Maximum Likelihood Transmission Image Reconstruction for Overlapping Transmission Beams

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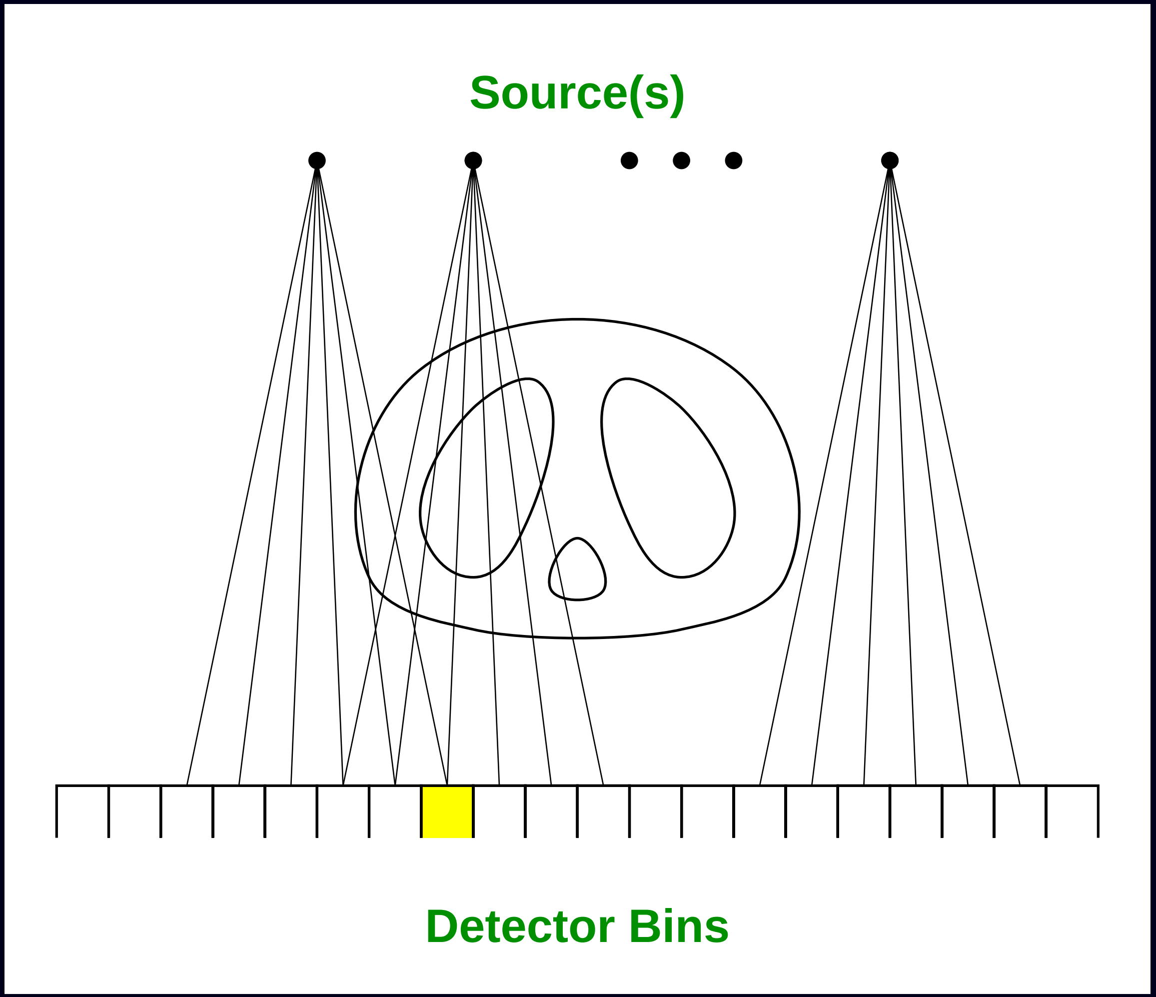
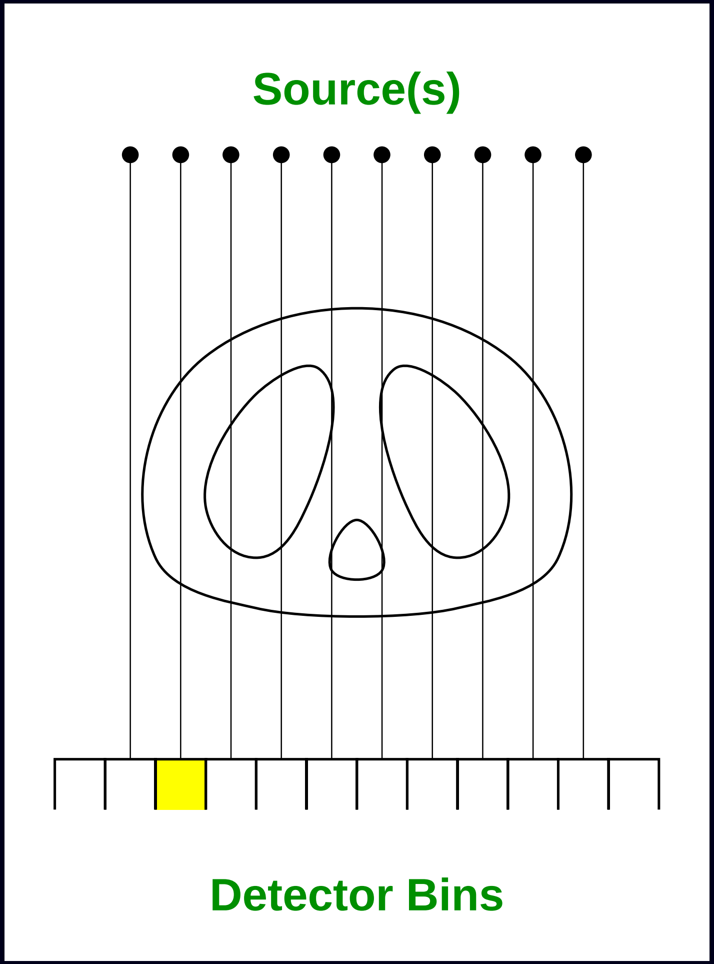
Problem Motivation

- Multiple line-source array
- Scanning line source



Multiplexing of transmitted photons onto individual detector elements.

