Medical image reconstruction using adaptive signal models

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Declaration: No relevant financial interests or relationships to disclose

Outline

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Overview of medical imaging:

$$\boxed{\text{Acquire Data}} \xrightarrow[\text{raw data}]{\boldsymbol{y}} \boxed{\text{Reconstruct Images}} \xrightarrow[\text{images}]{\hat{\boldsymbol{x}}} \boxed{\text{Process Images}} \rightarrow \boxed{\begin{array}{c}\text{Analyze}\\ \text{Diagnose}\\ \text{Quantify}\\ \text{Interpret}\\ \text{Intervene}\end{array}}$$

Most obvious place for machine learning is post-processing:

reconstructed images $\hat{\boldsymbol{x}}$ $\rightarrow$ $\boxed{\text{ML-based image processing (classification or regression)}}$ $\rightarrow$ interpretation

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(Many conference sessions; special issue of IEEE Trans. on Med. Imaging in May 2016 [1], ...)

Another (initially less obvious?) place for machine learning (multiple conference sessions):

$$\begin{array}{c} \text{raw data} \\ \boldsymbol{y} \end{array} \rightarrow \boxed{\begin{array}{c} \text{ML-based} \\ \text{image reconstruction} \end{array}} \rightarrow \begin{array}{c} \text{images} \\ \hat{\boldsymbol{x}} \end{array}$$

...

Another (initially less obvious?) place for machine learning (multiple conference sessions):

raw data $\boldsymbol{y}$ $\rightarrow$ $\boxed{\text{ML-based image reconstruction}}$ $\rightarrow$ images $\hat{\boldsymbol{x}}$

Possibly easier (than diagnosis) due to lower bar:

- current reconstruction methods based on simplistic image models;
- human eyes are better at detection than at solving inverse problems.

Machine learning in medical image reconstruction

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June 2018 special issue of IEEE Trans. on Medical Imaging [2]:

IEEE TRANSACTIONS ON MEDICAL IMAGING, VOL. 37, NO. 6, JUNE 2018 1289

Image Reconstruction Is a New Frontier of Machine Learning

Ge Wang, *Fellow, IEEE*, Jong Chu Ye, *Senior Member, IEEE*, Klaus Mueller, *Senior Member, IEEE*, and Jeffrey A. Fessler, *Fellow, IEEE*

A more speculative opportunity for machine learning:

raw data $\boldsymbol{y}$ $\rightarrow$ $\boxed{\text{ML-based ``magic''}}$ $\rightarrow$ interpretation

...

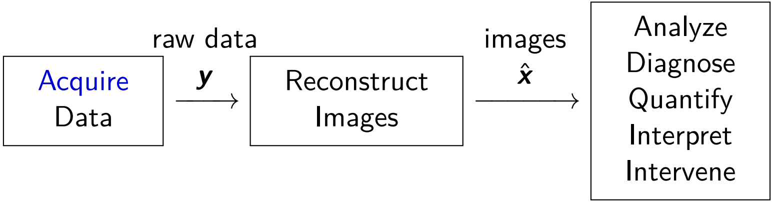
A more speculative opportunity for machine learning:

raw data $\boldsymbol{y}$ $\rightarrow$ ML-based "magic" $\rightarrow$ interpretation

- CT sinogram to vessel diameter [3]
- k-space to ???

See Wiro Niessen's keynote...

One more opportunity for ML in medical imaging:



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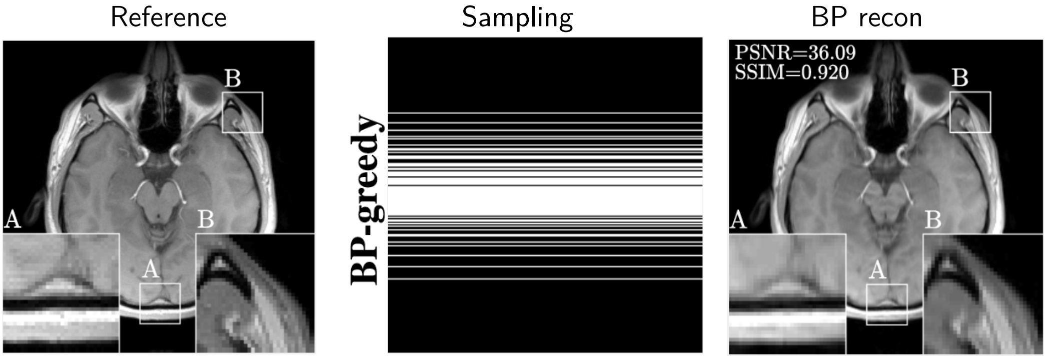
One more opportunity for ML in medical imaging:

$$\boxed{\text{Acquire Data}} \xrightarrow[\quad]{\text{raw data } \boldsymbol{y}} \boxed{\text{Reconstruct Images}} \xrightarrow[\quad]{\text{images } \hat{\boldsymbol{x}}} \boxed{\begin{array}{c}\text{Analyze}\\ \text{Diagnose}\\ \text{Quantify}\\ \text{Interpret}\\ \text{Intervene}\end{array}}$$

Choose best k-space phase encoding locations based on training images:

- "Learning-based compressive MRI" [4, 5]
  (Volkan Cevher group, June 2018 IEEE T-MI)
  Single coil only so far; perhaps hard to generalize to parallel MRI?
- Yue Cao and David Levin, MRM Sep. 1993 "Feature recognizing MRI" [6–8]

Adaptive phase-encode selection



Sampling designed to optimize PSNR for basis pursuit (BP) reconstruction using shearlet transform, at 25% sampling rate.

Sampling design considers both the training data and the reconstruction method.

No high spatial frequencies!?

(Images from Gözcü et al. [5].)

Outline

Image reconstruction background

- Forward problem (data acquisition):

  Scene Objects $\xrightarrow[]{\text{physics}\ \boldsymbol{x}}$ Imaging System $\rightarrow$ raw data $\boldsymbol{y}$

  SPECT, PET, X-ray CT, MRI, optical, ...

- Inverse problem (image formation):

  Acquire Data $\xrightarrow[]{\text{raw data}\ \boldsymbol{y}}$ Reconstruct Images $\rightarrow$ images $\hat{\boldsymbol{x}}$

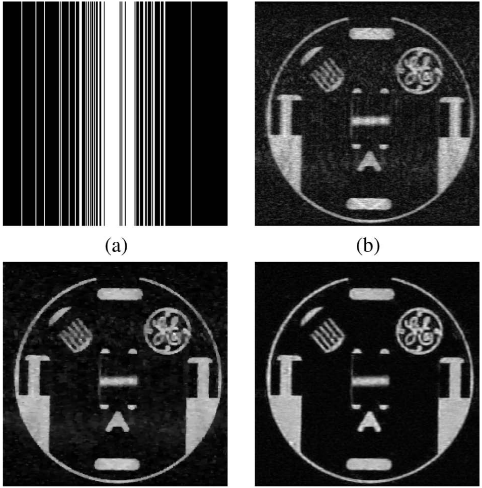
- Image reconstruction topics: physics models, measurement statistical models, regularization / object priors, optimization...

1. 70's "Analytical" methods (integral equations)
   FBP for SPECT / PET / X-ray CT, IFFT for MRI, ...
2. 80's Algebraic methods (as in "linear algebra")
   Solve $\boldsymbol{y} = \boldsymbol{A}\boldsymbol{x}$
3. 90's Statistical methods
   - LS / ML methods
   - Bayesian methods (Markov random fields, ...)
   - regularized methods
4. 00's Compressed sensing methods
   (mathematical sparsity models)
5. 10's Adaptive / data-driven methods
   machine learning, deep learning, ...

