Joint optimization of learning-based image reconstruction and sampling for MRI

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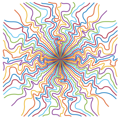
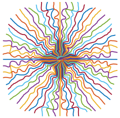


Brain



Knee

Acknowledgments:
Guanhua Wang, Tianrui Luo, Jon Nielsen, Doug Noll

Outline

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Medical imaging overview

raw data $\qquad$ images

Design Scan $\rightarrow$ Acquire Data $\xrightarrow{\boldsymbol{y}}$ Reconstruct Images $\xrightarrow{\hat{\boldsymbol{x}}}$ Process Images $\rightarrow$ Diagnose Interpret

Medical imaging overview

raw data $\quad\quad\quad\quad$ images

Design Scan $\rightarrow$ Acquire Data $\xrightarrow{\boldsymbol{y}}$ Reconstruct Images $\xrightarrow{\hat{\boldsymbol{x}}}$ **Process Images** $\rightarrow$ Diagnose Interpret

Most obvious place for machine learning is in post-processing (image analysis).
Numerous special issues and surveys in medical imaging journals, *e.g.*, [1–9].

Medical imaging overview

raw data ... images

Design Scan $\rightarrow$ Acquire Data $\xrightarrow{\boldsymbol{y}}$ Reconstruct Images $\xrightarrow{\hat{\boldsymbol{x}}}$ Process Images $\rightarrow$ Diagnose Interpret

Machine learning for scan design (k-space sampling):

- Choose best k-space phase encoding locations (usually Cartesian sampling)
- Uses "ground truth" (fully sampled) training images
- Hot topic in MRI research recently, *e.g.*, [10–15]
- Precursor by Yue Cao and David Levin, MRM Sep. 1993 [16–18]

Medical imaging overview

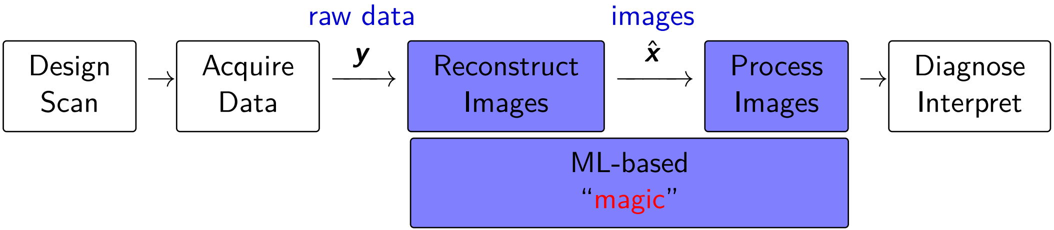
raw data　　　　images

Design Scan $\rightarrow$ Acquire Data $\xrightarrow{\boldsymbol{y}}$ **Reconstruct Images** $\xrightarrow{\hat{\boldsymbol{x}}}$ Process Images $\rightarrow$ Diagnose Interpret

Machine learning in medical image reconstruction:

- June 2018 special issue of IEEE Trans. on Medical Imaging [19].
- Surveys: [20–27]
- Possibly easier than diagnosis due to lower bar:
  - current reconstruction methods based on simplistic image models;
  - human eyes are better at detection than at solving inverse problems.

Medical imaging overview



A holy grail for machine learning in medical imaging?

- CT sinogram to vessel diameter [28, 29]
- k-space to prostate cancer detection and knee diagnosis [30]

Joint optimization of sampling and reconstruction

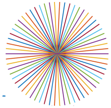
raw data $\quad$ images

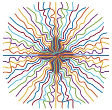
Design Scan $\rightarrow$ Acquire Data $\xrightarrow{\boldsymbol{y}}$ **Reconstruct Images** $\xrightarrow{\hat{\boldsymbol{x}}}$ Process Images $\rightarrow$ Diagnose Interpret

"B-spline parameterized joint optimization of reconstruction and k-space trajectories (BJORK) for accelerated 2D MRI," arXiv 2101.11369 [31] IEEE T-MI 2022 [32]
Guanhua Wang, T. Luo, J.-F. Nielsen, D. Noll, J. Fessler

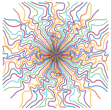
"Stochastic optimization of 3D non-Cartesian sampling trajectory (SNOPY)," MRM 2023 (in press)

Preview:





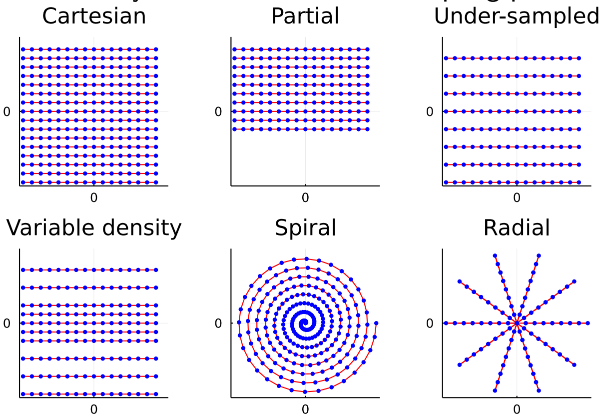
Knee



Brain

Related work: "PILOT" by Weiss et al. [33]; J-MoDL work of Aggarwal et al. [14].

All clinical MRI scans currently use "hand-crafted" sampling patterns:



- Reducing k-space sampling $\Longrightarrow$ reduced scan time / improved temporal resolution
- Under-sampled data benefits from advanced reconstruction methods

Outline

Overview:

- image-domain learning [34–36]...
- k-space or data-domain learning
  e.g., [37], [38], [39]
- transform learning (direct from k-space to image)
  e.g., AUTOMAP [40], [41–43]
- hybrid-domain learning (unrolled loop, *e.g.*, variational network)
  alternate between denoising/dealiasing and reconstruction from k-space
  e.g., [38, 44–48] ...

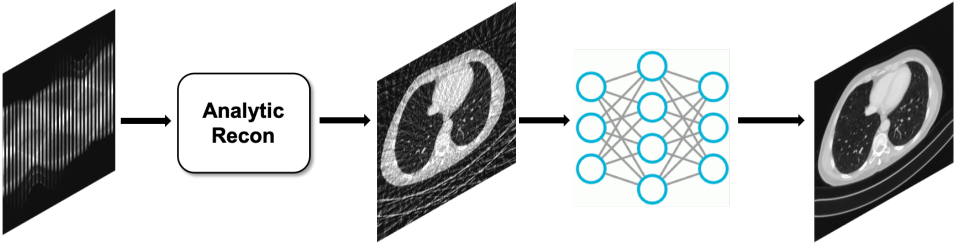


Figure courtesy of Jong Chul Ye, KAIST University.

