Joint optimization of learning-based image reconstruction and sampling for MRI

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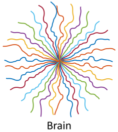
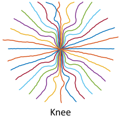
2021-07-20





Acknowledgments:
Guanhua Wang, Tianrui Luo, Jon Nielsen, Doug Noll

Outline

Introduction
- Machine learning in imaging
- MRI k-space sampling

Deep-learning approaches for image reconstruction

Supervised learning of k-space sampling

Joint optimization of k-space sampling and image reconstruction
- Problem formulation
- Results

Summary

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Medical imaging overview

raw data ... images

$$\boxed{\text{Design Scan}} \rightarrow \boxed{\text{Acquire Data}} \xrightarrow{\boldsymbol{y}} \boxed{\text{Reconstruct Images}} \xrightarrow{\hat{\boldsymbol{x}}} \boxed{\text{Process Images}} \rightarrow \boxed{\text{Diagnose Interpret}}$$

Medical imaging overview

raw data (y) and images ($\hat{x}$):

Design Scan $\rightarrow$ Acquire Data $\xrightarrow{\boldsymbol{y}}$ Reconstruct Images $\xrightarrow{\hat{\boldsymbol{x}}}$ **Process Images** $\rightarrow$ Diagnose Interpret

Most obvious place for machine learning is in post-processing (image analysis).
Numerous special issues and surveys in medical imaging journals, *e.g.*, [1–9].

Medical imaging overview

raw data $\boldsymbol{y}$, images $\hat{\boldsymbol{x}}$

Design Scan $\rightarrow$ Acquire Data $\xrightarrow{\boldsymbol{y}}$ Reconstruct Images $\xrightarrow{\hat{\boldsymbol{x}}}$ Process Images $\rightarrow$ Diagnose Interpret

Machine learning for scan design (k-space sampling):

- Choose best k-space phase encoding locations (usually Cartesian sampling)
- Uses “ground truth” (fully sampled) training images
- Hot topic in MRI research recently, *e.g.*, [10–15]
- Precursor by Yue Cao and David Levin, MRM Sep. 1993 [16–18]

Medical imaging overview

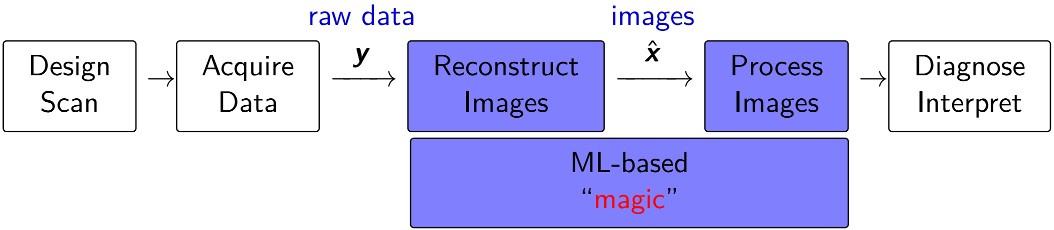
raw data $\quad\quad\quad\quad$ images

Design Scan $\rightarrow$ Acquire Data $\xrightarrow{\boldsymbol{y}}$ **Reconstruct Images** $\xrightarrow{\hat{\boldsymbol{x}}}$ Process Images $\rightarrow$ Diagnose Interpret

Machine learning in medical image reconstruction:

- June 2018 special issue of IEEE Trans. on Medical Imaging [19].
- Surveys: [20–27]
- Possibly easier than diagnosis due to lower bar:
  - current reconstruction methods based on simplistic image models;
  - human eyes are better at detection than at solving inverse problems.

Medical imaging overview



A holy grail for machine learning in medical imaging?

- CT sinogram to vessel diameter [28, 29]
- k-space to ???

