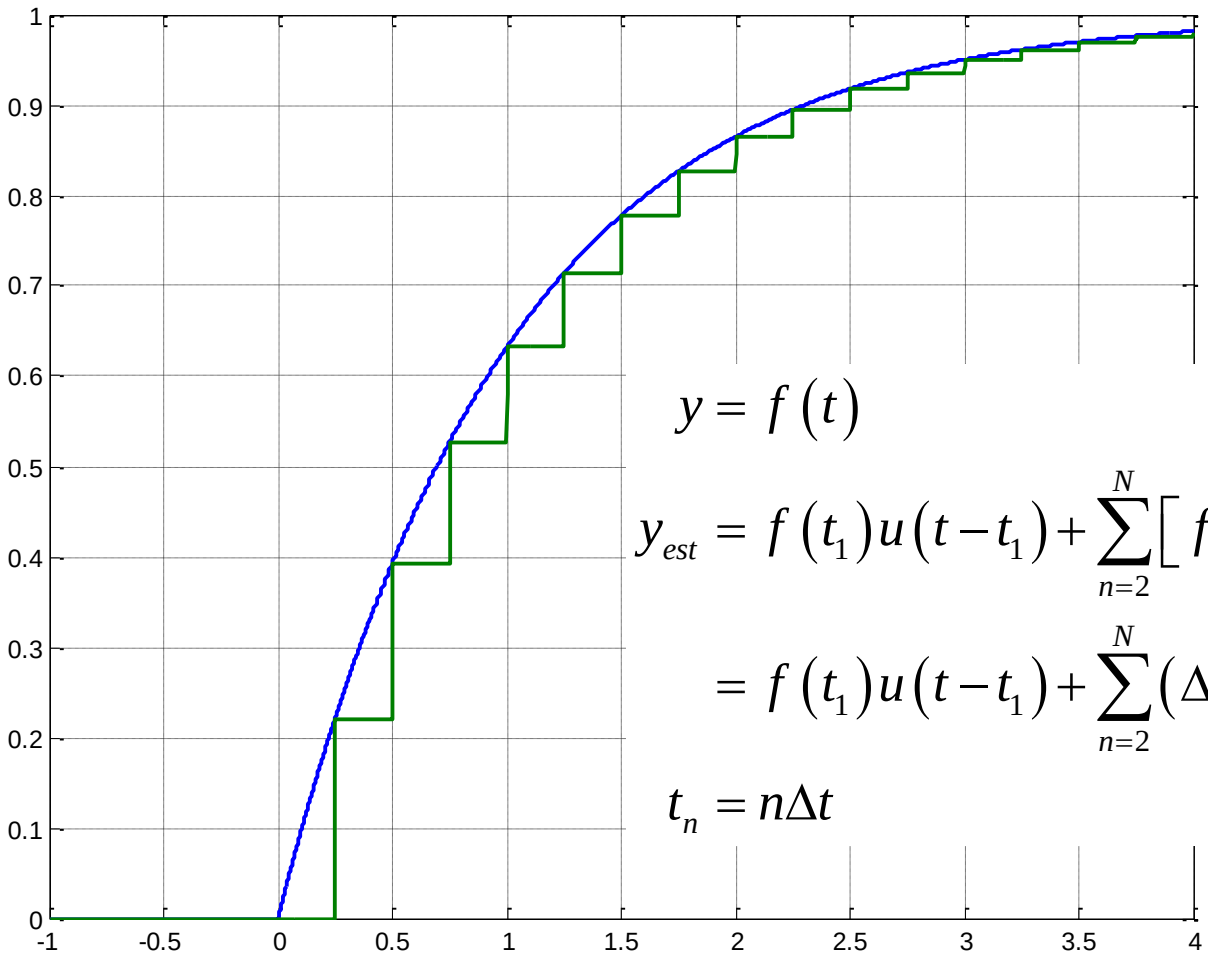
Unit Step & Time Shift



Example u(1-t)



Approximating Functions with Steps



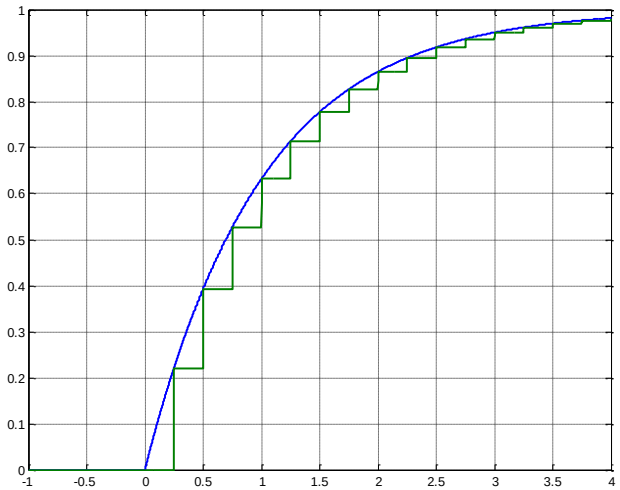
$$y = f(t)$$

$$y_{est} = f(t_1)u(t-t_1) + \sum_{n=2}^{N}\left[f(t_n) - f(t_{n-1})\right]u(t-t_n)$$

$$= f(t_1)u(t-t_1) + \sum_{n=2}^{N}(\Delta y_n)u(t-t_n)$$

$$t_n = n\Delta t$$

Approximating Response to Arbitrary Waveforms: Linearity and Superposition



$$y = f(t)$$

$$y_{est} = f(t_1)u(t-t_1) + \sum_{n=2}^{N}\left[f(t_n) - f(t_{n-1})\right]u(t-t_n)$$

$$= f(t_1)u(t-t_1) + \sum_{n=2}^{N}(\Delta y_n)u(t-t_n)$$

$$t_n = n\Delta t$$

Suppose that $v_{out} = g(t)u(t)$ is our linear circuit unit step response $\{\text{to } v_{in} = u(t)\}$

Then we can approximate the response to some arbitrary input $v_{in} = f(t)$ by

$$v_{out,est}(t) = g(t)u(t-t_1) + \sum_{n=2}^{N}\left[f(t_n) - f(t_{n-1})\right]g(t)u(t-t_n)$$

$$v_{out,est}(t) = f(t_1)u(t-t_1) + \sum_{n=2}^{N}(\Delta y_n)g(t)u(t-t_n)$$

$$t_n = n\Delta t$$

Energy Storage Elements

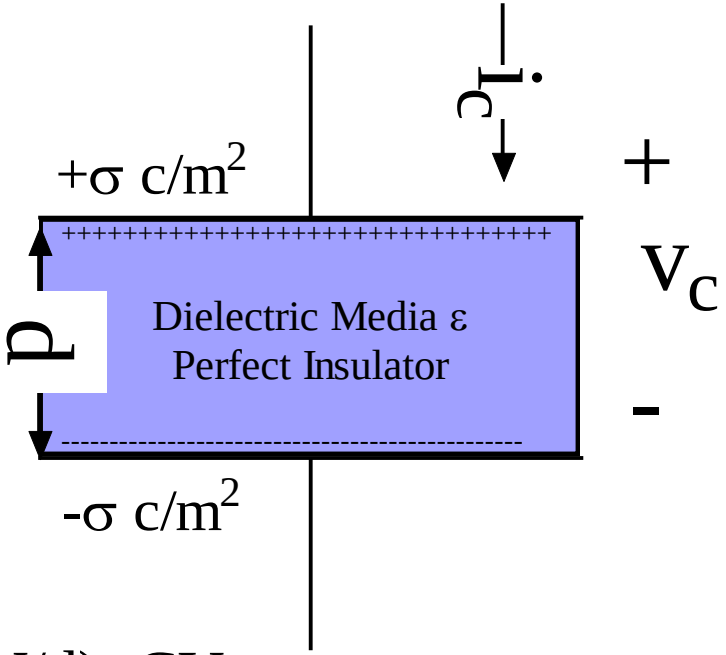
- Capacitors and Inductors Store Electromagnetic Energy
  - Useful in Power Conditioning (A.C. to D.C. Conversion)
  - Useful in Filtering Circuits
  - Useful in RF Matching Networks
- Takes Time To Change Energy Storage (unless you have infinite *power* available).
- The Capacitors and Inductors in a Circuit Are Strong Factors in Determining Frequency Response and Speed of All Real Circuits
- Parasitic (unavoidable but present) Elements in Transistor Circuits (a little in EECS 215, more in EECS 311, 312, model building in EECS 320)
- Delays on Real Wires (Transmission Lines, EECS 230, 330).