Conventional Parallel-Beam Transmission Scans



$$Y_i \sim \text{Poisson}\left\{ b_i \exp\left(-\sum_{j=1}^{p} a_{ij}\mu_j \right) + r_i \right\}$$

Each measurement Y_i is related to a single “line integral” through the object.

Conventional Transmission Scan Statistical Model for (non-overlapping) Parallel Beams

$$Y_i \sim \text{Poisson}\left\{ b_i \exp\left(-\sum_{j=1}^{p} a_{ij} \mu_j \right) + r_i \right\}, \; i = 1, \ldots, N$$

- N number of detector elements
- Y_i recorded counts by ith detector element
- b_i blank scan value for ith detector element
- a_{ij} length of intersection of ith ray with jth pixel
- μ_j linear attenuation coefficient of jth pixel
- r_i contribution of room background, scatter, and emission crosstalk

Conventional Maximum-Likelihood Reconstruction

$$\hat{\mu} = \arg\max_{\mu \geq 0} L(\mu) \quad \text{(Log-likelihood)}$$

$$L(\mu) = \sum_{i=1}^{N} Y_i \log \left[b_i \exp\left(-\sum_{j=1}^{p} a_{ij}\mu_j \right) + r_i \right] - \left[b_i \exp\left(-\sum_{j=1}^{p} a_{ij}\mu_j \right) + r_i \right]$$

Transmission ML Reconstruction Algorithms

- Conjugate gradient

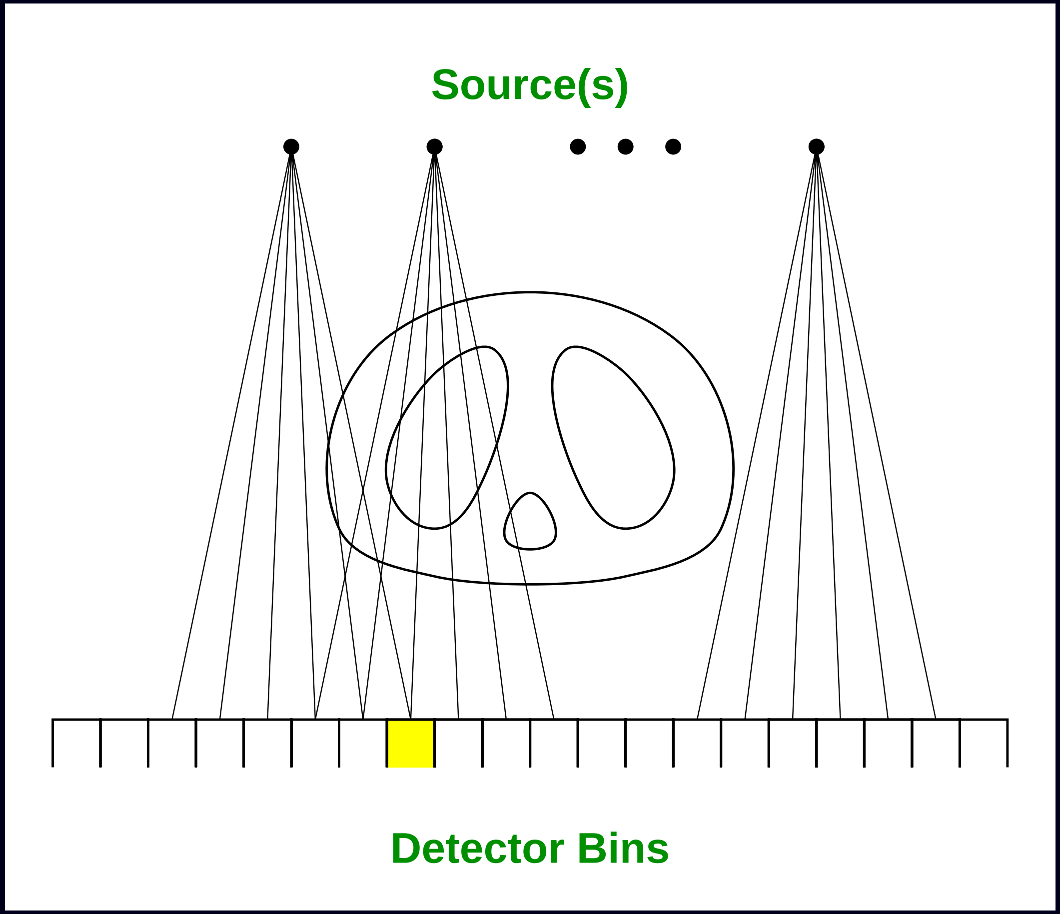
  Mumcuoğlu *et al.*, T-MI, Dec. 1994
- Paraboloidal surrogates coordinate ascent (PSCA)

  Erdoğan and Fessler, T-MI, 1999
- Ordered subsets separable paraboloidal surrogates

