Maximum Likelihood Transmission Image Reconstruction for Overlapping Transmission Beams

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Problem Motivation

- Multiple line-source array
- Scanning line source

Multiplexing of transmitted photons onto individual detector elements.
Each measurement $Y_i$ is related to a single “line integral” through the object.

$$Y_i \sim \text{Poisson} \left\{ b_i \exp \left( - \sum_{j=1}^{p} a_{ij}\mu_j \right) + r_i \right\}$$
Conventional Transmission Scan Statistical Model for (non-overlapping) Parallel Beams

\[ Y_i \sim \text{Poisson} \left\{ b_i \exp \left( - \sum_{j=1}^{p} a_{ij} \mu_j \right) + r_i \right\}, \quad i = 1, \ldots, N \]

- \( N \) number of detector elements
- \( Y_i \) recorded counts by \( i \)th detector element
- \( b_i \) blank scan value for \( i \)th detector element
- \( a_{ij} \) length of intersection of \( i \)th ray with \( j \)th pixel
- \( \mu_j \) linear attenuation coefficient of \( j \)th pixel
- \( r_i \) contribution of room background, scatter, and emission crosstalk
Conventional Maximum-Likelihood Reconstruction

\[ \hat{\mu} = \arg \max_{\mu \geq 0} L(\mu) \] (Log-likelihood)

\[
L(\mu) = \sum_{i=1}^{N} Y_i \log \left[ b_i \exp \left( - \sum_{j=1}^{p} a_{ij} \mu_j \right) + r_i \right] - \left[ b_i \exp \left( - \sum_{j=1}^{p} a_{ij} \mu_j \right) + r_i \right]
\]

Transmission ML Reconstruction Algorithms

- Conjugate gradient
  Mumcuoğlu et al., T-MI, Dec. 1994

- Paraboloidal surrogates coordinate ascent (PSCA)
  Erdoğan and Fessler, T-MI, 1999

- Ordered subsets separable paraboloidal surrogates
  Erdoğan et al., PMB, Nov. 1999

- Transmission expectation maximization (EM) algorithm
  Lange and Carson, JCAT, Apr. 1984
Overlapping-Beam Transmission Scans

\[ Y_i \sim \text{Poisson} \left\{ \sum_{m=1}^{M} b_{im} \exp \left( - \sum_{j=1}^{p} a_{ij}^m \mu_j \right) + r_i \right\} \]
Overlapping-Beam ML Reconstruction

\[ \hat{\mu} = \arg \max_{\mu \geq 0} L(\mu) \]

Log-likelihood:

\[
L(\mu) = \sum_{i=1}^{N} Y_i \log \left[ \sum_{m=1}^{M} b_{im} \exp \left( -\sum_{j=1}^{p} a_{ij}^m \mu_j \right) + r_i \right] - \left[ \sum_{m=1}^{M} b_{im} \exp \left( -\sum_{j=1}^{p} a_{ij}^m \mu_j \right) + r_i \right]
\]

Summations: detectors, sources, pixels

- \( N \) number of detector elements
- \( p \) number of pixels
- \( Y_i \) recorded counts by \( i \)th detector element
- \( \mu_j \) linear attenuation coefficient of \( j \)th pixel
- \( r_i \) contribution of background and emission crosstalk
- \( M \) number of sources
- \( b_{im} \) blank scan value for \( m \)th source to \( i \)th detector element
- \( a_{ij}^m \) length of intersection through \( j \)th pixel of the ray that connects \( m \)th source to \( i \)th detector
Optimization Transfer Illustrated

\[ \Phi(\mu) \text{ and } \phi(\mu;\mu^n) \]

Objective \( \Phi \)

Surrogate \( \phi \)
First Surrogate Function

- \( y \log x - x \) is concave in \( x \)
- Adapt De Pierro’s “multiplicative” convexity trick (T-MI, Jun. 1993)
- Move the summation over sources outside logarithm

\[
L(\mu) \geq Q_1(\mu; \mu^n) = \sum_{i=1}^{N} \sum_{m=1}^{M} \left( \frac{u_{im}^n}{\bar{y}_i^n} \right) \left[ y_i \log \left( \frac{u_{im}(\mu)}{u_{im}^n} \bar{y}_i^n \right) - \frac{u_{im}(\mu)}{u_{im}^n} \bar{y}_i^n \right]
\]

where

- \( \bar{y}_i^n \triangleq \bar{y}_i(\mu^n) \)
- \( u_{im}^n \triangleq u_{im}(\mu^n) \)
- \( u_{im}(\mu) \triangleq b_{im} \exp \left( -\sum_{j=1}^{p} a_{ij}^m \mu_j \right) + r_i/M \)

