RESPIRATORY MOTION ESTIMATION FROM SLOWLY ROTATING X-RAY PROJECTIONS

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ABSTRACT

As radiotherapy has become increasingly conformal, geometric uncertainties caused by breathing and organ motion have become an important issue. In this work, a nonrigid motion estimation method, which can estimate the motion history of the studied organs, is introduced. This motion information may increase the accuracy of the dose calculation in treatment planning.

In this approach, a set of projection views of the thorax are acquired by using a slowly rotating cone-beam CT scanner, such as a radiotherapy simulator. A thorax image of the same patient is also reconstructed using a conventional CT scanner under the breathhold condition. Then we design a parametric motion model based on B-splines to represent respiratory motion. The parameters are estimated by optimizing a cost function, which is the penalized least square error between the measured projections and the estimated projections. Preliminary simulation results on the 2D case show that there is good agreement between the estimated motion and the true motion. More complex simulation work will be done in the near future.

1. INTRODUCTION

Extensive research has been carried out on dynamically estimating cardiac motion [1], which greatly helps the diagnosis of cardiac dysfunction. However, as far as we know, there is little work on building a 4D respiratory motion model. Understanding the motion pattern of the thorax, especially the motion of the tumors inside the lung, is important for treatment planning. For example, knowledge of the space that tumors reach during a respiration cycle may guide delivery of the x-ray dose to focus more on tumors while sparing the normal adjacent tissue.

In our work, we propose a method to estimate respiratory motion from two measurements. One is a sequence of projection views acquired from a slow cone-beam CT scanner, which usually takes about one minute for one rotation. During the acquisition period, patients breathe freely, so the measured projections correspond to a moving object and contain breathing motion information, which make it reasonable to estimate respiratory motion using these projections. The other one is a reconstructed image of the thorax under the breathhold condition using a conventional CT scanner. This image serves as a reference image which will be deformed according to our estimated motion; then the projection of the deformed image is calculated and compared with the corresponding measured projection. As is well known, estimation is an inverse procedure aimed at recovering the unknown input from available measurements. Generally, for an iteratively solved estimation problem, there are three main tasks: define a suitable system model, choose a good cost function and select appropriate optimization algorithms. In our estimation problem, motion is defined by a parametric model based on B-splines. Cost function or similarity measure is the penalized least square error. The optimization algorithm can be a general numeric search method, such as the gradient descent method.

Instead of using projections, one may think that a feasible alternative is to estimate respiratory motion by registering a set of thorax CTs taken under the breathhold condition. There are at least two shortcomings to this idea. First, unnaturally controlled breathhold states tend to be discontinuous, and some people (like lung cancer patients) cannot control their respiration well. Second, patients may be required to be scanned dozens of times all at once to generate enough views for a respiration cycle, and this long-time exposure to X ray is not healthy. By contrast, in our method, patients breathe naturally and are only scanned for a few rotations, which overcomes those disadvantages.

The paper is organized as follows. First the theory is described, including the temporal motion model, similarity measure and optimization method. Then the preliminary simulation results are presented, followed by our conclusion and proposals for future work.

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2. THEORY

2.1. Temporal Motion Model

Let $\{f_{t_1}, \cdots, f_{t_m}, \cdots, f_{t_M}\}$ denote the M-frame moving image sequence, where f_{t_m} is the image at time t_m. Assuming these images are all a deformation of the reference image f_r, then there exists a correspondence between f_{t_m} and f_r:

$$\begin{aligned} f_{t_m}(x, y) &= \mathcal{W}_{\theta_m} f_r \\ &= f_r(T_x(x, y, \theta_m^{(x)}), T_y(x, y, \theta_m^{(y)})), \quad (1) \end{aligned}$$

where $\mathcal{W}_{\theta_m}$ is the warping operator controlled by parameter θ_m, $T_x(x, y, \theta_m^{(x)})$ and $T_y(x, y, \theta_m^{(y)})$ describe the deformation functions along x direction and y direction respectively, and vector θ_m is the parameter of the deformation functions. Nonrigid deformation is applied in our work. As described in [2], a smooth nonrigid deformation can be modelled using spline functions as follows:

$$T_x(x, y, t_m) = x + \sum_{i=1}^{I} \sum_{j=1}^{J} \theta_{m,ij}^{(x)} \beta^n(\frac{x}{\Delta x} - i) \beta^n(\frac{y}{\Delta y} - i)$$

$$T_x(x, y, t_m) = y + \sum_{i=1}^{I} \sum_{j=1}^{J} \theta_{m,ij}^{(y)} \beta^n(\frac{x}{\Delta x} - i) \beta^n(\frac{y}{\Delta y} - i),$$

where $\beta^n(x)$ is the spline function of degree n, $\theta^{(x)}$ and $\theta^{(y)}$ are spline coefficients, and Δx and Δy are constant scalars that control the width of the basis functions. The reasons we chose the B-spline function are its advantages of good accuracy and analytical derivatives [3].

Since the deformation of tissue over time is also continuous, a general temporal motion model can be further expressed, as depicted below:

$$\begin{aligned} T_x(x, y, t) &= x + \sum_{m=1}^{M} \sum_{i=1}^{I} \sum_{j=1}^{J} \theta_{mij}^{(x)} b(\frac{t}{\Delta t} - m) \\ & \quad \beta^n(\frac{x}{\Delta x} - i) \beta^n(\frac{y}{\Delta y} - i) \end{aligned}$$

$$\begin{aligned} T_y(x, y, t) &= y + \sum_{m=1}^{M} \sum_{i=1}^{I} \sum_{j=1}^{J} \theta_{mij}^{(y)} b(\frac{t}{\Delta t} - m) \\ & \quad \beta^n(\frac{x}{\Delta x} - i) \beta^n(\frac{y}{\Delta y} - i), \end{aligned}$$

where the basis function $b(t)$ can take many forms, such as the Dirac impulse function, rectangular function or spline function etc. In our current implementation, the rectangular function is chosen to be the time basis function for the reason of simplicity.

2.2. Measurement

As described before, the motion estimation are based on two measurements. One is a still thorax f_r, a reconstructed image using a conventional CT scanner under the breathhold condition. This image serves as a reference image and all the deformations are operated on this image. The other one is a projection sequence from a slow scanner:

$$\{p_{\phi_1}, \cdots, p_{\phi_m}, \cdots, p_{\phi_M}\},$$

where ϕ_i is the projection angle, and M is the total number of projections.

Let $\mathcal{A}_{\phi_m}$ denote the projection operator at angle ϕ_m, then

$$\begin{aligned} p_{\phi_m} &= \mathcal{A}_{\phi_m} f_{t_m} + n_m \\ &= \mathcal{A}_{\phi_m} \mathcal{W}_{\theta_m} f_r + n_m, \quad (2) \end{aligned}$$

where n_m is an additive Gaussian noise vector at time t_m. Based on the motion model defined in the previous section, the task of the motion estimation is to find the deformation parameters $\boldsymbol{\theta} = \{\theta_1, \cdots, \theta_m, \cdots, \theta_M\}$ using f_r from $\{p_{\phi_m}\}$.

2.3. Similarity Measure

The goal of our motion estimation is to find $\boldsymbol{\theta}$ that can make the calculated projections based on the deformed image f_r best match the measured projections. A straightforward metric to evaluate similarity is the least square error,

$$L(\boldsymbol{\theta}) = \frac{1}{2} \sum_{m=1}^{M} \left\| p_{\phi_m} - \mathcal{A}_{\phi_m} \mathcal{W}_{\theta_m} f_r \right\|^2,$$

where $\mathcal{W}_{\theta_m} f_r$ is the warped f_r as shown in (1). This cost function can also be derived from Maximum Likelihood estimation, under the assumption that the noises are independent and identical Gaussian processes.

Since estimation from finite data may yield unrealistic results, it is necessary to add temporal regularization to provide a practical solution. The regularized least square estimator of $\boldsymbol{\theta}$ contains two terms and is expressed as follows:

$$\begin{aligned} \widehat{\boldsymbol{\theta}} &= \arg\min_{\boldsymbol{\theta}} \psi(\boldsymbol{\theta}) \\ \psi(\boldsymbol{\theta}) &= L(\boldsymbol{\theta}) + \lambda R(\boldsymbol{\theta}), \end{aligned}$$

where $R(\boldsymbol{\theta})$ denotes the regularity function, and parameter λ controls the trade-off between the similarity term and the regularity term.