+ simple and fast
− aliasing is spatially widespread, requires deep network

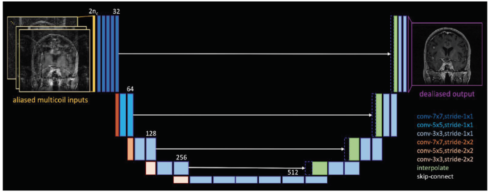
Dangers of image-domain learning: Method

Investigating Robustness to Unseen Pathologies in Model-Free Deep Multicoil Reconstruction

Gopal Nataraj[1] and Ricardo Otazo[1,2]
[1]Dept. of Medical Physics, Memorial Sloan Kettering Cancer Center
[2]Dept. of Radiology, Memorial Sloan Kettering Cancer Center

Introduction

Speed is often claimed as a key advantage of deep learning (DL) for undersampled parallel MRI reconstruction [1]. However, the only DL approach that to our knowledge has studied generalizability to pathologies unseen in training [2] requires repeated application of the MR acquisition model and its adjoint, just as in iterative methods. In contrast, model-free DL reconstruction has the potential to be much faster. Prior model-free DL work [3] proposes to learn a mapping directly from k-space, but with



[49] ISMRM 2020 Workshop on Data Sampling & Image Reconstruction

Dangers of image-domain learning: Result

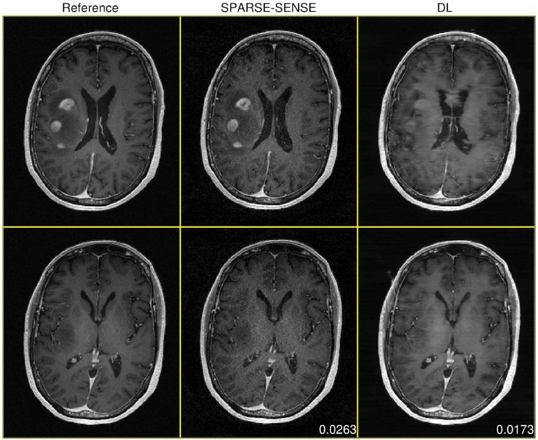


Figure 3: Reconstructions in a case of anaplastic astrocytoma, a rare malignant brain tumor. SPARSE-SENSE and DL reconstructions are from the same 4x-accelerated retrospectively undersampled acquisition. DL achieves lower whole-volume MAE than SPARSE-SENSE, but fails to properly reconstruct regions near the tumor.

Image-domain learning variations

- Use NN output as a “prior” for iterative reconstruction [34, 50]:

$$\hat{\boldsymbol{x}}_\beta = \arg\min_{\boldsymbol{x}} \|\boldsymbol{A}\boldsymbol{x} - \boldsymbol{y}\|_2^2 + \beta\, \|\boldsymbol{x} - \boldsymbol{x}_{\mathrm{NN}}\|_2^2 = (\boldsymbol{A}'\boldsymbol{A} + \beta\, \boldsymbol{I})^{-1}(\boldsymbol{A}'\boldsymbol{y} + \beta\, \boldsymbol{x}_{\mathrm{NN}})$$

  - For single-coil Cartesian case:
    - no iterations are needed (solve with FFTs)
    - $\lim_{\beta\to 0} \hat{\boldsymbol{x}}_\beta$ replaces missing k-space data with FFT of $\boldsymbol{x}_{\mathrm{NN}}$
  - Iterations needed for parallel MRI and/or non-Cartesian sampling (PCG)

- Learn residual (aliasing artifacts), then subtract [51, 52]

DL for IR: k-space / sinogram domain learning

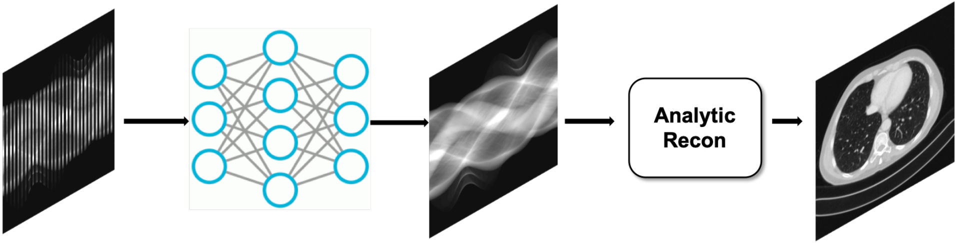


Figure courtesy of Jong Chul Ye, KAIST University.

+ simple and fast ("nonlinear GRAPPA")
+ "database-free" : learn from auto-calibration data [37], [38], [39]
− perhaps harder to represent local image features?

DL for IR: transform learning

Figure courtesy of Jong Chul Ye, KAIST University.

+ in principle, purely data driven; potential to avoid model mismatch
− high memory requirement for fully connected layers [40]

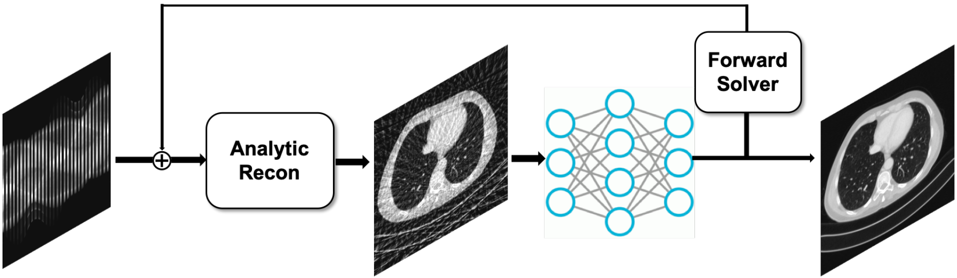
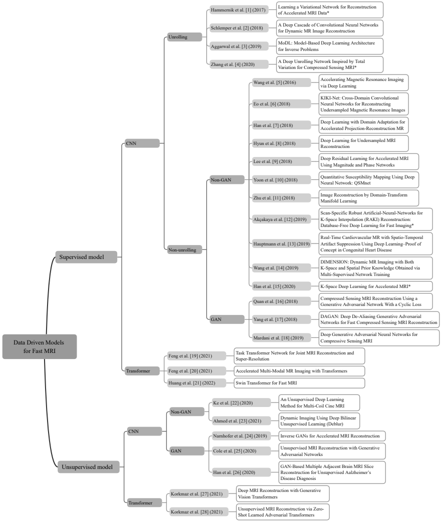


Figure courtesy of Jong Chul Ye, KAIST University.