Physics

- Detailed E&M from Maxwell's Equations
  - Phys 240
  - EECS 230
- Here We Will Use the Resulting Circuit Laws from This Physics

Ideal Capacitor Basics

$$i_c = C\frac{\partial V_c}{\partial t}$$



Charge on each plate Q=Aσ= Aε*E*=(AεV/d)=CV

Equal and opposite total charges on each plate

Total Device is Neutral

Device storage energy in E-field

Various types of capacitors

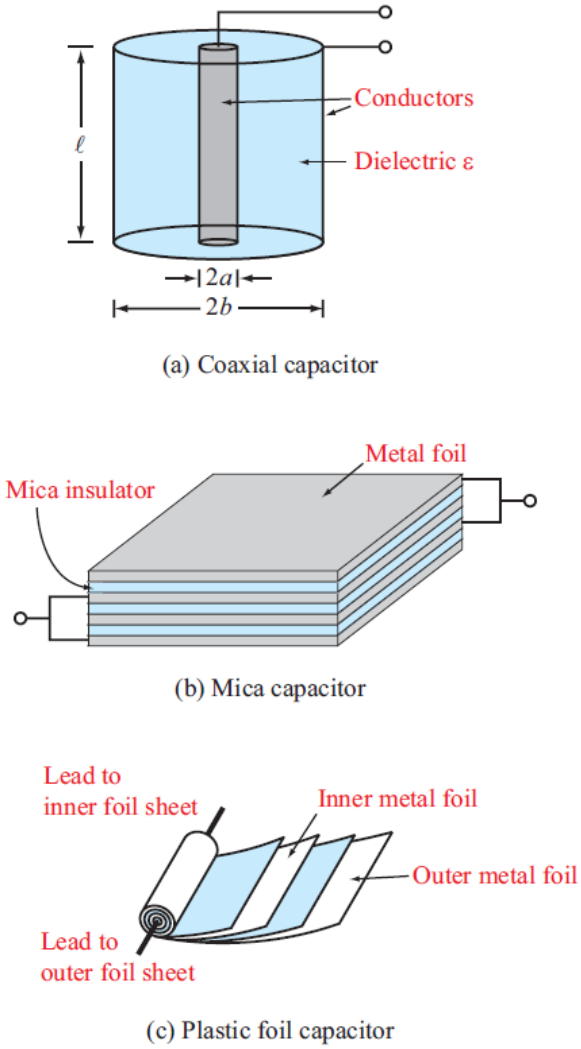


Figure 5-12: Various types of capacitors.

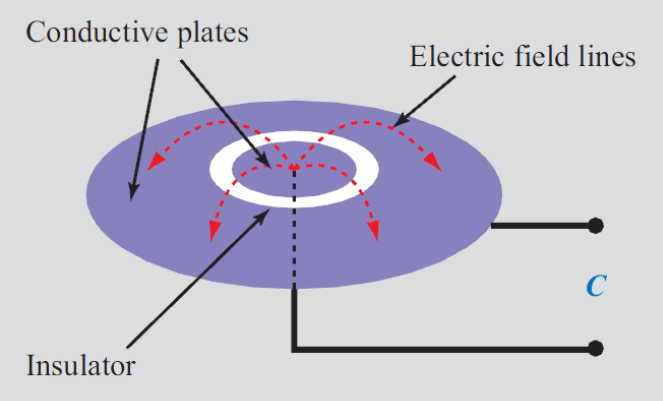


Figure TF9-4: Concentric-plate capacitor.

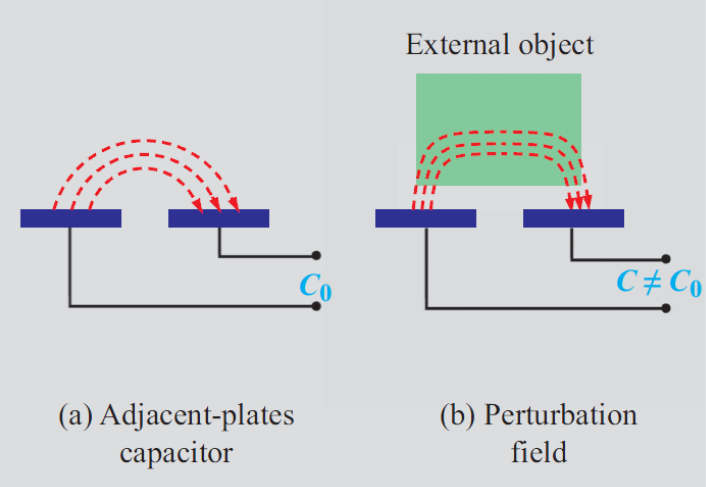


Figure TF9-5: (a) Adjacent-plates capacitor; (b) perturbation field.

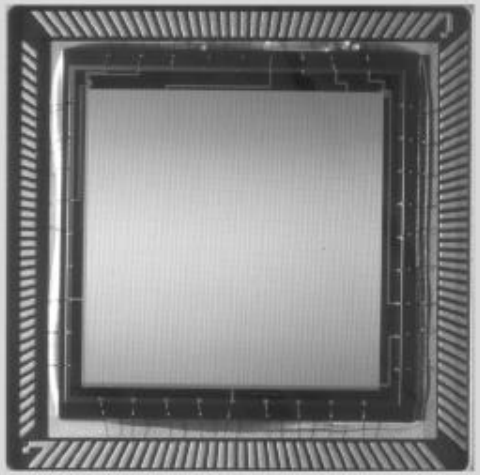
Capacitors in Fingerprint Imager

Fingerprint Imager *

An interesting extension of non-contact sensors is the development of a fingerprint imager consisting of a two-dimensional array of capacitive sensor cells, constructed to record an electrical representation of a fingerprint (B1 and B2). Each sensor cell is composed of an adjacent-plates capacitor connected to a capacitance measurement circuit (B3). The entire surface of the imager is covered by a thin layer of non-conductive oxide. When the finger is placed on the oxide surface, it perturbs the field lines of the individual sensor cells to varying degrees, depending on the distance between the ridges and valleys of the finger's surface from the sensor cells. Given that the dimensions of an individual sensor are on the order of 65 μm on the side, the imager is capable of recording a fingerprint image at a resolution corresponding to 400 dots per inch or better.

* Courtesy of Dr. M. Tartagni, University of Bologna, Italy

B2. Individual sensor cells



B1. 2-D capacitive sensor array

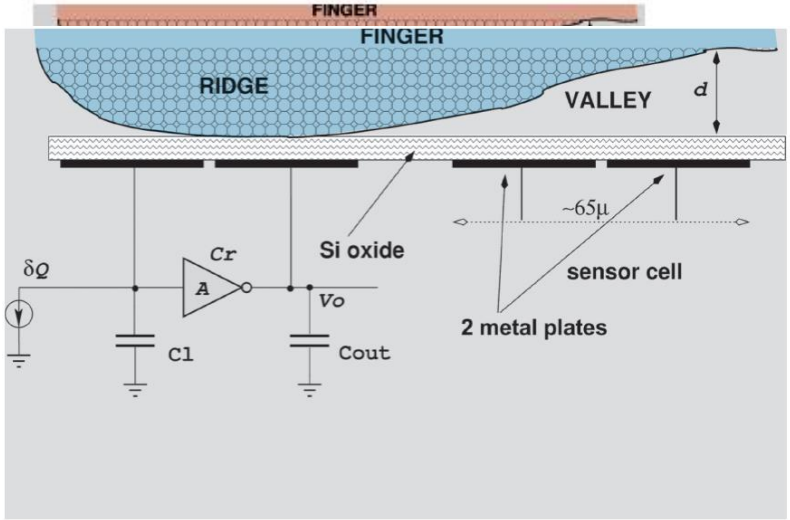
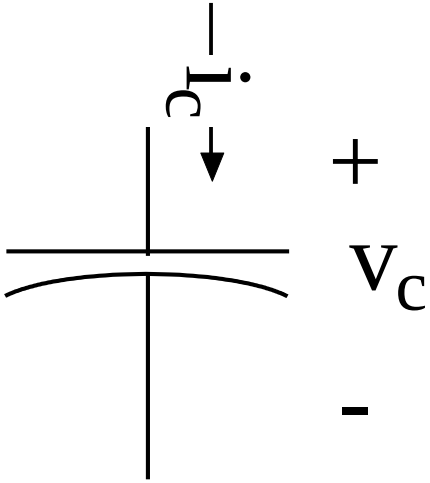


Figure TF9-7: Fingerprint representation. (Courtesy of Dr. M. Tartagni, University of Bologna, Italy.)