  Erdoğan *et al.*, PMB, Nov. 1999
- ~~Transmission expectation maximization (EM) algorithm~~

  Lange and Carson, JCAT, Apr. 1984

Overlapping-Beam Transmission Scans



$$Y_i \sim \text{Poisson}\left\{ \sum_{m=1}^{M} b_{im} \exp\left(-\sum_{j=1}^{p} a_{ij}^{m} \mu_j \right) + r_i \right\}$$

Overlapping-Beam ML Reconstruction

$$\hat{\mu} = \arg\max_{\mu \geq \underline{0}} L(\mu)$$

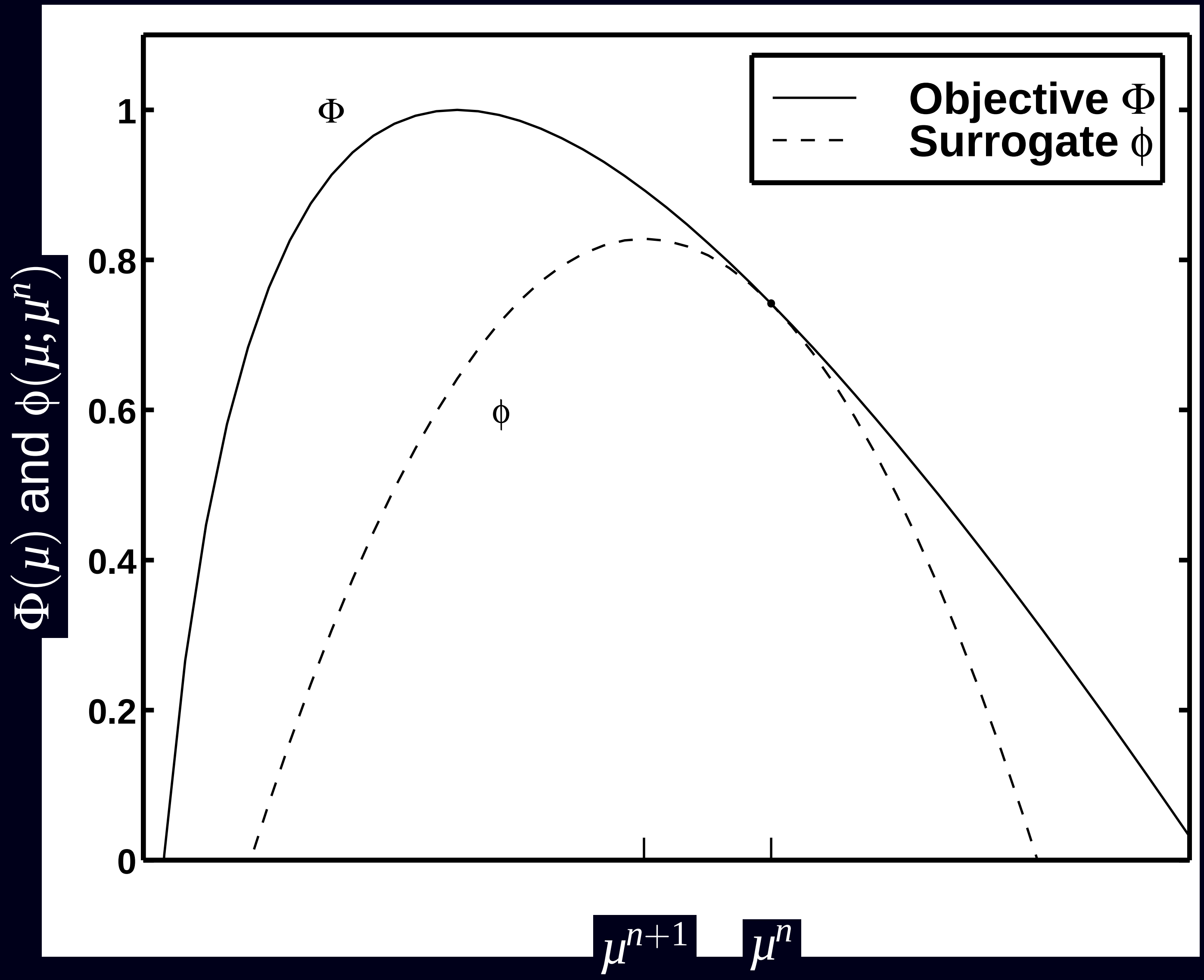
Log-likelihood:

$$L(\mu) = \sum_{i=1}^{N} Y_i \log\left[\sum_{m=1}^{M} b_{im} \exp\left(-\sum_{j=1}^{p} a_{ij}^{m}\mu_j\right) + r_i\right] - \left[\sum_{m=1}^{M} b_{im} \exp\left(-\sum_{j=1}^{p} a_{ij}^{m}\mu_j\right) + r_i\right]$$

Summations: detectors, sources, pixels

- N number of detector elements
- p number of pixels
- Y_i recorded counts by ith detector element
- μ_j linear attenuation coefficient of jth pixel
- r_i contribution of background and emission crosstalk
- M number of sources
- b_{im} blank scan value for mth source to ith detector element
- a_{ij}^{m} length of intersection through jth pixel of the ray that connects mth source to ith detector