+ physics-based use of k-space data & image-domain priors, *e.g.*, [38, 44–48, 53, 54] ...
+ interpretable connections to optimization approaches
+ best results in MRI recon challenges [55–57]
− more computation to due to "iterations" (hyper-layers) and repeated $\boldsymbol{A}\boldsymbol{x}$, $\boldsymbol{A}'\boldsymbol{r}$



Huang et al.., arXiv 2204.01706, Apr. 2022 [58]

Outline

Learning MRI sampling patterns I

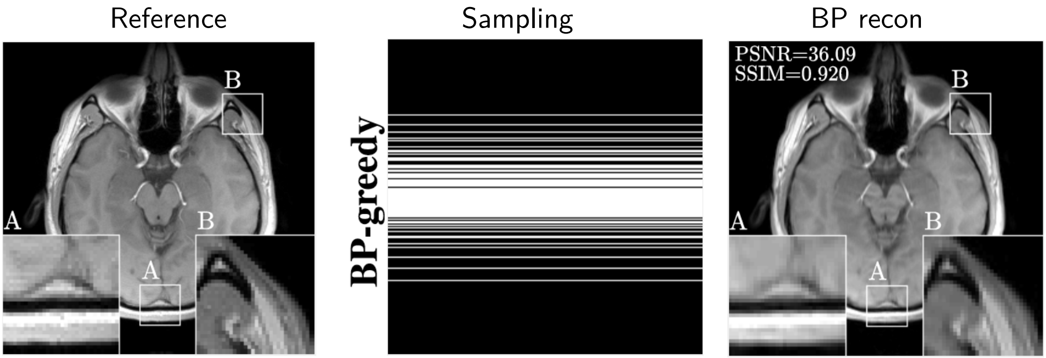
Pre-specified image reconstruction methods

Cartesian sampling pattern optimization

- Yue Cao & David Levin, MRM Sep. 1993 [16–18]
  Feature recognizing MRI
- Seeger et al., MRM 2010 [59]
  Single coil, 1D Cartesian, 2D spiral angles, CS-type recon, Bayesian information gain
- Ravishankar & Bresler, EMBS 2011 [10]
  Single coil, 1D & 2D sampling, DLMRI recon (DL = dictionary learning), weighted k-space loss
- Baldassarre ... Cevher, IEEE J-STSP 2016 [11]
  Single coil, 2D sampling, energy preserving criterion
- Sherry ... Ehrhardt, IEEE T-MI 2020 [60]
  Single coil, 2D sampling, various regularizers
- ...

Joint learning of sampling and reconstruction

- Gözcü . . . Cevher, IEEE T-MI 2018 [12]
  Single coil, 1D sampling, several fixed recon methods (TV, BP, BM3D, image-domain NN), image-domain training loss
- Aggarwal & Jacob IEEE J-STSP 2020 [14] (J-MoDL)
  Multi-coil, 1D (horizontal and vertical) sampling, MoDL recon
- Bahadir . . . Sabuncu, IEEE T-CI 2020 [15] (LOUPE)
  Single coil, 1D & 2D sampling, IFFT/U-Net recon
- Weiss et al., arXiv 1909.05773 (2019, 2020, 2021) (PILOT) [33]
  Single coil, Non-Cartesian sampling, IFFT/U-Net recon
- Wang . . . Fessler, ISMRM 2021, arXiv 2021 (BJORK) [31, 61]
  Multi-coil, Non-Cartesian sampling, MoDL-type recon
  Fast and efficient DFT Jacobian approximations [62, 63]



- Sampling designed to optimize PSNR for basis pursuit (BP) reconstruction using shearlet transform, at 25% sampling rate.
- Sampling design considers both the training data and the reconstruction method.
- No high spatial frequencies!?

(Images from Gözcü et al. [12].)

Outline

- MRI measurement model:

$$\boldsymbol{y} = \boldsymbol{A}(\boldsymbol{\omega})\boldsymbol{x} + \boldsymbol{\varepsilon}$$

  - $\boldsymbol{y} \in \mathbb{C}^M$: k-space data; $M \sim 10 - 30$K
  - $\boldsymbol{\omega} \in \mathbb{R}^{M \times 2}$: k-space sampling pattern ("trajectory"): 2D in BJORK, 3D in SNOPY
  - $\boldsymbol{x} \in \mathbb{C}^N$: unknown true image, $N \sim 100$K
  - $\boldsymbol{A}(\boldsymbol{\omega}) \in \mathbb{C}^{M \times N}$: encoding matrix (coil sensitivity, etc.)
  - $\boldsymbol{\varepsilon} \in \mathbb{C}^M$: measurement noise
- Reconstruction method:

$$\hat{\boldsymbol{x}} = f(\boldsymbol{y}; \boldsymbol{\omega}, \boldsymbol{\theta})$$

  - $\boldsymbol{\theta}$: model parameters of reconstruction method (e.g., CNN weights)
  - Deep iterative down-up CNN (DIDN) has $\sim$ 165M learned parameters [64]
- Image quality goal:

$$\hat{\boldsymbol{x}} = f(\boldsymbol{y}; \boldsymbol{\omega}, \boldsymbol{\theta}) = f(\boldsymbol{A}(\boldsymbol{\omega})\boldsymbol{x} + \boldsymbol{\varepsilon}; \boldsymbol{\omega}, \boldsymbol{\theta}) \approx \boldsymbol{x}$$

