Joint optimization of learning-based image reconstruction and sampling for MRI

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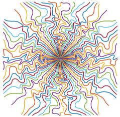
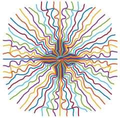

Brain


Knee

Acknowledgments:
Guanhua Wang, Tianrui Luo, Jon Nielsen, Doug Noll

Outline

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- Introduction
  - Machine learning in imaging
  - MRI k-space sampling
- Deep-learning approaches for image reconstruction
- Supervised learning of k-space sampling
- Joint optimization of k-space sampling and image reconstruction
- Summary
- Adaptive sampling
- Bibliography

Medical imaging overview

raw data images

Design Scan → Acquire Data $\xrightarrow{\boldsymbol{y}}$ Reconstruct Images $\xrightarrow{\hat{\boldsymbol{x}}}$ Process Images → Diagnose Interpret

Medical imaging overview

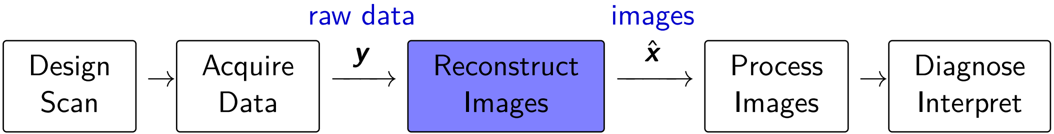
Most obvious place for machine learning is in post-processing (image analysis).
Numerous special issues and surveys in medical imaging journals, *e.g.*, [1–9].

Medical imaging overview

Machine learning for scan design (k-space sampling):

- Choose best k-space phase encoding locations (usually Cartesian sampling)
- Uses “ground truth” (fully sampled) training images
- Hot topic in MRI research recently, *e.g.*, [10–15]
- Precursor by Yue Cao and David Levin, MRM Sep. 1993 [16–18]

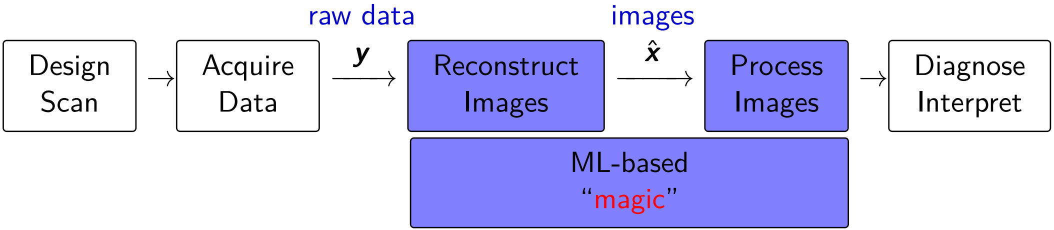
Medical imaging overview



Machine learning in medical image reconstruction:

- June 2018 special issue of IEEE Trans. on Medical Imaging [19].
- Surveys: [20–27]
- Possibly easier than diagnosis due to lower bar:
    - current reconstruction methods based on simplistic image models;
    - human eyes are better at detection than at solving inverse problems.

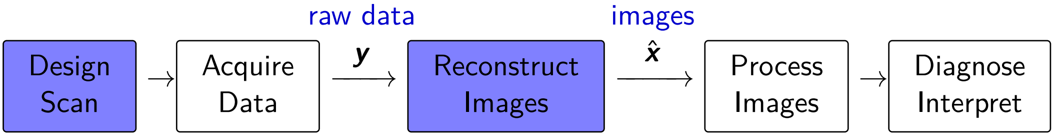
Medical imaging overview



A holy grail for machine learning in medical imaging?

- CT sinogram to vessel diameter [28, 29]
- k-space to ???

Joint optimization of sampling and reconstruction



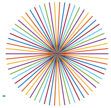
raw data $\quad$ images

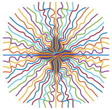
Design Scan → Acquire Data $\xrightarrow{\boldsymbol{y}}$ Reconstruct Images $\xrightarrow{\hat{\boldsymbol{x}}}$ Process Images → Diagnose Interpret

"B-spline parameterized joint optimization of reconstruction and k-space trajectories (BJORK) for accelerated 2D MRI," arXiv 2101.11369 [30] IEEE T-MI 2022 [31]
Guanhua Wang, T. Luo, J.-F. Nielsen, D. Noll, J. Fessler

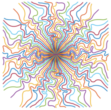
"Stochastic optimization of 3D non-Cartesian sampling trajectory (SNOPY)," MRM 2023 (in press)

Preview:





Knee

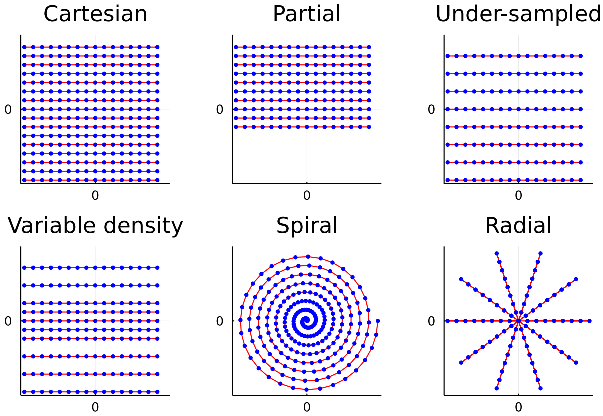


Brain

Related work: "PILOT" by Weiss et al. [32]; J-MoDL work of Aggarwal et al. [14].

MR sampling and under-sampling

All clinical MRI scans currently use “hand-crafted” sampling patterns:



- Reducing k-space sampling $\Longrightarrow$ reduced scan time / improved temporal resolution
- Under-sampled data benefits from advanced reconstruction methods

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Deep-learning approaches to image reconstruction

Overview:

- image-domain learning [33–35]...
- k-space or data-domain learning
  e.g., [36], [37], [38]
- transform learning (direct from k-space to image)
  e.g., AUTOMAP [39], [40–42]
- hybrid-domain learning (unrolled loop, *e.g.*, variational network)
  alternate between denoising/dealiasing and reconstruction from k-space
  e.g., [37, 43–47] ...

