A Temporal Model for Task-based Functional MRI Reconstruction
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Introduction
- Goal: better identify task-activated brain regions in task-based fMRI.
- Model: to separate task-correlated signal from non-task background.
- Novelty: use a priori knowledge of activation waveform shape, and temporal smoothness assumption of background.
- Merit: advance model-based reconstruction from undersampled k-space.

Problem Formulation
Reconstruct MR image series from undersampled k-space data:

\[ \text{argmin}_{\lambda} \frac{1}{2} \| EX - d \|^2 + \lambda R(X) \]

Where:
- \( E \): data acquisition operator (where \( N_v \) = number of voxels, \( N_t \) = number of time frames, \( N_s \) = number of k-space samples)
- \( d \): desired image series
- \( R(\cdot) \): regularizer with parameter \( \lambda \)

Existing Models
- Low-Rank Plus Sparse Decomposition (L+S) [1], [2]
  \[ \text{argmin}_{L,S} \frac{1}{2} \| EL + S - d \|^2 + \lambda_1 \| L \|_2 + \lambda_2 \| TS \|_2 \]
- Low-Rank Plus Task-Based Decomposition (L+UV) [3]
  \[ \text{argmin}_{L,U, V} \frac{1}{2} \| EL + UV - d \|^2 + \lambda_1 \| L \|_F \]

Proposed Model
Smooth Background Plus Spatial-Temporal Decomposition (B+UV)

\[ \text{argmin}_{B,U,V} \frac{1}{2} \| EB + UV - d \|^2 + \lambda_1 \| B \|_F \]

B: temporally smooth non-task background
U: estimated task spatial map
V: temporal basis with activation waveform

Optimization Algorithm
- Compatibility of vectorization with Kronecker product:
  \( \text{vec}(UV) = (V^T \otimes I) \text{vec}(U) \)
- Write \( E(UBV) = E(BV) \), \( X \sim [\text{vec}(B) ] \), \( D = [D_0] \), then (1) becomes
  \[ \min_{\lambda} \frac{1}{2} \| E \| X - d \|^2 + \frac{\lambda_1}{2} \| D X \|^2 \]
- Practical implementation: conjugate gradient (CG) method

Advantage over existing models:
- L+S: incoherence between \( L \) and \( S \) might not apply, and temporal Fourier sparsity assumption of \( S \) might not capture activation
- L+UV: both terms are low rank, might not separate signal from background
- B+UV: incoherence between smooth background signal \( B \) and task \( UV \)

Results
Simulated task: resting-state fMRI with 2 activated Gaussian regions of interest (ROI) in k-space, 32 coils, \( N_v = 100 \times 100, N_s = 300 \), 4× undersampling

![Figure 1](image1.png)

Finger Tapping Task: 3D task fMRI, 32 coils, \( N_v = 72 \times 48 \times 10, N_s = 235 \), 4× undersampling

![Figure 2](image2.png)

![Figure 3](image3.png)

![Figure 4](image4.png)

Conclusion
- Proposed B+UV model improves activation detection compared with existing fMRI models, as seen by higher AUC values.
- B+UV components separate task signal and non-task background.
- Solving B+UV is computationally advantageous with simple CG updates.

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References