Supervised approach

- Define training loss function such as $\ell(\hat{\boldsymbol{x}}, \boldsymbol{x}) = \|\hat{\boldsymbol{x}} - \boldsymbol{x}\|_1 + \alpha \|\hat{\boldsymbol{x}} - \boldsymbol{x}\|_2$
- Select N_{train} fully sampled training images $\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots$
- Jointly optimize k-space trajectory $\boldsymbol{\omega}$ and image reconstruction method $\boldsymbol{\theta}$

$$(\hat{\boldsymbol{\omega}}, \hat{\boldsymbol{\theta}}) = \arg\min_{\boldsymbol{\omega}, \boldsymbol{\theta}} \frac{1}{N_{\mathrm{train}}} \sum_{n=1}^{N_{\mathrm{train}}} \ell(f(\boldsymbol{A}(\boldsymbol{\omega})\boldsymbol{x}_n + \boldsymbol{\varepsilon}_n; \boldsymbol{\omega}, \boldsymbol{\theta}), \boldsymbol{x}_n)$$

-

Supervised approach

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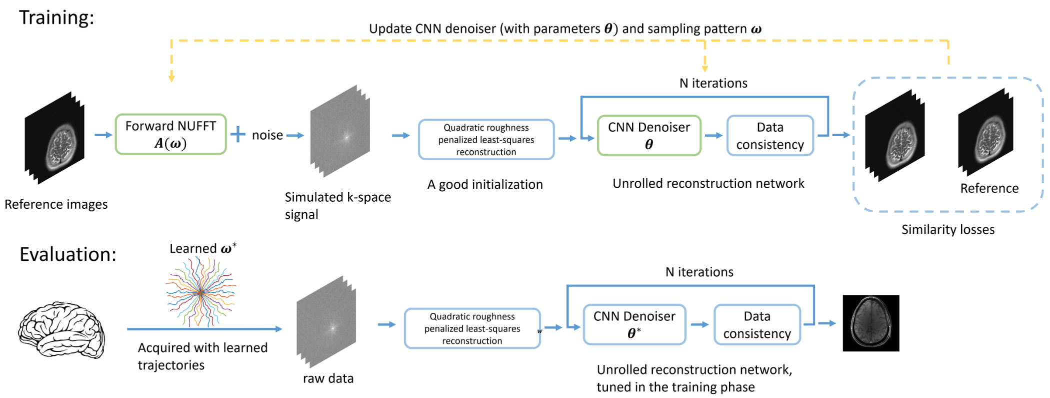
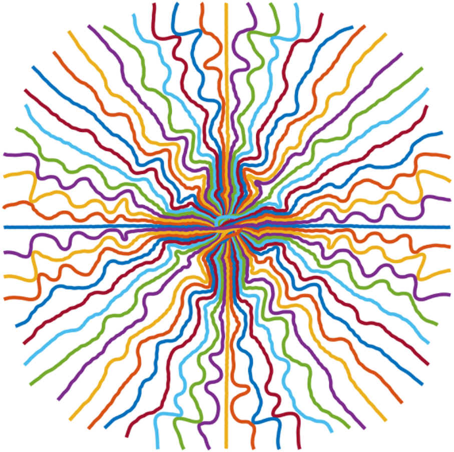
- Details:
  - Reconstruction using MoDL method [53]
  - Can use multiple noise realizations $\boldsymbol{\varepsilon}$ per training image
  - Enforce gradient amplitude and slew-rate limits for $\boldsymbol{\omega}$
  - Use B-spline parameterization of k-space trajectory
  - Coarse-to-fine search of trajectory to avoid poor local minimizers
  - Eddy current correction
  - Fast NUFFT Jacobian approximation [62, 63, 65]


Training:
Update CNN denoiser (with parameters $\boldsymbol{\theta}$) and sampling pattern $\boldsymbol{\omega}$
Reference images
Forward NUFFT $\boldsymbol{A}(\boldsymbol{\omega})$
+ noise
Simulated k-space signal
Quadratic roughness penalized least-squares reconstruction
A good initialization
N iterations
CNN Denoiser $\boldsymbol{\theta}$
Data consistency
Unrolled reconstruction network
Reference
Similarity losses
Evaluation:
Learned $\boldsymbol{\omega}^*$
Acquired with learned trajectories
raw data
Quadratic roughness penalized least-squares reconstruction
N iterations
CNN Denoiser $\boldsymbol{\theta}^*$
Data consistency
Unrolled reconstruction network, tuned in the training phase

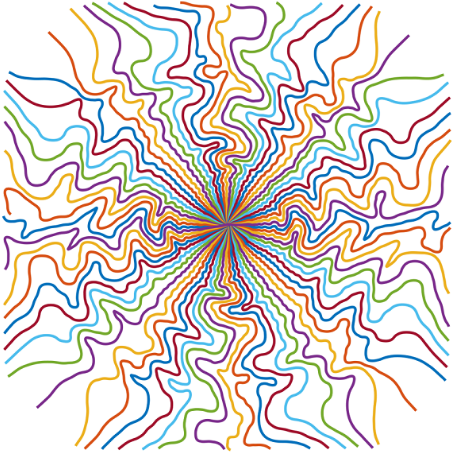
Data

- NYU/FAIR fastMRI brain and knee data
- 16/24/32 radial spokes of 1280 points for trajectory initialization ($\approx$ 10-20 $\times$ acceleration for 320 $\times$ 320 image)
- 22cm FOV, Gmax $=$ 5 Gauss/cm, slew rate $\leq$ 15 Gauss/cm/ms
- 5ms readout duration radial, 16ms spiral
- Comparison with SPARKLING approach of [66] using its default density

Trajectory can be tailored to anatomy



Knee



Brain

PSF results: 32 spokes

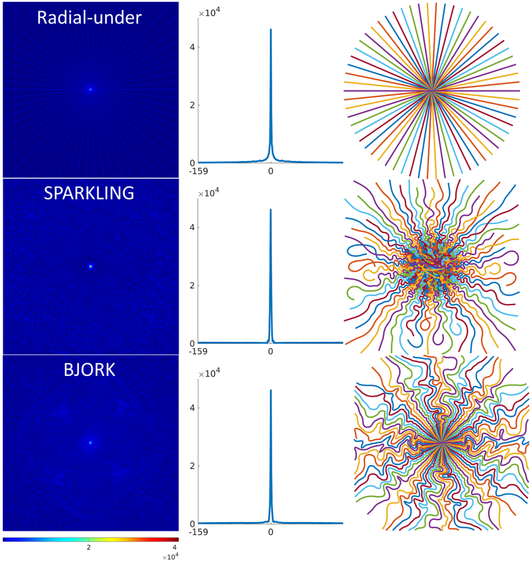
32-spoke results

FWHM (pixels):

