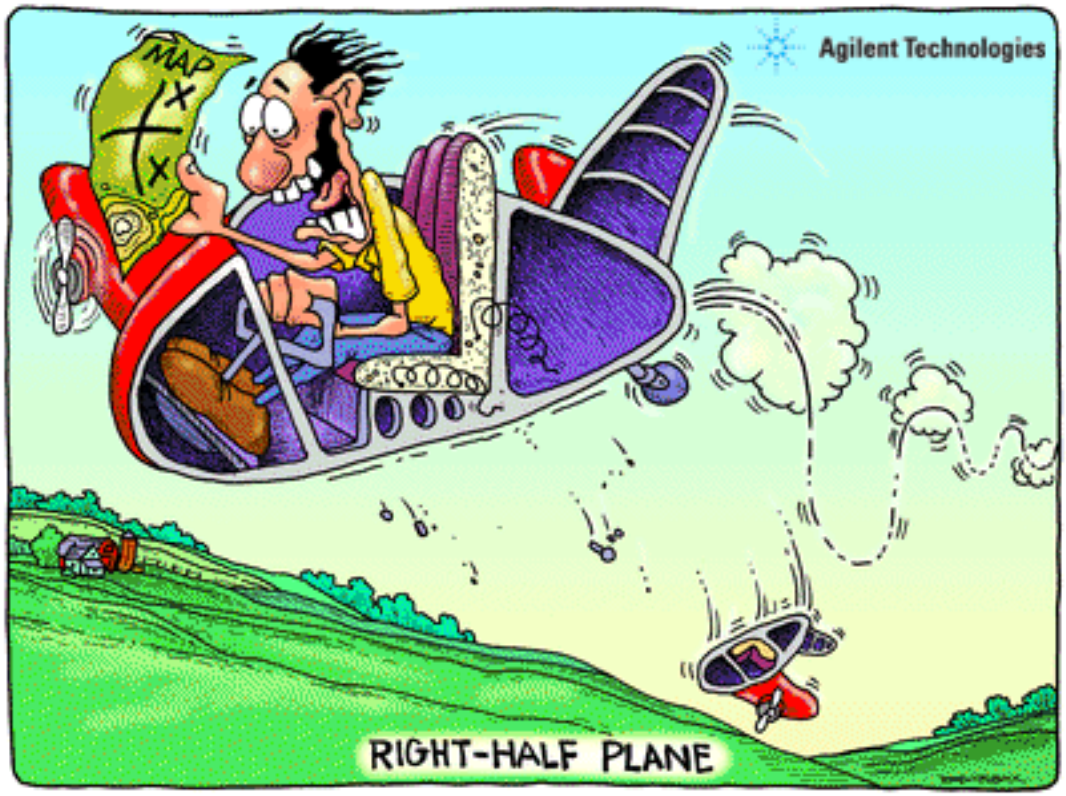
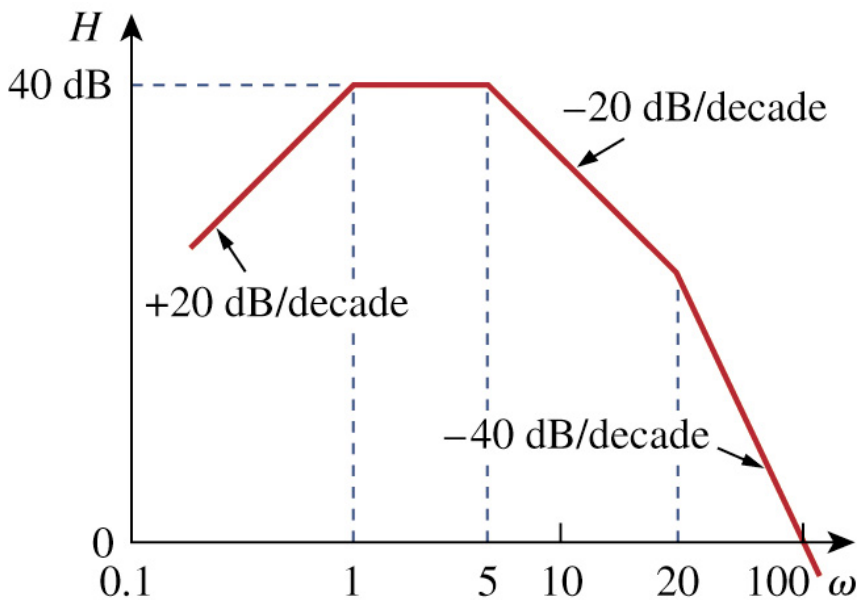


WHY DO WE ASSUME POLES ARE NEGATIVE?