(a) $4\times$ under-sampled MR k-space
(b) zero-filled reconstruction
(c) "compressed sensing"
reconstruction with TV regularization
(d) adaptive regularization
using dictionary learning

Ravishankar & Bresler, DLMRI, T-MI, May 2011, [9, Fig. 10]

DL = dictionary learning
(not "deep learning")

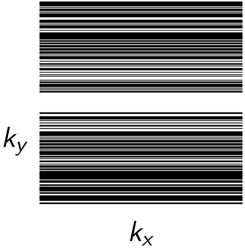

(a) (b) (c) (d)

Ill-posed inverse problems

$$\boldsymbol{y} = \boldsymbol{A}\boldsymbol{x} + \varepsilon$$

$\boldsymbol{y}$: measurements ε : noise
$\boldsymbol{x}$: unknown image $\boldsymbol{A}$: system matrix (typically wide)

- compressed sensing (*e.g.*, MRI)   ($\boldsymbol{A}$ "random" rows of DFT)



- deblurring (restoration)   ($\boldsymbol{A}$ Toeplitz)
- in-painting   ($\boldsymbol{A}$ subset of rows of $\boldsymbol{I}$)
- denoising (not ill posed)   ($\boldsymbol{A} = \boldsymbol{I}$)

Inverse problems via MAP estimation

Unknown image $\boldsymbol{x}$ → System model $\mathrm{p}(\boldsymbol{y} \mid \boldsymbol{x})$ → Data $\boldsymbol{y}$ → Estimator → Recon. image $\hat{\boldsymbol{x}}$

If we have a prior $\mathrm{p}(\boldsymbol{x})$, then the MAP estimate is:

$$\hat{\boldsymbol{x}} = \arg\max_{\boldsymbol{x}} \mathrm{p}(\boldsymbol{x} \mid \boldsymbol{y}) = \arg\max_{\boldsymbol{x}} \log \mathrm{p}(\boldsymbol{y} \mid \boldsymbol{x}) + \log \mathrm{p}(\boldsymbol{x}) \,.$$

For gaussian measurement errors and a linear forward model:

$$-\log \mathrm{p}(\boldsymbol{y} \mid \boldsymbol{x}) \equiv \frac{1}{2} \|\boldsymbol{y} - \boldsymbol{A}\boldsymbol{x}\|_{\boldsymbol{W}}^2$$

where $\|\boldsymbol{y}\|_{\boldsymbol{W}}^2 = \boldsymbol{y}' \boldsymbol{W} \boldsymbol{y}$

and $\boldsymbol{W}^{-1} = \mathrm{Cov}\{\boldsymbol{y} \mid \boldsymbol{x}\}$ is known
($\boldsymbol{A}$ from physics, $\boldsymbol{W}$ from statistics)

Priors for MAP estimation

- If all images $\boldsymbol{x}$ are "plausible" (have non-zero probability) then

$$\mathrm{p}(\boldsymbol{x}) \propto \mathrm{e}^{-\mathsf{R}(\boldsymbol{x})} \Longrightarrow -\log \mathrm{p}(\boldsymbol{x}) \equiv \mathsf{R}(\boldsymbol{x})$$

(from fantasy / imagination / wishful thinking / data)

- MAP $\equiv$ regularized weighted least-squares (WLS) estimation:

$$\begin{aligned}\hat{\boldsymbol{x}} &= \arg\max_{\boldsymbol{x}} \; \log \mathrm{p}(\boldsymbol{y} \mid \boldsymbol{x}) + \log \mathrm{p}(\boldsymbol{x}) \\ &= \arg\min_{\boldsymbol{x}} \; \frac{1}{2} \|\boldsymbol{y} - \boldsymbol{A}\boldsymbol{x}\|_{\boldsymbol{W}}^2 + \mathsf{R}(\boldsymbol{x})\end{aligned}$$

- A regularizer $\mathsf{R}(\boldsymbol{x})$, aka log prior, is essential for high-quality solutions to ill-conditioned / ill-posed inverse problems.
- Why ill-posed? Often high ambitions...

Non-adaptive regularizers

- Tikhonov regularization (IID gaussian prior)
- Markov random field (MRF) models
- Roughness penalty (cf MRF prior)
- Edge-preserving regularization
- Total-variation (TV) regularization
- Black-box denoiser like NLM, *e.g.*, plug-and-play ADMM [10]
- Sparsity in ambient space
- Sparsifying transforms: wavelets, curvelets, ...
- Graphical models
- ...

All "hand crafted" from statistical / mathematical models ...

Simpler methods for ML in image reconstruction

Many possible ways to use ML ideas in image reconstruction.

Basic "fast" methods:

- Enhance raw data (k-space, sinogram, ...)
- Enhance poorly reconstructed image
  - patch-based
  - image-based

Computation / quality trade-offs ?

...

Simpler methods for ML in image reconstruction

Many possible ways to use ML ideas in image reconstruction.

Basic "fast" methods:

- Enhance raw data (k-space, sinogram, ...)
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Computation / quality trade-offs ?

Basic "slow" methods:

- Auto-tune regularization parameter(s)
- Provide an initial image for "conventional" iterative reconstruction

May not fully exploit the potential of ML

- ML-based "prior" image for iterative reconstruction [11]:

$$\hat{\boldsymbol{x}} = \arg\min_{\boldsymbol{x}} \|\boldsymbol{A}\boldsymbol{x} - \boldsymbol{y}\|_2^2 + \beta \|\boldsymbol{x} - \boldsymbol{x}_{\text{prior}}\|_p^p$$

Fast for $p = 2$, but $p = 1$ more robust to errors in prior image
Reminiscent of U. Wisconsin's PICCS methods, *e.g.*, [12]

Advanced "fast" methods for ML-based IR

- ML-based "prior" image for iterative reconstruction [11]:

  $$\hat{\boldsymbol{x}} = \arg\min_{\boldsymbol{x}} \|\boldsymbol{A}\boldsymbol{x} - \boldsymbol{y}\|_2^2 + \beta \|\boldsymbol{x} - \boldsymbol{x}_{\text{prior}}\|_p^p$$

  Fast for $p = 2$, but $p = 1$ more robust to errors in prior image
  Reminiscent of U. Wisconsin's PICCS methods, *e.g.*, [12]

- Unrolled loop (recurrent NN) with learned components [13–16]

- ML-based nonlinear encoder, *e.g.*, autoencoder or generative adversarial network (GAN) [17, 18]: nonlinear generalizations of subspace models
- learn G: maps low-dimensional latent parameter $\boldsymbol{z}$ into high-dimensional image $\boldsymbol{x}$

▶ Synthesis form [19]:

$$\hat{\boldsymbol{x}} = G(\hat{\boldsymbol{z}}), \qquad \hat{\boldsymbol{z}} = \arg\min_{\boldsymbol{z}} \|\boldsymbol{A} G(\boldsymbol{z}) - \boldsymbol{y}\|_2^2$$

Challenges: $\hat{\boldsymbol{x}} \in \text{Range}(G)$, non-convex minimization

- ML-based nonlinear encoder, *e.g.*, autoencoder or generative adversarial network (GAN) [17, 18]: nonlinear generalizations of subspace models
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Challenges: $\hat{\boldsymbol{x}} \in \text{Range}(G)$, non-convex minimization