Ideal Capacitor Basics

$$i_c = C\frac{\partial V_c}{\partial t}$$



C in Farads 1F=1 C/V

V_c must be continuous function of time t. Otherwise i_c is infinite (& power is infinite).

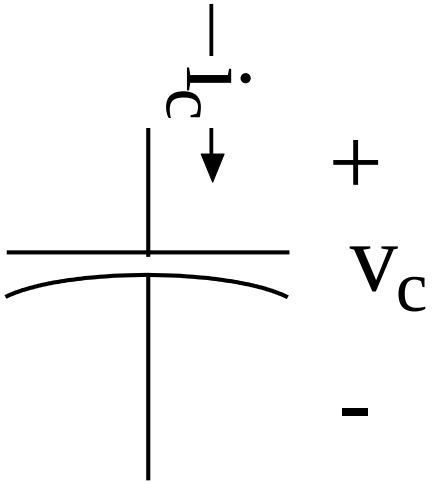
Voltage across a capacitor cannot change instantly !

ONLY exception: Idealized problem with a PERFECT V-source connected directly across a capacitor

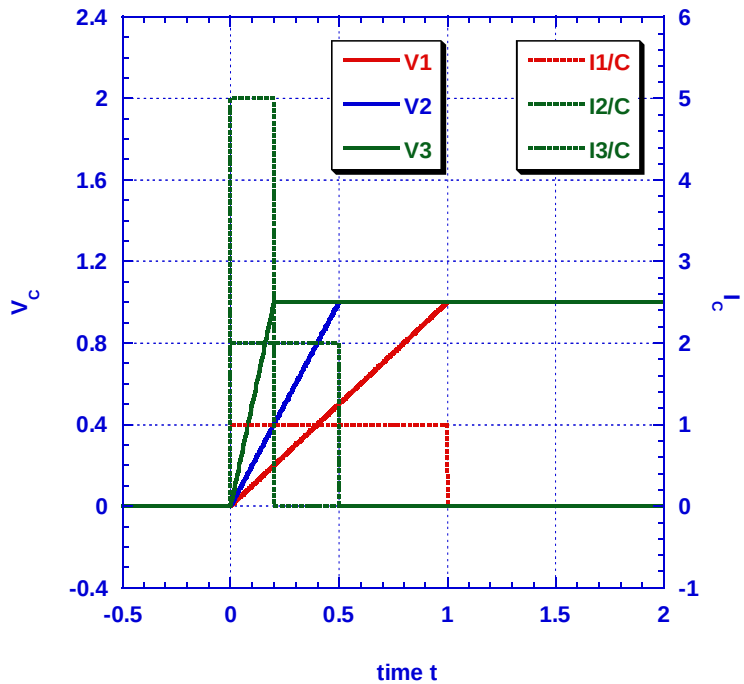
At DC, i_c=0→Capacitor looks like OPEN Circuit

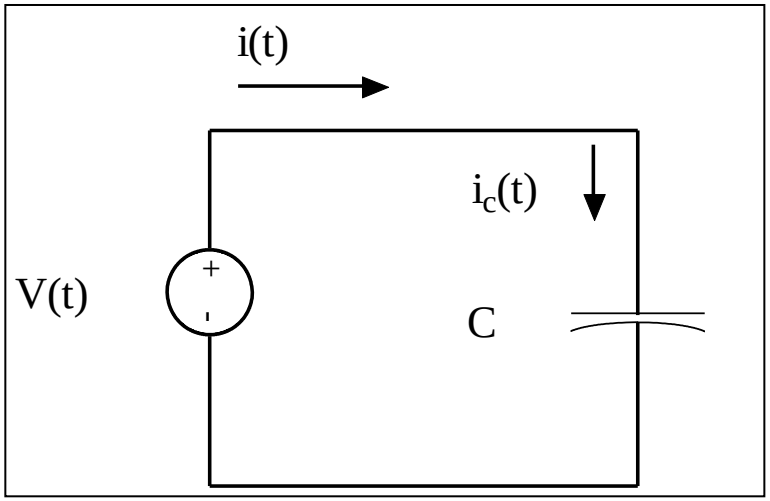
Ideal Capacitor Basics

$$i_c = C\frac{\partial V_c}{\partial t}$$



Voltage Ramp Into Capacitor





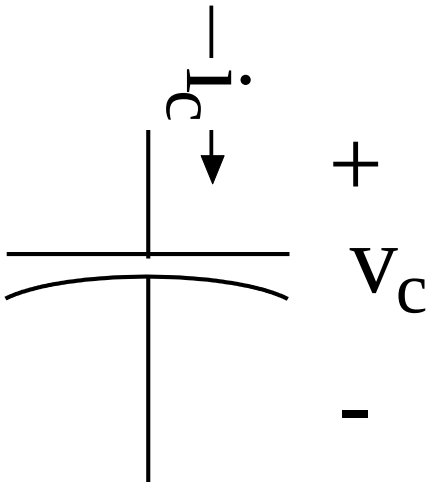
Limit: As Ramp Rate $\rightarrow\infty$ $i_c\rightarrow\infty$ at t=0 only (unit impulse of charge)

Infinite Instantaneous Power, but Finite Total Energy

Stored Energy in Capacitor

Work w to charge a capacitor

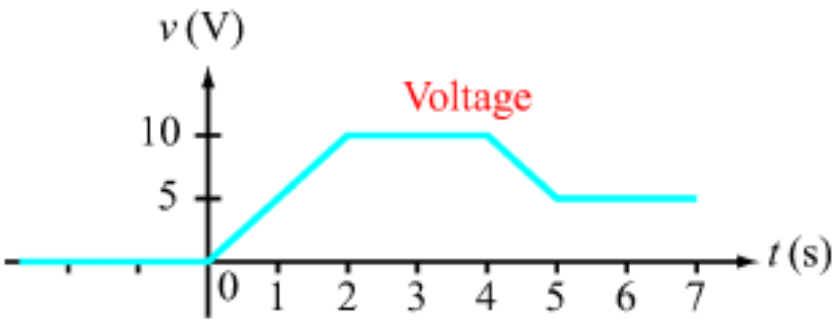
$$
\begin{aligned}
p(t) &= i_c(t)V_c(t) \\
&= C\left(\frac{\partial V_c}{\partial t}\right)V_c \\
w &= \int_0^t C\left(\frac{\partial V_c}{\partial t}\right)V_c \partial t \\
&= \int_{V_c(0)}^{V_c(t)} CV_c \partial V_c \\
&= \frac{1}{2}C\left[\left(V_c(t)\right)^2 - \left(V_c(0)\right)^2\right]
\end{aligned}
$$