FA13, Fred Terry

Interpreting Bode Plots



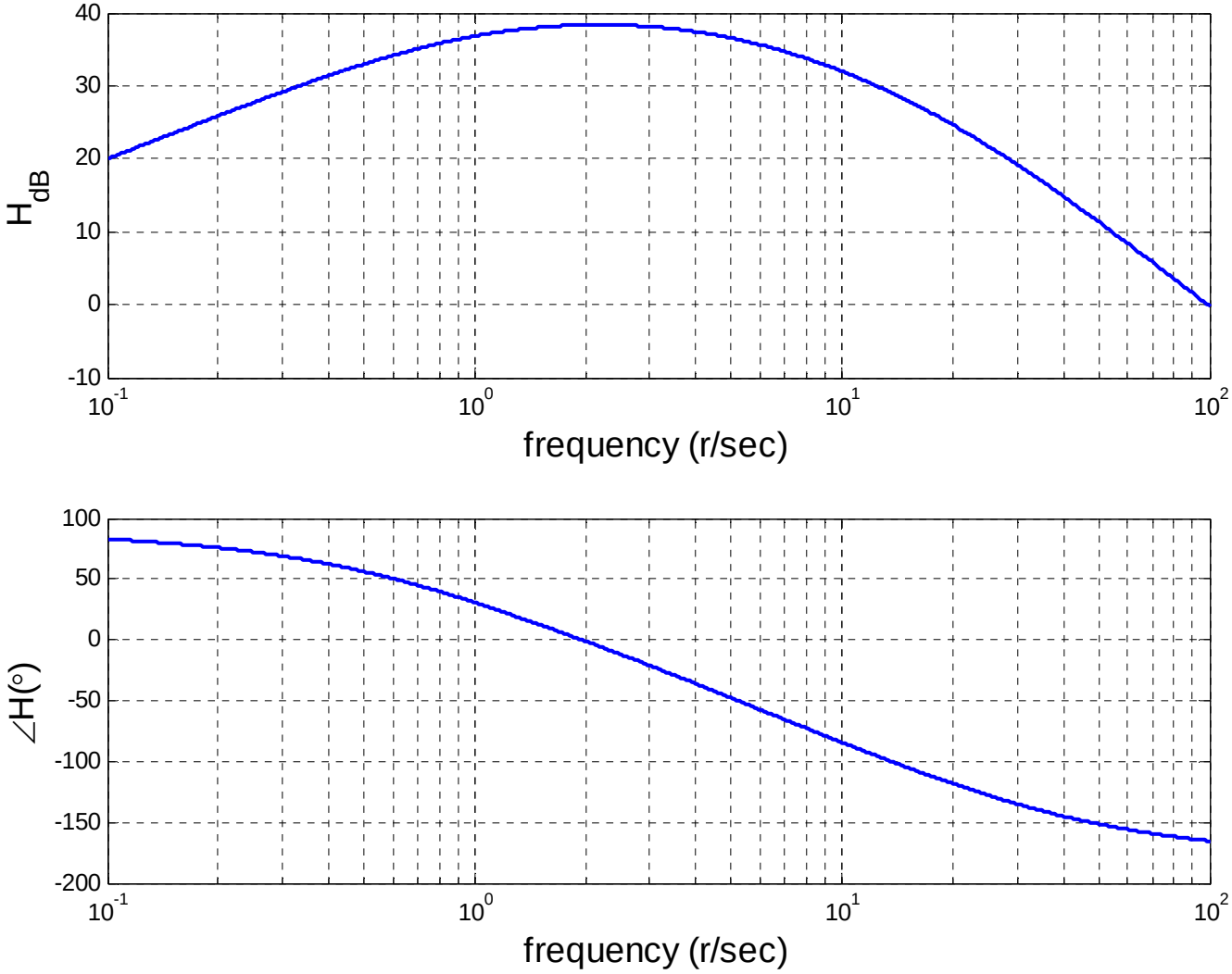
Zeros	Poles
s=0	s=-1
Gain	s=-5
	s=-20

40 dB = 100

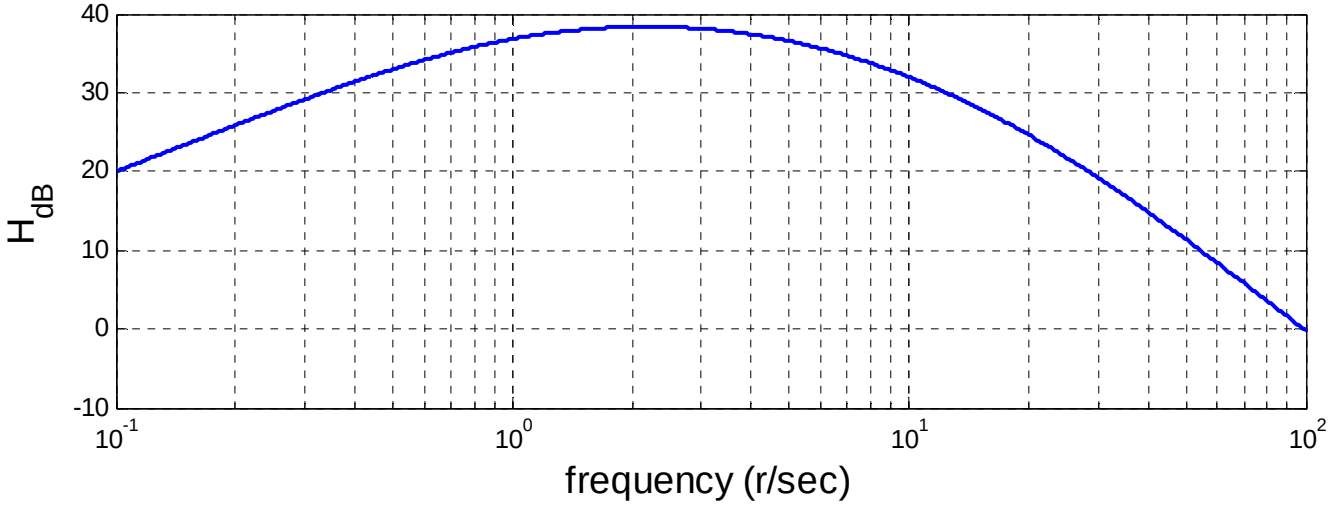
$$H(s) = \frac{100s}{\left(1+\frac{s}{1}\right)\left(1+