Open problems in signal processing: Medical imaging

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Medical imaging overview

System (sensor) $\xrightarrow[\boldsymbol{y}]{\text{Data}}$ Image reconstruction $\xrightarrow[\hat{\boldsymbol{x}}]{\text{Images}}$ Image processing $\rightarrow$?

- Image reconstruction goals
  - Produce "better" images from same data
  - Produce "good" images from less data or noisier data (*cf.* data used by conventional algorithms)
- Image reconstruction challenges
  - Accurate physics/statistics models for system/sensor
  - Best/suitable signal models
  - Fast computation / optimization
  - Characterization / performance guarantees
- Image processing goals and challenges
  - ?

Model-based image reconstruction

$$\xrightarrow[\boldsymbol{x}]{\text{Object}} \boxed{\text{System}} \xrightarrow[\boldsymbol{y}]{\text{Data}} \boxed{\text{Estimator}} \xrightarrow[\hat{\boldsymbol{x}}]{\text{Image}}$$

Typical "modern" formulation (MBIR) [1–5]:

$$\hat{\boldsymbol{x}} = \arg\min_{\boldsymbol{x}} \Psi(\boldsymbol{x}), \quad \Psi(\boldsymbol{x}) = -\log \mathrm{p}(\boldsymbol{y} \,|\, \boldsymbol{x}) + \mathrm{R}(\boldsymbol{x})$$

"Bayesian / variational / statistical / regularized / iterative / ..."

Active research topics:

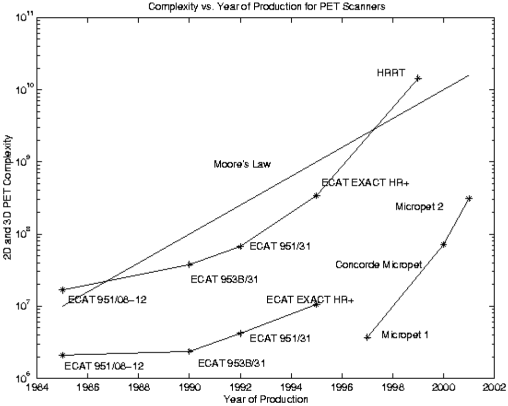
- $\mathrm{p}(\boldsymbol{y} \,|\, \boldsymbol{x})$: physics / statistics models (computation trade-offs)
- $\mathrm{R}(\boldsymbol{x})$: regularizer / prior information / signal models
- $\arg\min$: optimization algorithms
- $\hat{\boldsymbol{x}}$: characterization / performance guarantees

(Clinical "breakthrough" ≈20 years ago in PET, ≈4 in CT, ≈0 from now in MRI)

Breakthroughs / impact I

What is the most important breakthrough in your field in the past 10 years and how did this affect your field?

1. Advances in computing power (but Moore's law insufficient)
   $\Longrightarrow$ Clinical adoption of MBIR methods in PET and CT.



(Courtesy of R. Leahy)

Breakthroughs / impact II

2. Advances in optimization algorithms
   - incremental gradients / ordered subsets [6–8]
   - non-smooth problems: (AL/ADMM, proximal splitting, majorization, ...) [9, 10]
   - momentum methods (*e.g.*, FISTA) [11, 12]
   - non-convex problems (...)

   $\Longrightarrow$ OS made MBIR for PET clinically feasible.

3. Signal models based on sparsity (compressed sensing ...) [13–20]

   $\Longrightarrow$ Emboldened research on highly under-sampled problems.

Dynamic MRI overview

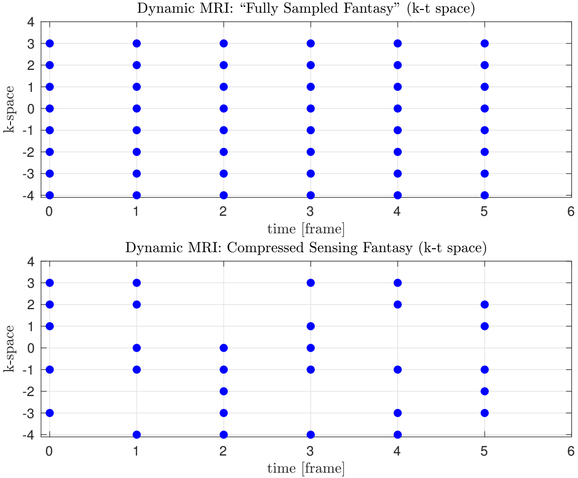
"dynamic" = changing over time = motion [21–24]

- Nuisance motions:
  - Breathing
  - Cardiac
  - Peristalsis
  - Tremors
  - Kids ...