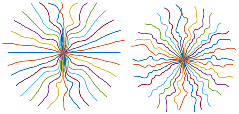
Joint optimization of sampling and reconstruction

Design Scan $\rightarrow$ Acquire Data $\xrightarrow[\text{raw data}]{\boldsymbol{y}}$ Reconstruct Images $\xrightarrow[\text{images}]{\hat{\boldsymbol{x}}}$ Process Images $\rightarrow$ Diagnose Interpret

"B-spline parameterized joint optimization of reconstruction and k-space trajectories (BJORK) for accelerated 2D MRI," arXiv 2101.11369 [30]
Guanhua Wang, T. Luo, J.-F. Nielsen, D. Noll, J. Fessler

Preview:

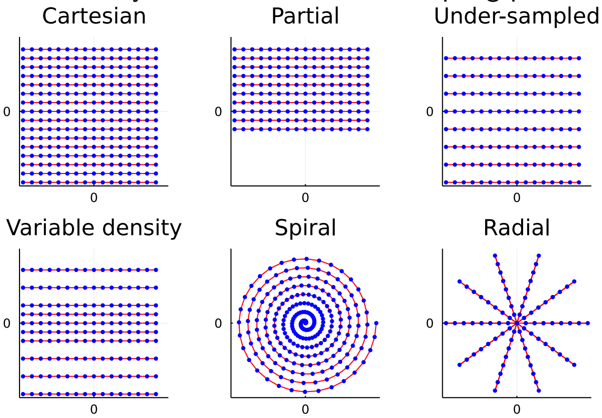
Init.



Knee Brain

Related work: "PILOT" by Weiss et al. [31]; J-MoDL work of Aggarwal et al. [14]

All clinical MRI scans currently use "hand-crafted" sampling patterns:



- Reducing k-space sampling $\Longrightarrow$ reduced scan time / improved temporal resolution
- Under-sampled data benefits from advanced reconstruction methods

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Overview:

- image-domain learning [32–34]...
- k-space or data-domain learning
  e.g., [35], [36], [37]
- transform learning (direct from k-space to image)
  e.g., AUTOMAP [38], [39–41]
- hybrid-domain learning (unrolled loop, *e.g.*, variational network)
  alternate between denoising/dealiasing and reconstruction from k-space
  e.g., [36, 42–46] ...

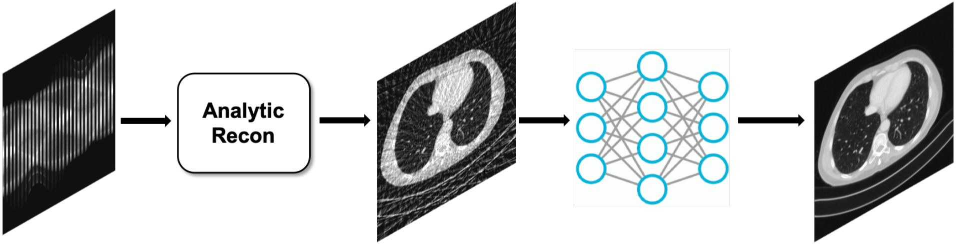


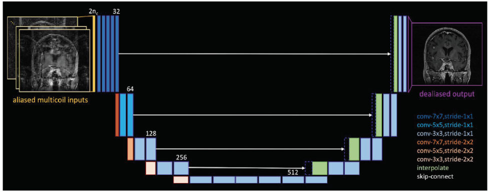
Figure courtesy of Jong Chul Ye, KAIST University.

+ simple and fast
– aliasing is spatially widespread, requires deep network

Investigating Robustness to Unseen Pathologies in Model-Free Deep Multicoil Reconstruction

Gopal Nataraj[1] and Ricardo Otazo[1,2]
[1]Dept. of Medical Physics, Memorial Sloan Kettering Cancer Center
[2]Dept. of Radiology, Memorial Sloan Kettering Cancer Center

Introduction
Speed is often claimed as a key advantage of deep learning (DL) for undersampled parallel MRI reconstruction [1]. However, the only DL approach that to our knowledge has studied generalizability to pathologies unseen in training [2] requires repeated application of the MR acquisition model and its adjoint, just as in iterative methods. In contrast, model-free DL reconstruction has the potential to be much faster. Prior model-free DL work [3] proposes to learn a mapping directly from k-space, but with