\frac{s}{5}\right)\left(1+\frac{s}{20}\right)} = \frac{100s}{\left(1-\frac{s}{p_1}\right)\left(1-\frac{s}{p_2}\right)\left(1-\frac{s}{p_3}\right)}$$

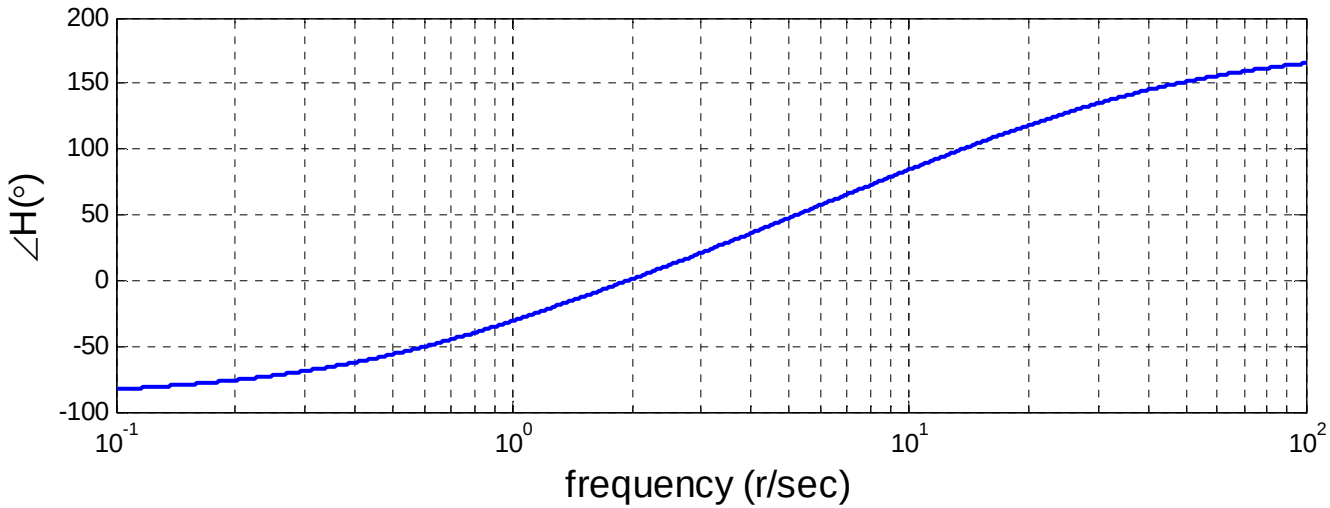
$$= \frac{10^4 s}{(s-p_1)(s-p_2)(s-p_3)}$$