Optimization Transfer Illustrated



First Surrogate Function

- $y \log x - x$ is concave in x
- Adapt De Pierro's "multiplicative" convexity trick (T-MI, Jun. 1993)
- Move the summation over sources outside logarithm

$$L(\mu) \geq Q_1(\mu; \mu^n) = \sum_{i=1}^{N} \sum_{m=1}^{M} \left(\frac{u_{im}^n}{\bar{y}_i^n} \right) \left[y_i \log \left(\frac{u_{im}(\mu)}{u_{im}^n} \bar{y}_i^n \right) - \left(\frac{u_{im}(\mu)}{u_{im}^n} \bar{y}_i^n \right) \right]$$

where

- $\bar{y}_i^n \triangleq \bar{y}_i(\mu^n)$
- $u_{im}^n \triangleq u_{im}(\mu^n)$
- $u_{im}(\mu) \triangleq b_{im} \exp\left(-\sum_{j=1}^{p} a_{ij}^m \mu_j \right) + r_i / M$

Q_1 still difficult to maximize

Second Surrogate Function

- $y \log(be^{-l} + r) - (be^{-l} + r)$ has a parabola surrogate: q_{im}^n
- Optimum curvature of parabola derived by Erdoğan (T-MI, 1999).
- Replace nonquadratic surrogate with paraboloidal surrogate

$$Q_1(\mu; \mu^n) \geq Q_2(\mu; \mu^n) = \sum_{i=1}^{N} \sum_{m=1}^{M} \left(\frac{u_{im}^n}{\bar{y}_i^n} \right) q_{im}^n \left(\sum_{j=1}^{p} a_{ij}^m \mu_j \right)$$

- q_{im}^n is a simple quadratic function
- Iterative algorithm:

$$\boxed{\mu^{n+1} = \arg\max_{\mu \geq \underline{0}} Q_2(\mu; \mu^n)}$$

- Maximizing $Q_2(\mu; \mu^n)$ over μ is equivalent to (reweighted) least-squares.
- Natural algorithms
  - Conjugate gradient
  - Coordinate ascent

(Optional) Third Surrogate Function

- Parabolas are convex functions
- Apply De Pierro's "additive" convexity trick (T-MI, Mar. 1995)
- Move summation over pixels outside quadratic

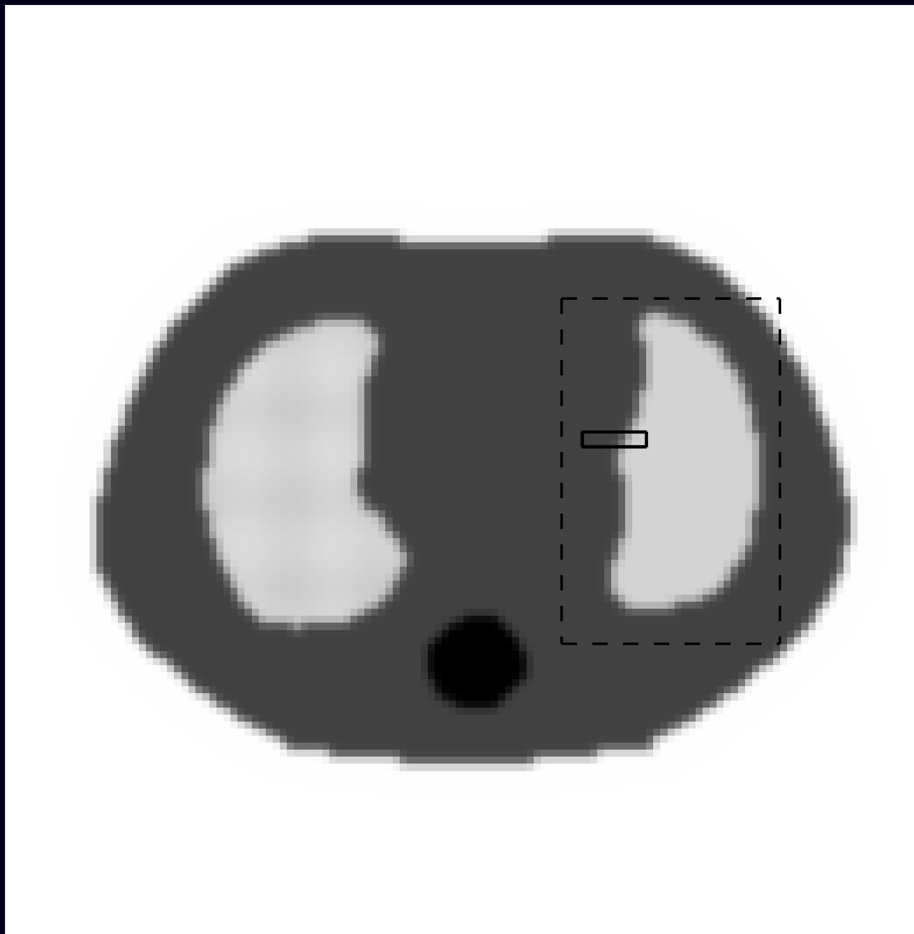
$$Q_2(\mu;\mu^n) \geq Q_3(\mu;\mu^n) = \sum_{i=1}^{N}\sum_{m=1}^{M}\sum_{j=1}^{p}\frac{u_{im}^n a_{ij}^m}{\bar{y}_i^n \gamma_i^m} q_{im}^n(\gamma_i^m(\mu_j - \mu_j^n) + [A^m \mu^n]_i)$$

- $\gamma_i^m \triangleq \sum_{j=1}^{p} a_{ij}^m$
- Separable paraboloidal surrogate function $\Rightarrow$ trivial to maximize (cf EM)

