Learning-based image reconstruction

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Outline

Medical imaging overview

raw data $\quad\quad$ images

Design Scan $\rightarrow$ Acquire Data $\xrightarrow{\boldsymbol{y}}$ Reconstruct Images $\xrightarrow{\hat{\boldsymbol{x}}}$ Process Images $\rightarrow$ Diagnose Interpret

Medical imaging overview

raw data / images

Design Scan $\rightarrow$ Acquire Data $\xrightarrow{\boldsymbol{y}}$ Reconstruct Images $\xrightarrow{\hat{\boldsymbol{x}}}$ **Process Images** $\rightarrow$ Diagnose Interpret

Most obvious place for machine learning is in post-processing (image analysis).
Numerous special issues and surveys in medical imaging journals, *e.g.*, [1–9].

Medical imaging overview

raw data $\qquad\qquad$ images

Design Scan $\rightarrow$ Acquire Data $\xrightarrow{\boldsymbol{y}}$ Reconstruct Images $\xrightarrow{\hat{\boldsymbol{x}}}$ Process Images $\rightarrow$ Diagnose Interpret

Machine learning for scan design
Choose best k-space phase encoding locations based on training images
Hot topic in MRI recently [10–15].
Precursor by Yue Cao and David Levin, MRM Sep. 1993 [16–18].

Medical imaging overview

raw data $\quad\quad\quad$ images

Design Scan $\rightarrow$ Acquire Data $\xrightarrow{\boldsymbol{y}}$ Reconstruct Images $\xrightarrow{\hat{\boldsymbol{x}}}$ Process Images $\rightarrow$ Diagnose Interpret

Machine learning in medical image reconstruction
June 2018 special issue of IEEE Trans. on Medical Imaging [19].
Surveys: [20–27]
Possibly easier than diagnosis due to lower bar:

- current reconstruction methods based on simplistic image models;
- human eyes are better at detection than at solving inverse problems.

Medical imaging overview

raw data — images

Design Scan $\rightarrow$ Acquire Data $\xrightarrow{\boldsymbol{y}}$ Reconstruct Images $\xrightarrow{\hat{\boldsymbol{x}}}$ Process Images $\rightarrow$ Diagnose Interpret

ML-based "magic"

A holy grail for machine learning in medical imaging?

- CT sinogram to vessel diameter [28, 29]
- k-space to ???

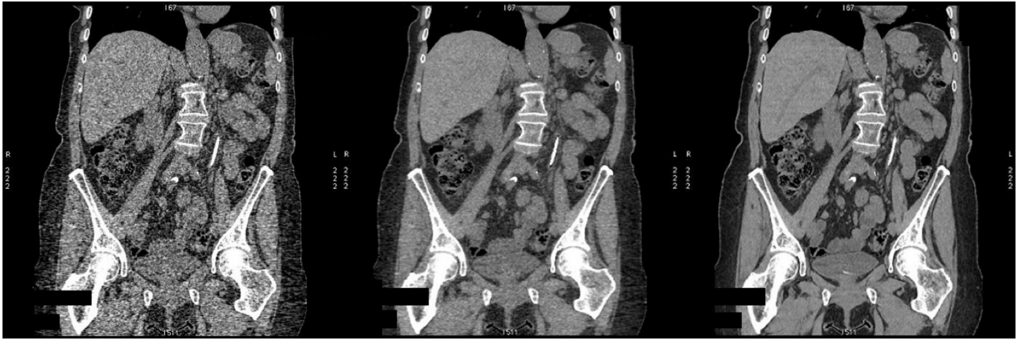
Outline

Generations of medical image reconstruction methods

1. 70's "Analytical" methods (integral equations)
   FBP for SPECT / PET / X-ray CT, IFFT for MRI, ...
2. 80's Algebraic methods (as in "linear algebra")
   Solve $\boldsymbol{y} = \boldsymbol{A}\boldsymbol{x}$
3. 90's Statistical methods
   - LS / ML methods based on imaging physics ("model based")
   - Bayesian methods (Markov random fields, ...)
   - regularized methods
4. 00's Compressed sensing methods
   (mathematical sparsity models)
5. 10's Adaptive / data-driven methods
   machine learning, deep learning, ...

