REGULARIZED METHODS FOR TOPOLOGY-PRESERVING SMOOTH NONRIGID IMAGE REGISTRATION USING B-SPLINE BASIS

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ABSTRACT

B-splines are a convenient tool for nonrigid registration, but ensuring invertibility can be challenge. This paper describes a new penalty method that is devised to enforce a sufficient condition for local invertibility and smoothness of nth order B-spline based deformations. Traditional direct Jacobian penalty methods penalize negative Jacobian determinant values only at grid points. In contrast, our new penalty method enforces the sufficient condition for invertibility directly on the B-spline coefficients by using a modified quadratic penalty function so that it enforces invertibility globally over a 3D continuous domain. This approach also saves computation time and memory compared to using Jacobian determinant values. We apply this method to 3D CT images of a thorax at inhale and exhale.

Index Terms— B-splines, 3D nonrigid image registration, topology preserving, penalty method

1. INTRODUCTION

Nonrigid image registration enables more flexible matching of local details between two images than rigid registration. B-spline bases are attractive for nonrigid registration because of their compact support, smoothness and fast interpolation schemes [1]. However high degrees of freedom in deformation can lead to unrealistic transformation results such as folding in the absence of appropriate constraints [2].

There have been efforts to regularize this problem by using some reasonable assumptions. Rueckert *et al.* [3] penalized the bending energy of the deformation directly, assuming that the local deformation of tissues should be smooth. Rohlfing *et al.* [4] used an incompressibility constraint, assuming that local deformations should be volume preserving.

Topology preservation has been another reasonable constraint for image registration and preserving local invertibility is one way to achieve it. One way to guarantee the local invertibility is to constrain the Jacobian determinant of the transformation to be positive by a penalty method [5]. However this approach only constrains each discrete grid point and the local invertibility is not strictly guaranteed on the whole continuous domain. Furthermore, penalizing the Jacobian determinant significantly increases the computation time for B-spline based image registration.

There has been some research on methods that enforce the local invertibility globally. Noblet *et al.* [6] devised a scheme to constrain the Jacobian determinant values of transformation to be positive on continuous domain in 3D. Rohde *et al.* [7] suggested a sufficient condition for local invertibility derived using Neuman series. Motivated by [7], Kim *et al.* [8] suggested similar sufficient conditions for cubic B-spline based transformation and implemented a constrained minimization algorithm using Dykstra's cyclic projection method. That algorithm was fairly slow.

This paper describes a new penalty function approach that is based on an extended version of Kim's sufficient condition for local invertibility. We implemented it with a simple and fast quadratic-like penalty function and compared it with a variant of traditional Jacobian penalty method [5, 8]. The new method is at least as effective at ensuring local invertibility and is much faster.

This work was supported in part by NIH/NCI grant 1P01 CA87634.

2. METHOD

2.1. Sufficient condition for local invertibility

A nonrigid transformation T in 3D can be represented as

$$T(\underline{r}) = \underline{r} + \underline{d}(\underline{r}), \qquad (1)$$

where $\underline{r} = (x, y, z)$. We model the 3D deformation $\underline{d} = (d^x, d^y, d^z)$ using tensor-product nth order B-splines as follows:

$$d^l(\underline{r}) = \sum_{i,j,k} c^l_{i,j,k} \beta^n\Big(\frac{x}{m_x} - i\Big)\beta^n\Big(\frac{y}{m_y} - j\Big)\beta^n\Big(\frac{z}{m_z} - k\Big),$$

where $l \in \{x, y, z\}$ and β^n is a nth order B-spline basis. The goal in image registration is to estimate the B-spline coefficients $\{c^l_{i,j,k}\}$ by maximizing a similarity metric. Often we would like to ensure that the coefficients correspond to an invertible transformation T.

Kim *et al.* [8] proposed sufficient conditions for local invertibility in 3D case with cubic B-spline basis case by two propositions. Our first proposition is an extended version of their first proposition.

Proposition 1. *In (1), suppose that* $\left|\frac{\partial}{\partial q} d^l(\underline{r})\right| \leq k_l < \frac{1}{2}$ *where* $l \in \{x, y, z\}$, $q \in \{x, y, z\}$ *and* $l \neq q$. *Also suppose that* $-k_l \leq \frac{\partial}{\partial l} d^l(\underline{r}) \leq K_l$ *where* $l \in \{x, y, z\}$. *Then* $1 - (k_x + k_y + k_z) \leq \det J_T(\underline{r}) \leq (1 + K_x)(1 + K_y)(1 + K_z) + (1 + K_x)k_y k_z + k_x(1 + K_y)k_z + k_x k_y(1 + K_z)$ *for* $\forall \underline{r} = (x, y, z)$ *where* J_T *is the Jacobian matrix of transformation* T.