[47] ISMRM 2020 Workshop on Data Sampling & Image Reconstruction

Dangers of image-domain learning: Result

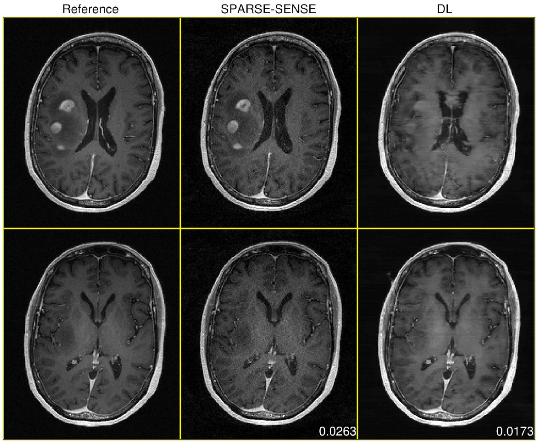


Figure 3: Reconstructions in a case of anaplastic astrocytoma, a rare malignant brain tumor. SPARSE-SENSE and DL reconstructions are from the same 4x-accelerated retrospectively undersampled acquisition. DL achieves lower whole-volume MAE than SPARSE-SENSE, but fails to properly reconstruct regions near the tumor.

- Use NN output as a “prior” for iterative reconstruction [32, 48]:

$$\hat{\boldsymbol{x}}_\beta = \arg\min_{\boldsymbol{x}} \|\boldsymbol{A}\boldsymbol{x} - \boldsymbol{y}\|_2^2 + \beta \|\boldsymbol{x} - \boldsymbol{x}_{\text{NN}}\|_2^2 = (\boldsymbol{A}'\boldsymbol{A} + \beta\boldsymbol{I})^{-1}(\boldsymbol{A}'\boldsymbol{y} + \beta\boldsymbol{x}_{\text{NN}})$$

  - For single-coil Cartesian case:
    - no iterations are needed (solve with FFTs)
    - $\lim_{\beta\to 0} \hat{\boldsymbol{x}}_\beta$ replaces missing k-space data with FFT of $\boldsymbol{x}_{\text{NN}}$
  - Iterations needed for parallel MRI and/or non-Cartesian sampling (PCG)

- Learn residual (aliasing artifacts), then subtract [49, 50]

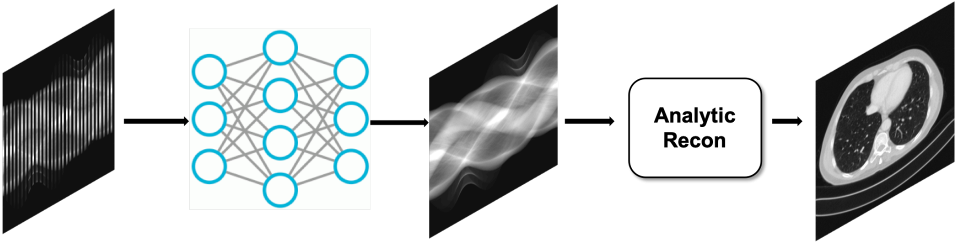


Figure courtesy of Jong Chul Ye, KAIST University.

+ simple and fast ("nonlinear GRAPPA")
+ "database-free" : learn from auto-calibration data [35], [36], [37]
− perhaps harder to represent local image features?

DL for IR: transform learning

Figure courtesy of Jong Chul Ye, KAIST University.

+ in principle, purely data driven; potential to avoid model mismatch
− high memory requirement for fully connected layers [38]

DL for IR: hybrid domain learning

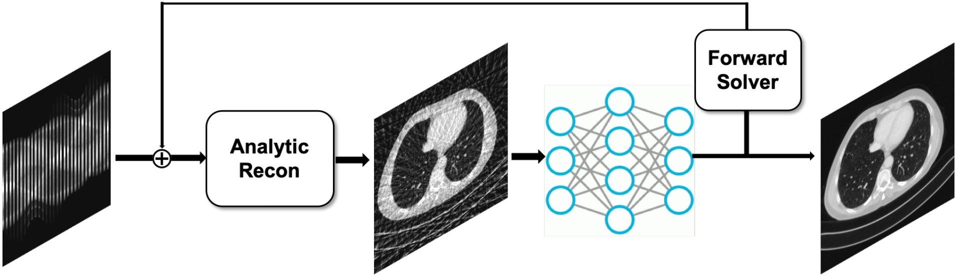


Figure courtesy of Jong Chul Ye, KAIST University.