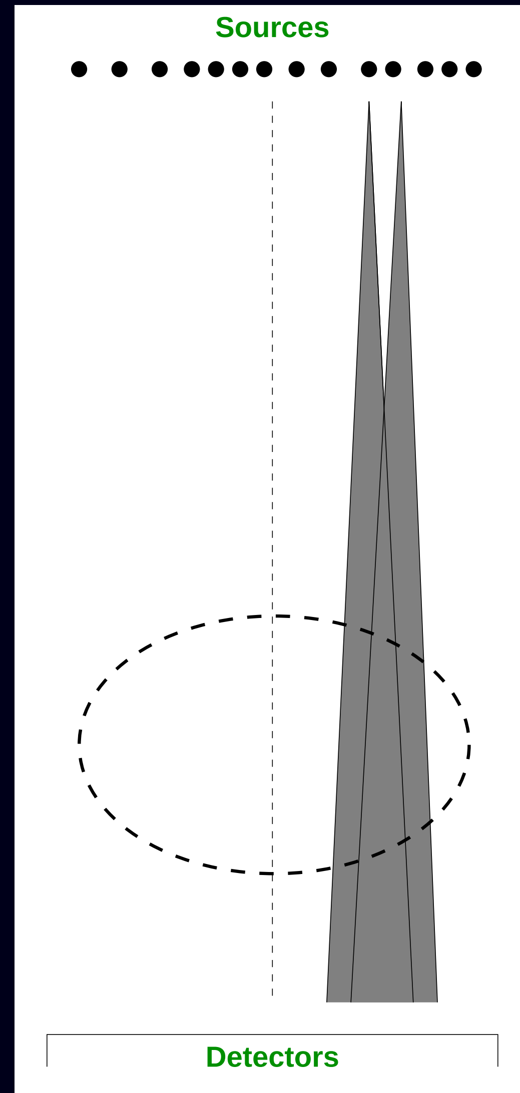
$$\mu_j^{n+1} = \left[\mu_j^n + \frac{1}{d_j(\mu^n)}\frac{\partial}{\partial \mu_j}L(\mu^n)\right]_+$$

- $d_j(\mu)$ related to parabola curvatures (not to Fisher information)
- Natural starting point for forming ordered-subsets variation

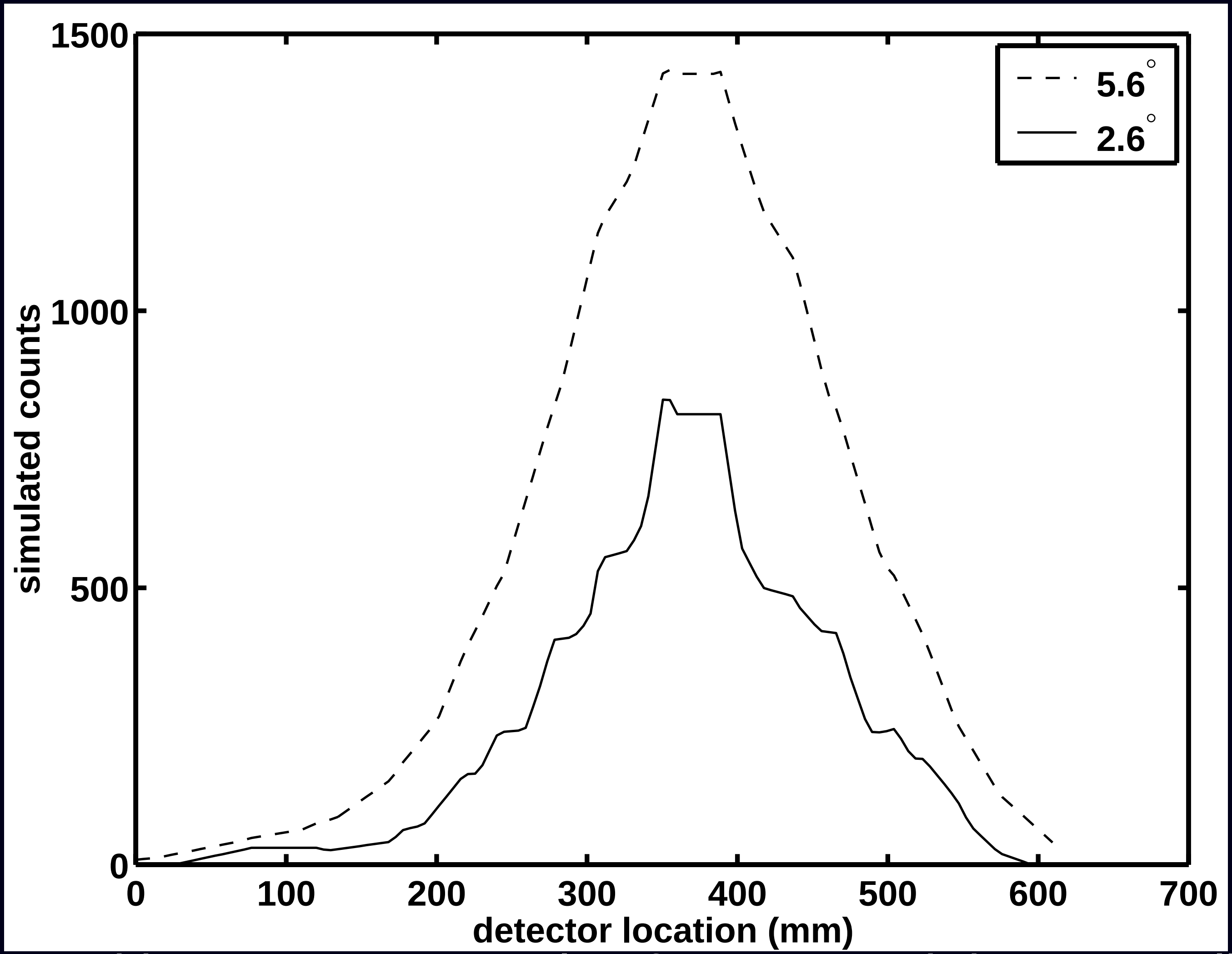
Simulation



Digital Thorax Phantom
128×128 3.56mm pixels



Source Collimation: Blank-scan profiles



(Results vary with acceptance angle of source and detector collimators.)