Two important milestones for clinical CT

- Model-based image reconstruction (MBIR) FDA approved circa 2012 [30]

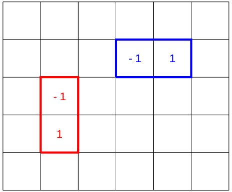


- Deep-learning image reconstruction FDA approved 2019 [31, 32]

- Learn models (sparsifying transform or dictionary) for image patches from training data
  - interpretable (?) optimization formulations
  - local prior information only (patch size)
  - perhaps slower computation due to optimization iterations
- Train neural network (aka **deep learning**)
  - less interpretable
  - possibly more global prior information
  - slow training, but perhaps faster computation after trained

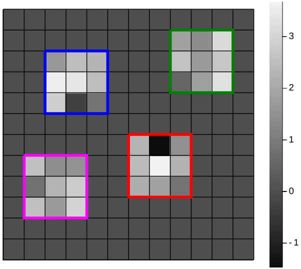
Outline

Anisotropic discrete TV regularizer:
$R(\boldsymbol{x}) = \|\boldsymbol{T}\boldsymbol{x}\|_1$
where $\boldsymbol{T}$ is finite-differences
$\equiv$ patches of size 2×1.



Larger patches provide more context for distinguishing signal from noise.

cf. CNN approaches



Patch-based regularizers:
- synthesis models
- analysis methods

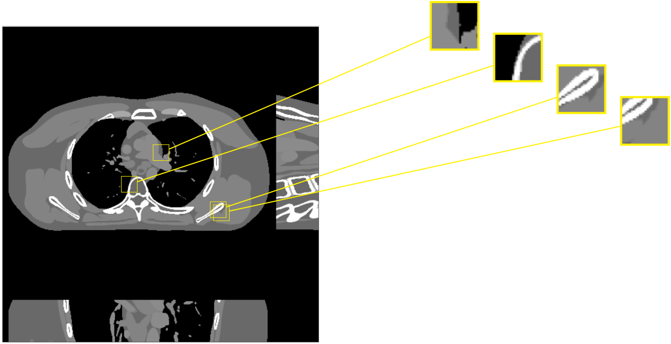
X-ray CT with learned sparsifying transforms

- Data
  - Population adaptive methods
  - Patient adaptive methods
- Spatial structure
  - Patch-based models
  - Convolutional models
- Regularizer formulation
  - Synthesis (dictionary) approach
  - Analysis (sparsifying transform) approach

Assumption: if $\boldsymbol{x}$ is a plausible image, then each patch transform $\boldsymbol{T}\boldsymbol{P}_m\boldsymbol{x}$ is sparse.

- $\boldsymbol{P}_m\boldsymbol{x}$ extracts the mth of M patches from $\boldsymbol{x}$
- $\boldsymbol{T}$ is a (often square) sparsifying transform matrix.

What $\boldsymbol{T}$?



Sparsifying transform learning (population adaptive)

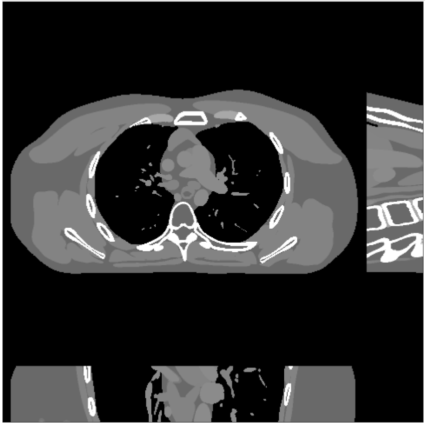
Given training images $\boldsymbol{x}_1, \ldots, \boldsymbol{x}_L$ from a representative population, find transform $\boldsymbol{T}_*$ that best sparsifies their patches:

$$\boldsymbol{T}_* = \underset{\boldsymbol{T} \text{ unitary}}{\arg\min} \min_{\{\boldsymbol{z}_{l,m}\}} \sum_{l=1}^{L} \sum_{m=1}^{M} \|\boldsymbol{T}\boldsymbol{P}_m \boldsymbol{x}_l - \boldsymbol{z}_{l,m}\|_2^2 + \alpha \|\boldsymbol{z}_{l,m}\|_0$$