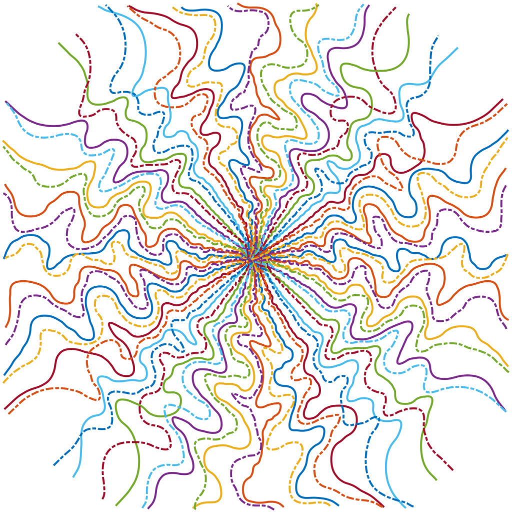
1.5

2.1

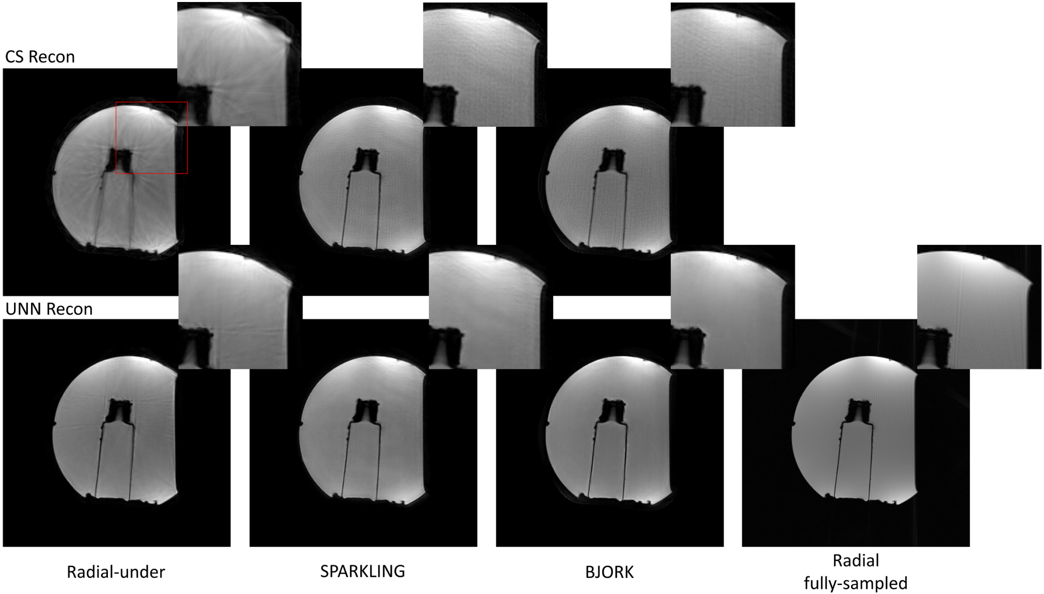
1.6



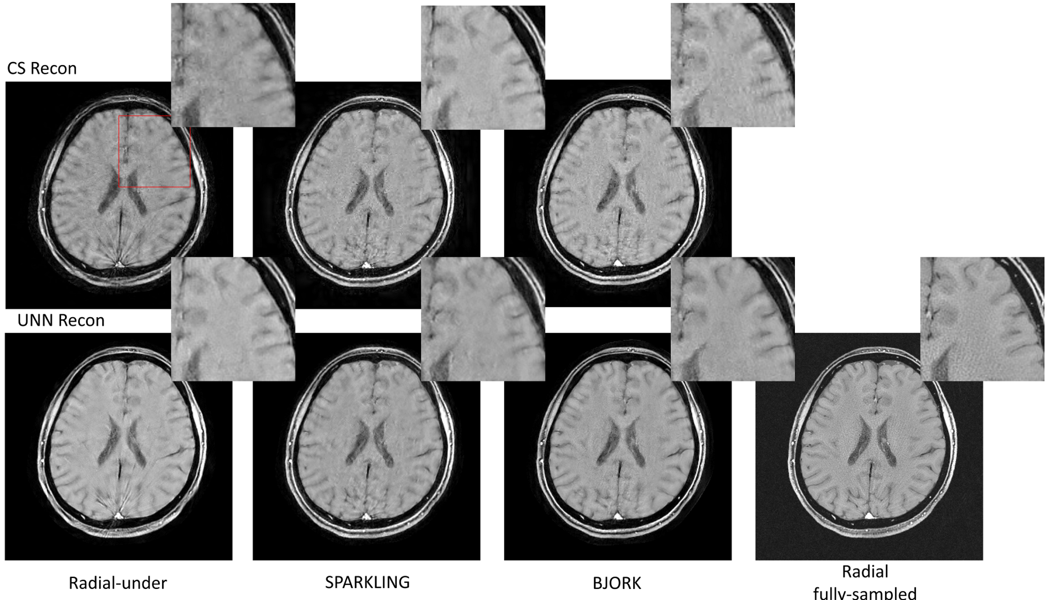
Learning about spectral conjugate symmetry



Prospectively under-sampled MRI phantom study



Prospective in-vivo study (GE scanner, 32 shot)



Quantitative simulation results: PSNR

PSNR (in dB):

		Standard	SPARKLING	BJORK
radial-like Ns=16	UNN	32.7	33.9	**34.3**
	CS	31.7	33.6	**34.1**
radial-like Ns=24	UNN	34.1	35.0	**35.6**
	CS	33.3	34.6	**35.1**
radial-like Ns=32	UNN	35.0	36.0	**36.9**
	CS	33.9	35.7	**36.3**
spiral-like Ns=8	UNN	40.9	41.7	**41.9**
	CS	39.9	40.4	**40.7**

Ns: the number of shots or spokes.