\( Q_1 \) still difficult to maximize
Second Surrogate Function

- $y \log(be^{-l} + r) - (be^{-l} + r)$ has a parabola surrogate: $q_{im}^n$
- Optimum curvature of parabola derived by Erdoğan (T-MI, 1999).
- Replace nonquadratic surrogate with paraboloidal surrogate

$$Q_1(\mu; \mu^n) \geq Q_2(\mu; \mu^n) = \sum_{i=1}^{N} \sum_{m=1}^{M} \left( \frac{u_{im}^n}{\bar{y}_i^n} \right) q_{im}^n \left( \sum_{j=1}^{p} a_{ij}^n \mu_j \right)$$

- $q_{im}^n$ is a simple quadratic function
- Iterative algorithm:

$$\mu^{n+1} = \arg \max_{\mu \geq 0} Q_2(\mu; \mu^n)$$

- Maximizing $Q_2(\mu; \mu^n)$ over $\mu$ is equivalent to (rewighted) least-squares.
- Natural algorithms
  - Conjugate gradient
  - Coordinate ascent
(Optional) Third Surrogate Function

- Parabolas are convex functions
- Apply De Pierro’s “additive” convexity trick (T-MI, Mar. 1995)
- Move summation over pixels outside quadratic

\[ Q_2(\mu; \mu^n) \geq Q_3(\mu; \mu^n) = \sum_{i=1}^{N} \sum_{m=1}^{M} \sum_{j=1}^{p} u_{im}^{n} a_{ij}^{m} q_{im}^{n} (\gamma_{m}^{n}(\mu_{j} - \mu_{j}^{n}) + [A_{m}^{n} \mu_{j}^{n}]) \]

- \[ \gamma_{i}^{m} \triangleq \sum_{j=1}^{p} a_{ij}^{m} \]
- Separable paraboloidal surrogate function \( \Rightarrow \) trivial to maximize (cf EM)

\[ \mu_{j}^{n+1} = \left[ \mu_{j}^{n} + \frac{1}{d_{j}(\mu^{n})} \frac{\partial}{\partial \mu_{j}} L(\mu^{n}) \right]_{+} \]

- \( d_{j}(\mu) \) related to parabola curvatures (not to Fisher information)
- Natural starting point for forming ordered-subsets variation
Simulation

Digital Thorax Phantom
$128 \times 128$ 3.56mm pixels
Source Collimation: Blank-scan profiles

(Results vary with acceptance angle of source and detector collimators.)
Reconstruction Algorithms

• Filtered backprojection (FBP)
  Based on usual idealized parallel, non-overlapping, line-integral model
  Requires subtraction of emission crosstalk before taking logarithm

• Parallel-beam method
  Penalized-likelihood based on usual non-overlapping strip-integral model
  Effect of emission crosstalk built into statistical model

• Proposed method
  Penalized-likelihood based on overlapping system/statistical model

Variables

• Source strength
• Crosstalk background level
• Source collimation angle
• Desired target spatial resolution
Simulation Results

- Parallel-beam Method
  - Parallel algorithm

- Proposed Method
  - Proposed algorithm

- FBP

- ±3.6° source collimation
- 497,000 transmitted counts + 263,000 emission crosstalk counts
- Resolution matched to 6.8 pixels FWHM effective Gaussian width
Noise vs Collimation

- Resolution matched to 4.7 pixels FWHM
- Similar curves for other target resolutions
(Each point is for noise-minimizing collimator resolution)
Summary

- New algorithm for overlapping beams
- Intrinsically monotonic (no line searches, no divergence)
- Convergence to a local maximizer
- Requires separate blank scan / system matrix for each source
  - increased memory and computational requirements
- Improved resolution/noise tradeoff over parallel algorithm
- Allows increased acceptance angles : higher transmitted/crosstalk ratio
- Acceptance angle currently limited by detector collimation.
Collimation vs Resolution

Proposed algorithm
Parallel algorithm
Noiseless Data Reconstructions ($\beta \approx 0$)

Parallel algorithm

Proposed algorithm