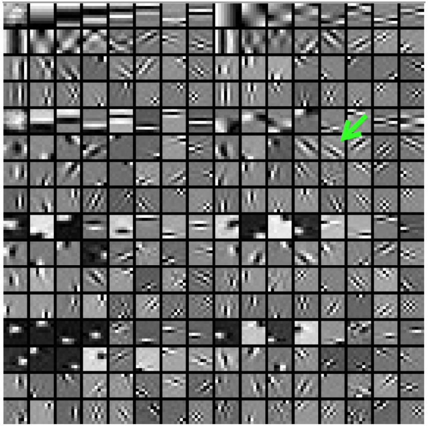
- Encourage aggregate sparsity, not patch-wise sparsity (cf K-SVD [33])
- Non-convex due to unitary constraint and $\|\cdot\|_0$
- Efficient alternating minimization algorithm [34]
  - $\boldsymbol{z}$ update : simple hard thresholding
  - $\boldsymbol{T}$ update : orthogonal Procrustes problem (SVD)
  - Subsequence convergence guarantees [34]

Example of learned sparsifying transform

3D X-ray training data (XCAT phantom)



Parts of learned sparsifier $\boldsymbol{T}_*$



(2D slices in x-y, x-z, y-z, from 3D image volume)

$8 \times 8 \times 8$ patches $\Longrightarrow$ $\boldsymbol{T}_*$ is $8^3 \times 8^3 = 512 \times 512$

top 8×8 slice of 256 of the 512 rows of $\boldsymbol{T}_*$ $\uparrow$

Regularizer based on learned sparsifying transform

Regularized inverse problem [35]:

$$\hat{\boldsymbol{x}} = \arg\min_{\boldsymbol{x}} \|\boldsymbol{A}\boldsymbol{x} - \boldsymbol{y}\|_{\boldsymbol{W}}^2 + \beta\,\mathrm{R}(\boldsymbol{x})$$

$$\mathrm{R}(\boldsymbol{x}) = \min_{\{\boldsymbol{z}_m\}} \sum_{m=1}^{M} \|\boldsymbol{T}_* \boldsymbol{P}_m \boldsymbol{x} - \boldsymbol{z}_m\|_2^2 + \alpha\,\|\boldsymbol{z}_m\|_0\,.$$

$\boldsymbol{T}_*$ adapted to population training data

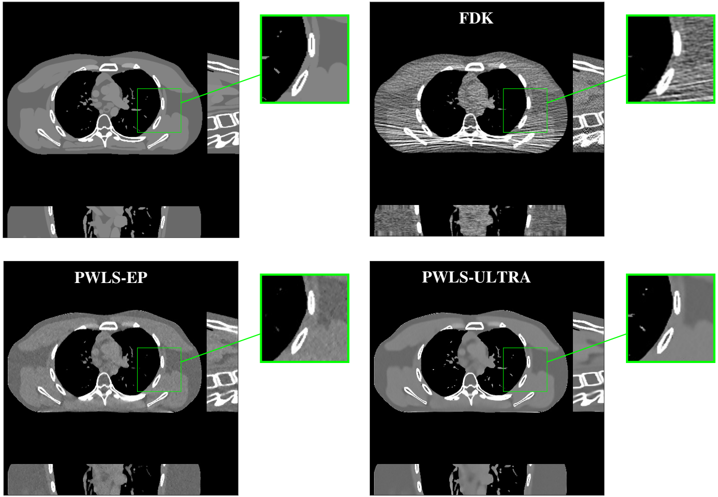
Alternating minimization optimizer:

- $\boldsymbol{z}_m$ update : simple hard thresholding
- $\boldsymbol{x}$ update : quadratic problem (many options)

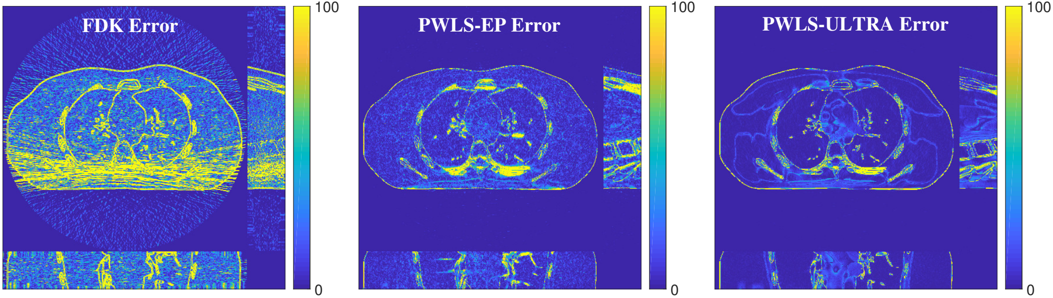
  Linearized augmented Lagrangian method (LALM) [36]