DL for IR: image-domain learning

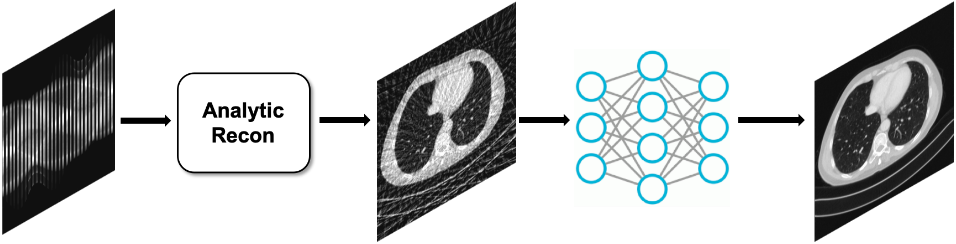
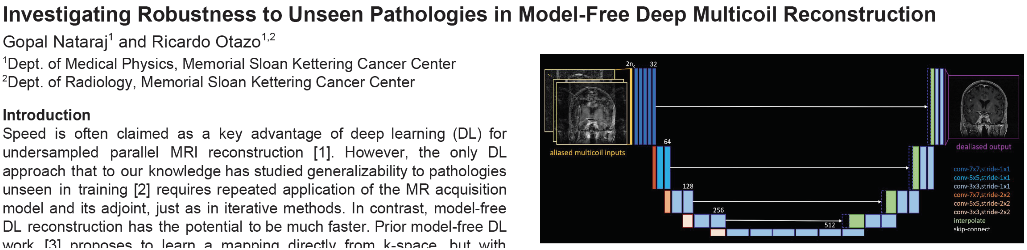


Figure courtesy of Jong Chul Ye, KAIST University.

- \+ simple and fast
- − aliasing is spatially widespread, requires deep network

Dangers of image-domain learning: Method



Investigating Robustness to Unseen Pathologies in Model-Free Deep Multicoil Reconstruction

Gopal Nataraj[1] and Ricardo Otazo[1,2]

[1]Dept. of Medical Physics, Memorial Sloan Kettering Cancer Center
[2]Dept. of Radiology, Memorial Sloan Kettering Cancer Center

Introduction

Speed is often claimed as a key advantage of deep learning (DL) for undersampled parallel MRI reconstruction [1]. However, the only DL approach that to our knowledge has studied generalizability to pathologies unseen in training [2] requires repeated application of the MR acquisition model and its adjoint, just as in iterative methods. In contrast, model-free DL reconstruction has the potential to be much faster. Prior model-free DL work [3] proposes to learn a mapping directly from k-space, but with

[48] ISMRM 2020 Workshop on Data Sampling & Image Reconstruction

Dangers of image-domain learning: Result

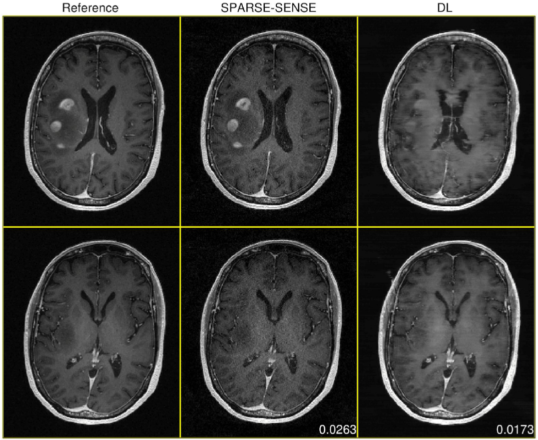


Figure 3: Reconstructions in a case of anaplastic astrocytoma, a rare malignant brain tumor. SPARSE-SENSE and DL reconstructions are from the same 4x-accelerated retrospectively undersampled acquisition. DL achieves lower whole-volume MAE than SPARSE-SENSE, but fails to properly reconstruct regions near the tumor.

Image-domain learning variations

- Use NN output as a “prior” for iterative reconstruction [33, 49]:

$$\hat{\boldsymbol{x}}_\beta = \arg\min_{\boldsymbol{x}} \|\boldsymbol{A}\boldsymbol{x} - \boldsymbol{y}\|_2^2 + \beta \|\boldsymbol{x} - \boldsymbol{x}_{\text{NN}}\|_2^2 = (\boldsymbol{A}'\boldsymbol{A} + \beta \boldsymbol{I})^{-1}(\boldsymbol{A}'\boldsymbol{y} + \beta \boldsymbol{x}_{\text{NN}})$$

  - For single-coil Cartesian case:
    - no iterations are needed (solve with FFTs)
    - $\lim_{\beta \to 0} \hat{\boldsymbol{x}}_\beta$ replaces missing k-space data with FFT of $\boldsymbol{x}_{\text{NN}}$
  - Iterations needed for parallel MRI and/or non-Cartesian sampling (PCG)

- Learn residual (aliasing artifacts), then subtract [50, 51]

DL for IR: k-space / sinogram domain learning

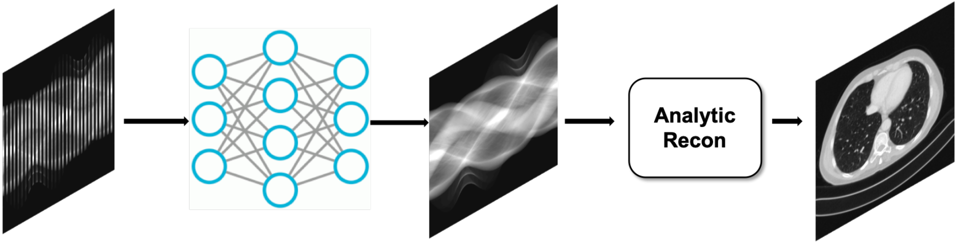


Figure courtesy of Jong Chul Ye, KAIST University.

+ simple and fast (“nonlinear GRAPPA”)
+ “database-free” : learn from auto-calibration data [36], [37], [38]
− perhaps harder to represent local image features?

DL for IR: transform learning

Figure courtesy of Jong Chul Ye, KAIST University.

+ in principle, purely data driven; potential to avoid model mismatch
– high memory requirement for fully connected layers [39]

DL for IR: hybrid domain learning (unrolled loop)

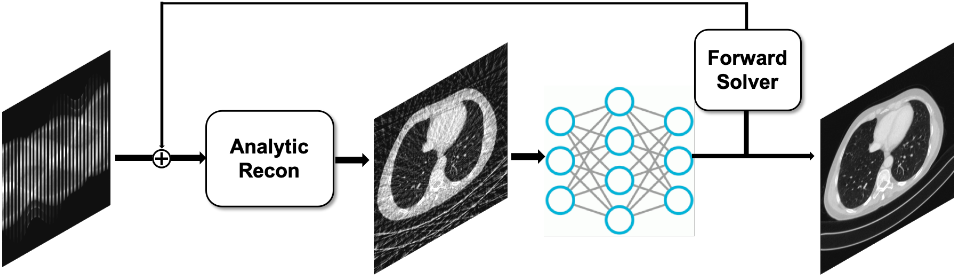
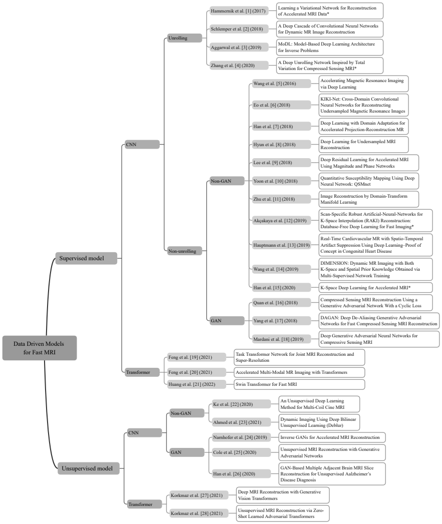


Figure courtesy of Jong Chul Ye, KAIST University.