Reconstruction Algorithms

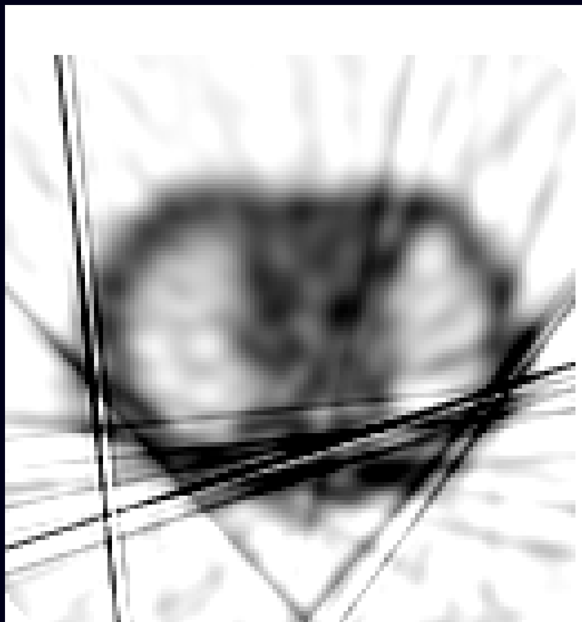
- Filtered backprojection (FBP)
  Based on usual idealized parallel, non-overlapping, line-integral model
  Requires subtraction of emission crosstalk before taking logarithm
- Parallel-beam method
  Penalized-likelihood based on usual non-overlapping strip-integral model
  Effect of emission crosstalk built into statistical model
- Proposed method
  Penalized-likelihood based on overlapping system/statistical model

Variables

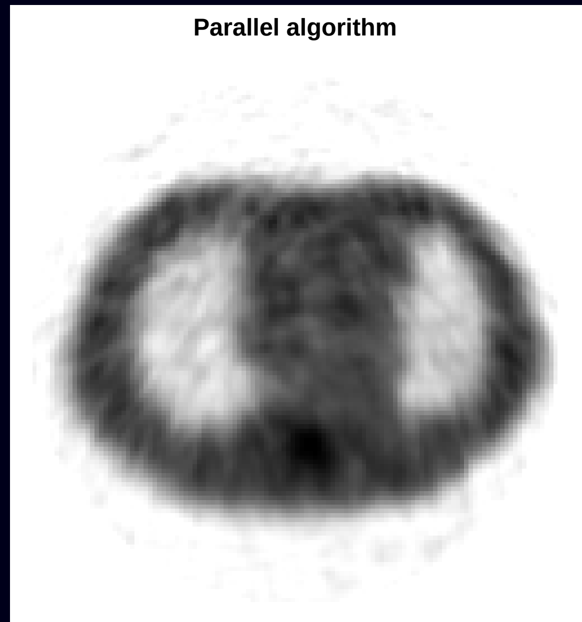
- Source strength
- Crosstalk background level
- Source collimation angle
- Desired target spatial resolution