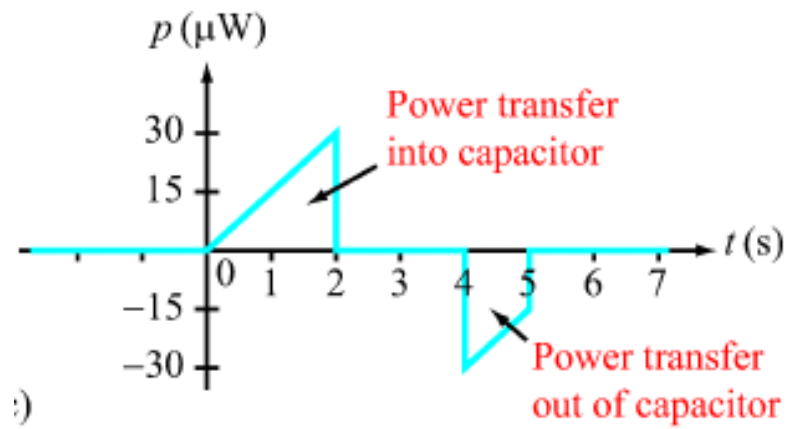


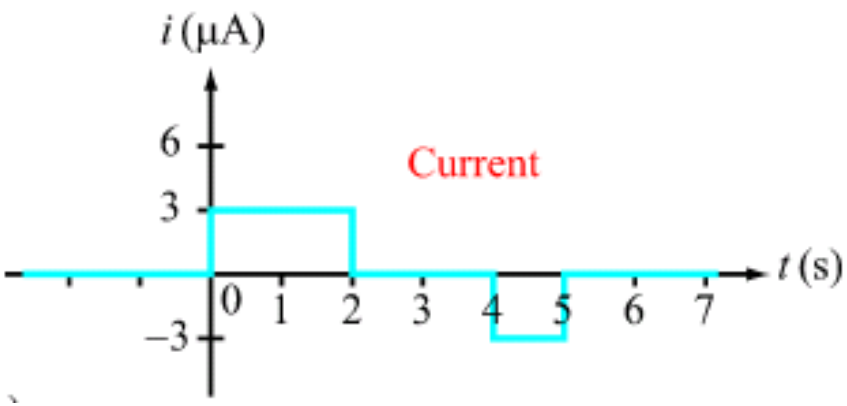
Total Stored Energy in Capacitor

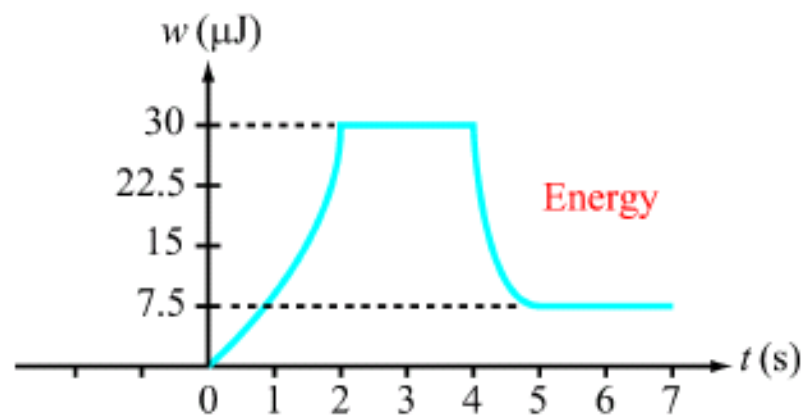
$$E_c = \frac{1}{2}CV_c^2$$

Capacitor Response



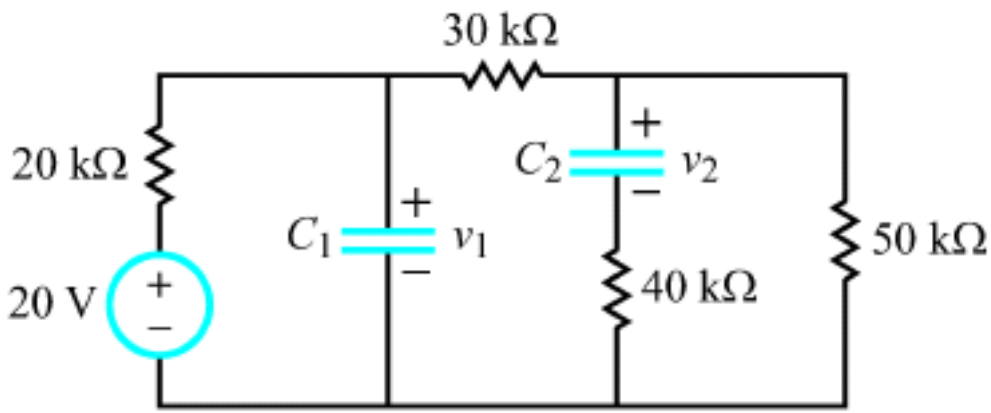


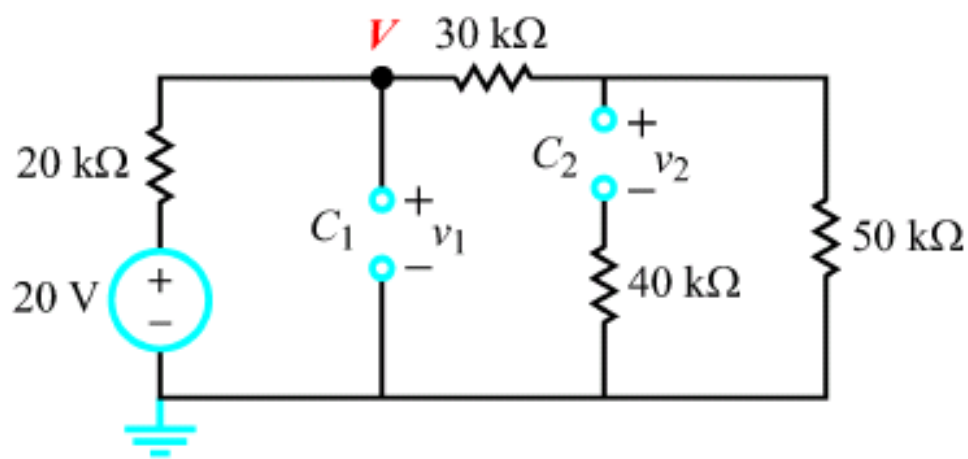




RC Circuits at DC

- No current flow through capacitors: open circuit





Parallel Capacitors

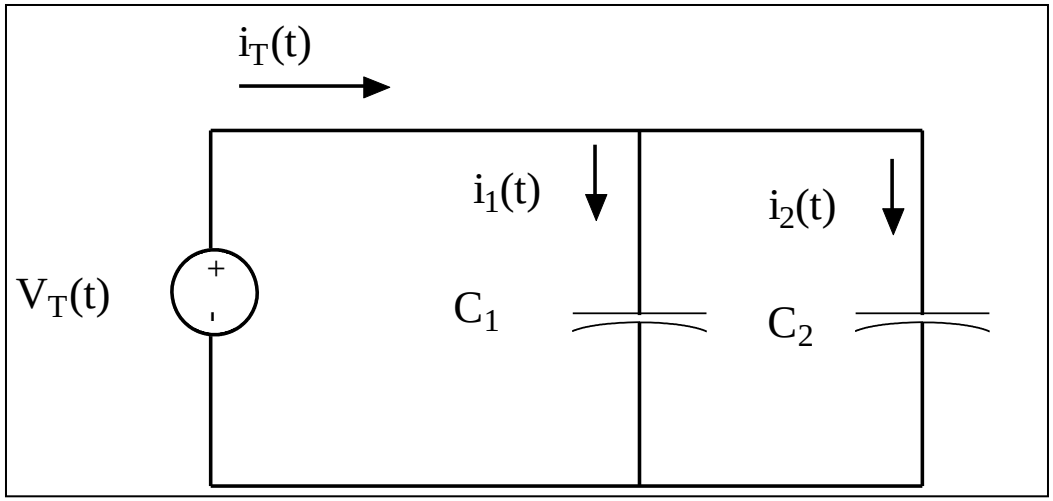
Parallel Capacitors Add

$$i_T = i_1 + i_2$$

$$= C_1 \frac{\partial V_{c1}}{\partial t} + C_2 \frac{\partial V_{c2}}{\partial t}$$

$$= \left(C_1 + C_2\right) \frac{\partial V_T}{\partial t}$$

$$= C_{eff} \frac{\partial V_T}{\partial t}$$



$$C_{eff} = C_1 + C_2$$

$$N - Caps - in - parallel:$$

$$C_{eff} = \sum_{i=1}^{N} C_i$$

Series Capacitors

Series Capacitors Add Reciprocally

$$V_T = V_{c1} + V_{c2}$$

$$i_T(t) = C_1 \frac{\partial V_{c1}}{\partial t} = C_2 \frac{\partial V_{c2}}{\partial t}$$