Example: low-dose 3D X-ray CT simulation

X. Zheng, S. Ravishankar, Y. Long, JF: IEEE T-MI, June 2018 [35].



3D X-ray CT simulation Error maps



	X-ray Intensity	FDK	EP	ST $\boldsymbol{T}_*$	ULTRA	ULTRA-$\{\tau_j\}$
RMSE in HU	1×10^4	67.8	34.6	32.1	30.7	**29.2**
	5×10^3	89.0	41.1	37.3	35.7	**34.2**

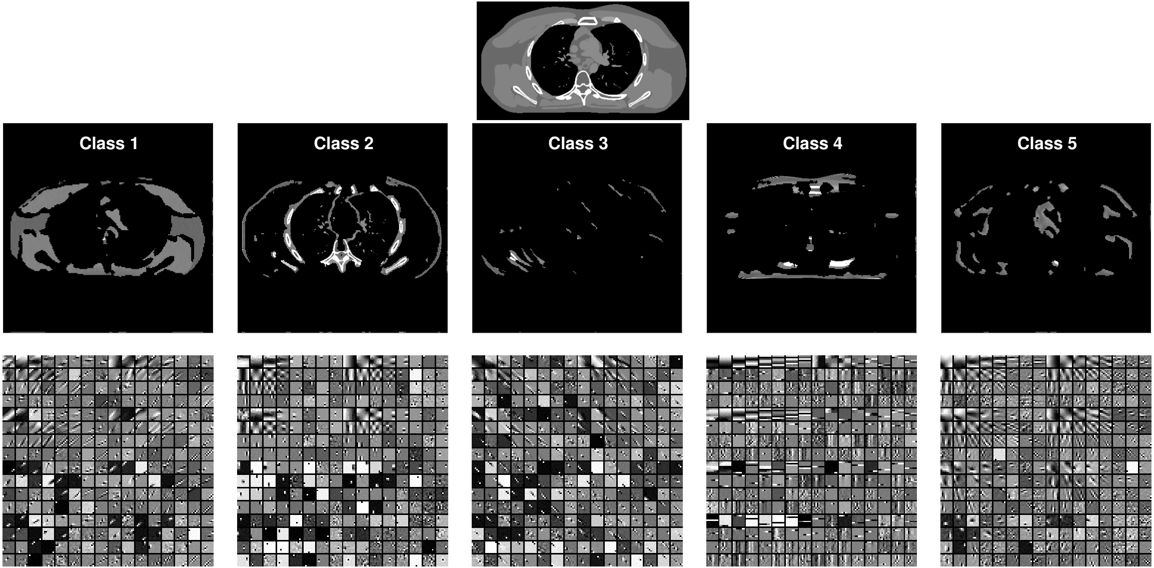
- Physics / statistics provides dramatic improvement
- Data adaptive regularization further reduces RMSE

Given training images $\boldsymbol{x}_1, \ldots, \boldsymbol{x}_L$ from a representative population, find a set of transforms $\left\{ \hat{\boldsymbol{T}}_k \right\}_{k=1}^{K}$ that best sparsify image patches:

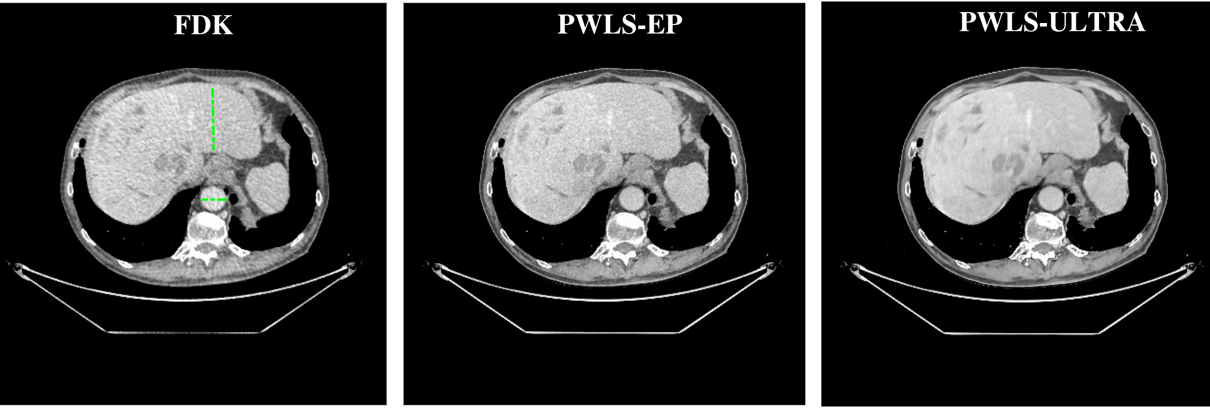
$$\left\{ \hat{\boldsymbol{T}}_k \right\} = \mathop{\arg\min}_{\{\boldsymbol{T}_k \text{ unitary}\}} \min_{\{\boldsymbol{z}_{l,m}\}} \sum_{l=1}^{L} \sum_{m=1}^{M} \left(\min_{k \in \{1,\ldots,K\}} \|\boldsymbol{T}_k \boldsymbol{P}_m \boldsymbol{x}_l - \boldsymbol{z}_{l,m}\|_2^2 + \alpha \|\boldsymbol{z}_{l,m}\|_0 \right)$$

- Joint unsupervised clustering / sparsification
- Further nonconvexity due to clustering
- Efficient alternating minimization algorithm [37]

Example: 3D X-ray CT learned set of transforms



Example: 3D X-ray CT ULTRA for chest scan



Zheng et al., IEEE T-MI, June 2018 [35] (Special issue on machine learning for image reconstruction)

Matlab code: `http://web.eecs.umich.edu/~fessler/irt/reproduce/`

`https://github.com/xuehangzheng/PWLS-ULTRA-for-Low-Dose-3D-CT-Image-Reconstruction`

Outline

Overview:

- image-domain learning [38–40]...
- k-space or data-domain learning
  e.g., [41], [42], [43]
- transform learning (direct from k-space to image)
  e.g., AUTOMAP [44], [45–47]
- hybrid-domain learning (unrolled loop, *e.g.*, variational network)
  alternate between denoising/dealiasing and reconstruction from k-space
  e.g., [42, 48–52] ...

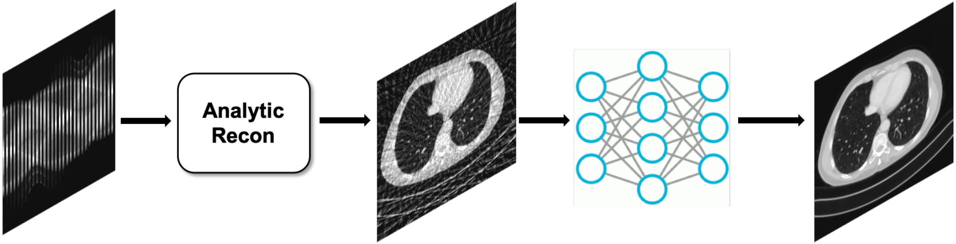


Figure courtesy of Jong Chul Ye, KAIST University.

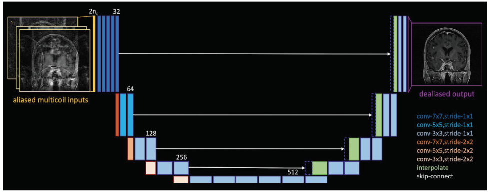
+ simple and fast
− aliasing is spatially widespread, requires deep network

Dangers of image-domain learning I

Investigating Robustness to Unseen Pathologies in Model-Free Deep Multicoil Reconstruction

Gopal Nataraj[1] and Ricardo Otazo[1,2]
[1]Dept. of Medical Physics, Memorial Sloan Kettering Cancer Center
[2]Dept. of Radiology, Memorial Sloan Kettering Cancer Center

Introduction
Speed is often claimed as a key advantage of deep learning (DL) for undersampled parallel MRI reconstruction [1]. However, the only DL approach that to our knowledge has studied generalizability to pathologies unseen in training [2] requires repeated application of the MR acquisition model and its adjoint, just as in iterative methods. In contrast, model-free DL reconstruction has the potential to be much faster. Prior model-free DL work [3] proposes to learn a mapping directly from k-space, but with