Quantitative simulation results: SSIM

SSIM:

		Standard	SPARKLING	BJORK
radial-like Ns=16	UNN	0.940	0.946	**0.950**
	CS	0.911	0.936	**0.938**
radial-like Ns=24	UNN	0.950	0.955	**0.959**
	CS	0.929	0.943	**0.948**
radial-like Ns=32	UNN	0.957	0.963	**0.968**
	CS	0.932	0.946	**0.956**
spiral-like Ns=8	UNN	0.986	0.989	**0.990**
	CS	0.976	0.978	**0.981**

Gradient constraints

MRI gradient amplifiers have maximum amplitude and slew rate

- gradient amplitude is 1st derivative of k-space trajectory:

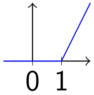
$$\left\| \boldsymbol{D}_1 \boldsymbol{\omega}_d \right\|_\infty \leq g_{\max}$$

- slew rate is 2nd derivative of k-space trajectory:

$$\left\| \boldsymbol{D}_2 \boldsymbol{\omega}_d \right\|_\infty \leq s_{\max}$$

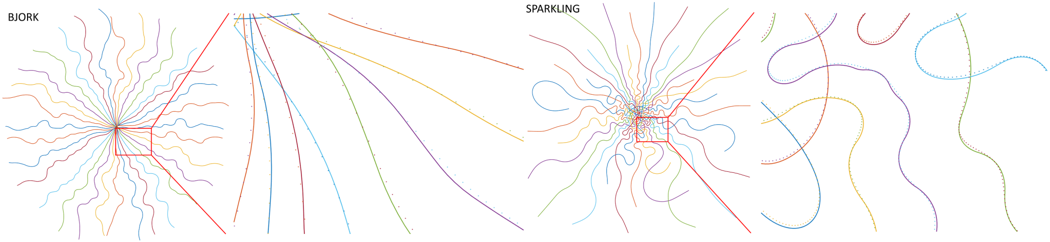
- Box constraints relaxed to penalty functions that rise rapidly above 1, on:

$$\left\| \boldsymbol{D}_1 \boldsymbol{\omega}_d \right\|_\infty / g_{\max} \text{ and } \left\| \boldsymbol{D}_2 \boldsymbol{\omega}_d \right\|_\infty / s_{\max}$$

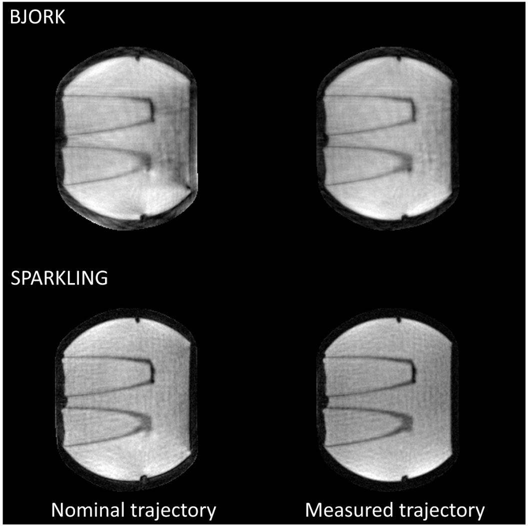


Facilitates (sub)gradient-based optimization using Adam

Trajectory calibration



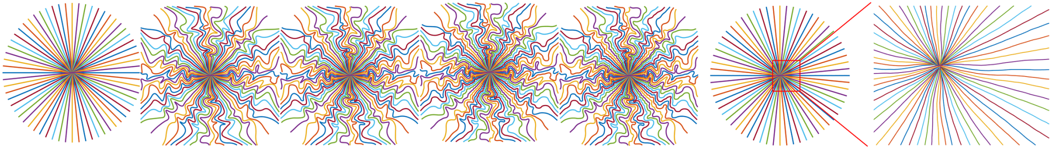
Eddy current compensation



Coarse-to-fine parameterization and evolution

Quadratic B-spline kernels for non-Cartesian k-space trajectory:

$$\omega_d = \boldsymbol{B}\boldsymbol{c}_d, \qquad d = 1, 2, \boldsymbol{c}_d \in \mathbb{R}^{M/\text{Decim}}$$



Init. SSIM=0.957 | Decim=32 SSIM=0.963 | Decim=16 SSIM=0.965 | Decim=8 SSIM=0.967 | Decim=4 SSIM=0.968 | Nonparametric SSIM=0.959

Highly non-convex problem in ω.

- Coarse-to-fine search may find better local minimizers
- However, parameterization/decimation does not save much computation

Unrolled-loop image reconstruction method

Motivated by model-based image reconstruction with variable splitting of the form

$$\begin{aligned}\hat{\boldsymbol{x}} &= \arg\min_{\boldsymbol{x}} \|\boldsymbol{A}(\omega)\boldsymbol{x} - \boldsymbol{y}\|_2^2 + R(\boldsymbol{x}) \\ &= \arg\min_{\boldsymbol{x}} \min_{\boldsymbol{z}} \|\boldsymbol{A}(\omega)\boldsymbol{x} - \boldsymbol{y}\|_2^2 + R(\boldsymbol{z}), \quad \text{s.t. } \boldsymbol{z} = \boldsymbol{x}\end{aligned}$$

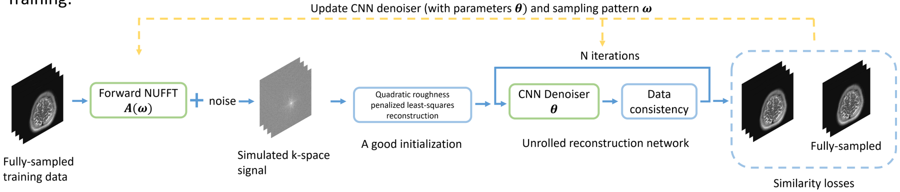
Alternating minimization:

$$\begin{aligned}\boldsymbol{x}_{t+1} &= \arg\min_{\boldsymbol{x}} \|\boldsymbol{A}(\omega)\boldsymbol{x} - \boldsymbol{y}\|_2^2 + \mu \|\boldsymbol{x} - \boldsymbol{z}_t\|_2^2 && \text{(data consistency, solved via CG)} \\ \boldsymbol{z}_{t+1} &= \arg\min_{\boldsymbol{z}} R(\boldsymbol{z}) + \mu \|\boldsymbol{x}_{t+1} - \boldsymbol{z}\|_2^2 && \text{(denoising)} \\ \text{``} &= \text{''} \mathcal{D}_{\boldsymbol{\theta}}(\boldsymbol{x}_{t+1}) && \text{(CNN denoiser)}\end{aligned}$$

- CNN weights $\boldsymbol{\theta}$ shared across iterations, per MODL [53]
- 6 outer iterations for results shown, with augmented Lagrangian parameter $\mu = 2$