$$H(s) = \frac{10^4 s}{(s-p_1)(s-p_2)(s-p_3)} \quad p_1 = -1 \;\; p_2 = -5 \;\; p_3 = -20$$



$$H(s) = \frac{10^4 s}{(s-p_1)(s-p_2)(s-p_3)} \quad p_1 = +1 \;\; p_2 = +5 \;\; p_3 = +20$$

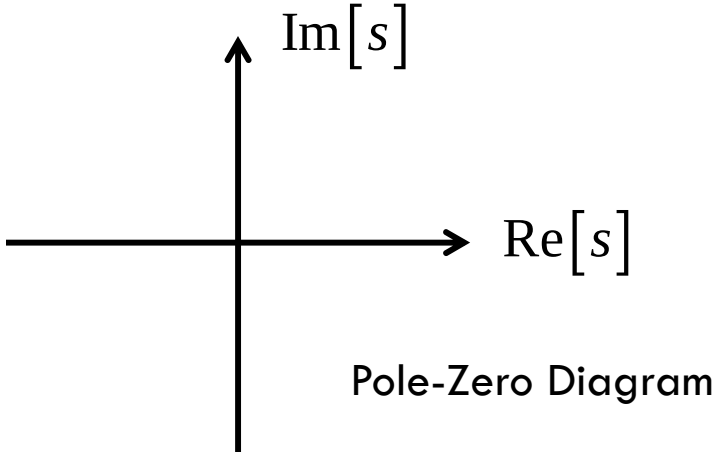




Stable System

- For a system to be stable, the poles must line in the left half-plane
  - The Real Part of the Poles Must Be Negative
- This Does Not Have To Be True for the Zeros, but It Helps, Particularly if the System Is Truly Reciprocal (System Run Backwards, Noise At Output)

$$H(s) = \frac{N(s)}{D(s)}$$

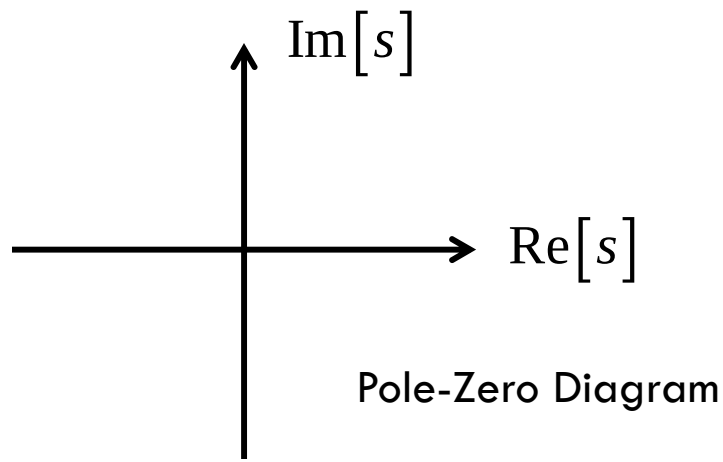


Pole-Zero Diagram

Example 1: 1 Real Pole

$$H(s) = \frac{1}{s+a}$$

$$pole \quad p = -a$$



Unit Step Response

$$v(t) = L^{-1}\left[\frac{1}{s+a}\left(\frac{1}{s}\right)\right] = L^{-1}\left[\frac{1}{s+a}\left(\frac{1}{s}\right)\right] = L^{-1}\left[\frac{1}{a}\left(\frac{1}{s} - \frac{1}{s+a}\right)\right]$$