[53] ISMRM 2020 Workshop on Data Sampling & Image Reconstruction

Dangers of image-domain learning II

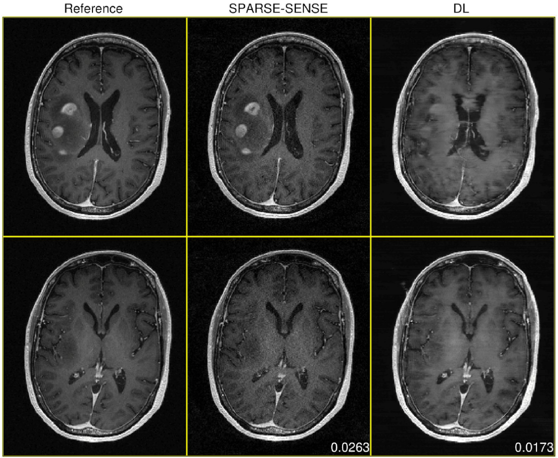


Figure 3: Reconstructions in a case of anaplastic astrocytoma, a rare malignant brain tumor. SPARSE-SENSE and DL reconstructions are from the same 4x-accelerated retrospectively undersampled acquisition. DL achieves lower whole-volume MAE than SPARSE-SENSE, but fails to properly reconstruct regions near the tumor.

Image-domain learning variations

- Use NN output as a "prior" for iterative reconstruction [38, 54]:

$$\hat{\boldsymbol{x}}_\beta = \arg\min_{\boldsymbol{x}} \|\boldsymbol{A}\boldsymbol{x} - \boldsymbol{y}\|_2^2 + \beta \|\boldsymbol{x} - \boldsymbol{x}_{\mathrm{NN}}\|_2^2 = (\boldsymbol{A}'\boldsymbol{A} + \beta \boldsymbol{I})^{-1}(\boldsymbol{A}'\boldsymbol{y} + \beta \boldsymbol{x}_{\mathrm{NN}})$$

  - For single-coil Cartesian case:
    - no iterations are needed (solve with FFTs)
    - $\lim_{\beta \to 0} \hat{\boldsymbol{x}}_\beta$ replaces missing k-space data with FFT of $\boldsymbol{x}_{\mathrm{NN}}$
  - Iterations needed for parallel MRI and/or non-Cartesian sampling (PCG)

- Learn residual (aliasing artifacts), then subtract [55, 56]

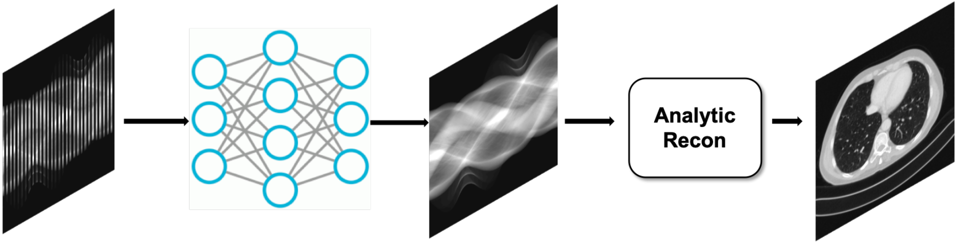


Figure courtesy of Jong Chul Ye, KAIST University.

+ simple and fast ("nonlinear GRAPPA")
+ "database-free" : learn from auto-calibration data
− perhaps harder to represent local image features?

DL for IR: transform learning

Figure courtesy of Jong Chul Ye, KAIST University.

+ in principle, purely data driven; potential to avoid model mismatch
− high memory requirement for fully connected layers

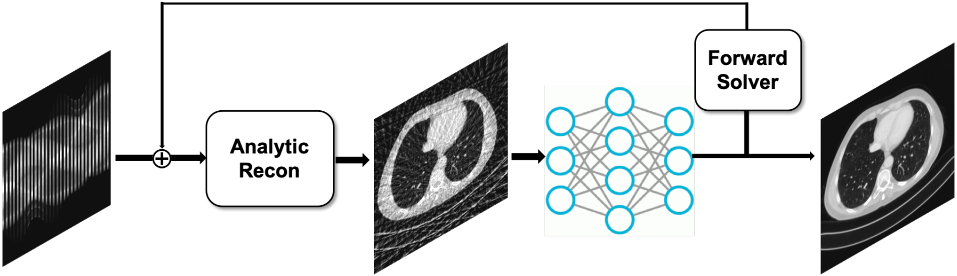


Figure courtesy of Jong Chul Ye, KAIST University.