Efficient NUFFT backpropagation I

Training:



Data consistency block has steps like

$$\boldsymbol{x}_{t+1} = \boldsymbol{x}_t + \alpha \left(\boldsymbol{A}'(\boldsymbol{\omega}) \left(\boldsymbol{A}(\boldsymbol{\omega}) \boldsymbol{x} - \boldsymbol{y} \right) + \mu (\boldsymbol{x} - \boldsymbol{z}_t) \right)$$

$\boldsymbol{A}(\boldsymbol{\omega})$ is dense and huge:

$$a_{ij} = \mathrm{e}^{-\imath \vec{\omega}_i \cdot \vec{r}_j} \tag{1}$$

- Fast approach to $\boldsymbol{A}(\boldsymbol{\omega})\boldsymbol{x}$ uses NUFFT approximation:
  zero-padding, over-sampled FFT, interpolation [67, 68].
- Backpropagation for $\boldsymbol{\omega}$ update through NUFFT steps via autodifferentiation is slow.

Efficient NUFFT backpropagation II

Derive Jacobian matrix for exact form (1):

$$\frac{\partial}{\partial \boldsymbol{\omega}_d} \boldsymbol{A}(\boldsymbol{\omega})\boldsymbol{x} = -\imath \, \mathrm{Diag}\{\boldsymbol{A}(\boldsymbol{\omega})(\boldsymbol{x} \odot \boldsymbol{r}_d)\} \, .$$

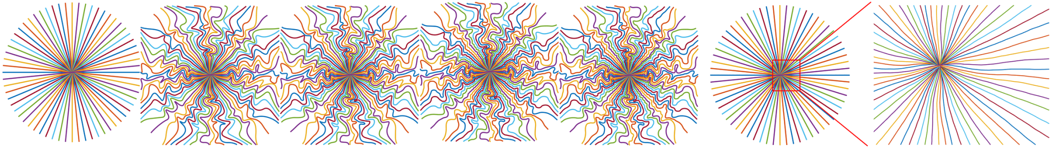
Applying this Jacobian to a vector $\boldsymbol{v} \in \mathbb{C}^M$ during backpropagation yields

$$\left(\frac{\partial}{\partial \boldsymbol{\omega}_d} \boldsymbol{A}(\boldsymbol{\omega})\boldsymbol{x}\right) \boldsymbol{v} = -\imath \, \mathrm{Diag}\{\boldsymbol{A}(\boldsymbol{\omega})(\boldsymbol{x} \odot \boldsymbol{r}_d)\} \, \boldsymbol{v} = -\imath \, (\boldsymbol{A}(\boldsymbol{\omega})(\boldsymbol{x} \odot \boldsymbol{r}_d)) \odot \boldsymbol{v} \, .$$

Implemented efficiently using NUFFT applied to $\boldsymbol{x} \odot \boldsymbol{r}_d$

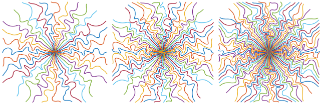
Similar idea for Jacobian of adjoint of $\boldsymbol{A}$.
Even more important is accurately approximating Jacobian of CG solve of $(\boldsymbol{A}'\boldsymbol{A} + \mu \boldsymbol{I})^{-1}$

Evolution with improved Jacobians



Init. SSIM=0.957 | Decim=32 SSIM=0.963 | Decim=16 SSIM=0.965 | Decim=8 SSIM=0.967 | Decim=4 SSIM=0.968 | Nonparametric SSIM=0.959

Different acceleration factors:



16 spokes | 24 spokes | 32 spokes

Cross-contrast comparison

- Each contrast has 4500 training slices, 500 test slices
- No extra noise in training
- Testing variance is 10^{-3} mean test signal
- SSIM values

test \ training	T1w	T2w	FLAIR
T1w+noise	0.981	0.980	0.981
T2w+noise	0.951	0.953	0.953
FLAIR+noise	0.974	0.974	0.975

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- Approximately constant in each row!

Outline

Summary / future directions

- Machine learning methods have much potential for both scan design and image reconstruction
- Quantitative results in paper demonstrate synergy of jointly optimizing both
- Anatomy specific trajectories: pro or con?
- Self-supervised methods when training data unavailable
- Extension to 3D accepted to MRM [69]
  Also control of peripheral nerve stimulation (PNS)
- Extension to 3D+time is planned (and challenging)

2D code for BJORK: `https://github.com/guanhuaw/Bjork`

Outline

Patient-specific adaptive sampling

- Goal: shorten MRI scan by adaptive sampling
  "Adaptive sampling for linear sensing systems via Langevin dynamics"
  Guanhua Wang, D Noll, J Fessler, arXiv 2302.13468 2023 [70]

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Patient-specific adaptive sampling

- Goal: shorten MRI scan by adaptive sampling
  "Adaptive sampling for linear sensing systems via Langevin dynamics"
  Guanhua Wang, D Noll, J Fessler, arXiv 2302.13468 2023 [70]

- Overview:
  - Pick image prior $\mathrm{p}(\boldsymbol{x})$
  - Collect (incomplete) k-space data
  - Sample repeatedly from the posterior $\hat{\boldsymbol{x}} \sim \mathrm{p}(\boldsymbol{x}|\boldsymbol{y})$
  - Predict missing measurements $\hat{\boldsymbol{y}} = \boldsymbol{A}\,\hat{\boldsymbol{x}}$
  - Select new k-space samples where posterior variance is highest
  - Repeat

Related image-domain adaptive sampling: Godaliyadda et al., ICASSP 2014 & IEEE T-CI 2018 [13, 71]

Adaptive sampling algorithm

Require: Score function $f_{\boldsymbol{\theta}}(\boldsymbol{x}) \approx \nabla \log p(\boldsymbol{x})$ (score matching or hand crafted)

Acquire initial k-space measurements $\boldsymbol{y}^0$

for $k = 1$ to N_{add} **do**

  for $i = 1$ to N_{sample} **do**

    for $t = 1$ to N_{step} **do**

      Initialize $\tilde{\boldsymbol{x}}_0$; sample from posterior via Langevin MC:

      $\tilde{\boldsymbol{x}}_t = \tilde{\boldsymbol{x}}_{t-1} + \mu_t f_{\boldsymbol{\theta}}(\tilde{\boldsymbol{x}}_{t-1}) - \mu_t \eta_t \boldsymbol{A}'(\boldsymbol{A}\tilde{\boldsymbol{x}}_{t-1} - \boldsymbol{y}^{(k)}) + \sqrt{2\mu_t}\,\mathcal{N}(0, 1)$

    end for

    $\hat{\boldsymbol{x}}_i^{(k)} = \tilde{\boldsymbol{x}}_{N_{\text{add}}}$

    $\hat{\boldsymbol{y}}_i^{(k)} = \boldsymbol{A}\,\hat{\boldsymbol{x}}_i^{(k)}$

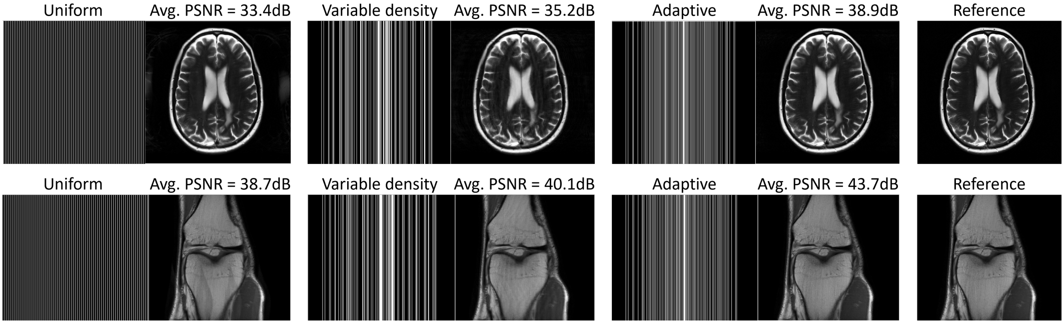
  end for

  $l = \arg\max_{n \in 1,2,\ldots N} \operatorname{Var}\{[\hat{\boldsymbol{y}}_1^{(k)}]_n, \ldots, [\hat{\boldsymbol{y}}_{N_{\text{sample}}}^{(k)}]_n\}$

  Acquire measurement index l, concatenate with previous: $\boldsymbol{y}^{(k)} = [\boldsymbol{y}^{(k-1)},\ y_l]$.

end for

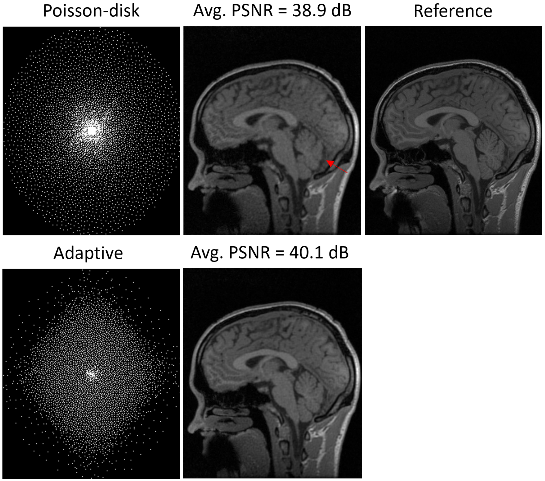
Preliminary results: 1D sampling



- $10\times$ acceleration, $N_{\mathrm{add}} = 50$, $N_{\mathrm{step}} = 200$, $N_{\mathrm{sample}} = 8 - 10$
- PSNR averaged over 20 test cases
- Hand-crafted roughness regularizer: $\nabla \log \mathrm{p}(\boldsymbol{x}) = \nabla \frac{\beta}{2} \|\boldsymbol{T}\boldsymbol{x}\|_2^2 = \beta\, \boldsymbol{T}' \boldsymbol{T}\boldsymbol{x}$

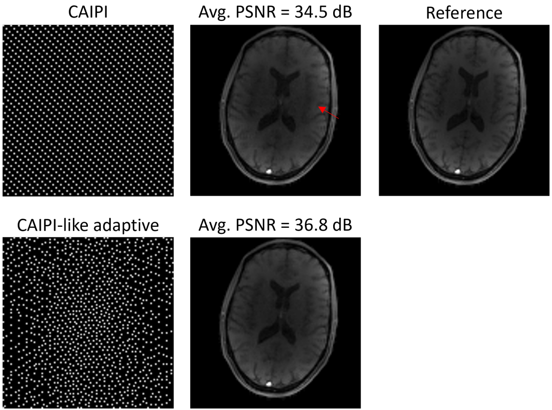
Preliminary results: 2D sampling

- $12\times$ acceleration
- Hand-crafted roughness regularizer
- PSNR for 10 test cases



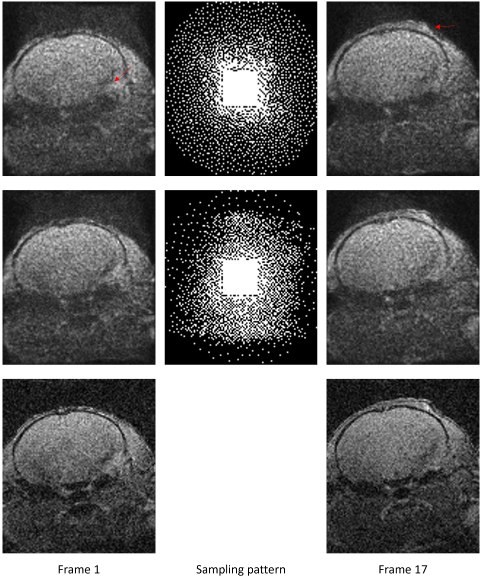
Preliminary results: 2D sampling

- $10\times$ acceleration
- U-Net noise-conditional score model
  Song et al., ICLR 2021 [72]
- Trained with fastMRI data
- PSNR for 16 test cases:
  out-of-distribution GRE images



Preliminary results: DCE mouse brain

- 4× acceleration
- U-Net score model
- Very out-of-distribution!
- Adaptive sampling
  - optimized with 1st frame
  - applied to 17th frame
- Top to bottom:
  - Poisson disk
  - Adaptive
  - Reference



Adapative sampling future work

- Compare patient adaptive to population adaptive
- Accelerate sampling process, *e.g.*, using a latent space [73–75]
- Find compelling applications. Dynamic imaging?
- Better criteria than posterior marginal variances?
- ...

Talk and code available online at
`http://web.eecs.umich.edu/~fessler`

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