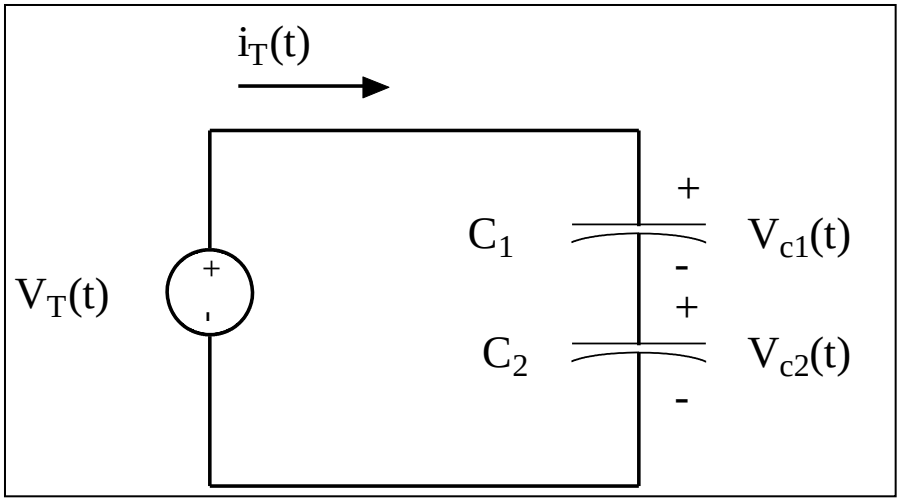
$$\frac{\partial V_T}{\partial t} = \frac{\partial V_{c1}}{\partial t} + \frac{\partial V_{c2}}{\partial t}$$

$$= \frac{i_T}{C_1} + \frac{i_T}{C_2}$$

$$= \left[\frac{1}{C_1} + \frac{1}{C_2}\right] i_T$$

$$i_T(t) = C_{eff} \frac{\partial V_T}{\partial t}$$

$$= \left[\frac{1}{C_1} + \frac{1}{C_2}\right]^{-1} \left(\frac{\partial V_T}{\partial t}\right)$$



$$C_{eff} = \left[C_1^{-1} + C_2^{-1}\right]^{-1}$$

$$N - Caps - in - series:$$

$$C_{eff} = \left[\sum_{i=1}^{N} C_i^{-1}\right]^{-1}$$

Series Capacitors: Capacitive V-Divider

$$V_T = V_{c1} + V_{c2}$$

$$Q_1 = Q_2$$

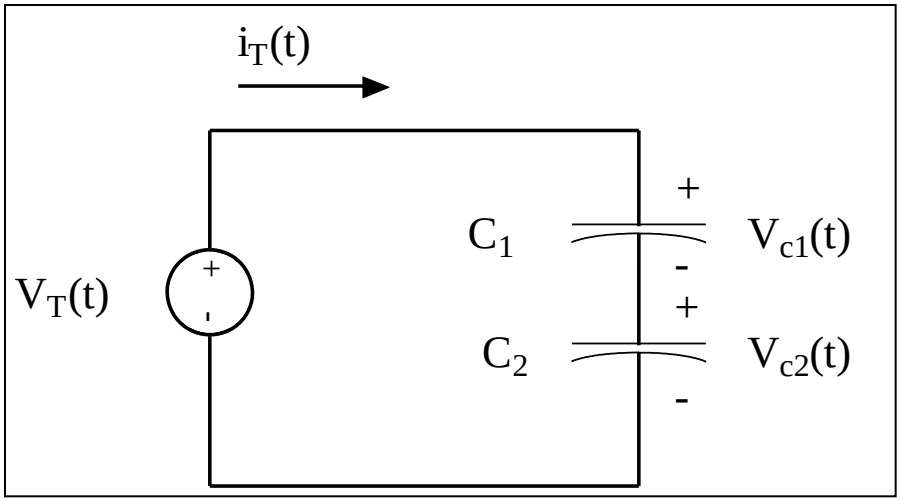
$$C_1 V_{c1} = C_2 V_{c2}$$

$$V_{c2} = \left(\frac{C_1}{C_2}\right) V_{c1}$$

$$V_T = V_{c1}\left(1 + \frac{C_1}{C_2}\right)$$

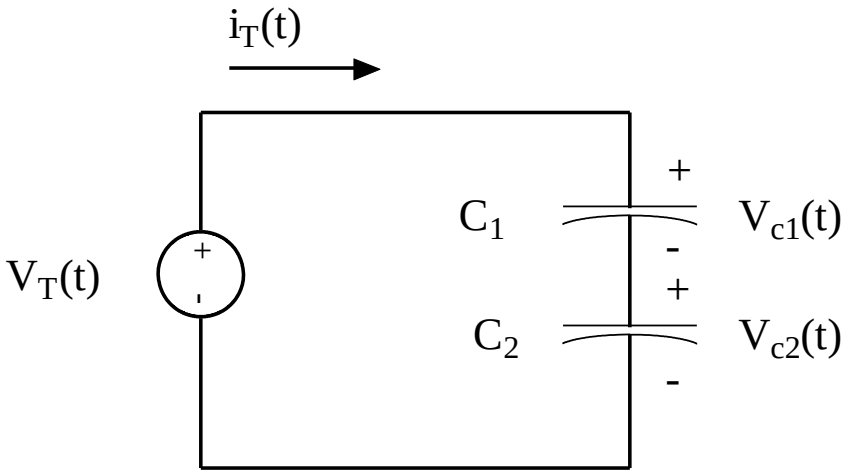
$$V_{c1} = \frac{V_T}{1 + \frac{C_1}{C_2}} = \left[\frac{C_2}{C_1 + C_2}\right] V_T$$

$$V_{c2} = \left[\frac{C_1}{C_1 + C_2}\right] V_T$$

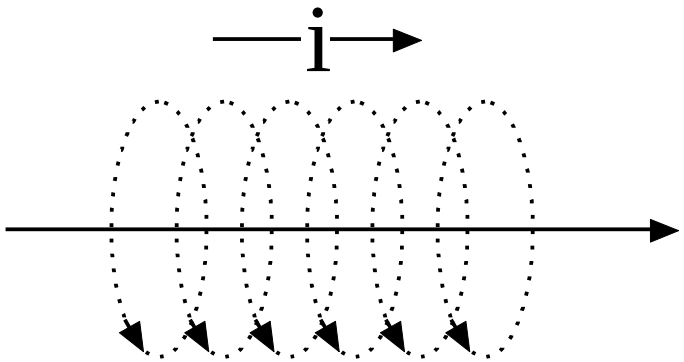


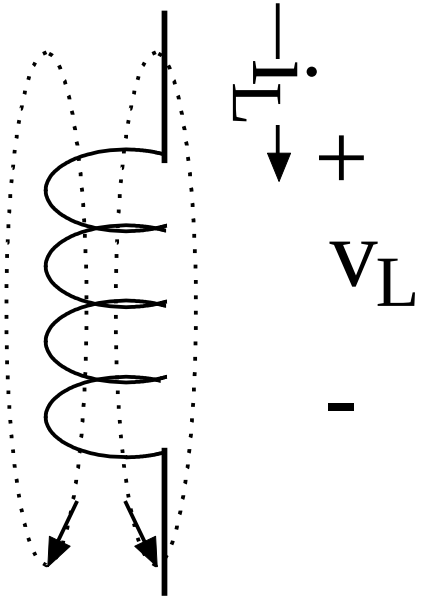
What would you get for 3 Cap's in Series?

Series Capacitors: Capacitive V-Divider



Ideal Inductor Basics





$$i = \oint \vec{H} \bullet \partial \vec{l}$$

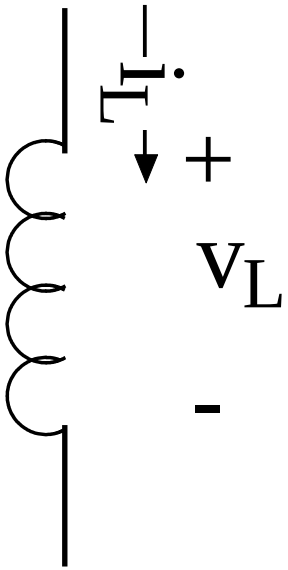
$$= \int_0^{2\pi} \vec{H}(r) r \partial \theta$$

Magnetic Energy Storage tied to CURRENT

Changing H-field requires Changing I

Ideal Inductor Basics

$$V_L = L\frac{\partial i_L}{\partial t}$$



L in Henries 1H= 1V-s/A

i_L must be continuous function of time t. Otherwise V_L is infinite (& power is infinite).