▶ Regularizer form:

$$\hat{\boldsymbol{x}} = \arg\min_{\boldsymbol{x}} \|\boldsymbol{A}\boldsymbol{x} - \boldsymbol{y}\|_2^2 + \beta R_{\text{encoder}}(\boldsymbol{x})$$

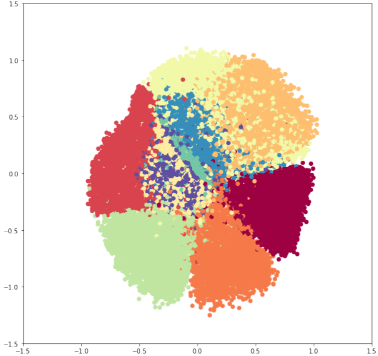
$$R_{\text{encoder}}(\boldsymbol{x}) = \min_{\boldsymbol{z}} \|\boldsymbol{x} - G(\boldsymbol{z})\|_p^p$$

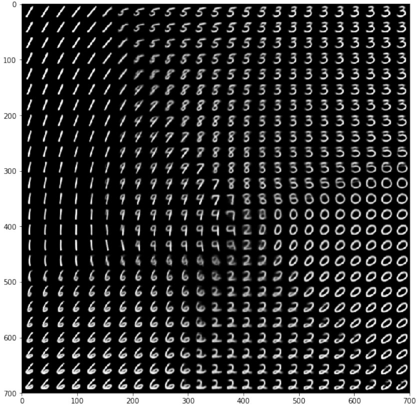
Expensive non-convex double minimization, but more robust to encoder?

From jupyter notebook for [20] (13 layer CNN with $\approx$ 300K learned parameters) at

`https://github.com/skolouri/swae/blob/master/MNIST_SlicedWassersteinAutoEncoder_Circle.ipynb`

$\boldsymbol{z} \in \mathbb{R}^2 \quad \mapsto \quad \boldsymbol{x} = G(\boldsymbol{z}) \in \mathbb{R}^{28 \times 28}$



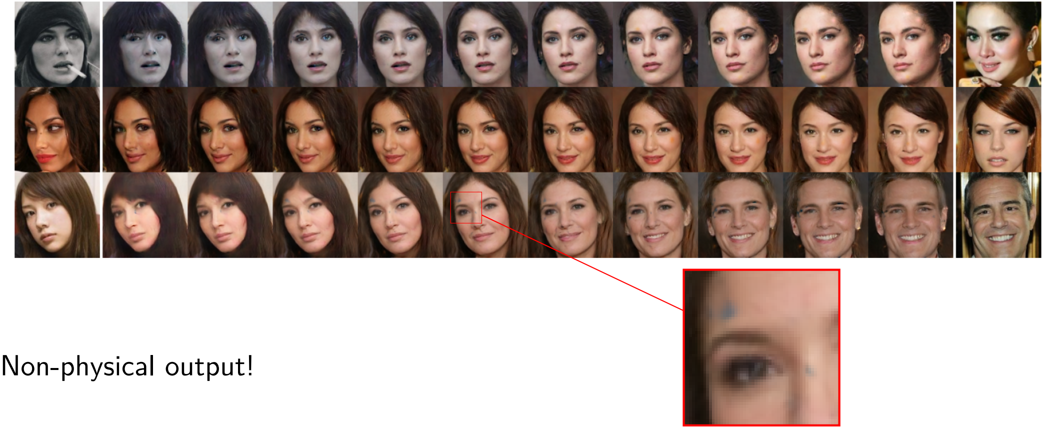


Where is 4?

From Google's [21]:

Much more realistic than linear interpolation (averaging).
"setting a new milestone in visual quality" [21].

From Google's [21]:



Non-physical output!

Outline

- Data
  - Population adaptive methods (*e.g.*, X-ray CT)
  - Patient adaptive methods (*e.g.*, dynamic MRI?)
- Spatial structure
  - Patch-based models
  - Convolutional models
- Regularizer formulation
  - Synthesis (dictionary) approach
  - Analysis (sparsifying transforms) approach

Many options...

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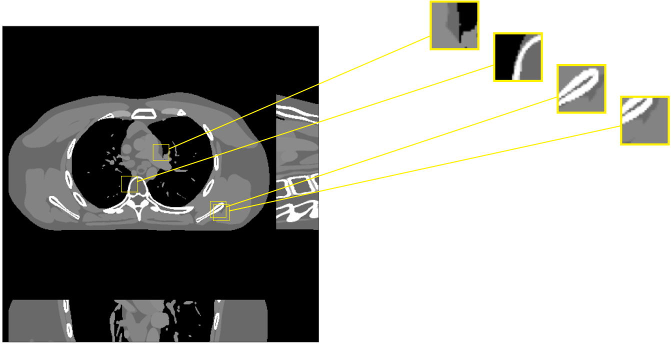
Patch-wise transform sparsity model

Assumption: if $\boldsymbol{x}$ is a plausible image, then each $\boldsymbol{\Omega} \boldsymbol{P}_m \boldsymbol{x}$ is sparse.

- $\boldsymbol{P}_m \boldsymbol{x}$ extracts the mth of M patches from $\boldsymbol{x}$
- $\boldsymbol{\Omega}$ is a square sparsifying transform matrix



Sparsifying transform learning (population adaptive)

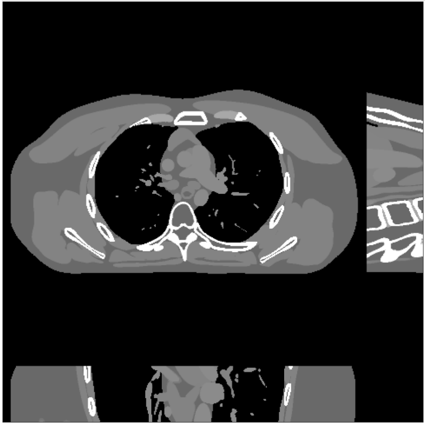
Given training images $\boldsymbol{x}_1, \ldots, \boldsymbol{x}_L$ from a representative population, find transform $\boldsymbol{\Omega}_*$ that best sparsifies their patches:

$$\boldsymbol{\Omega}_* = \underset{\boldsymbol{\Omega} \text{ unitary}}{\arg\min} \min_{\{\boldsymbol{z}_{l,m}\}} \sum_{l=1}^{L} \sum_{m=1}^{M} \|\boldsymbol{\Omega} \boldsymbol{P}_m \boldsymbol{x}_l - \boldsymbol{z}_{l,m}\|_2^2 + \alpha \|\boldsymbol{z}_{l,m}\|_0$$

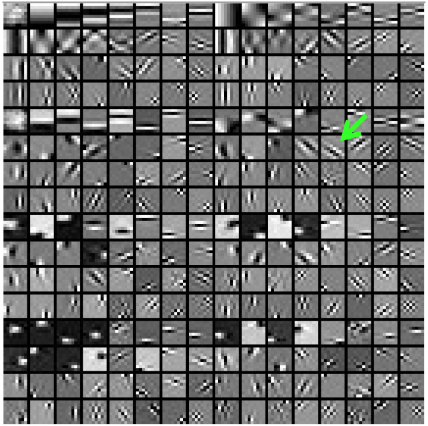
- Encourage aggregate sparsity, not patch-wise sparsity (cf K-SVD [22])
- Non-convex due to unitary constraint and $\|\cdot\|_0$
- Efficient alternating minimization algorithm [23]
  - $\boldsymbol{z}$ update is simply hard thresholding
  - $\boldsymbol{\Omega}$ update is an orthogonal Procrustes problem (SVD)
  - Subsequence convergence guarantees [23]