Simulation Results

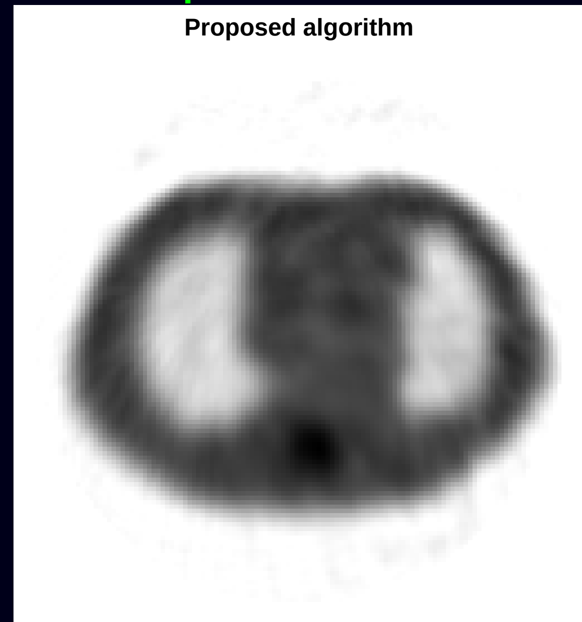
FBP



Parallel-beam Method

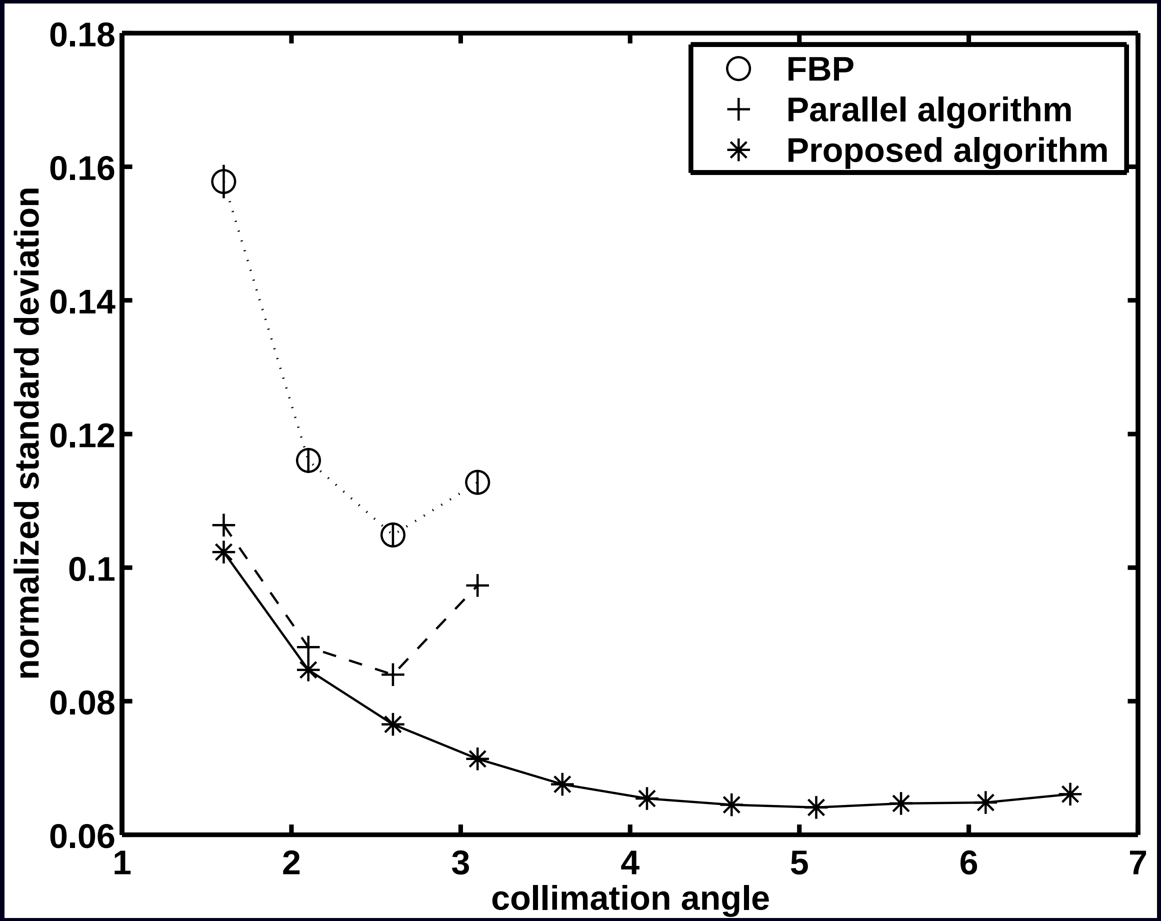


Proposed Method



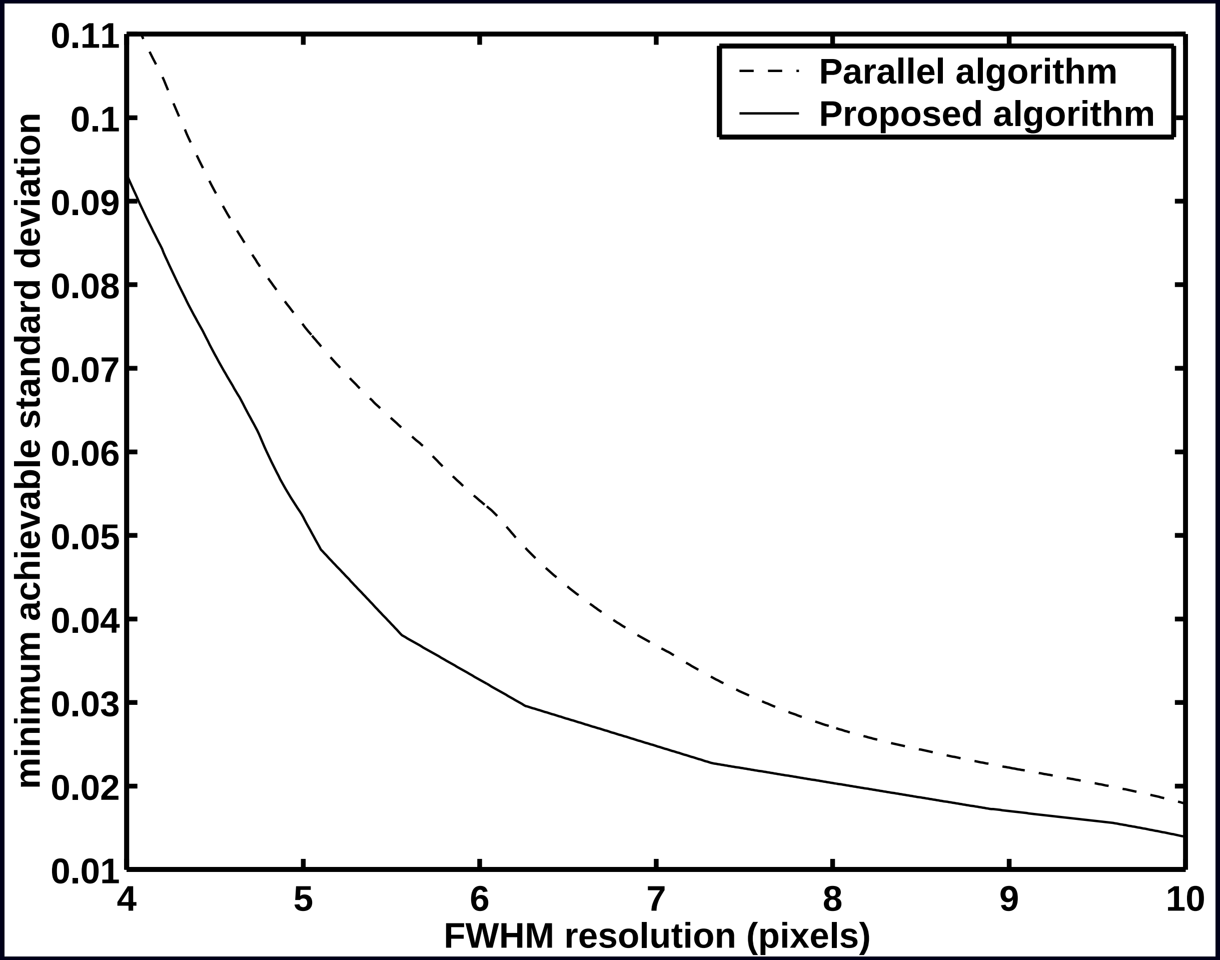
- $\pm 3.6^\circ$ source collimation
- 497,000 transmitted counts + 263,000 emission crosstalk counts
- Resolution matched to 6.8 pixels FWHM effective Gaussian width