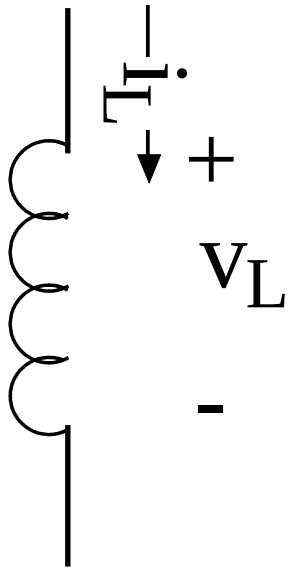
Current through an inductor cannot change instantly!

ONLY exception: Idealized problem with a PERFECT I-source connected directly to one node of an inductor

At DC, V_L=0→Inductor looks like Short Circuit

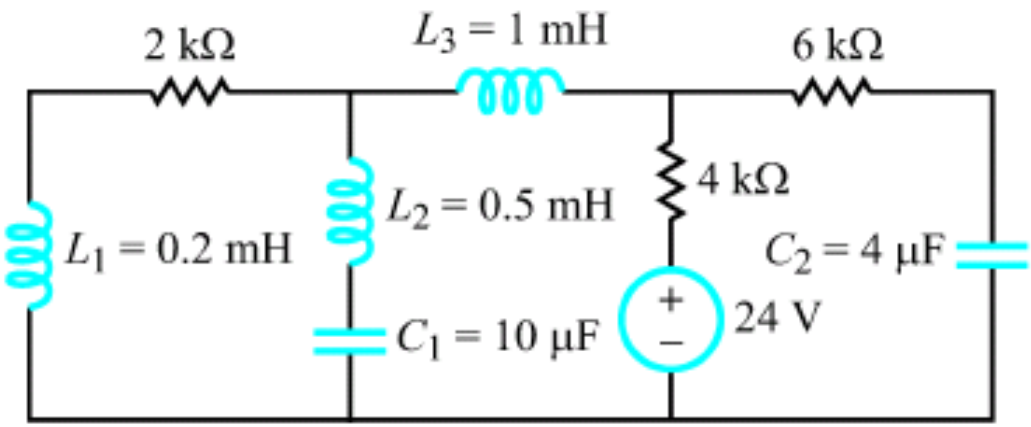
Ideal Inductor Basics

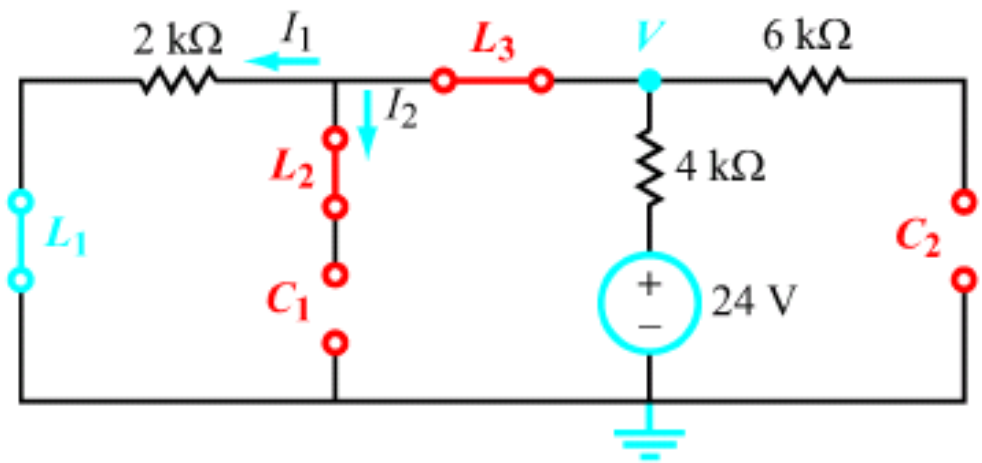
$$V_L = L\frac{\partial i_L}{\partial t}$$



RL Circuits at DC

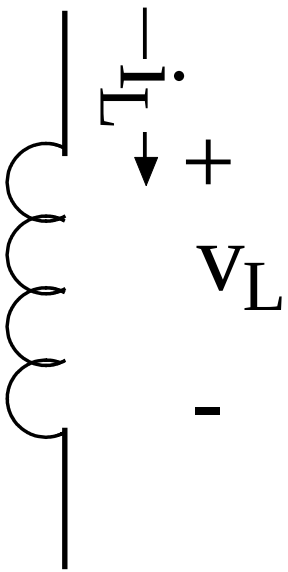
- No voltage across inductors: short circuit





Stored Energy in Inductor

Work w to energize an inductor



$$
\begin{aligned}
p(t) &= i_L(t)V_L(t) \\
&= L\left(\frac{\partial i_L}{\partial t}\right)i_L \\
w &= \int_0^t L\left(\frac{\partial i_L}{\partial t}\right)i_L \partial t \\
&= \int_{i_L(0)}^{i_L(t)} L i_L \partial i_L \\
&= \frac{1}{2}L\left[\left(i_L(t)\right)^2 - \left(i_L(0)\right)^2\right]
\end{aligned}
$$

Total Stored Energy in Inductor

$$E_L = \frac{1}{2}Li_L^2$$

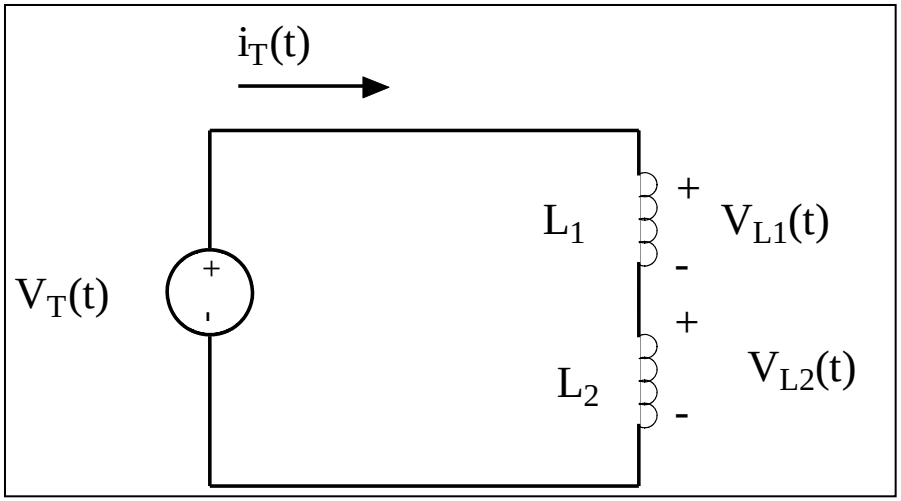
Series Inductors

Series Inductors Add

$$V_T = V_{L1} + V_{L2}$$

$$= L_1 \frac{\partial i_{L1}}{\partial t} + L_2 \frac{\partial i_{L2}}{\partial t}$$

$$= \left(L_1 + L_2\right) \frac{\partial i_T}{\partial t}$$



$$Leff = \left(L_1 + L_2\right)$$

$$N - Inductors - in - series:$$

$$L_{eff} = \left[\sum_{i=1}^{N} L_i\right]$$

Parallel Inductors

Parallel Inductors Add Reciprocally

$$i_T = i_1 + i_2$$

$$\frac{\partial i_T}{\partial t} = \frac{\partial i_1}{\partial t} + \frac{\partial i_2}{\partial t}$$