+ physics-based use of k-space data & image-domain priors, *e.g.*, [36, 42–46, 51, 52] ...
+ interpretable connections to optimization approaches
− more computation to due to "iterations" (layers) and repeated $\boldsymbol{Ax}$, $\boldsymbol{A}' \boldsymbol{r}$

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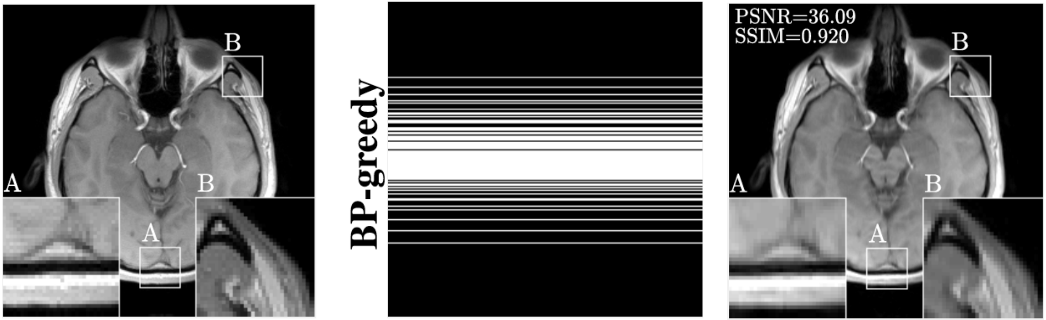

- Sampling designed to optimize PSNR for basis pursuit (BP) reconstruction using shearlet transform, at 25% sampling rate.
- Sampling design considers both the training data and the reconstruction method.
- No high spatial frequencies!?

(Images from Gözcü et al. [12].)

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Problem formulation

- MRI measurement model:

$$\boldsymbol{y} = \boldsymbol{A}(\boldsymbol{\omega})\boldsymbol{x} + \boldsymbol{\varepsilon}$$

  - $\boldsymbol{y} \in \mathbb{C}^M$: k-space data; $M \sim 10 - 30\text{K}$
  - $\boldsymbol{\omega} \in \mathbb{R}^{M \times 2}$: k-space sampling pattern ("trajectory"): 2D in this work
  - $\boldsymbol{x} \in \mathbb{C}^N$: unknown true image, $N \sim 100\text{K}$
  - $\boldsymbol{A}(\boldsymbol{\omega}) \in \mathbb{C}^{M \times N}$: encoding matrix (coil sensitivity, etc.)
  - $\boldsymbol{\varepsilon} \in \mathbb{C}^M$: measurement noise
- Reconstruction method:

$$\hat{\boldsymbol{x}} = f(\boldsymbol{y}; \boldsymbol{\omega}, \boldsymbol{\theta})$$

  - $\boldsymbol{\theta}$: model parameters of reconstruction method (e.g., CNN weights)
  - Deep iterative down-up CNN (DIDN) has $\sim$ 165M learned parameters [53]
- Image quality goal:

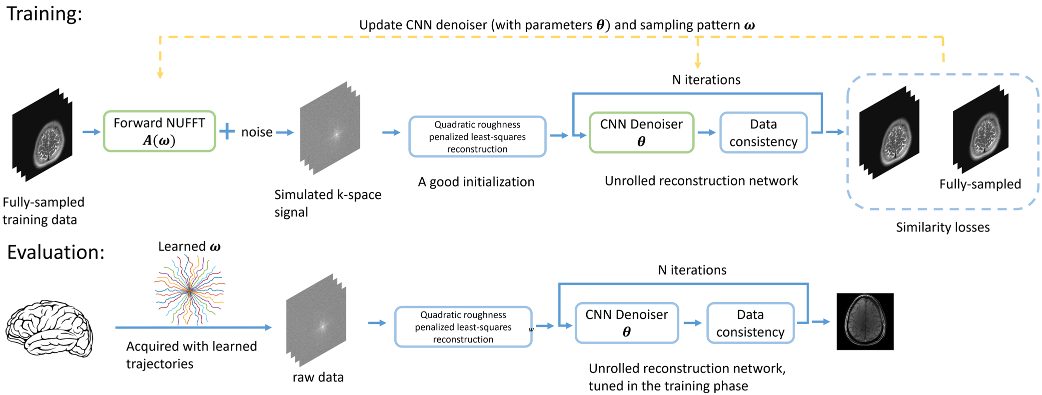
$$\hat{\boldsymbol{x}} = f(\boldsymbol{y}; \boldsymbol{\omega}, \boldsymbol{\theta}) = f(\boldsymbol{A}(\boldsymbol{\omega})\boldsymbol{x} + \boldsymbol{\varepsilon}; \boldsymbol{\omega}, \boldsymbol{\theta}) \approx \boldsymbol{x}$$