+ physics-based use of k-space data & image-domain priors
+ interpretable connections to optimization approaches
− more computation to due to "iterations" (layers) and repeated $\boldsymbol{A}\boldsymbol{x}$, $\boldsymbol{A}'\boldsymbol{r}$

Cost function for convolutional sparsity regularization:

$$\arg\min_{\boldsymbol{x}} \frac{1}{2} \|\boldsymbol{A}\boldsymbol{x} - \boldsymbol{y}\|_{\boldsymbol{W}}^2 + \beta \left(\min_{\zeta} \sum_{k=1}^{K} \frac{1}{2} \|\boldsymbol{h}_k * \boldsymbol{x} - \zeta_k\|_2^2 + \alpha \|\zeta_k\|_1 \right)$$

Alternating minimization, aka block coordinate descent (BCD), updates:

$$\text{Sparse code:} \qquad \zeta_k^{(n+1)} = \mathrm{soft}\{\boldsymbol{h}_k * \boldsymbol{x}^{(n)}, \alpha\}$$

$$\text{Image: } \boldsymbol{x}^{(n+1)} = \arg\min_{\boldsymbol{x}} F(\boldsymbol{x}; \boldsymbol{y}, \boldsymbol{z}^{(n)})$$

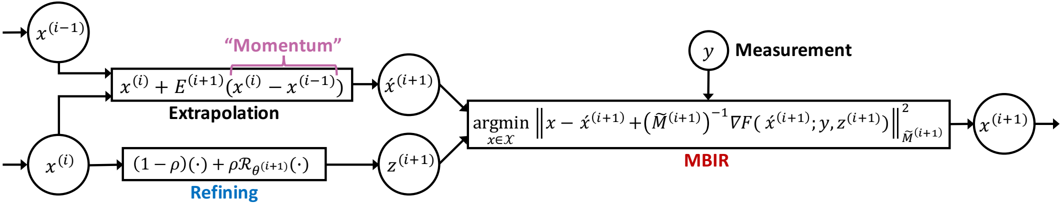
$$F(\boldsymbol{x}; \boldsymbol{y}, \boldsymbol{z}^{(n)}) \triangleq \frac{1}{2} \|\boldsymbol{A}\boldsymbol{x} - \boldsymbol{y}\|_{\boldsymbol{W}}^2 + \beta \left(\sum_{k=1}^{K} \frac{1}{2} \left\|\boldsymbol{h}_k * \boldsymbol{x} - \zeta_k^{(n+1)}\right\|_2^2 + \alpha \left\|\zeta_k^{(n+1)}\right\|_1 \right)$$

$$= \frac{1}{2} \|\boldsymbol{A}\boldsymbol{x} - \boldsymbol{y}\|_{\boldsymbol{W}}^2 + \beta \frac{1}{2} \left\|\boldsymbol{x} - \boldsymbol{z}^{(n)}\right\|_2^2 \quad \text{(quadratic but } \textit{large} \implies \text{majorize)}$$

$$\boldsymbol{z}^{(n)} = \mathcal{R}(\boldsymbol{z}^{(n)}) = \sum_{k=1}^{K} \mathrm{flip}(\boldsymbol{h}_k) * \mathrm{soft}\{\boldsymbol{h}_k * \boldsymbol{x}^{(n)}\} \quad \text{(denoise} \implies \text{learn)}$$

Momentum-Net overview

Unrolled loop network with momentum and quadratic majorizer [57, 58]:



- Diagonal majorizer for CT: $\boldsymbol{M} = \mathrm{Diag}\{\boldsymbol{A}' \boldsymbol{W} \boldsymbol{A} \boldsymbol{1}\} + \beta \boldsymbol{I} \succeq \boldsymbol{A}' \boldsymbol{W} \boldsymbol{A} + \beta \boldsymbol{I}$
- Learn image mapper ("refiner") $\mathcal{R}$ from training data (supervised).
  cf CNN: filter $\to$ threshold $\to$ filter