- \+ physics-based use of k-space data & image-domain priors, *e.g.*, [37, 43–47, 52, 53] ...
- \+ interpretable connections to optimization approaches
- \+ best results in MRI recon challenges [54–56]
- – more computation to due to "iterations" (hyper-layers) and repeated $\boldsymbol{A}\boldsymbol{x}$, $\boldsymbol{A}'\boldsymbol{r}$

DL for MRI: a taxonomy



- Data Driven Models for Fast MRI
  - Supervised model
    - CNN
      - Unrolling
        - Hammernik et al. [1] (2017) — Learning a Variational Network for Reconstruction of Accelerated MRI Data*
        - Schlemper et al. [2] (2018) — A Deep Cascade of Convolutional Neural Networks for Dynamic MR Image Reconstruction
        - Aggarwal et al. [3] (2019) — MoDL: Model-Based Deep Learning Architecture for Inverse Problems
        - Zhang et al. [4] (2020) — A Deep Unrolling Network Inspired by Total Variation for Compressed Sensing MRI*
      - Non-unrolling
        - Non-GAN
          - Wang et al. [5] (2016) — Accelerating Magnetic Resonance Imaging via Deep Learning
          - Eo et al. [6] (2018) — KIKI-Net: Cross-Domain Convolutional Neural Networks for Reconstructing Undersampled Magnetic Resonance Images
          - Han et al. [7] (2018) — Deep Learning with Domain Adaptation for Accelerated Projection-Reconstruction MR
          - Hyun et al. [8] (2018) — Deep Learning for Undersampled MRI Reconstruction
          - Lee et al. [9] (2018) — Deep Residual Learning for Accelerated MRI Using Magnitude and Phase Networks
          - Yoon et al. [10] (2018) — Quantitative Susceptibility Mapping Using Deep Neural Network: QSMnet
          - Zhu et al. [11] (2018) — Image Reconstruction by Domain-Transform Manifold Learning
          - Akçakaya et al. [12] (2019) — Scan-Specific Robust Artificial-Neural-Networks for K-Space Interpolation (RAKI) Reconstruction: Database-Free Deep Learning for Fast Imaging*
          - Hauptmann et al. [13] (2019) — Real-Time Cardiovascular MR with Spatio-Temporal Artifact Suppression Using Deep Learning–Proof of Concept in Congenital Heart Disease
          - Wang et al. [14] (2019) — DIMENSION: Dynamic MR Imaging with Both K-Space and Spatial Prior Knowledge Obtained via Multi-Supervised Network Training
          - Han et al. [15] (2020) — K-Space Deep Learning for Accelerated MRI*
        - GAN
          - Quan et al. [16] (2018) — Compressed Sensing MRI Reconstruction Using a Generative Adversarial Network With a Cyclic Loss
          - Yang et al. [17] (2018) — DAGAN: Deep De-Aliasing Generative Adversarial Networks for Fast Compressed Sensing MRI Reconstruction
          - Mardani et al. [18] (2019) — Deep Generative Adversarial Neural Networks for Compressive Sensing MRI
    - Transformer
      - Feng et al. [19] (2021) — Task Transformer Network for Joint MRI Reconstruction and Super-Resolution
      - Feng et al. [20] (2021) — Accelerated Multi-Modal MR Imaging with Transformers
      - Huang et al. [21] (2022) — Swin Transformer for Fast MRI
  - Unsupervised model
    - CNN
      - Non-GAN
        - Ke et al. [22] (2020) — An Unsupervised Deep Learning Method for Multi-Coil Cine MRI
        - Ahmed et al. [23] (2021) — Dynamic Imaging Using Deep Bilinear Unsupervised Learning (Deblur)
      - GAN
        - Narnhofer et al. [24] (2019) — Inverse GANs for Accelerated MRI Reconstruction
        - Cole et al. [25] (2020) — Unsupervised MRI Reconstruction with Generative Adversarial Networks
        - Han et al. [26] (2020) — GAN-Based Multiple Adjacent Brain MRI Slice Reconstruction for Unsupervised Aalzheimer's Disease Diagnosis
    - Transformer
      - Korkmaz et al. [27] (2021) — Deep MRI Reconstruction with Generative Vision Transformers
      - Korkmaz et al. [28] (2021) — Unsupervised MRI Reconstruction via Zero-Shot Learned Adversarial Transformers

Huang et al.., arXiv 2204.01706, Apr. 2022 [57]

Outline

Learning MRI sampling patterns I

Pre-specified image reconstruction methods

Cartesian sampling pattern optimization

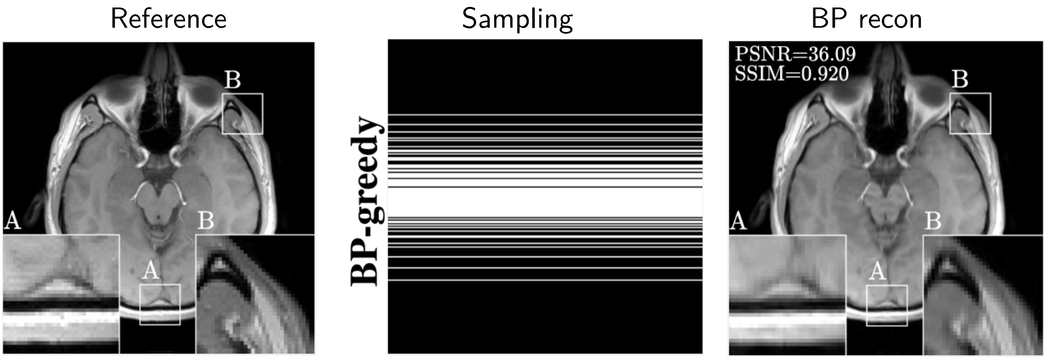
- Yue Cao & David Levin, MRM Sep. 1993 [16–18]
  Feature recognizing MRI
- Seeger et al., MRM 2010 [58]
  Single coil, 1D Cartesian, 2D spiral angles, CS-type recon, Bayesian information gain
- Ravishankar & Bresler, EMBS 2011 [10]
  Single coil, 1D & 2D sampling, DLMRI recon (DL = dictionary learning), weighted k-space loss
- Baldassarre . . . Cevher, IEEE J-STSP 2016 [11]
  Single coil, 2D sampling, energy preserving criterion
- Sherry . . . Ehrhardt, IEEE T-MI 2020 [59]
  Single coil, 2D sampling, various regularizers
- . . .