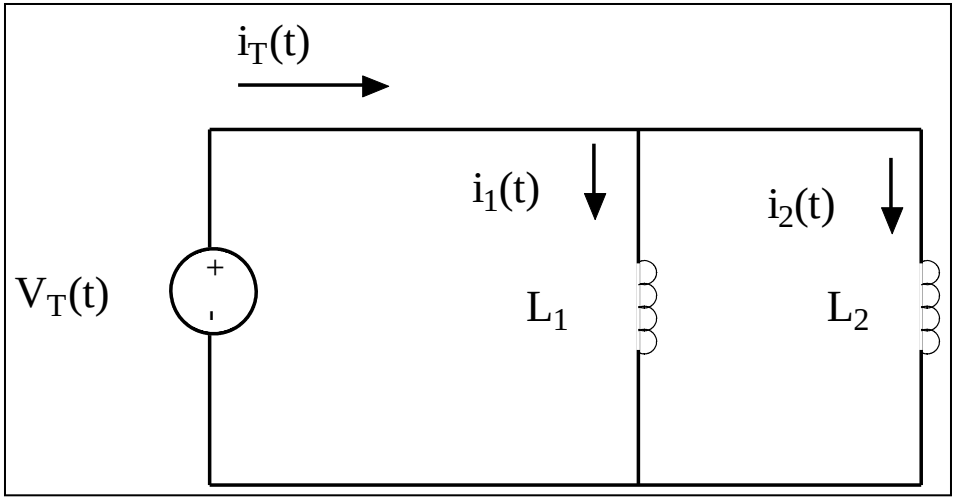
$$V_T = L_1 \frac{\partial i_1}{\partial t} = L_2 \frac{\partial i_2}{\partial t}$$

$$\frac{\partial i_1}{\partial t} = \frac{V_T}{L_1}$$

$$\frac{\partial i_2}{\partial t} = \frac{V_T}{L_2}$$

$$\frac{\partial i_T}{\partial t} = \frac{\partial i_1}{\partial t} + \frac{\partial i_2}{\partial t} = \left[\frac{1}{L_1} + \frac{1}{L_2}\right] V_T$$

$$V_T = \left[\frac{1}{L_1} + \frac{1}{L_2}\right]^{-1} \left(\frac{\partial i_T}{\partial t}\right)$$



$$L_{eff} = \left[\frac{1}{L_1} + \frac{1}{L_2}\right]^{-1}$$

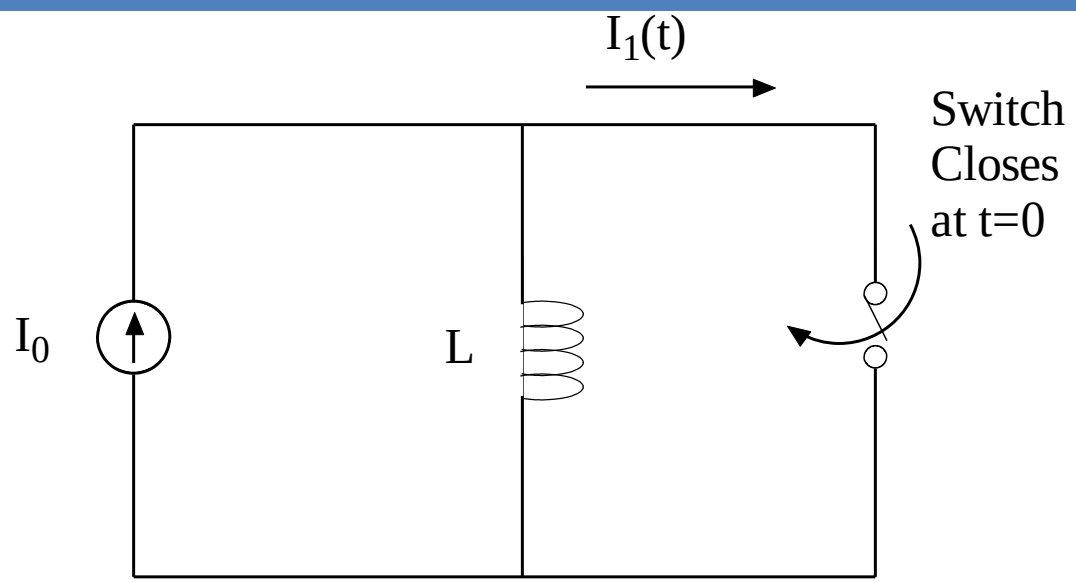
$$N - Inductors - in - parallel:$$

$$L_{eff} = \left[\sum_{i=1}^{N} L_i^{-1}\right]^{-1}$$

Example

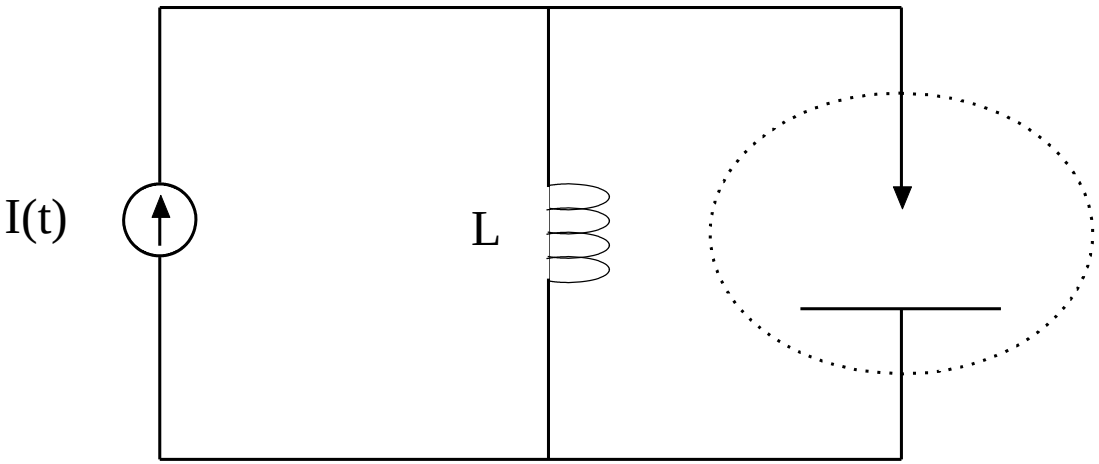
Suppose I_0 is constant and non-zero.

What is $I_1(t)$ for $t>0$?

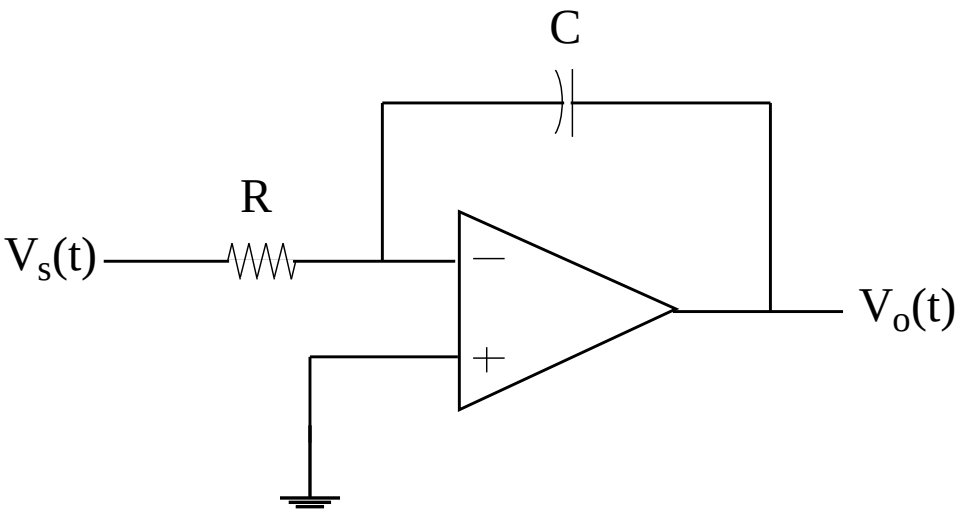


Example: Spark Gap

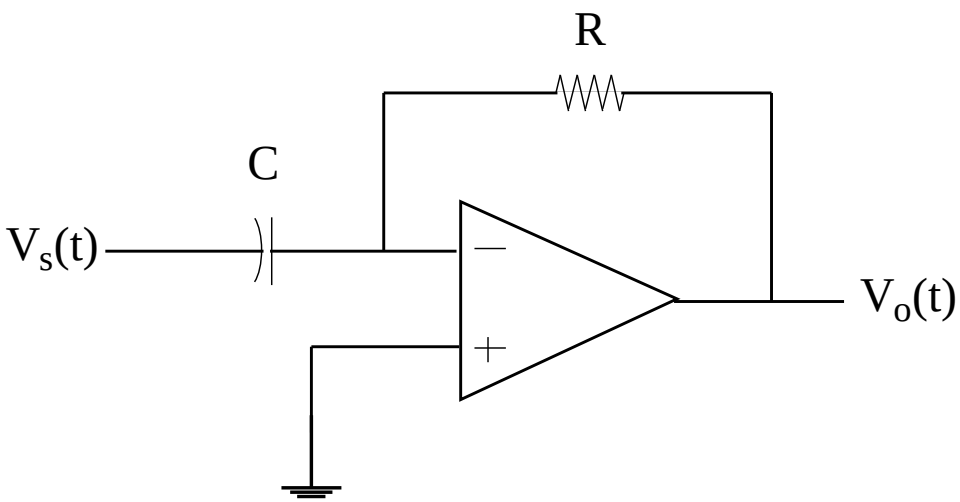
Suppose I(t) = constant t<0 but ramps to 0 for t>0 at rate B A/s