Example of learned sparsifying transform

3D X-ray training data



Parts of learned sparsifier $\boldsymbol{\Omega}_*$



(2D slices in x-y, x-z, y-z, from 3D image volume)

$8 \times 8 \times 8$ patches $\Longrightarrow$ $\boldsymbol{\Omega}_*$ is $8^3 \times 8^3 = 512 \times 512$

top 8×8 slice of 256 of the 512 rows of $\boldsymbol{\Omega}_*$ $\uparrow$

Regularized inverse problem [24]:

$$\hat{\boldsymbol{x}} = \arg\min_{\boldsymbol{x}} \|\boldsymbol{A}\boldsymbol{x} - \boldsymbol{y}\|_{\boldsymbol{W}}^2 + \beta\, \mathsf{R}(\boldsymbol{x})$$

$$\mathsf{R}(\boldsymbol{x}) = \min_{\{\boldsymbol{z}_m\}} \sum_{m=1}^{M} \|\boldsymbol{\Omega}_* \boldsymbol{P}_m \boldsymbol{x} - \boldsymbol{z}_m\|_2^2 + \alpha \|\boldsymbol{z}_m\|_0 .$$

$\boldsymbol{\Omega}_*$ adapted to population training data

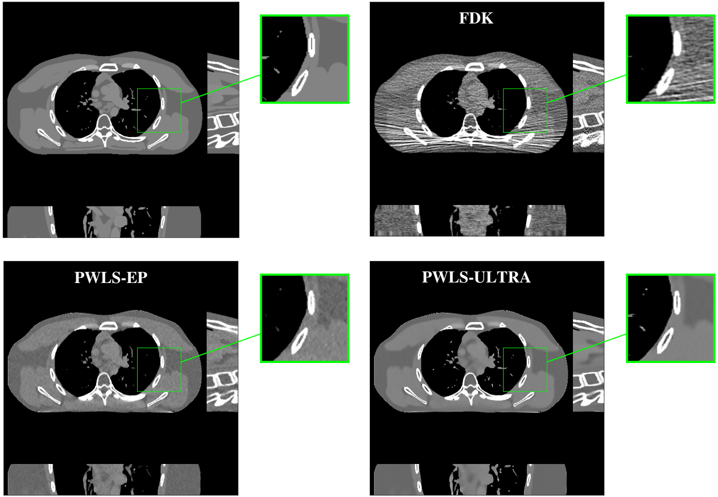
Alternating minimization optimizer:

- $\boldsymbol{z}_m$ update is simple hard thresholding
- $\boldsymbol{x}$ update is a quadratic problem: many options

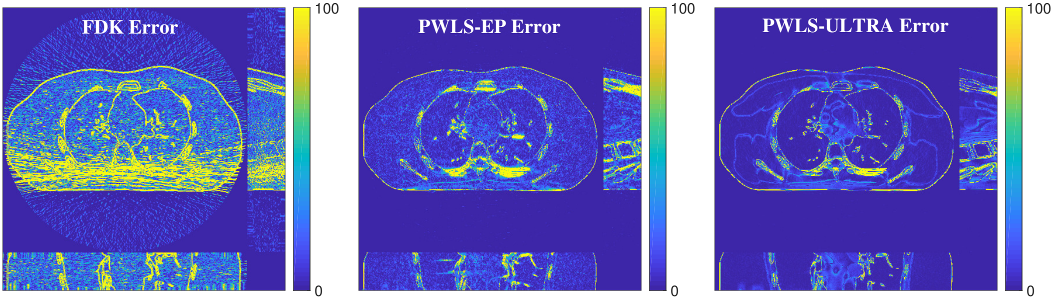
  Linearized augmented Lagrangian method (LALM) [25]

Example: low-dose 3D X-ray CT simulation

X. Zheng, S. Ravishankar, Y. Long, JF:
IEEE T-MI, June 2018 [24]



3D X-ray CT simulation Error maps



	X-ray Intensity	FDK	EP	ST Ω_*	ULTRA	ULTRA-$\{\tau_j\}$
RMSE in HU	1×10^4	67.8	34.6	32.1	30.7	**29.2**
	5×10^3	89.0	41.1	37.3	35.7	**34.2**

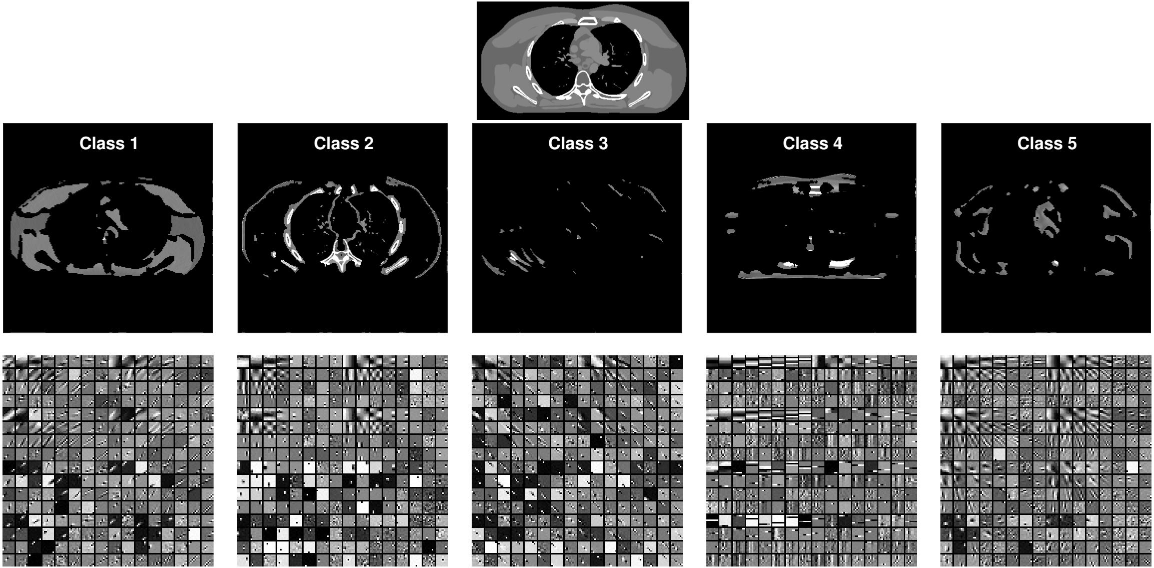
- Physics / statistics provides dramatic improvement
- Data adaptive regularization further reduces RMSE

Given training images $\boldsymbol{x}_1, \ldots, \boldsymbol{x}_L$ from a representative population, find a set of transforms $\left\{\hat{\boldsymbol{\Omega}}_k\right\}_{k=1}^{K}$ that best sparsify image patches:

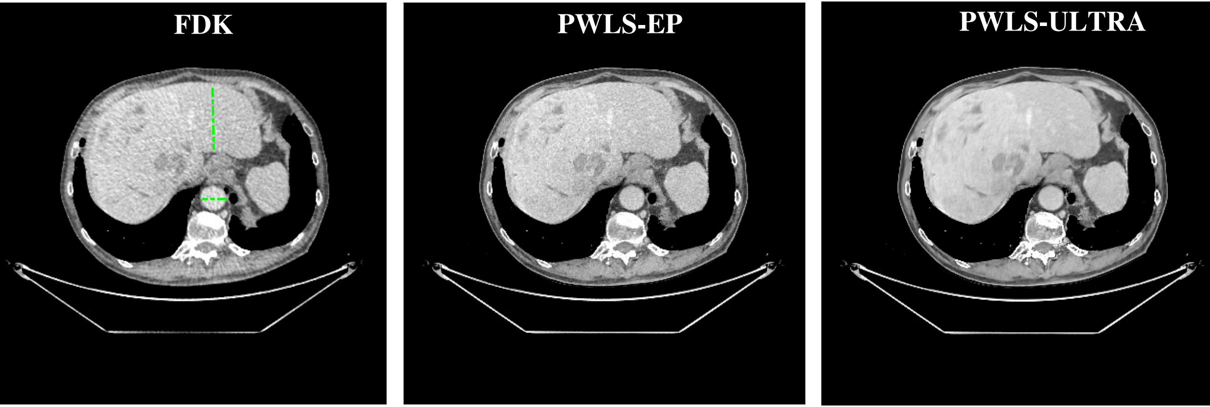
$$\left\{\hat{\boldsymbol{\Omega}}_k\right\} = \underset{\{\boldsymbol{\Omega}_k \text{ unitary}\}}{\arg\min} \; \min_{\left\{k_{l,m}\in\{1,\ldots,K\}\right\}} \; \min_{\left\{\boldsymbol{z}_{l,m}\right\}} \sum_{l=1}^{L} \sum_{m=1}^{M} \left\| \boldsymbol{\Omega}_{k_{l,m}} \boldsymbol{P}_m \boldsymbol{x}_l - \boldsymbol{z}_{l,m} \right\|_2^2 + \alpha \left\| \boldsymbol{z}_{l,m} \right\|_0$$

- Joint unsupervised clustering / sparsification
- Further nonconvexity due to clustering
- Efficient alternating minimization algorithm [26]

Example: 3D X-ray CT learned set of transforms



Example: 3D X-ray CT ULTRA for chest scan



Zheng et al., IEEE T-MI, June 2018 [24]

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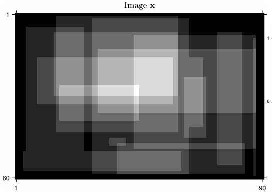
Drawback of basic patch-based methods:
$512 \times 512 \times 512$ 3D X-ray CT image volume
$8 \times 8 \times 8$ patches
$\Longrightarrow 512^3 \cdot 8^3 \cdot 4 = 256$ Gbyte of patch data for stride=1

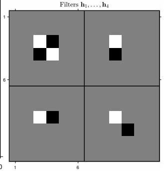
Convolutional sparsity model

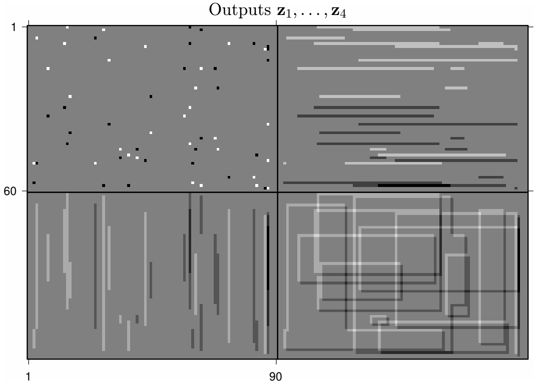
Assumption: There is a set of filters $\{\boldsymbol{h}_k\}_{k=1}^K$ such that the images $\{\boldsymbol{h}_k * \boldsymbol{x}\}$ are sparse for a plausible image $\boldsymbol{x}$.

- For more plausible images, $\{\boldsymbol{h}_k * \boldsymbol{x}\}$ is more sparse.
- $*$ denotes convolution
- Inherently shift invariant and no patches

Example (hand crafted filters):







Sparsifying filter learning (population adaptive)

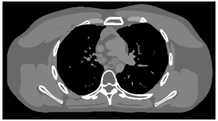
Given training images $\boldsymbol{x}_1, \ldots, \boldsymbol{x}_L$ from a representative population, find filters $\left\{ \hat{\boldsymbol{h}}_k \right\}_{k=1}^{K}$ that best sparsify them:

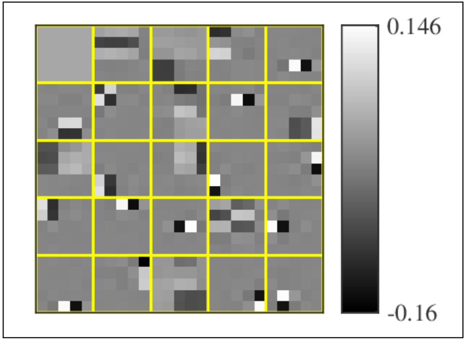
$$\left\{ \hat{\boldsymbol{h}}_k \right\} = \underset{\{\boldsymbol{h}_k\} \in \mathcal{H}}{\arg\min} \; \min_{\{\boldsymbol{z}_{l,k}\}} \sum_{l=1}^{L} \sum_{k=1}^{K} \left\| \boldsymbol{h}_k * \boldsymbol{x}_l - \boldsymbol{z}_{l,k} \right\|_2^2 + \alpha \left\| \boldsymbol{z}_{l,k} \right\|_0$$

- To encourage filter diversity:
  - $\mathcal{H} = \{ \boldsymbol{H} : \boldsymbol{H}\boldsymbol{H}' = \boldsymbol{I} \}, \; \boldsymbol{H} = [\boldsymbol{h}_1 \; \ldots \; \boldsymbol{h}_K]$
  - *cf.* tight-frame condition $\sum_{k=1}^{K} \|\boldsymbol{h}_k * \boldsymbol{x}\|_2^2 \propto \|\boldsymbol{x}\|_2^2$
- Encourage aggregate sparsity, period
- Non-convex due to constraint $\mathcal{H}$ and $\|\cdot\|_0$
- Efficient alternating minimization algorithm [27]
  - $\boldsymbol{z}$ update is simply hard thresholding
  - Filter update uses diagonal majorizer, proximal map (SVD)
  - Subsequence convergence guarantees [27]

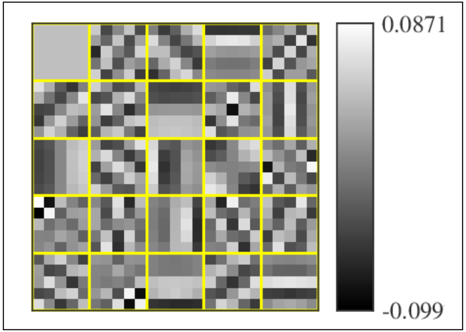
Examples of learned sparsifying filters

2D X-ray CT training data and learned 5×5 sparsifying filters $\left\{\hat{\boldsymbol{h}}_k\right\}$ [27]:





$\alpha = 10^{-4}$



$\alpha = 2 \times 10^{-3}$

Regularizer based on learned sparsifying filters

Regularized inverse problem [27]:

$$\hat{\boldsymbol{x}} = \underset{\boldsymbol{x} \succeq \boldsymbol{0}}{\arg\min} \|\boldsymbol{A}\boldsymbol{x} - \boldsymbol{y}\|_{\boldsymbol{W}}^2 + \beta \, \mathsf{R}(\boldsymbol{x})$$

$$\mathsf{R}(\boldsymbol{x}) = \underset{\{\boldsymbol{z}_k\}}{\arg\min} \sum_{k=1}^{K} \left\| \hat{\boldsymbol{h}}_k * \boldsymbol{x} - \boldsymbol{z}_k \right\|_2^2 + \alpha \|\boldsymbol{z}_k\|_0 \,.$$