This result suggested that local invertibility can be controlled by the first derivatives of deformations. Kim *et al.* proved this proposition only for the case $K_l = k_l$. That restriction meant that the upper bound of the Jacobian determinant of transformation was determined by the lower bound of it. In contrast, our Proposition 1 enables us to design the upper bound independently.

Kim *et al.* showed a second proposition about the relationship between the first partial derivative of deformation and adjacent deformation coefficients for the cubic B-spline basis case. As shown in Appendix A, this second proposition is also valid for the general nth order B-spline basis ($n \geq 1$).

Proposition 2. *If* $-b \leq c^l_{i+1,j,k} - c^l_{i,j,k} \leq B$ *for* $\forall i, j, k$, *then* $-\frac{b}{m_x} \leq \frac{\partial}{\partial x} d^l(\underline{r}) \leq \frac{B}{m_x}$. *Similarly, if* $-b \leq c^l_{i,j+1,k} - c^l_{i,j,k} \leq B$ *for* $\forall i, j, k$, *then* $-\frac{b}{m_y} \leq \frac{\partial}{\partial y} d^l(\underline{r}) \leq \frac{B}{m_y}$ *and if* $-b \leq c^l_{i,j,k+1} - c^l_{i,j,k} \leq B$ *for* $\forall i, j, k$, *then* $-\frac{b}{m_z} \leq \frac{\partial}{\partial z} d^l(\underline{r}) \leq \frac{B}{m_z}$.

These two propositions show that one can obtain a transformation T that is everywhere locally invertible by maximizing a similarity metric subject to constraints on the differences between adjacent deformation coefficients. Kim *et al.* used Dykstra's cyclic projection algorithm for optimization, but it was slow.

2.2. New penalty design

For faster registration, we propose to relax the constraints in Proposition 2 by using penalty functions instead. The proposed penalty function are defined as

$$p(t) = \begin{cases} \frac{1}{2}(t + \zeta_1)^2, & t \leq -\zeta_1 \\ 0, & -\zeta_1 < t \leq \zeta_2 \\ \frac{1}{2}(t - \zeta_2)^2, & \text{otherwise}, \end{cases}$$

as illustrated in Fig. 1. The argument t denotes a difference between two adjacent deformation coefficients.

This function does not strictly constrain the sufficient condition, but its first and second derivatives are simple and convenient for use in optimization algorithms such as conjugate gradient. The final new penalty function is $R(\underline{c}) = \sum_{l \in \{x,y,z\}} \sum_{i,j,k} \{p^l_x(c^l_{i+1,j,k} - c^l_{i,j,k}) + p^l_y(c^l_{i,j+1,k} - c^l_{i,j,k}) + p^l_z(c^l_{i,j,k+1} - c^l_{i,j,k})\}$.

Note that choosing $\zeta_1 = \zeta_2 = 0$ would correspond to a quadratic roughness penalty over B-spline coefficients, which is akin to the volume preserving constraint $\det J_T(\underline{r}) = 1$ for $\forall \underline{r}$.

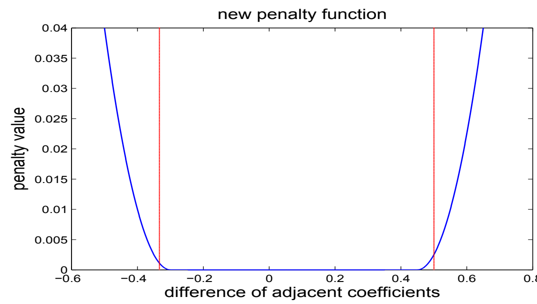


Fig. 1. A modified quadratic penalty function (solid) and real constraints (dashed).

2.3. Discussion and comparison

We compare this new penalty method with a variant of direct Jacobian penalty method [5, 8].

The direct Jacobian penalty method uses the exact condition for the local invertibility, so its solution space is larger. However, our new method has a smaller solution space partially because it constrains the first derivatives of the deformation. This constraint encourages smoothness inherently which seems appropriate in medical imaging [3]. Invertibility constraints alone do not guarantee smoothness [9] and one could achieve both invertibility and smoothness by adding a roughness penalty to the Jacobian penalty.

The direct Jacobian penalty method is applied only on discrete grid points of the domain, so it does not enforce local invertibility on the whole continuous domain. However the new penalty method can enforce the constraint on the entire continuous domain.