Example: Integrator



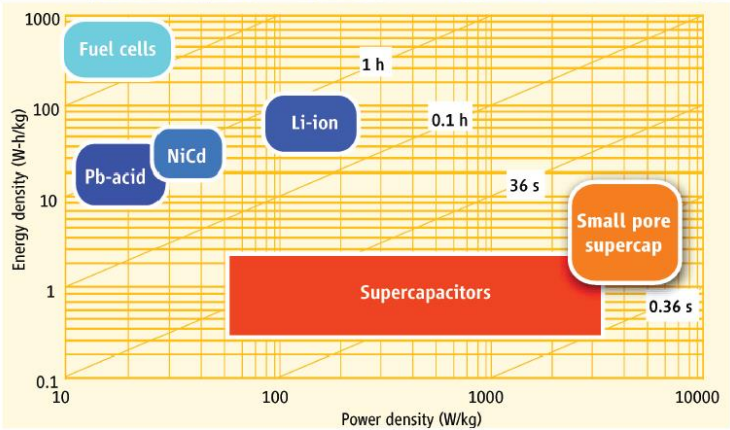
Example: Differentiator



Tech Brief 11: Supercapacitors

A new generation of capacitor technologies, termed ***supercapacitors or ultracapacitors, is narrowing the gap between*** capacitors and batteries. These capacitors can have sufficiently high energy densities to approach within 10 percent of battery storage densities, and additional improvements may increase this even more. Importantly, supercapacitors can absorb or release energy much faster than a chemical battery of identical volume. This helps immensely during recharging. Moreover, most batteries can be recharged only a few hundred times before they are degraded completely; supercapacitors can be charged and discharged millions of times before they wear out. Supercapacitors also have a much smaller environmental footprint than conventional chemical batteries, making them particularly attractive for green energy solutions.

On demand. New supercapacitors store less charge than batteries but can supply it more quickly, making them ideal for hybrid cars.



A supercapacitor is a "hybrid" battery/capacitor

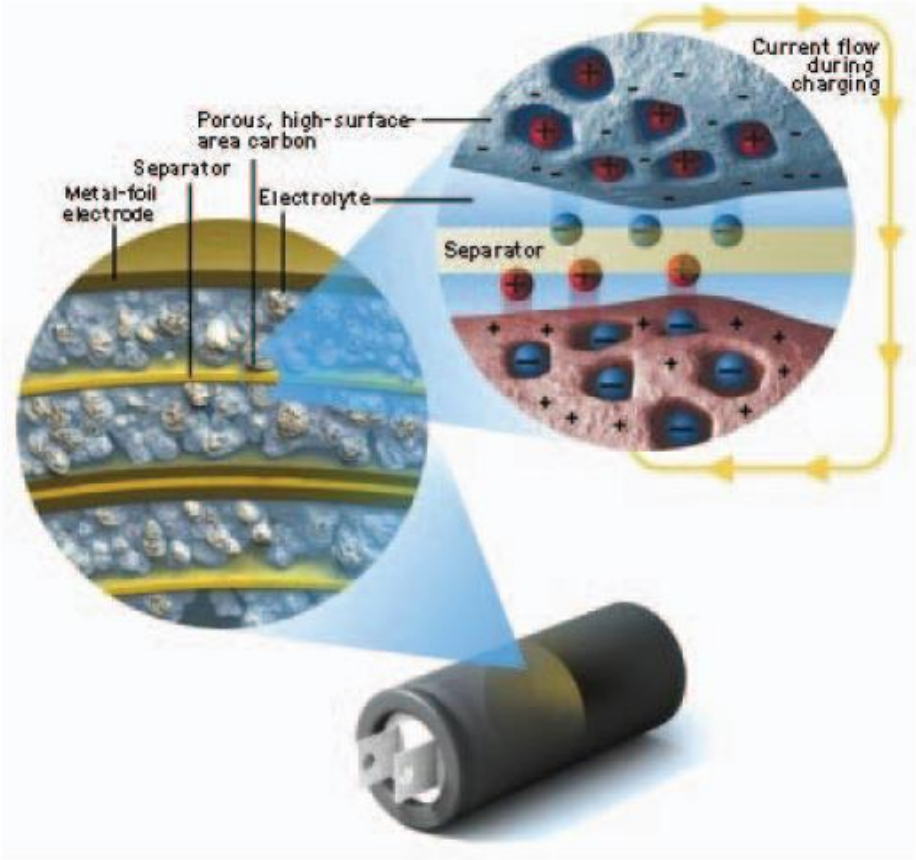


Figure TF8-1: Cross-sectional view of an electrochemical double-layer capacitor (EDLC), otherwise known as a supercapacitor. (Courtesy of Ultracapacitor.org.)

Users of Supercapacitors

Figure TF8-2: Examples of systems that use supercapacitors. (Courtesy of Railway Gazette International; BMW; NASA; Applied Innovative Technologies.)

Energy Comparison

Energy Storage Devices

Feature	Traditional Capacitor	Supercapacitor	Battery
Energy density W' (Wh/kg)	$\sim 10^{-2}$	1 to 10	5 to 150
Power density P' (W/kg)	1,000 to 10,000	1,000 to 5,000	10 to 500
Charge and discharge rate T	10^{-3} sec	$\sim$ 1 sec to 1 min	$\sim$ 1 to 5 hrs
Cycle life N_c	∞	$\sim 10^6$	$\sim 10^3$

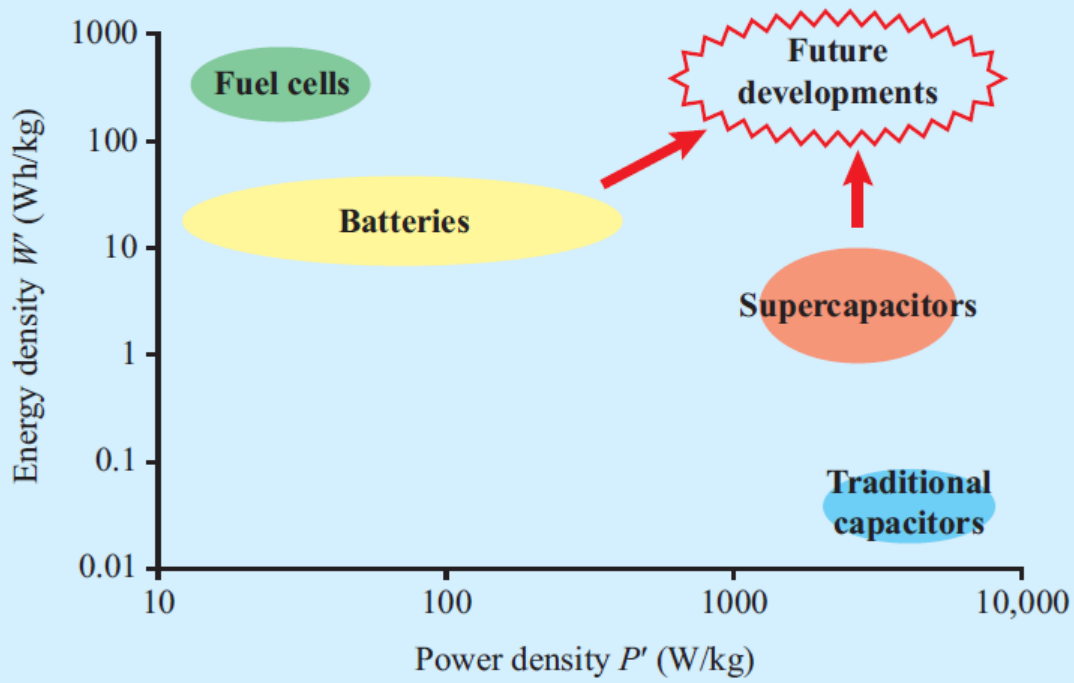


Figure TF8-3: Comparison of energy storage devices.