Learning MRI sampling patterns II

Joint learning of sampling and reconstruction

- Gözcü . . . Cevher, IEEE T-MI 2018 [12]
  Single coil, 1D sampling, several fixed recon methods (TV, BP, BM3D, image-domain NN), image-domain training loss
- Aggarwal & Jacob IEEE J-STSP 2020 [14] (J-MoDL)
  Multi-coil, 1D (horizontal and vertical) sampling, MoDL recon
- Bahadir . . . Sabuncu, IEEE T-CI 2020 [15] (LOUPE)
  Single coil, 1D & 2D sampling, IFFT/U-Net recon
- Weiss et al., arXiv 1909.05773 (2019, 2020, 2021) (PILOT) [32]
  Single coil, Non-Cartesian sampling, IFFT/U-Net recon
- Wang . . . Fessler, ISMRM 2021, arXiv 2021 (BJORK) [30, 60]
  Multi-coil, Non-Cartesian sampling, MoDL-type recon
  Fast and efficient DFT Jacobian approximations [61, 62]

Adaptive phase-encode selection



- Sampling designed to optimize PSNR for basis pursuit (BP) reconstruction using shearlet transform, at 25% sampling rate.
- Sampling design considers both the training data and the reconstruction method.
- No high spatial frequencies!?

(Images from Gözcü et al. [12].)

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  - Problem formulation
  - Results
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Problem formulation

- MRI measurement model:

$$\boldsymbol{y} = \boldsymbol{A}(\boldsymbol{\omega})\boldsymbol{x} + \boldsymbol{\varepsilon}$$

  - $\boldsymbol{y} \in \mathbb{C}^M$: k-space data; $M \sim 10 - 30\text{K}$
  - $\boldsymbol{\omega} \in \mathbb{R}^{M \times 2}$: k-space sampling pattern ("trajectory"): 2D in BJORK, 3D in SNOPY
  - $\boldsymbol{x} \in \mathbb{C}^N$: unknown true image, $N \sim 100\text{K}$
  - $\boldsymbol{A}(\boldsymbol{\omega}) \in \mathbb{C}^{M \times N}$: encoding matrix (coil sensitivity, etc.)
  - $\boldsymbol{\varepsilon} \in \mathbb{C}^M$: measurement noise
- Reconstruction method:

$$\hat{\boldsymbol{x}} = f(\boldsymbol{y}; \boldsymbol{\omega}, \boldsymbol{\theta})$$

  - $\boldsymbol{\theta}$: model parameters of reconstruction method (e.g., CNN weights)
  - Deep iterative down-up CNN (DIDN) has $\sim$ 165M learned parameters [63]
- Image quality goal:

$$\hat{\boldsymbol{x}} = f(\boldsymbol{y}; \boldsymbol{\omega}, \boldsymbol{\theta}) = f(\boldsymbol{A}(\boldsymbol{\omega})\boldsymbol{x} + \boldsymbol{\varepsilon}; \boldsymbol{\omega}, \boldsymbol{\theta}) \approx \boldsymbol{x}$$

Supervised approach

- Define training loss function such as $\ell(\hat{\boldsymbol{x}}, \boldsymbol{x}) = \|\hat{\boldsymbol{x}} - \boldsymbol{x}\|_1 + \alpha \|\hat{\boldsymbol{x}} - \boldsymbol{x}\|_2$
- Select N_{train} fully sampled training images $\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots$
- Jointly optimize k-space trajectory $\boldsymbol{\omega}$ and image reconstruction method $\boldsymbol{\theta}$

$$(\hat{\boldsymbol{\omega}}, \hat{\boldsymbol{\theta}}) = \operatorname*{arg\,min}_{\boldsymbol{\omega}, \boldsymbol{\theta}} \frac{1}{N_{\mathrm{train}}} \sum_{n=1}^{N_{\mathrm{train}}} \ell(f(\boldsymbol{A}(\boldsymbol{\omega})\boldsymbol{x}_n + \boldsymbol{\varepsilon}_n; \boldsymbol{\omega}, \boldsymbol{\theta}), \boldsymbol{x}_n)$$

-

Supervised approach

- Define training loss function such as $\ell(\hat{\boldsymbol{x}}, \boldsymbol{x}) = \|\hat{\boldsymbol{x}} - \boldsymbol{x}\|_1 + \alpha \|\hat{\boldsymbol{x}} - \boldsymbol{x}\|_2$
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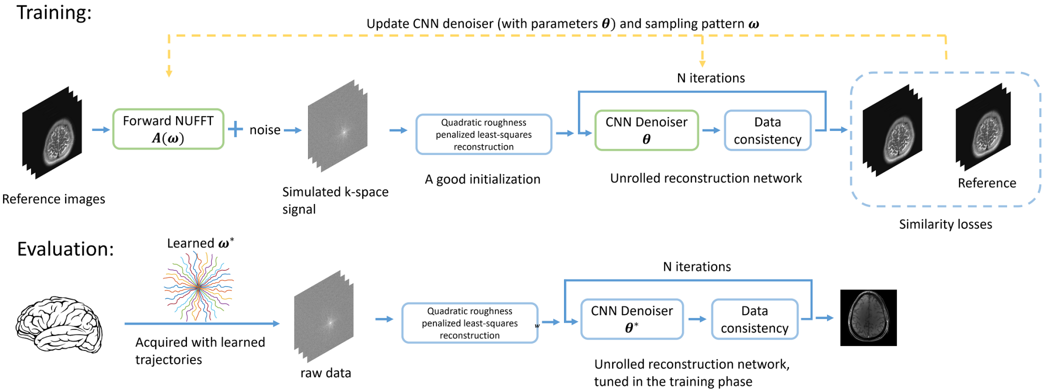
- Details:
  - Reconstruction using MoDL method [52]
  - Can use multiple noise realizations $\boldsymbol{\varepsilon}$ per training image
  - Enforce gradient amplitude and slew-rate limits for $\boldsymbol{\omega}$
  - Use B-spline parameterization of k-space trajectory
  - Coarse-to-fine search of trajectory to avoid poor local minimizers
  - Eddy current correction
  - Fast NUFFT Jacobian approximation [61, 62, 64]

BJORK Diagram



Training:

Update CNN denoiser (with parameters $\boldsymbol{\theta}$) and sampling pattern $\boldsymbol{\omega}$

Reference images

Forward NUFFT $\boldsymbol{A}(\boldsymbol{\omega})$

\+ noise

Simulated k-space signal

Quadratic roughness penalized least-squares reconstruction

A good initialization

N iterations

CNN Denoiser $\boldsymbol{\theta}$

Data consistency

Unrolled reconstruction network

Reference

Similarity losses

Evaluation:

Learned $\boldsymbol{\omega}^*$

Acquired with learned trajectories

raw data

Quadratic roughness penalized least-squares reconstruction

N iterations

CNN Denoiser $\boldsymbol{\theta}^*$

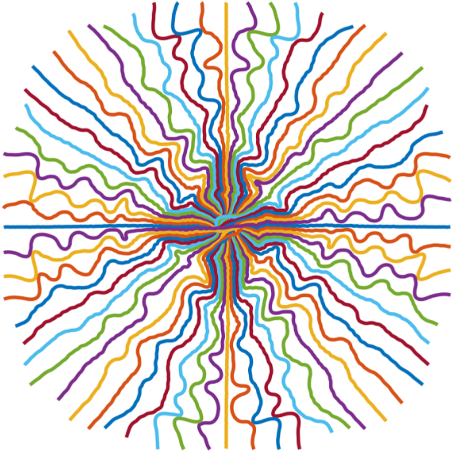
Data consistency

Unrolled reconstruction network, tuned in the training phase

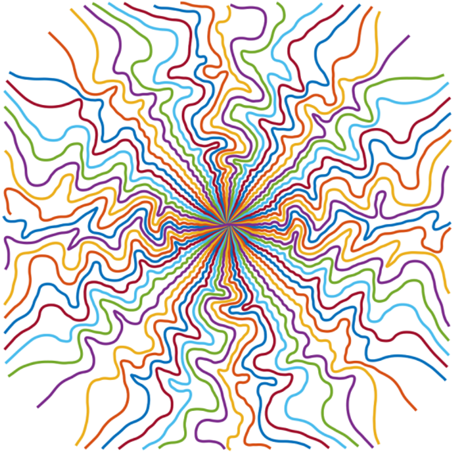
Data

- NYU/FAIR fastMRI brain and knee data
- 16/24/32 radial spokes of 1280 points for trajectory initialization ($\approx$ 10-20 × acceleration for 320×320 image)
- 22cm FOV, Gmax $=$ 5 Gauss/cm, slew rate $\leq$ 15 Gauss/cm/ms
- 5ms readout duration radial, 16ms spiral
- Comparison with SPARKLING approach of [65] using its default density

Trajectory can be tailored to anatomy



Knee



Brain

PSF results: 32 spokes

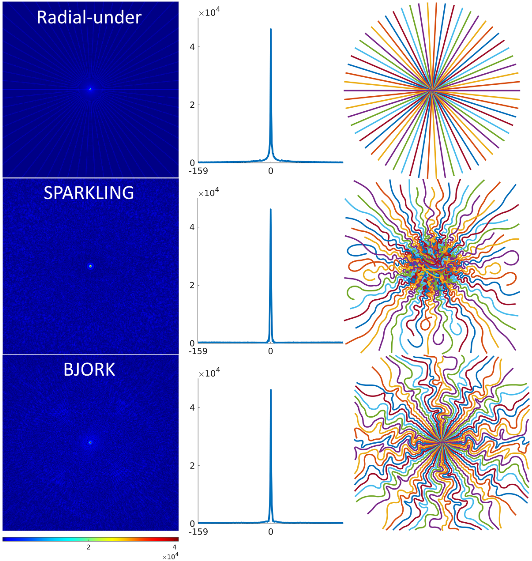
32-spoke results

FWHM (pixels):

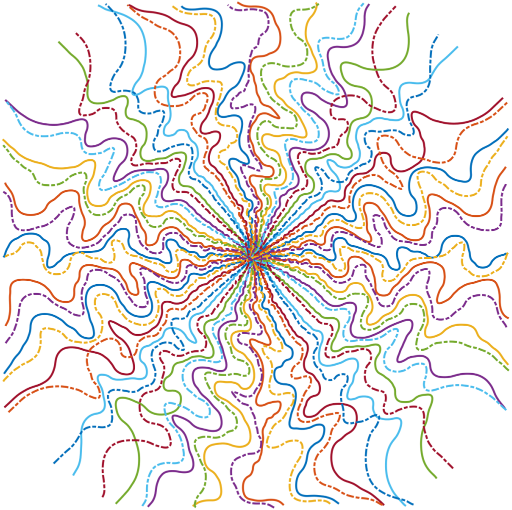
1.5

2.1

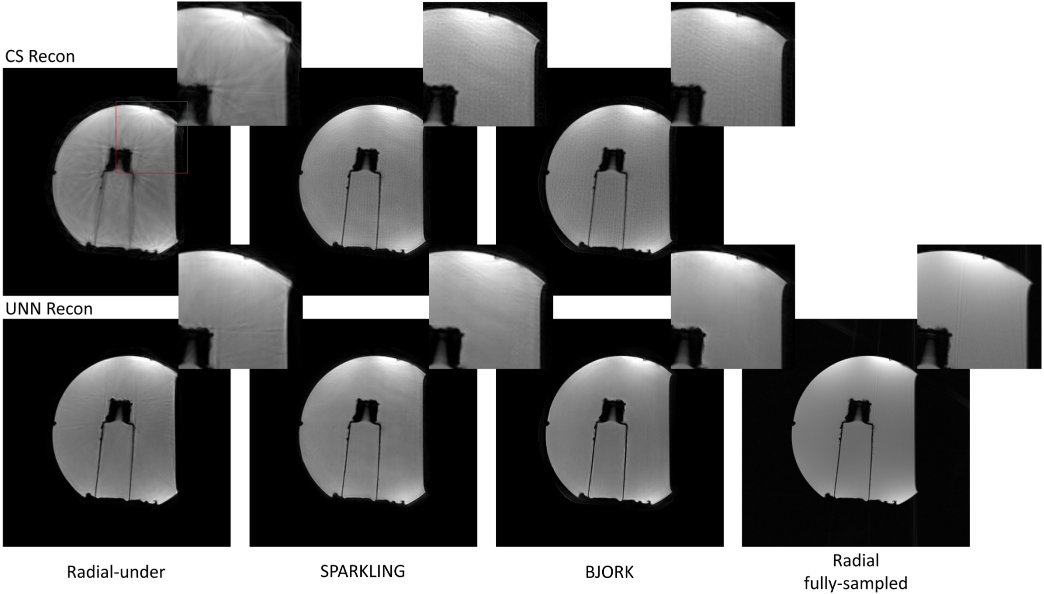
1.6



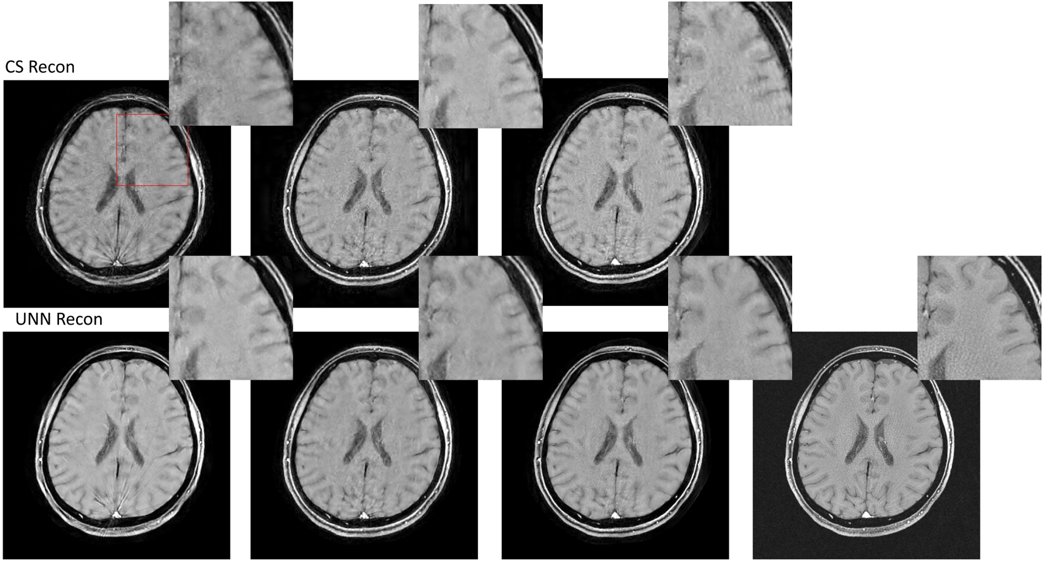
Learning about spectral conjugate symmetry