- Define training loss function such as $\ell(\hat{\boldsymbol{x}}, \boldsymbol{x}) = \|\hat{\boldsymbol{x}} - \boldsymbol{x}\|_1 + \alpha \|\hat{\boldsymbol{x}} - \hat{\boldsymbol{x}}\|_2$
- Select N_{train} fully sampled training images $\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots$
- Jointly optimize k-space trajectory $\boldsymbol{\omega}$ and image reconstruction method $\boldsymbol{\theta}$

$$(\hat{\boldsymbol{\omega}}, \hat{\boldsymbol{\theta}}) = \arg\min_{\boldsymbol{\omega}, \boldsymbol{\theta}} \frac{1}{N_{\text{train}}} \sum_{n=1}^{N_{\text{train}}} \ell(f(\boldsymbol{A}(\boldsymbol{\omega})\boldsymbol{x}_n + \boldsymbol{\varepsilon}_n; \boldsymbol{\omega}, \boldsymbol{\theta}), \boldsymbol{x}_n)$$

- Details:
  - Reconstruction using MoDL method [51]
  - Can use multiple noise realizations $\boldsymbol{\varepsilon}$ per training image
  - Enforce gradient amplitude and slew-rate limits for $\boldsymbol{\omega}$
  - Use B-spline parameterization of k-space trajectory
  - Coarse-to-fine search of trajectory to avoid poor local minimizers
  - Eddy current correction
  - Fast NUFFT Jacobian approximation [54]

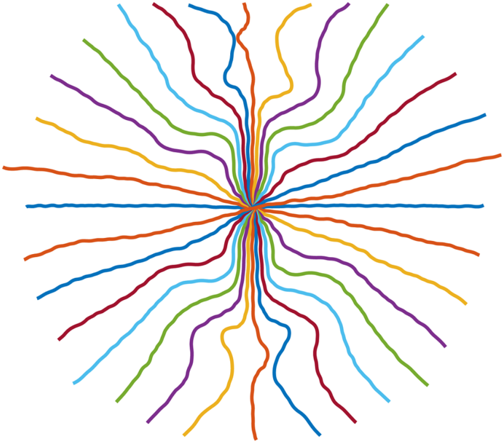
BJORK Diagram



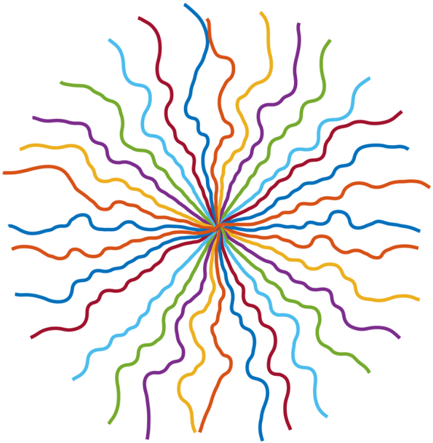
Data

NYU/FAIR fastMRI brain and knee data
16 radial spokes of 640 points for initialization
22cm FOV, Gmax $=$ 5 Gauss/cm, slew rate $\leq$ 15 Gauss/cm/ms
2.5ms readout duration radial, 16ms spiral
Comparison with SPARKLING approach of [55] using its default density