Lastly, the new penalty method is faster and more memory-efficient than the direct Jacobian penalty method. The direct Jacobian penalty method involves the calculation of the Jacobian determinant, which requires additional interpolations beyond the interpolations needed for the data fitting term. In general, B-spline interpolations dominate the cpu time for calculating gradients of the cost function and storing additional interpolations require lots of additional memory. In contrast, the new penalty method simply reuses the calculations needed for common quadratic roughness penalties only over B-spline deformation coefficients and requires much smaller additional memory.

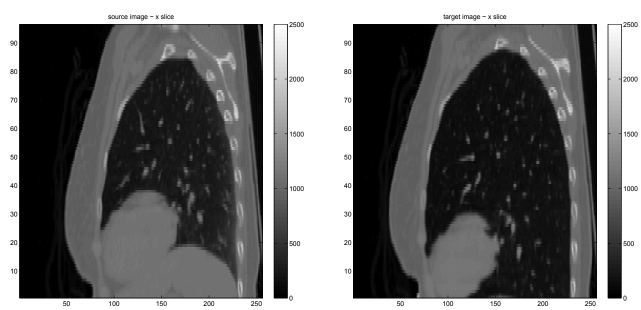
3. SIMULATION

We applied this new penalty method to register inhale and exhale 3D breath-hold CT images of a real patient. The image size was $396 \times 256 \times 96$ as in Fig. 2. The sum of squared difference was used for data fitting term. We used the conjugate gradient method for optimization and determined each step size by the first step of Newton's method.

We tuned the regularization parameter experimentally to achieve the minimum value of data fitting term such that all Jacobian determinant values on image grid are positive. We applied a multiresolution scheme. For the first 3 levels of multiresolution, knot spacing was every 8 voxels for downsampled images and then for the last level of multiresolution the knot spacing was every 4 voxels in each direction.

Fig. 3 shows the results for both methods. On almost all voxels, the determinant of deformation Jacobian values were positive. Fig. 3 (b), (c) and (d) show the effect of regularization comparing to Fig. 3 (a). Fig. 3 (b) and (c) show that a roughness penalty helps preserve some details inside lung such as lung nodule which are weak features. These show that local invertibility alone does not ensure smoothness [9], which is one of the reasons that roughness penalties are often used for nonrigid image registration in medical imaging [3, 4]. Yet the new penalty method ensures smoothness implicitly since it controls the magnitude of the first order derivative.

The new penalty method was much faster and more memory efficient than direct Jacobian penalty method. If one uses the sum of squared error as a data fitting term and penalizes negative Jacobian determinant values on each image grid point in 3D cubic B-spline case, then the interpolations needed for gradients of the direct Jacobian penalty function requires about 1.8 times more operations than the interpolations needed for gradient of the data fitting term. In our implementation the direct Jacobian penalty method was about 4 times slower than our new penalty method. The direct Jacobian penalty method requires 9 times of the size of 3D image to store each partial derivatives for deformations in each direction, whereas the new penalty method requires much smaller additional memory.



(a) source image (exhale) (b) target image (inhale)

Fig. 2. Real 3D CT source and target images.

4. CONCLUSION

We apply the invertibility sufficient condition of transformation to the image registration problem by using quadratic-like penalty approach. This approach provides not only an invertible deformation, but also a smooth deformation. Over direct Jacobian penalty method, this approach has the advantages such as enforcing the invertibility on the continuous domain as well as memory-efficient, faster computation because it does not require interpolation for Jacobian values. However, we observed some bone warping result in each deformed image so the natural further research would be to use a rigidity penalty term such as [10]. Comparing with constrained optimization methods such as [6] would also be interesting. More quantitative study is also necessary.

A. APPENDIX

Proof sketch of proposition 1

$\det J_T(\underline{r})$ is an affine function of $\frac{\partial}{\partial q} d^l(\underline{r})$ for fixed q and l. This implies that both max and min of $\det J_T(\underline{r})$ are achieved at either max or min of $\frac{\partial}{\partial q} d^l(\underline{r})$. Thus, the min and max of $\det J_T(\underline{r})$ over $\forall \underline{r}$ is the same as the min and max of $\det J_T(\underline{r})$ at $\min \frac{\partial}{\partial q} d^l(\underline{r})$ or $\max \frac{\partial}{\partial q} d^l(\underline{r})$ which are finite sets. One may easily get the global min and max of $\det J_T(\underline{r})$ from the given restrictive range of each $\frac{\partial}{\partial q} d^l(\underline{r})$.