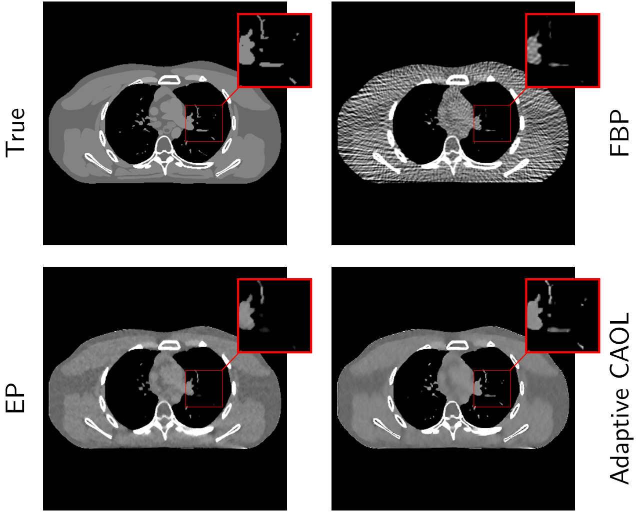
$\left\{ \hat{\boldsymbol{h}}_k \right\}$ adapted to population training data

Block proximal gradient with majorizer (BPG-M) optimizer:

- $\boldsymbol{z}_k$ update is simple hard thresholding
- $\boldsymbol{x}$ update is a quadratic problem: diagonal majorizer

I. Y. Chun, JF, 2018, arXiv 1802.05584 [27]

Example: sparse-view 2D X-ray CT simulation



Quantitative results

123 views (out of usual 984) $\Longrightarrow$ $8\times$ dose reduction

RMSE (in HU):

FBP	82.8
EP	40.8
Adaptive filters	**35.2**

- Physics / statistics provides dramatic improvement
- Data-adaptive regularization further reduces RMSE

Extension to multiple layers (cf CNN) I

Convolutional sparsity model: $\boldsymbol{h}_k * \boldsymbol{x}$ is sparse for $k = 1, \ldots, K_1$

Learning 1 "layer" of filters:

$$\{\hat{\boldsymbol{h}}_k^{[1]}\} = \operatorname*{arg\,min}_{\{\boldsymbol{h}_k^{[1]}\} \in \mathcal{H}} \min_{\{\boldsymbol{z}_{l,k}^{[1]}\}} \sum_{l=1}^{L} \sum_{k=1}^{K_1} \left\| \boldsymbol{h}_k^{[1]} * \boldsymbol{x}_l - \boldsymbol{z}_{l,k}^{[1]} \right\|_2^2 + \alpha \left\| \boldsymbol{z}_{l,k}^{[1]} \right\|_0$$

Learning 2 layers of filters [27]:

$$\left(\{\hat{\boldsymbol{h}}_k^{[1]}\}, \{\hat{\boldsymbol{h}}_k^{[2]}\}\right) = \mathop{\arg\min}_{\{\boldsymbol{h}_k^{[1]}\},\{\boldsymbol{h}_k^{[2]}\}\in\mathcal{H}} \min_{\{\boldsymbol{z}_{l,k}^{[1]}\}} \min_{\{\boldsymbol{z}_{l,k}^{[2]}\}} \sum_{l=1}^{L}\sum_{k=1}^{K_1} \left\|\boldsymbol{h}_k^{[1]} * \boldsymbol{x}_l - \boldsymbol{z}_{l,k}^{[1]}\right\|_2^2 + \alpha \left\|\boldsymbol{z}_{l,k}^{[1]}\right\|_0 + \sum_{l=1}^{L}\sum_{k=1}^{K_2} \left\|\boldsymbol{h}_k^{[2]} * \left(\boldsymbol{P}_k \boldsymbol{z}_l^{[1]}\right) - \boldsymbol{z}_{l,k}^{[2]}\right\|_2^2 + \alpha \left\|\boldsymbol{z}_{l,k}^{[2]}\right\|_0$$

Here $\boldsymbol{P}_k$ is a pooling operator for the output of first layer

Block proximal gradient with majorizer (BPG-M) optimizer

I. Y. Chun, JF, 2018, arXiv 1802.05584 [27]

Use multi-level learned filters as (interpretable?) regularizer for CT.

Outline

MR with adapted dictionary

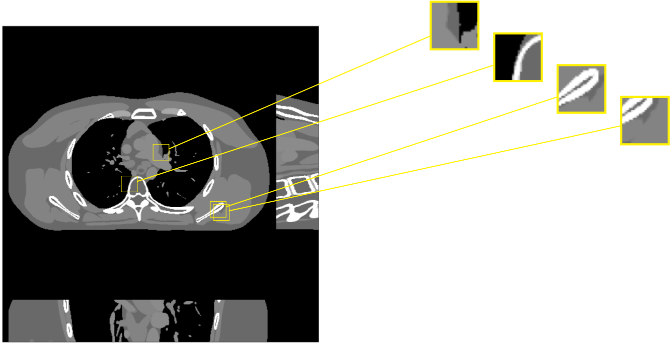
- Data
  - Population adaptive methods
  - Patient adaptive methods
- Spatial structure
  - Patch-based models
  - Convolutional models
- Regularizer formulation
  - Synthesis (dictionary) approach
  - Analysis (sparsifying transform) approach

Assumption: if $\boldsymbol{x}$ is a plausible image, then each patch has

$$\boldsymbol{P}_m \boldsymbol{x} \approx \boldsymbol{D} \boldsymbol{z}_m,$$

for a sparse coefficient vector $\boldsymbol{z}_m$. (Synthesis approach.)

- $\boldsymbol{P}_m \boldsymbol{x}$ extracts the mth of M patches from $\boldsymbol{x}$
- $\boldsymbol{D}$ is a (typically overcomplete) dictionary for patches



Dictionary-blind MR image reconstruction:

$$\hat{\boldsymbol{x}} = \arg\min_{\boldsymbol{x}} \frac{1}{2} \|\boldsymbol{y} - \boldsymbol{A}\boldsymbol{x}\|_2^2 + \beta\, \mathsf{R}(\boldsymbol{x})$$

$$\mathsf{R}(\boldsymbol{x}) = \min_{\boldsymbol{D} \in \mathcal{D}} \min_{\boldsymbol{Z}' \in \mathcal{C}} \sum_{m=1}^{M} \left(\|\boldsymbol{P}_m \boldsymbol{x} - \boldsymbol{D}\boldsymbol{z}_m\|_2^2 + \lambda^2 \|\boldsymbol{z}_m\|_0 \right)$$

where $\boldsymbol{P}_m$ extracts mth of M image patches.

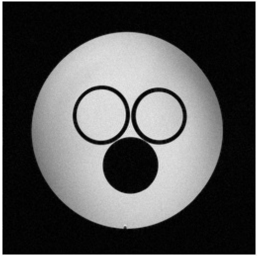
In words: of the many images...