  $\Longrightarrow$ Faster scans (shorter time) can help reduce motion blur
- Motions of interest (true "dynamic" scans):
  - Vocalization (for speech studies)
  - Cardiac (for function)
  - Joint articulation (musculoskeletal scans)
  - Contrast agent (blood flow / perfusion)
  - Diffusion

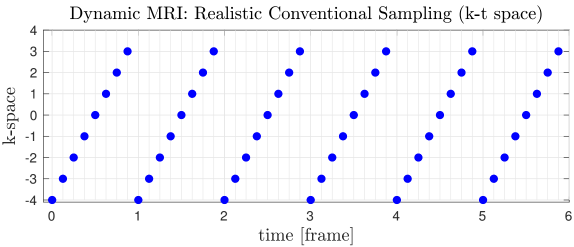
$\Longrightarrow$ Trade-offs between temporal resolution and spatial resolution

Dynamic MRI sampling: Fantasy edition



- Scan "twice as fast" !?
- Matrix completion problem!? $\Longrightarrow$... robust PCA (L+S) ...
  [25, 26]

Dynamic MRI sampling: Reality



- All 3D dynamic MRI data is *inherently under-sampled*
- No real "fully sampled" data exists, now or ever
- Unlikely to satisfy any "matrix completion" sufficient conditions
  (N measurements but N^2 unknowns per frame)
- Retrospective "under sampling" of "fully sampled" dynamic data is dubious
- Opportunity: powerful signal models needed for reconstruction from such data
- Challenge: validation of signal models given such highly incomplete data
  (low-rank / locally low rank / tensors / wavelets / non-local patches / ...)

Regularization / signal models

- Edge-preserving roughness penalties / Markov random fields:

$$\mathrm{R}(\boldsymbol{x}) = \beta \sum_{j=1}^{n_{\mathrm{p}}} \sum_{k \in \mathcal{N}_J} \psi(x_j - x_k) \,.$$

- Sparsity (analysis form): $\mathrm{R}(\boldsymbol{x}) = \beta \left\| \boldsymbol{W} \boldsymbol{x} \right\|_1 .$
- Sparsity (synthesis form): $\boldsymbol{x} = \boldsymbol{D} \boldsymbol{z}, \quad \|\boldsymbol{z}\|_0 \leq k$, $\boldsymbol{D}$ “known”

$$\mathrm{R}(\boldsymbol{x}) = \min_{\boldsymbol{z}} \beta \left\| \boldsymbol{x} - \boldsymbol{D} \boldsymbol{z} \right\|_2^2 + \alpha \left\| \boldsymbol{z} \right\|_p$$

- Sparsity of patches in adapted (learned) patch dictionary:

$$\mathrm{R}(\boldsymbol{x}) = \min_{\boldsymbol{D} \in \mathcal{D}} \min_{\boldsymbol{Z}} \; \beta \sum_k \left\| \boldsymbol{P}_k \boldsymbol{x} - \boldsymbol{D} \boldsymbol{z}_k \right\|_2^2 + \alpha \left\| \boldsymbol{Z} \right\|_p$$

- Dynamic problems: low-rank / locally low-rank, tensors, ...

Image reconstruction algorithm generations

- Analytical reconstruction methods (classical):
  idealized mathematical imaging system models [27]
  e.g., CT inverse Radon transform, MR inverse FFT

- Model-based image reconstruction ("recent"):
  - physics and statistics models
  - mathematical signal models

- Data-driven image reconstruction (emerging):
  parts of reconstruction algorithm learned from *training data*

Data-driven image reconstruction: transform learning

- Training stage
  Learn sparsifying transform $\hat{\boldsymbol{\Omega}}$
  from training data (image patches) $\{\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, \}$:

$$\hat{\boldsymbol{\Omega}} \triangleq \arg\min_{\boldsymbol{\Omega}} \min_{\boldsymbol{Z}} \sum_k \|\boldsymbol{\Omega} \boldsymbol{x}_k - \boldsymbol{z}_k\|_2^2 + \lambda \|\boldsymbol{Z}\|_p .$$

  Efficient methods with some convergence guarantees
  Sai Ravishankar & Yoram Bresler, 2012-2015 [28–37].