Noise vs Collimation



- Resolution matched to 4.7 pixels FWHM
- Similar curves for other target resolutions

Noise vs Resolution

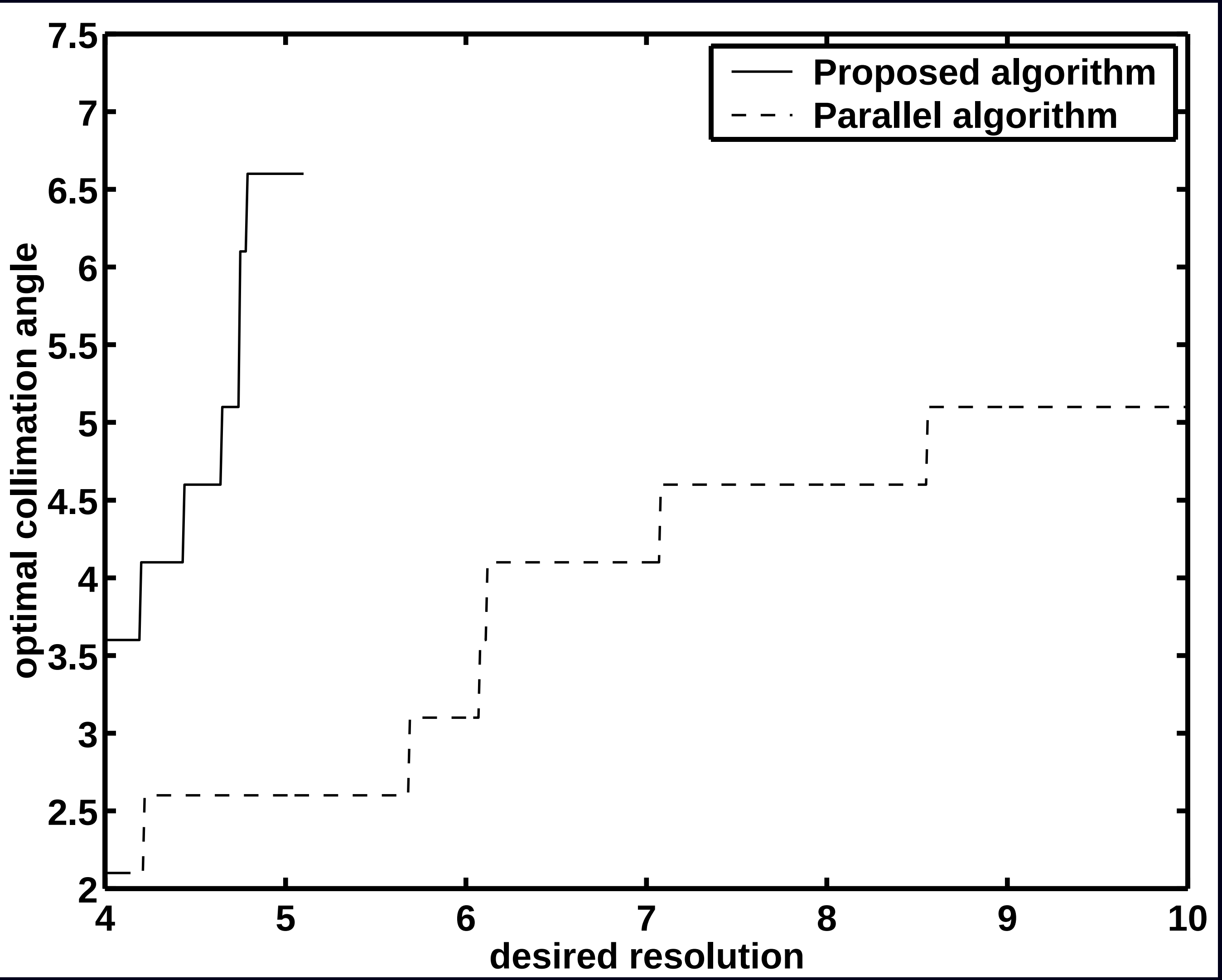


(Each point is for noise-minimizing collimator resolution)

Summary

- New algorithm for overlapping beams
- Intrinsically monotonic (no line searches, no divergence)
- Convergence to a local maximizer
- Requires separate blank scan / system matrix for each source
  ∴ increased memory and computational requirements
- Improved resolution/noise tradeoff over parallel algorithm
- Allows increased acceptance angles ∴ higher transmitted/crosstalk ratio
- Acceptance angle currently limited by detector collimation.

Collimation vs Resolution



Noiseless Data Reconstructions ($\beta \approx 0$)

Parallel algorithm

Proposed algorithm