Trajectory is tailored to anatomy

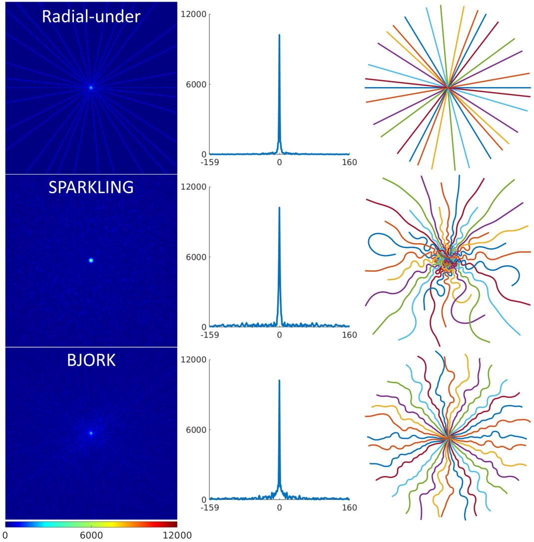


Knee



Brain

PSF results


Radial-under
SPARKLING
BJORK
12000
6000
0
-159
0
160
12000
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-159
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12000
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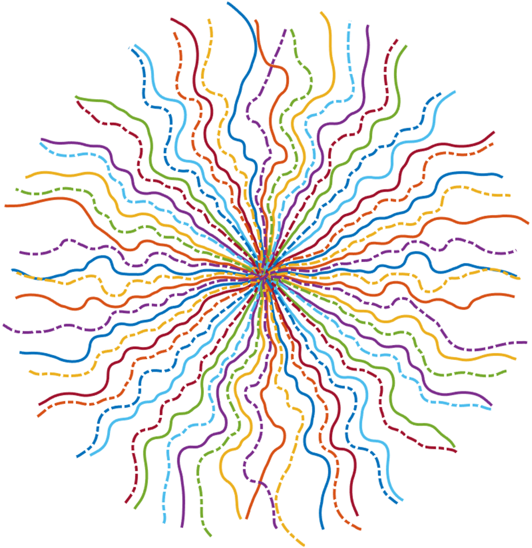
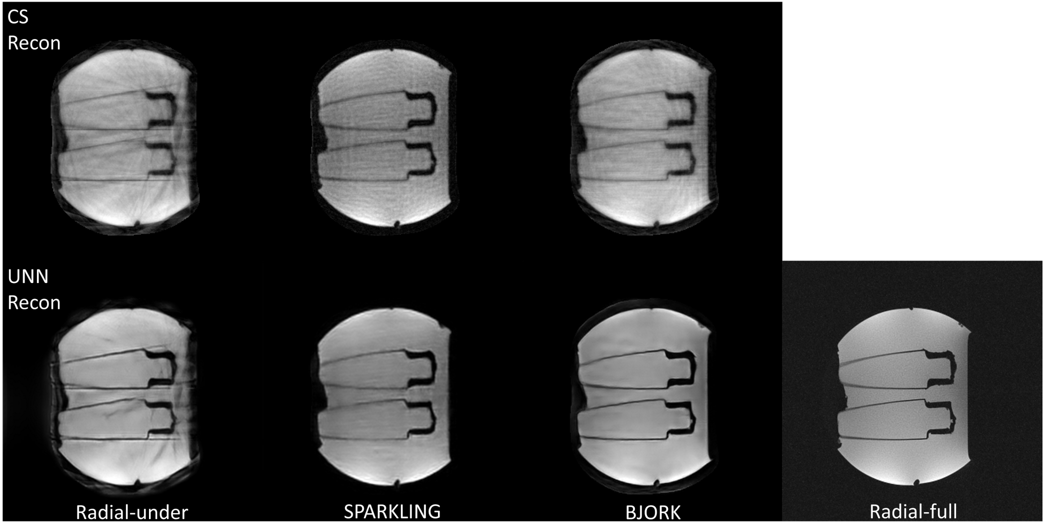
160
0
6000
12000