Alternating (nested) minimization:

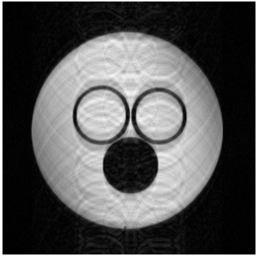
- Fixing $\boldsymbol{x}$ and $\boldsymbol{D}$, update each row of $\boldsymbol{Z} = [\boldsymbol{z}_1 \ \ldots \ \boldsymbol{z}_M]$ sequentially via hard-thresholding.
- Fixing $\boldsymbol{x}$ and $\boldsymbol{Z}$, update $\boldsymbol{D}$ using SOUP-DIL [28].
- Fixing $\boldsymbol{Z}$ and $\boldsymbol{D}$, updating $\boldsymbol{x}$ is a quadratic problem.
  - Efficient FFT solution for single-coil Cartesian MRI.
  - Use CG for non-Cartesian and/or parallel MRI.
- Non-convex, but monotone decreasing and some convergence theory [28].

2D CS MRI results I

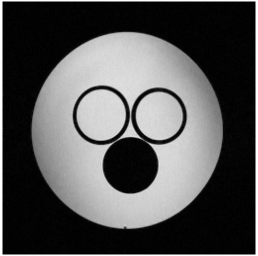
Fully Sampled



Zero-Filled



SOUP-DILLO-MRI

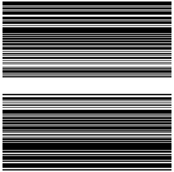


6×6 patches

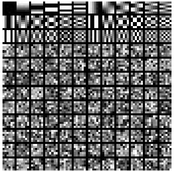
$\boldsymbol{D} \in \mathbb{C}^{6^2 \times 144}$

[28]

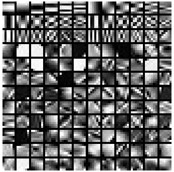
Sampling ($2.5\times$)



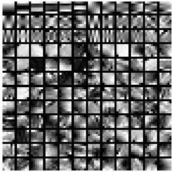
Initial $\boldsymbol{D}$

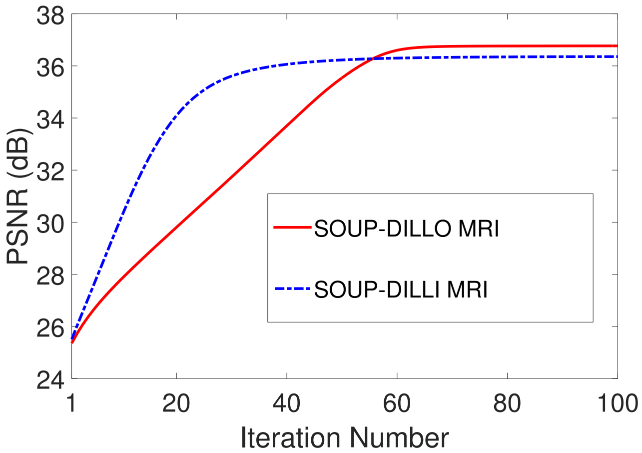


Learned real$\{\boldsymbol{D}\}$



imag$\{\boldsymbol{D}\}$



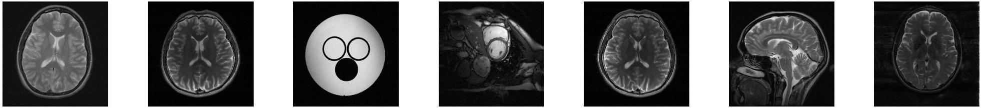


(SNR compared to fully sampled image.)

Using $\|\boldsymbol{z}_m\|_0$ leads to higher SNR than $\|\boldsymbol{z}_m\|_1$.

Adaptive case is non-convex anyway...

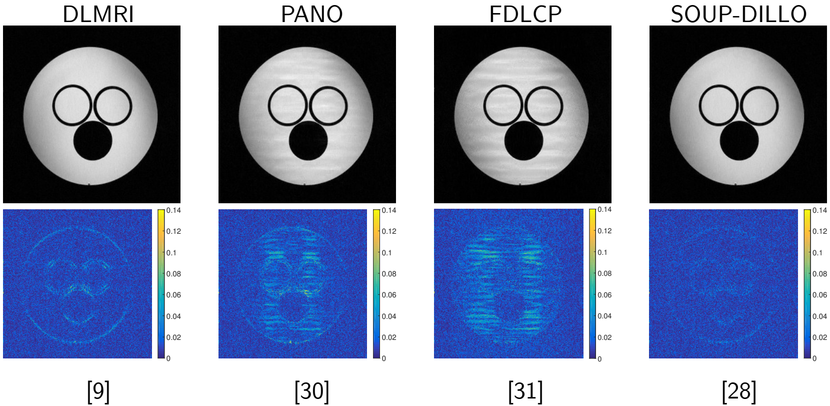
2D CS MRI results III



(a) (b) (c) (d) (e) (f) (g)

Im.	Samp.	Acc.	0-fill	Sparse MRI	PANO	DLMRI	SOUP-DILLI	SOUP-DILLO
a	Cart.	7x	27.9	28.6	**31.1**	**31.1**	30.8	**31.1**
b	Cart.	2.5x	27.7	31.6	41.3	40.2	38.5	**42.3**
c	Cart.	2.5x	24.9	29.9	34.8	36.7	36.6	**37.3**
c	Cart.	4x	25.9	28.8	**32.3**	32.1	32.2	**32.3**
d	Cart.	2.5x	29.5	32.1	36.9	38.1	36.7	**38.4**
e	Cart.	2.5x	28.1	31.7	40.0	38.0	37.9	**41.5**
f	2D rand.	5x	26.3	27.4	30.4	30.5	30.3	**30.6**
g	Cart.	2.5x	32.8	39.1	41.6	41.7	42.2	**43.2**
Ref.				[29]	[30]	[9]	[28]	[28]

2D CS MRI results IV



Summary: 2D static MR reconstruction from under-sampled data with adaptive dictionary learning and convergent algorithm, faster than K-SVD approach of DLMRI.

Summary of patch-based, data-driven adaptive regularizers

Use training data to learn:

- dictionary $\boldsymbol{D}$ (for patches)
- sparsifying transform(s) $\boldsymbol{\Omega}$ (for patches)
- or convolutional versions thereof [32, 33]

ML-based regularized optimization problem using M image patches:

$$\hat{\boldsymbol{x}} = \arg\min_{\boldsymbol{x}} \|\boldsymbol{A}\boldsymbol{x} - \boldsymbol{y}\|_2^2 + \beta R_{\mathrm{ML}}(\boldsymbol{x})$$

$$R_{\mathrm{ML-DL}}(\boldsymbol{x}) = \min_{\{\boldsymbol{z}_m\}} \sum_{m=1}^{M} \|\boldsymbol{P}_m \boldsymbol{x} - \boldsymbol{D}\boldsymbol{z}_m\|_2^2 + \alpha \|\boldsymbol{z}_m\|_0$$

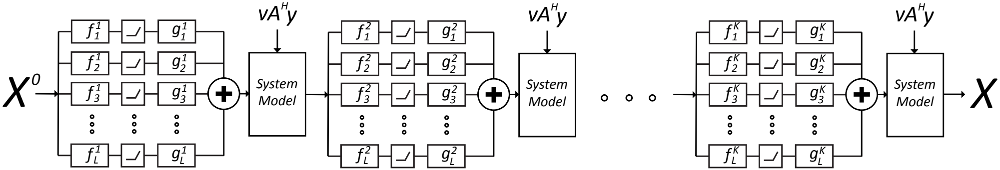
$$R_{\mathrm{ML-ST}}(\boldsymbol{x}) = \min_{\{\boldsymbol{z}_m\}} \sum_{m=1}^{M} \|\boldsymbol{\Omega}\boldsymbol{P}_m \boldsymbol{x} - \boldsymbol{z}_m\|_2^2 + \alpha \|\boldsymbol{z}_m\|_0$$

Alternative: blind adaptive learned dictionary [9] or learned sparsifying transform [34].
Double minimization (so very "deep?") More interpretable than CNNs?