As a motion problem, it is desirable to take the temporal smoothness of deformation as a regularity term. Here we approximate the smoothness of the temporal deformation as the smoothness of the parameter vector $\boldsymbol{\theta}$. As a result, $R(\theta)$ can take the form of $\left\|\boldsymbol{C\theta}\right\|^2$, where $\boldsymbol{C}$ is a differencing matrix.

2.4. Optimization

The gradient descent method is chosen to be our optimization algorithm. This method updates variables by the following scheme:

$$\boldsymbol{\theta}^{n+1} = \boldsymbol{\theta}^n - \alpha\nabla(\boldsymbol{\theta}^n), \quad (3)$$

where α is the step size, $\nabla(\boldsymbol{\theta}^n)$ is the gradient of $\psi(\boldsymbol{\theta})$ with respect to $\boldsymbol{\theta}^n$. Because of the linearity of the projection operator and the close form of the B-spline functions, the gradients can be calulated analytically using the chain rule.

A faster convergent algorithm, the preconditioned gradient descent (PGD) method, can also be used as the optimization algorithm, which has the following iterative form,

$$\boldsymbol{\theta}^{n+1} = \boldsymbol{\theta}^n - \alpha P\nabla(\boldsymbol{\theta}^n), \quad (4)$$

where P is a preconditioning matrix. The ideal preconditioning matrix P_0 would satisfy $P_0H = I$, where H is the Hessian of $\psi(\boldsymbol{\theta})$, and I is the identity matrix. Since it is very hard to compute H^{-1}, we select $P = (\operatorname{diag}\{H\})^{-1}$ to approximate H^{-1}.

3. SIMULATION RESULTS

In our simulation, the reference image f_r is a 128×128 thorax phantom (Fig. 1) [4]. We simulate a parallel-beam slow scanner to generate the data $\{p_{\phi_m}\}_{m=1}^{m=M}$ (Fig. 2), which contain $M = 15$ projections at angles uniformly spaced over 180^o. The detector size is 160-pixel. During the scanning process, the thorax phantom moves according to an artificial motion represented by the temporal motion model described in Section 2.1. Here the deformation of each frame is defined by cubic splines with a control grid of 2×2 points, which means that there are 4×2 parameters for each frame. Since there are 15 frames, the motion model is a 120-parameter function. The reason we start our simulation with a rather idealistic condition is that knowledge of the ground truth enables us better to evaluate and analyze the performance of the proposed method.

Fig. 3, Fig. 4, and Fig. 5 display the estimation results. Each parameter is initialized to be a deviation from its true value. The deviation is randomly distributed between ± 8 pixels (1 pixel corresponds to about 0.3cm). As can be seen in Fig. 4, the estimated values of most parameters are fairly close to their true values. We also examine two points from the image (marked in Fig. 1) and compare the estimated displacement with their true displacement. The comparison shown in Fig. 5 illustrates that there is good agreement between the true movement and the movement found by the algorithm. The largest error is around 1 pixel. It is necessary to investigate this issue further in order to ascertain whether the algorithm can be improved, and if so, how to achieve the improvement.

4. CONCLUSION

In this paper, we proposed a nonrigid motion estimation method from a sequence of slowly rotating projection views. Cubic B-spline functions are applied as the basis of our parametric temporal motion model. Preliminary results presented in Section 3 show the potential of this method. More simulation work will be undertaken in the near future, including, for example, experimenting on different time basis functions and penalty functions, testing the performance of the algorithm under noisy situations, and extending the current 2D implementation to the 3D case. Finally, the algorithm will be applied to real lung data.