Prospectively under-sampled MRI phantom study



Prospective in-vivo study (GE scanner, 32 shot)



Quantitative simulation results: PSNR

PSNR (in dB):

		Standard	SPARKLING	BJORK
radial-like Ns=16	UNN	32.7	33.9	**34.3**
	CS	31.7	33.6	**34.1**
radial-like Ns=24	UNN	34.1	35.0	**35.6**
	CS	33.3	34.6	**35.1**
radial-like Ns=32	UNN	35.0	36.0	**36.9**
	CS	33.9	35.7	**36.3**
spiral-like Ns=8	UNN	40.9	41.7	**41.9**
	CS	39.9	40.4	**40.7**

Ns: the number of shots or spokes.

Quantitative simulation results: SSIM

SSIM:

		Standard	SPARKLING	BJORK
radial-like Ns=16	UNN	0.940	0.946	**0.950**
	CS	0.911	0.936	**0.938**
radial-like Ns=24	UNN	0.950	0.955	**0.959**
	CS	0.929	0.943	**0.948**
radial-like Ns=32	UNN	0.957	0.963	**0.968**
	CS	0.932	0.946	**0.956**
spiral-like Ns=8	UNN	0.986	0.989	**0.990**
	CS	0.976	0.978	**0.981**

Gradient constraints

MRI gradient amplifiers have maximum amplitude and slew rate

- gradient amplitude is 1st derivative of k-space trajectory:

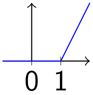
$$\left\| \boldsymbol{D}_1 \boldsymbol{\omega}_d \right\|_\infty \leq g_{\max}$$

- slew rate is 2nd derivative of k-space trajectory:

$$\left\| \boldsymbol{D}_2 \boldsymbol{\omega}_d \right\|_\infty \leq s_{\max}$$

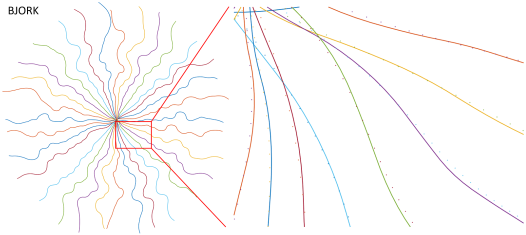
- Box constraints relaxed to penalty functions that rise rapidly above 1, on:

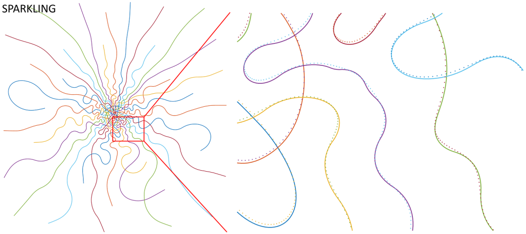
$$\left\| \boldsymbol{D}_1 \boldsymbol{\omega}_d \right\|_\infty / g_{\max} \text{ and } \left\| \boldsymbol{D}_2 \boldsymbol{\omega}_d \right\|_\infty / s_{\max}$$



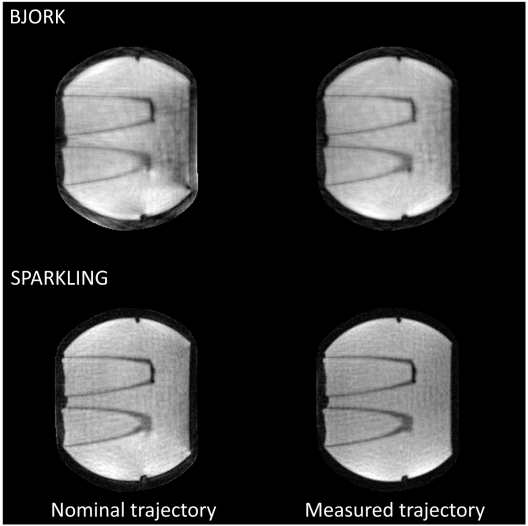
Facilitates (sub)gradient-based optimization using Adam

Trajectory calibration





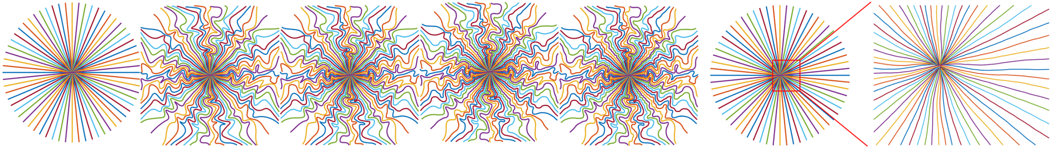
Eddy current compensation



Coarse-to-fine parameterization and evolution

Quadratic B-spline kernels for non-Cartesian k-space trajectory:

$$\boldsymbol{\omega}_d = \boldsymbol{B}\boldsymbol{c}_d, \qquad d = 1, 2, \boldsymbol{c}_d \in \mathbb{R}^{M/\text{Decim}}$$



Init.
SSIM=0.957

Decim=32
SSIM=0.963

Decim=16
SSIM=0.965

Decim=8
SSIM=0.967

Decim=4
SSIM=0.968

Nonparametric
SSIM=0.959

Highly non-convex problem in $\boldsymbol{\omega}$.

- Coarse-to-fine search may find better local minimizers
- However, parameterization/decimation does not save much computation

Unrolled-loop image reconstruction method

Motivated by model-based image reconstruction with variable splitting of the form

$$\begin{aligned}\hat{\boldsymbol{x}} &= \arg\min_{\boldsymbol{x}} \|\boldsymbol{A}(\omega)\boldsymbol{x} - \boldsymbol{y}\|_2^2 + R(\boldsymbol{x}) \\ &= \arg\min_{\boldsymbol{x}} \min_{\boldsymbol{z}} \|\boldsymbol{A}(\omega)\boldsymbol{x} - \boldsymbol{y}\|_2^2 + R(\boldsymbol{z}), \quad \text{s.t. } \boldsymbol{z} = \boldsymbol{x}\end{aligned}$$