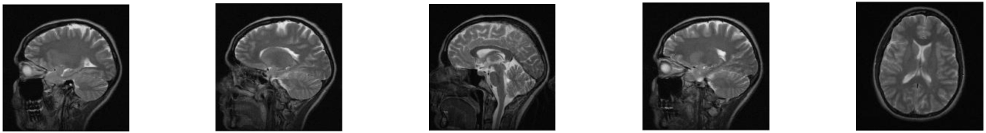
Training an unrolled loop I

Unrolled loop method with 20 layers trained with $1.3 \cdot 10^6$ MR image 8×8 patches

Ravishankar et al., ISBI 2018 [15]



Tested with 5 different MR images:



Results:

Undersampling	Image	Zero-filled	Sparse MRI	UTMRI	Unrolled
3.3×	1	25.6	26.7	28.3	28.2
	2	25.2	26.6	27.9	27.8
	3	26.0	27.3	29.3	28.9
	4	25.4	26.7	28.2	28.1
	5	27.2	28.9	30.6	30.3
Avg. PSNR change	-	-	**1.36**	**2.98**	**2.78**
5×	1	24.7	25.9	27.6	27.5
	2	24.2	25.5	27.2	27.0
	3	24.9	26.3	28.5	28.0
	4	24.4	25.7	27.6	27.4
	5	26.2	27.9	29.8	29.5
Avg. PSNR change	-	-	**1.38**	**3.26**	**3.0**
Approx recon time	-	-	**100s**	**240s**	**50s**

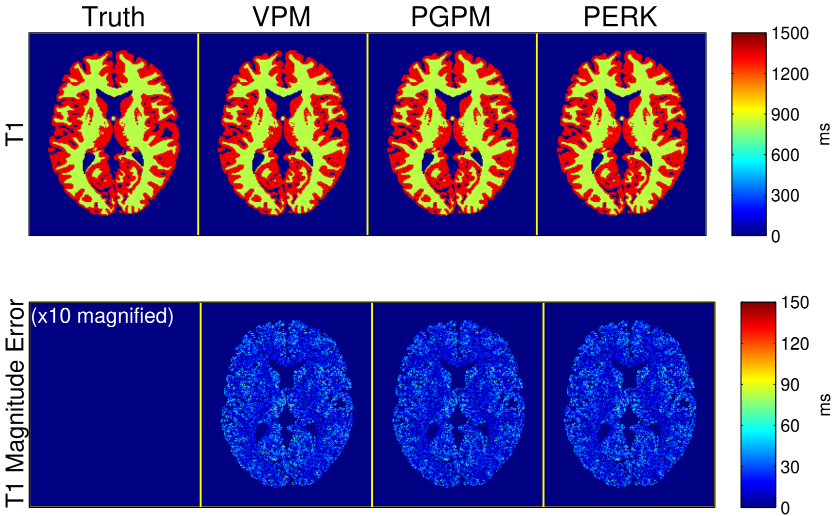
Sparse MRI [35] total variation (TV) and wavelets
UTMRI [26] (union of learned sparsifying transforms): adaptive, not "deep"

Shallow machine learning for qMRI

Quantitative MRI: images $\rightarrow$ $\boxed{\text{estimation}}$ $\rightarrow$ parameters (T1,T2,...)

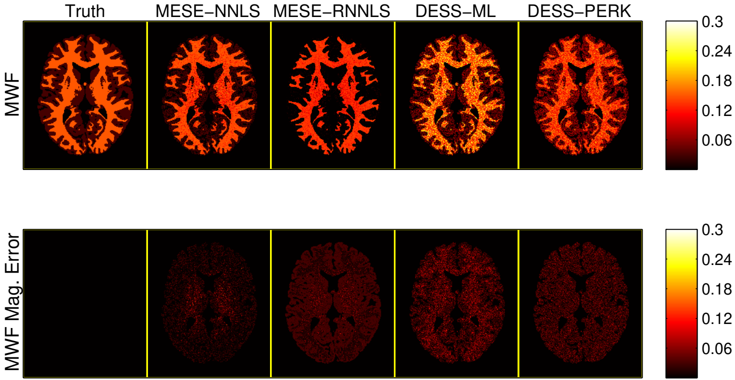
- Traditional nonlinear estimation methods:
  - nonlinear least squares
  - dictionary matching (quantized maximum likelihood via variable projection)
- Machine-learning methods
  - deep neural network regression [36–39]
    Requires long training times
  - parameter estimation via kernel regression (PERK)
    Gopal Nataraj et al., ISBI 2017, IEEE T-MI 2018 [40, 41]

Parameter estimation via kernel regression (PERK) example



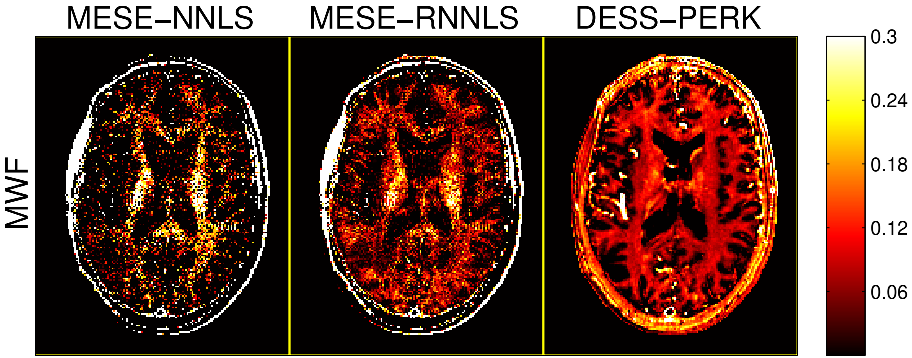
PERK applied to myelin water imaging

6 parameters (T1 slow/fast, T2 slow/fast, M_0, fast fraction)
Estimated from 3 optimized dual-echo steady state (DESS) scans [42]



PERK training: 33.8s, testing 0.99s / slice

PERK in vivo myelin water fraction results



MESE scan took 32m (16m $\times$2)
DESS scan took 3m15s
Take away: "traditional" machine learning is still useful...

Summary

- Machine learning has great potential for medical imaging
- Much excitement but many challenges
- Image reconstruction seems especially suitable for ML ideas
- Data-driven, adaptive regularizers beneficial for low-dose CT and under-sampled MRI
- More comparisons between model-based methods with adaptive regularizers and CNN-based methods needed
- Machine learning tools like kernel regression remain useful

Recommended reading (incomplete lists)

- Overviews: [43–45]
- Generative models: [20, 46]:
- Deep learning myths [47]
- NN complexity analysis / function approximation [48–50] [51]
- Application to MR fingerprinting [36, 39]
- MR reconstruction / enhancement using CNN [16, 52–59]
- Dynamic MR reconstruction using CNN [60]
- ...

Resources

Talk and code available online at
`http://web.eecs.umich.edu/~fessler`

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