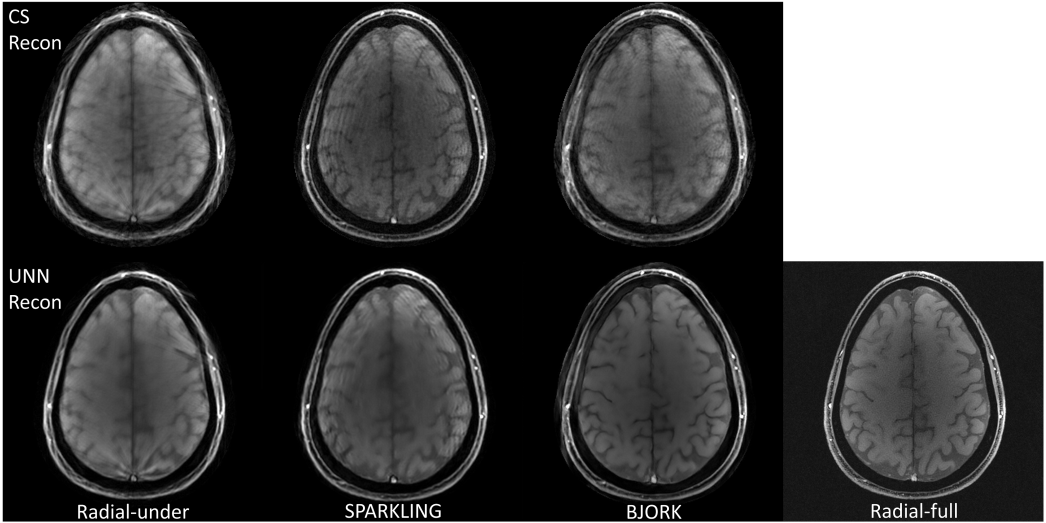


Phantom study



(no phantoms in training data!)

Prospective in-vivo study (GE scanner)



Gradient constraints

MRI gradient amplifiers have maximum amplitude and slew rate

- gradient amplitude is 1st derivative of k-space trajectory:

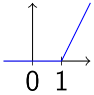
$$\left\| \boldsymbol{D}_1 \boldsymbol{\omega}_d \right\|_\infty \leq g_{\max}$$

- slew rate is 2nd derivative of k-space trajectory:

$$\left\| \boldsymbol{D}_2 \boldsymbol{\omega}_d \right\|_\infty \leq s_{\max}$$

- Box constraints relaxed to penalty functions that rise rapidly above 1 on

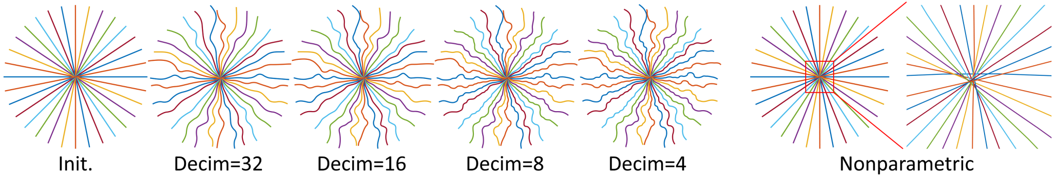
$$\left\| \boldsymbol{D}_1 \boldsymbol{\omega}_d \right\|_\infty / g_{\max} \text{ and } \left\| \boldsymbol{D}_2 \boldsymbol{\omega}_d \right\|_\infty / s_{\max}$$



Facilitates (sub)gradient-based optimization using Adam

Quadratic B-spline kernels for non-Cartesian k-space trajectory:

$$\boldsymbol{\omega}_d = \boldsymbol{B}\boldsymbol{c}_d, \qquad d = 1, 2, \boldsymbol{c}_d \in \mathbb{R}^{M/\text{Decim}}$$



Highly non-convex problem in $\boldsymbol{\omega}$.

- Coarse-to-fine search may find better local minimizers
- However, parameterization/decimation does not save much computation

Unrolled-loop image reconstruction method

Motivated by model-based image reconstruction with variable splitting of the form

$$\hat{\boldsymbol{x}} = \arg\min_{\boldsymbol{x}} \|\boldsymbol{A}(\boldsymbol{\omega})\boldsymbol{x} - \boldsymbol{y}\|_2^2 + R(\boldsymbol{x})$$
$$= \arg\min_{\boldsymbol{x}} \min_{\boldsymbol{z}} \|\boldsymbol{A}(\boldsymbol{\omega})\boldsymbol{x} - \boldsymbol{y}\|_2^2 + R(\boldsymbol{z}), \quad \text{s.t. } \boldsymbol{z} = \boldsymbol{x}$$