5. REFERENCES

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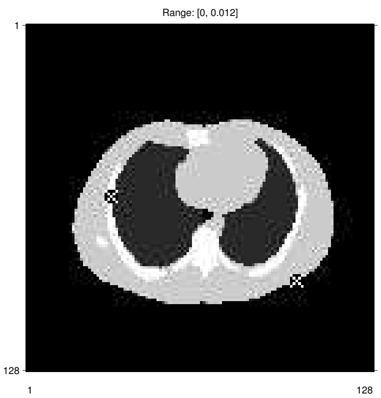

Fig. 1. Image f, marked points are (100,90) and (32,64) respectively

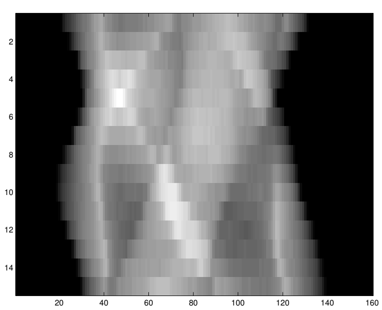

Fig. 2. Projection $\{p_{\phi_m}\}_{m=1}^{15}$

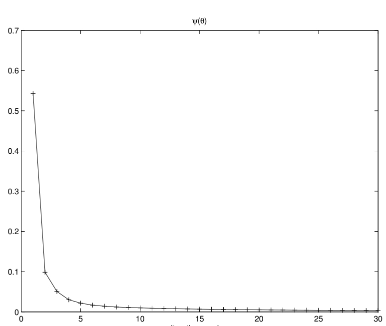

Fig. 3. Cost function

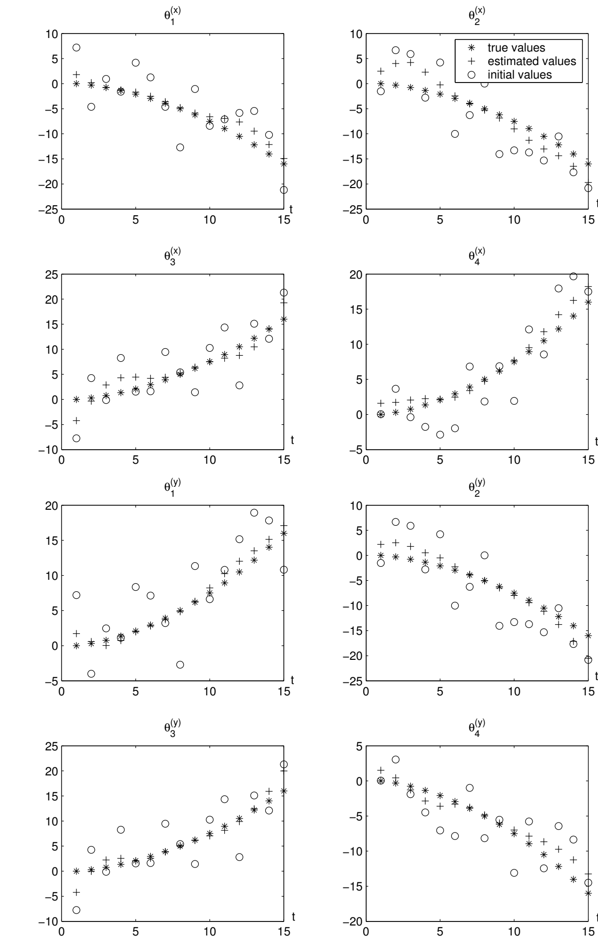

Fig. 4. Comparison of the the estimated motion parameters with their true values

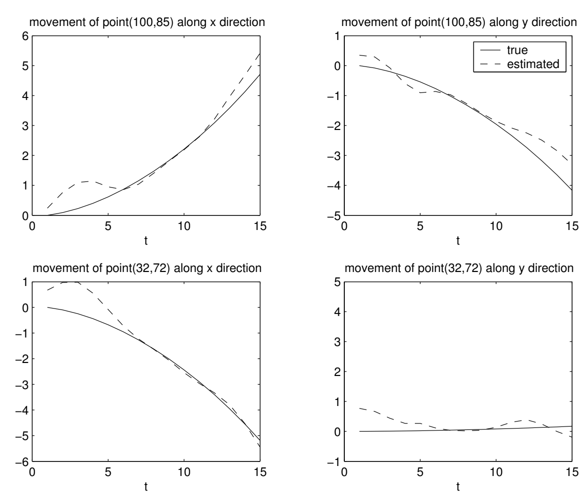

Fig. 5. Comparison of the true movement and the estimated movement of point(100,90),(32,64) as marked in Fig. 1