- Image reconstruction stage:

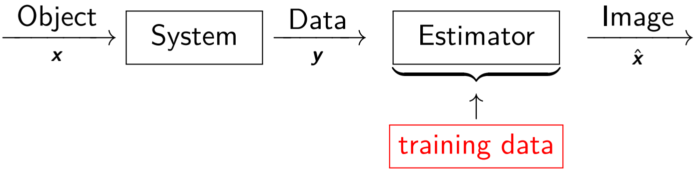
$$\arg\min_{\boldsymbol{x}} \Psi(\boldsymbol{x}), \quad \Psi(\boldsymbol{x}) \triangleq -\log \mathrm{p}(\boldsymbol{y} \mid \boldsymbol{x}) + \mathrm{R}(\boldsymbol{x})$$

$$\mathrm{R}(\boldsymbol{x}) \triangleq \min_{\boldsymbol{Z}} \sum_j \left\| \hat{\boldsymbol{\Omega}} \boldsymbol{P}_j \boldsymbol{x} - \boldsymbol{z}_j \right\|_2^2 + \lambda \|\boldsymbol{Z}\|_p .$$

  Regularizer based on training data [38, 39] (!)
- Adaptive (blind) versions [40–44]
- Synthesis (dictionary) variant [45–47]

Data-driven image reconstruction: algorithm design

New paradigm:

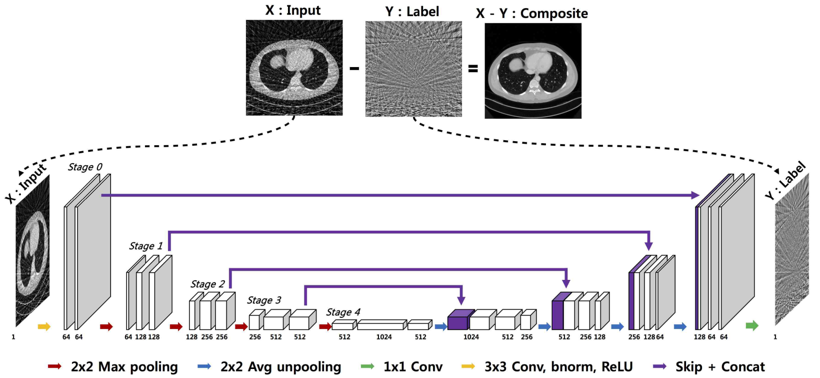


- Recent papers (mostly using "deep" convolutional neural networks): [48–57] ("deep imaging" ?)
- Many more to appear in 2017, *e.g.*, [58].

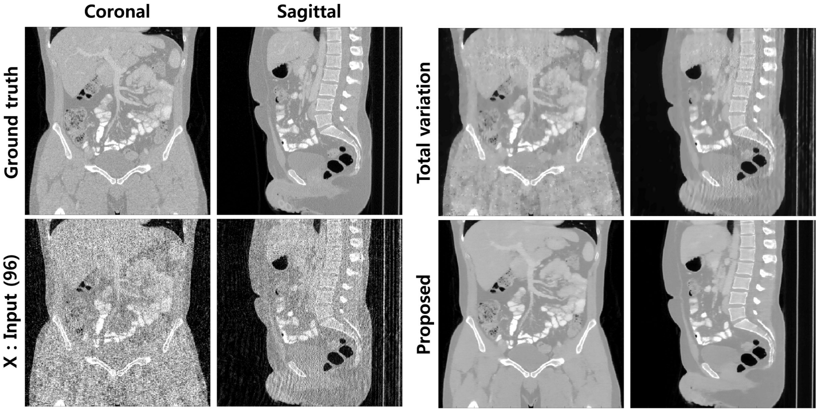
Sparse-view CT "reconstruction" with learned streak removal

Han et al. 2016 [51]

Streak-estimation stages learned from (fully sampled) training data



Data-driven image reconstruction: Results



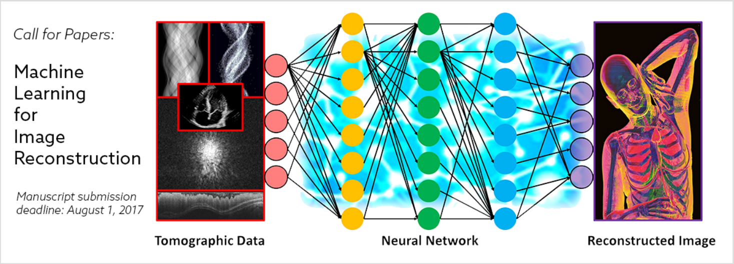
[51, Fig. 7], Han et al. 2016

Data-driven image reconstruction: Challenges

- Slow learning from training data: O(days)
  (But very fast processing after training, *cf.* iterative MBIR)
- Re-training for different imaging conditions
- Non-convexity / nonlinearity
- Characterization / performance guarantees

(Job security for signal processors...)

Special issue of IEEE Transactions on Medical Imaging:



https://ieee-tmi.org

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