Momentum-Net benefits

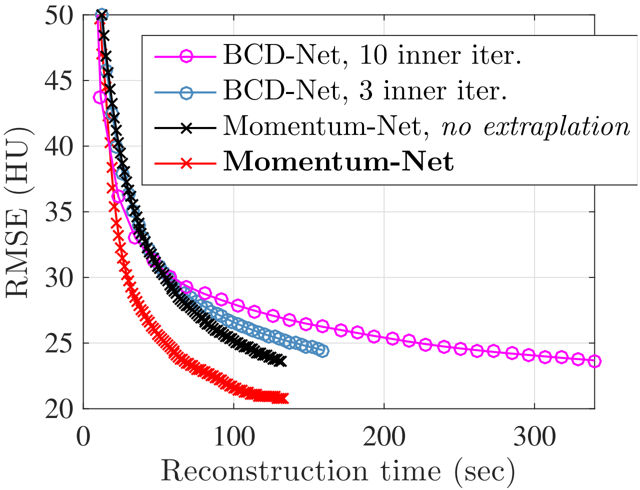
- Image mapper $\mathcal{R}$ is shallow
  $\Longrightarrow$ less risk of over-fitting / hallucination
- Momentum accelerates convergence $\Longrightarrow$ fewer "layers" (outer iterations)
- First unrolled loop approach to have convergence theory
  (under suitable assumptions on $\mathcal{R}$)
- Image update uses original measurements $\boldsymbol{y}$ and imaging physics $\boldsymbol{A}$

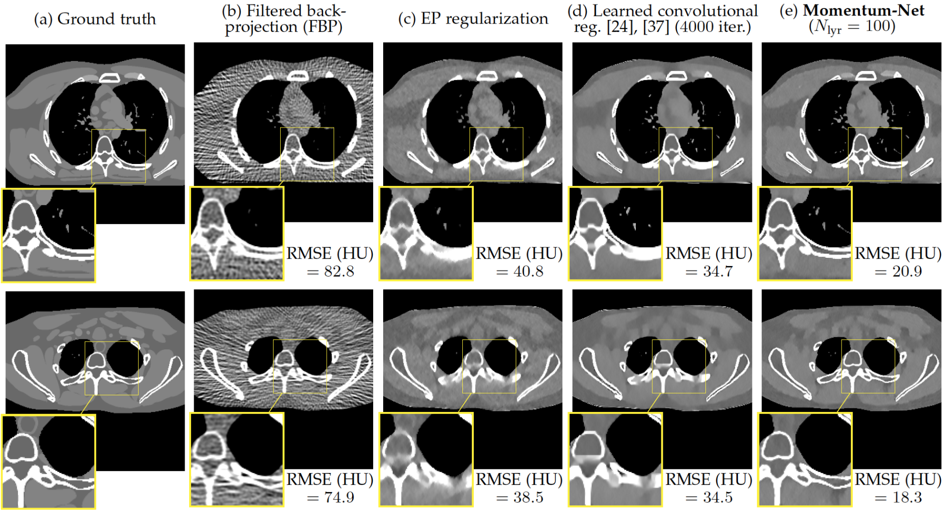
[57, 58] Il Yong Chun, Zhengyu Huang, Hongki Lim, J A Fessler
Momentum-Net: Fast and convergent iterative neural network for inverse problems
http://arxiv.org/abs/1907.11818,

IEEE Tr. on PAMI, 2020 http://doi.org/10.1109/TPAMI.2020.3012955

Momentum-Net preliminary results

Illustration of benefits of momentum:





Sparse-view CT with 123/984 views, $I_0 = 10^5$, 800-1200 mod. HU display.

Outline

Summary

- CT image reconstruction has evolved greatly in the 50+ years since Allan Cormack's seminal papers [59, 60]
  - physics
  - statistics
  - regularization and optimization
  - data adaptive methods inspired by machine learning
- Machine learning has great potential for medical imaging
- Much excitement but many challenges
- Image reconstruction seems especially suitable for ML ideas
- Data-driven, adaptive regularizers beneficial for low-dose CT
- More comparisons between model-based methods with adaptive regularizers and CNN-based methods needed

Talk and code available online at
`http://web.eecs.umich.edu/~fessler`

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