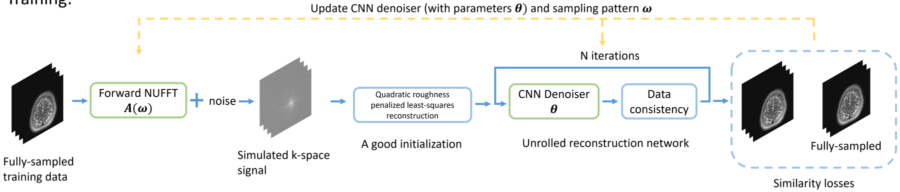
Alternating minimization:

$$\boldsymbol{x}_{t+1} = \arg\min_{\boldsymbol{x}} \|\boldsymbol{A}(\boldsymbol{\omega})\boldsymbol{x} - \boldsymbol{y}\|_2^2 + \mu \|\boldsymbol{x} - \boldsymbol{z}_t\|_2^2 \quad \text{(data consistency, solved via CG)}$$
$$\boldsymbol{z}_{t+1} = \arg\min_{\boldsymbol{z}} R(\boldsymbol{z}) + \mu \|\boldsymbol{x}_{t+1} - \boldsymbol{z}\|_2^2 \quad \text{(denoising)}$$
$$\text{``} = \text{''} \mathcal{D}_{\boldsymbol{\theta}}(\boldsymbol{x}_{t+1}) \quad \text{(CNN denoiser)}$$

- CNN weights $\boldsymbol{\theta}$ shared across iterations, per MODL [51]
- 6 outer iterations for results shown, with augmented Lagrangian parameter $\mu = 2$

Efficient NUFFT backpropagation I

Training:



Data consistency block has steps like

$$\boldsymbol{x}_{t+1} = \boldsymbol{x}_t + \alpha \left(\boldsymbol{A}'(\boldsymbol{\omega}) \left(\boldsymbol{A}(\boldsymbol{\omega}) \boldsymbol{x} - \boldsymbol{y} \right) + \mu (\boldsymbol{x} - \boldsymbol{z}_t) \right)$$

$\boldsymbol{A}(\boldsymbol{\omega})$ is dense and huge:

$$a_{ij} = \mathrm{e}^{-\imath \vec{\omega}_i \cdot \vec{r}_j} \tag{1}$$

- Fast approach to $\boldsymbol{A}(\boldsymbol{\omega})\boldsymbol{x}$ uses NUFFT approximation:
  zero-padding, over-sampled FFT, interpolation [56, 57].
- Backpropagation for $\boldsymbol{\omega}$ update through NUFFT steps via autodifferentiation is slow.

Derive Jacobian matrix for exact form (1):

$$\frac{\partial}{\partial \omega_d} \boldsymbol{A}(\boldsymbol{\omega})\boldsymbol{x} = -\imath \operatorname{Diag}\{\boldsymbol{A}(\boldsymbol{\omega})(\boldsymbol{x} \odot \boldsymbol{r}_d)\}.$$

Applying this Jacobian to a vector $\boldsymbol{v} \in \mathbb{C}^M$ during backpropagation yields

$$\left(\frac{\partial}{\partial \omega_d} \boldsymbol{A}(\boldsymbol{\omega})\boldsymbol{x}\right) \boldsymbol{v} = -\imath \operatorname{Diag}\{\boldsymbol{A}(\boldsymbol{\omega})(\boldsymbol{x} \odot \boldsymbol{r}_d)\} \boldsymbol{v} = -\imath \left(\boldsymbol{A}(\boldsymbol{\omega})(\boldsymbol{x} \odot \boldsymbol{r}_d)\right) \odot \boldsymbol{v}.$$

Implemented efficiently using NUFFT applied to $\boldsymbol{x} \odot \boldsymbol{r}_d$

Similar idea for Jacobian of adjoint of $\boldsymbol{A}$.

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Summary / future directions

- Machine learning methods have much potential for both scan design and image reconstruction
- Quantitative results in paper demonstrate synergy of jointly optimizing both
- Anatomy specific trajectories: pro or con?
- Self-supervised methods when training data unavailable
- Extensions to 3D and 3D+time are planned, and challenging

Talk and code available online at
`http://web.eecs.umich.edu/~fessler`

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