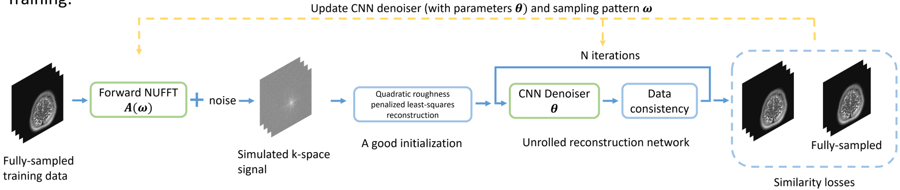
Alternating minimization:

$$\begin{aligned}\boldsymbol{x}_{t+1} &= \arg\min_{\boldsymbol{x}} \|\boldsymbol{A}(\omega)\boldsymbol{x} - \boldsymbol{y}\|_2^2 + \mu \|\boldsymbol{x} - \boldsymbol{z}_t\|_2^2 && \text{(data consistency, solved via CG)} \\ \boldsymbol{z}_{t+1} &= \arg\min_{\boldsymbol{z}} R(\boldsymbol{z}) + \mu \|\boldsymbol{x}_{t+1} - \boldsymbol{z}\|_2^2 && \text{(denoising)} \\ \text{``} &= \text{''} \mathcal{D}_{\boldsymbol{\theta}}(\boldsymbol{x}_{t+1}) && \text{(CNN denoiser)}\end{aligned}$$

- CNN weights $\boldsymbol{\theta}$ shared across iterations, per MODL [52]
- 6 outer iterations for results shown, with augmented Lagrangian parameter $\mu = 2$

Efficient NUFFT backpropagation I

Training:



Update CNN denoiser (with parameters $\boldsymbol{\theta}$) and sampling pattern $\boldsymbol{\omega}$

Fully-sampled training data

Forward NUFFT $\boldsymbol{A}(\boldsymbol{\omega})$

+ noise

Simulated k-space signal

Quadratic roughness penalized least-squares reconstruction

A good initialization

N iterations

CNN Denoiser $\boldsymbol{\theta}$

Data consistency

Unrolled reconstruction network

Fully-sampled

Similarity losses

Data consistency block has steps like

$$\boldsymbol{x}_{t+1} = \boldsymbol{x}_t + \alpha \left(\boldsymbol{A}'(\boldsymbol{\omega}) \left(\boldsymbol{A}(\boldsymbol{\omega}) \boldsymbol{x} - \boldsymbol{y} \right) + \mu (\boldsymbol{x} - \boldsymbol{z}_t) \right)$$

$\boldsymbol{A}(\boldsymbol{\omega})$ is dense and huge:

$$a_{ij} = \mathrm{e}^{-\imath \vec{\omega}_i \cdot \vec{r}_j} \qquad (1)$$

- Fast approach to $\boldsymbol{A}(\boldsymbol{\omega})\boldsymbol{x}$ uses NUFFT approximation:
  zero-padding, over-sampled FFT, interpolation [66, 67].
- Backpropagation for $\boldsymbol{\omega}$ update through NUFFT steps via autodifferentiation is slow.

Efficient NUFFT backpropagation II

Derive Jacobian matrix for exact form (1):

$$\frac{\partial}{\partial \omega_d} \boldsymbol{A}(\boldsymbol{\omega})\boldsymbol{x} = -\imath \, \mathrm{Diag}\{\boldsymbol{A}(\boldsymbol{\omega})(\boldsymbol{x} \odot \boldsymbol{r}_d)\} \, .$$

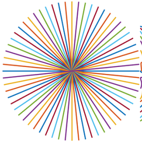
Applying this Jacobian to a vector $\boldsymbol{v} \in \mathbb{C}^M$ during backpropagation yields

$$\left(\frac{\partial}{\partial \omega_d} \boldsymbol{A}(\boldsymbol{\omega})\boldsymbol{x}\right) \boldsymbol{v} = -\imath \, \mathrm{Diag}\{\boldsymbol{A}(\boldsymbol{\omega})(\boldsymbol{x} \odot \boldsymbol{r}_d)\} \, \boldsymbol{v} = -\imath \, (\boldsymbol{A}(\boldsymbol{\omega})(\boldsymbol{x} \odot \boldsymbol{r}_d)) \odot \boldsymbol{v} \, .$$

Implemented efficiently using NUFFT applied to $\boldsymbol{x} \odot \boldsymbol{r}_d$

Similar idea for Jacobian of adjoint of $\boldsymbol{A}$.
Even more important is accurately approximating Jacobian of CG solve of $(\boldsymbol{A}'\boldsymbol{A} + \mu \boldsymbol{I})^{-1}$

Evolution with improved Jacobians



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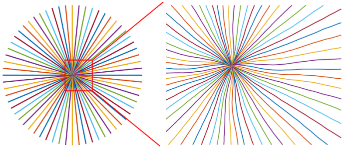
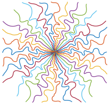
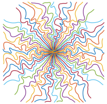


Nonparametric
SSIM=0.959

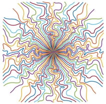
Different acceleration factors:



16 spokes



24 spokes



32 spokes

Cross-contrast comparison

- Each contrast has 4500 training slices, 500 test slices
- No extra noise in training
- Testing variance is 10^{-3} mean test signal
- SSIM values

training / test	T1w	T2w	FLAIR
T1w+noise	0.981	0.980	0.981
T2w+noise	0.951	0.953	0.953
FLAIR+noise	0.974	0.974	0.975

Cross-contrast comparison

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- Approximately constant in each row!

Outline

Introduction

Deep-learning approaches for image reconstruction

Supervised learning of k-space sampling

Joint optimization of k-space sampling and image reconstruction

Summary

Adaptive sampling

Bibliography

Summary / future directions

- Machine learning methods have much potential for both scan design and image reconstruction
- Quantitative results in paper demonstrate synergy of jointly optimizing both
- Anatomy specific trajectories: pro or con?
- Self-supervised methods when training data unavailable
- Extension to 3D accepted to MRM [68]
  Also control of peripheral nerve stimulation (PNS)
- Extension to 3D+time is planned (and challenging)

2D code for BJORK: `https://github.com/guanhuaw/Bjork`

Outline

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- Adaptive sampling
- Bibliography

Patient-specific adaptive sampling

- Goal: shorten MRI scan by adaptive sampling
  "Adaptive sampling for linear sensing systems via Langevin dynamics"
  Guanhua Wang, D Noll, J Fessler, arXiv 2302.13468 2023 [69]
-

Patient-specific adaptive sampling

- Goal: shorten MRI scan by adaptive sampling
  "Adaptive sampling for linear sensing systems via Langevin dynamics"
  Guanhua Wang, D Noll, J Fessler, arXiv 2302.13468 2023 [69]