Proof of proposition 2

For $d(x) = \sum_i c_i \beta^n(x/m_x - i)$, by using [11] $\frac{\partial}{\partial x}\beta^n(x) = \beta^{n-1}(x + 1/2) - \beta^{n-1}(x - 1/2)$. Thus,

$$\begin{aligned}\frac{\partial}{\partial x} d(x) &= \sum_i c_i \frac{\partial}{\partial x} \beta^n(x/m_x - i) \\ &= \sum_i (c_i - c_{i-1}) \beta^{n-1}(x/m_x - i + 1/2)/m_x.\end{aligned}$$

By using the given $c^l_{i+1,j,k} - c^l_{i,j,k} \geq -b$ and the property $\sum_i \beta^n(x/m_x - i) = 1$, we have the bounds

$$\begin{aligned}\frac{\partial}{\partial x} d^l(\underline{r}) &\geq \sum_i \sum_j \sum_k (c^l_{i,j,k} - c^l_{i-1,j,k}) \beta^{n-1}(x/m_x - i \\ &\qquad + 1/2)\beta^n(y/m_y - j)\beta^n(z/m_z - k)/m_x \\ &\geq -b/m_x \sum_i \beta^{n-1}(x/m_x - i + 1/2) \\ &\qquad \sum_j \beta^n(y/m_y - j) \sum_k \beta^n(z/m_z - k) \\ &\geq -b/m_x.\end{aligned}$$

Similarly, $\frac{\partial}{\partial x} d^l(\underline{r}) \leq B/m_x$ and other directions y, z can be proved in a similar way.$\square$

B. ACKNOWLEDGEMENT

The authors thank Dr. Michael Unser's research group for sharing their B-spline interpolation and pyramid codes, Dr. Marc Kessler for the 3D CT data and Dr. James Balter for discussions about image registration.

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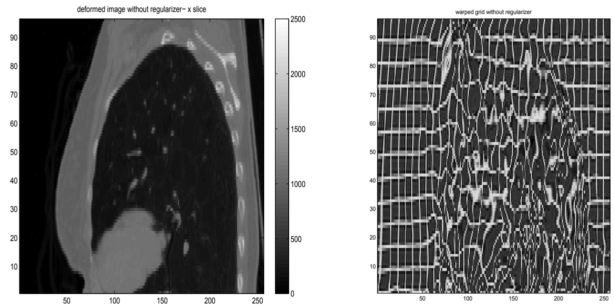
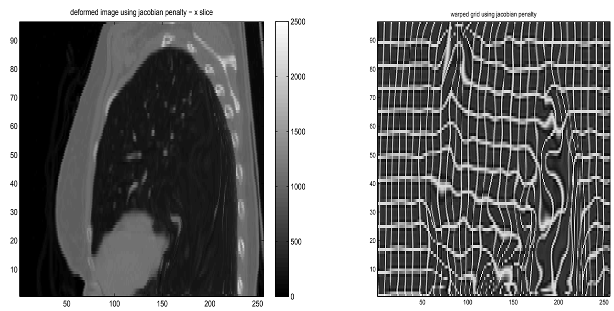
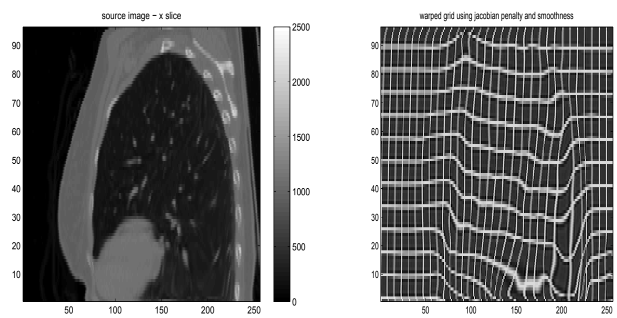


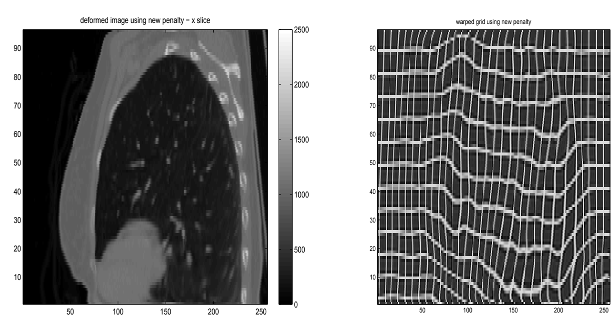
(a) no regularizer



(b) direct Jacobian penalty method (no roughness penalty)



(c) direct Jacobian penalty method (with roughness penalty)



(d) new penalty method

Fig. 3. Deformed images (left) and their warped grids (right).