$$v(t) = \frac{1}{a}\left(1 - e^{-at}\right)u(t)$$

Bounded Function if $a > 0$ $(p < 0)$

Exponentially Headed for ∞ if $a < 0$ $(p > 0)$

Example 2: 1 Real Pole, 1 Real Zero

$$H(s) = \frac{s}{s+a}$$

$$pole \quad p = -a, \;\; zero \;\; z = 0$$

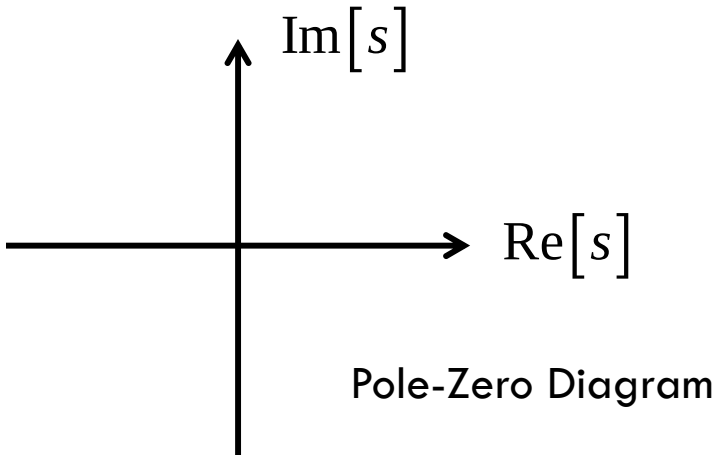
Unit Step Response

$$v(t) = L^{-1}\left[\frac{s}{s+a}\left(\frac{1}{s}\right)\right] = L^{-1}\left[\frac{1}{s+a}\right]$$

$$v(t) = e^{-at}u(t)$$

Bounded Function if $a > 0$ ($p < 0$)

Exponentially Headed for -∞ if $a < 0$ ($p > 0$)



Pole-Zero Diagram

Example 3: Complex Conjugate Poles

$$H(s) = \frac{1}{(s+a+jb)(s+a-jb)}$$

$$poles \quad p_1 = (-a-jb),\ p_2 = (-a+jb)$$

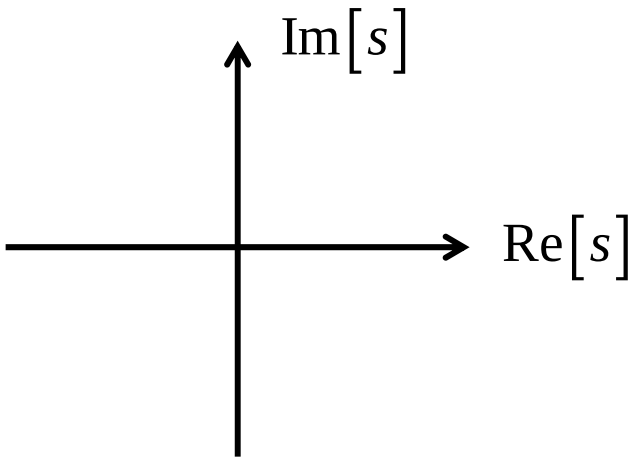
Unit Step Response

$$v(t) = L^{-1}\left[\frac{1}{(s+a+jb)(s+a-jb)}\left(\frac{1}{s}\right)\right]$$

$$v(t) = \left\{\frac{1}{a^2+b^2} + \frac{e^{-at}}{\sqrt{a^2+b^2}}\cos\left(bt + \frac{\pi}{2} + \angle(a+jb)\right)\right\}u(t)$$

Bounded Function if $a > 0$ ($p < 0$)

Exponentially Growing Oscillations if $a < 0$ ($p > 0$)



Pole-Zero Diagram