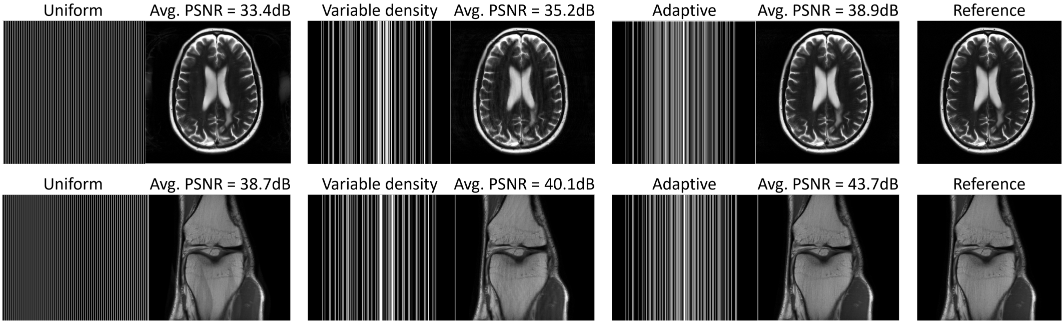
- Overview:
  - Pick image prior $\mathrm{p}(\boldsymbol{x})$
  - Collect (incomplete) k-space data
  - Sample repeatedly from the posterior $\hat{\boldsymbol{x}} \sim \mathrm{p}(\boldsymbol{x}|\boldsymbol{y})$
  - Predict missing measurements $\hat{\boldsymbol{y}} = \boldsymbol{A}\,\hat{\boldsymbol{x}}$
  - Select new k-space samples where posterior variance is highest
  - Repeat

Related image-domain adaptive sampling: Godaliyadda et al., ICASSP 2014 & IEEE T-CI 2018 [13, 70]

Adaptive sampling algorithm

```
Require: Score function $f_{\theta}(x) \approx \nabla \log p(x)$ (score matching or hand crafted)
Acquire initial k-space measurements $y^0$
for $k = 1$ to $N_{\text{add}}$ do
    for $i = 1$ to $N_{\text{sample}}$ do
        for $t = 1$ to $N_{\text{step}}$ do
            Initialize $\tilde{x}_0$; sample from posterior via Langevin MC:
            $\tilde{x}_t = \tilde{x}_{t-1} + \mu_t f_{\theta}(\tilde{x}_{t-1}) - \mu_t \eta_t A'(A\tilde{x}_{t-1} - y^{(k)}) + \sqrt{2\mu_t}\,\mathcal{N}(0, 1)$
        end for
        $\hat{x}_i^{(k)} = \tilde{x}_{N_{\text{add}}}$
        $\hat{y}_i^{(k)} = A\,\hat{x}_i^{(k)}$
    end for
    $l = \arg\max_{n \in 1,2,\ldots N} \operatorname{Var}\{[\hat{y}_1^{(k)}]_n, \ldots, [\hat{y}_{N_{\text{sample}}}^{(k)}]_n\}$
    Acquire measurement index $l$, concatenate with previous: $y^{(k)} = [y^{(k-1)},\ y_l]$.
end for
```

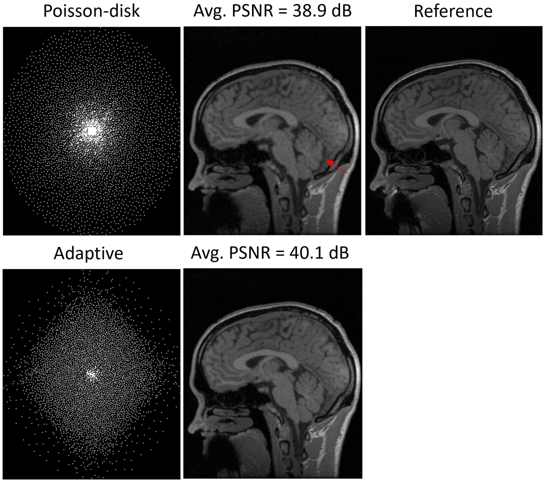
Preliminary results: 1D sampling



- $10\times$ acceleration, $N_{\mathrm{add}} = 50$, $N_{\mathrm{step}} = 200$, $N_{\mathrm{sample}} = 8 - 10$
- PSNR averaged over 20 test cases
- Hand-crafted roughness regularizer: $\nabla \log \mathrm{p}(\boldsymbol{x}) = \nabla \frac{\beta}{2} \|\boldsymbol{T}\boldsymbol{x}\|_2^2 = \beta \, \boldsymbol{T}' \boldsymbol{T}\boldsymbol{x}$

Preliminary results: 2D sampling

- 12× acceleration
- Hand-crafted roughness regularizer
- PSNR for 10 test cases



Poisson-disk

Avg. PSNR = 38.9 dB

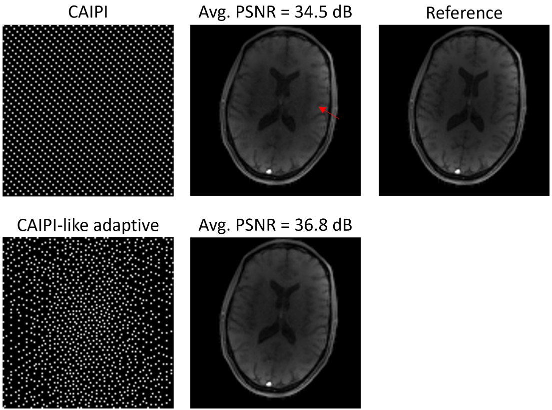
Reference

Adaptive

Avg. PSNR = 40.1 dB

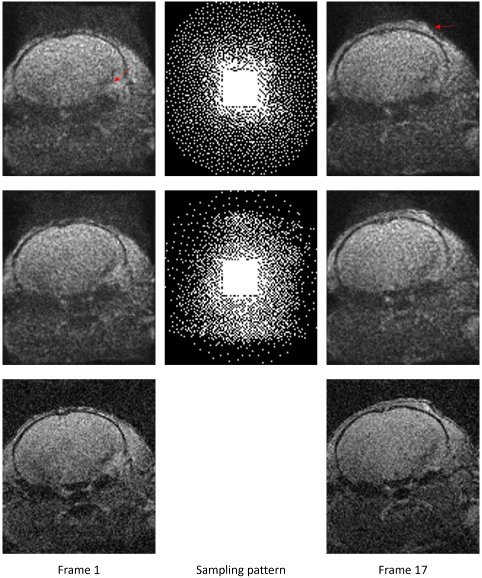
Preliminary results: 2D sampling

- 10× acceleration
- U-Net noise-conditional score model
  Song et al., ICLR 2021 [71]
- Trained with fastMRI data
- PSNR for 16 test cases:
  out-of-distribution GRE images



Preliminary results: DCE mouse brain

- 4× acceleration
- U-Net score model
- Very out-of-distribution!
- Adaptive sampling
  - optimized with 1st frame
  - applied to 17th frame
- Top to bottom:
  - Poisson disk
  - Adaptive
  - Reference



Adapative sampling future work

- Compare patient adaptive to population adaptive
- Accelerate sampling process, *e.g.*, using a latent space [72–74]
- Find compelling applications. Dynamic imaging?
- Better criteria than posterior marginal variances?
- . . .

Resources

Talk and code available online at
`http://web.eecs.umich.edu/~fessler`

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