Limits of dose reduction in CT: Statistical reconstruction methods

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- Meng Wu, Stanford
- ...

Why statistical/iterative methods for CT?

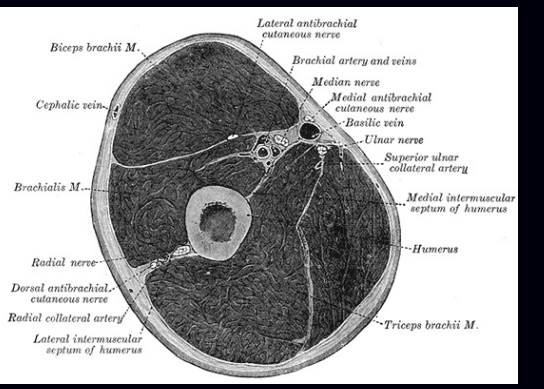
- Accurate physics models
  - X-ray spectrum, beam-hardening, scatter, ...
    $\Longrightarrow$ reduced artifacts? quantitative CT?
  - X-ray detector spatial response, focal spot size, ...
    $\Longrightarrow$ improved spatial resolution?
  - detector spectral response (*e.g.*, photon-counting detectors)
    $\Longrightarrow$ improved contrast?

- Nonstandard geometries
  - transaxial truncation (wide patients)
  - long-object problem in helical CT
  - irregular sampling in "next-generation" geometries
  - coarse angular sampling in image-guidance applications
  - limited angular range (tomosynthesis)
  - "missing" data, *e.g.*, bad pixels in flat-panel systems

- Appropriate models of (data dependent) measurement statistics
  - weighting reduces influence of photon-starved rays (*cf.* FBP)
    $\Longrightarrow$ reducing image noise or X-ray dose

and more...

- Object constraints / priors
  - nonnegativity
  - object support
  - piecewise smoothness
  - object sparsity (*e.g.*, angiography)
  - sparsity in some basis
  - motion models
  - dynamic models
  - ...



Henry Gray, Anatomy of the Human Body, 1918, Fig. 413.

These constraints may help reduce image artifacts or noise or dose.

Disadvantages?

- Computation time (super computer)
- Must reconstruct entire FOV
- Complexity of models and software
- Algorithm nonlinearities
  - Difficult to analyze resolution/noise properties (*cf.* FBP)
  - Tuning parameters
  - Challenging to characterize performance / assess image quality

"Iterative" vs "Statistical"

- Traditional *successive substitutions* iterations
  - *e.g.*, Joseph and Spital (JCAT, 1978) bone correction
  - usually only one or two "iterations"
  - not statistical

- Algebraic reconstruction methods
  - Given sinogram data $\boldsymbol{y}$ and system model $\boldsymbol{A}$, reconstruct object $\boldsymbol{x}$ by "solving" $\boldsymbol{y} = \boldsymbol{A}\boldsymbol{x}$
  - ART, SIRT, SART, ...
  - iterative, but typically not statistical $\Longrightarrow$ limited (if any) dose reduction
  - Iterative filtered back-projection (FBP):

$$\boldsymbol{x}^{(n+1)} = \boldsymbol{x}^{(n)} + \underbrace{\alpha}_{\text{step size}} \text{FBP}(\underbrace{\boldsymbol{y}}_{\text{data}} - \underbrace{\boldsymbol{A}\boldsymbol{x}^{(n)}}_{\text{forward project}})$$

- Statistical reconstruction methods now come in several flavors:
  - Image domain
  - Sinogram domain
  - Fully statistical (both)
  - Hybrid methods (*e.g.*, AIR, SPIE 7961-18, 2011, Bruder *et al.*)

  Examine these in more detail next to consider *dose*.

"Statistical" methods: Image domain

- Denoising methods

$$\text{sinogram } \boldsymbol{y} \rightarrow \boxed{\text{FBP}} \rightarrow \text{noisy reconstruction } \tilde{\boldsymbol{x}} \rightarrow \boxed{\text{iterative denoiser}} \rightarrow \text{final image } \hat{\boldsymbol{x}}$$

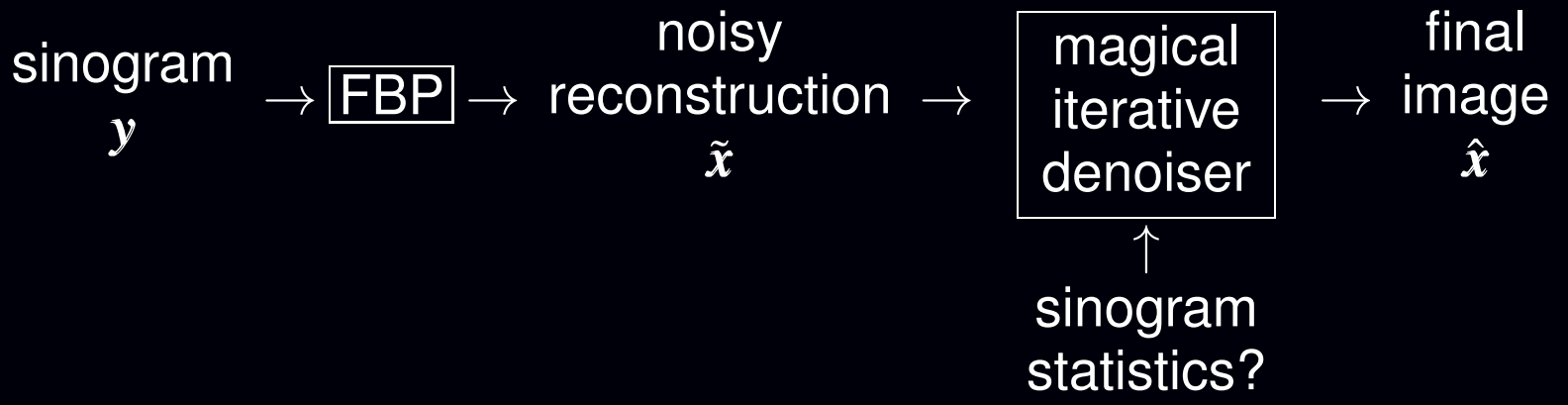
  - Relatively fast, even if iterative
  - Remarkable advances in denoising methods in last decade

Zhu & Milanfar, T-IP, Dec. 2010, using "steering kernel regression" (SKR) method

Challenges:

  - Typically assume *white noise*
  - Streaks in low-dose FBP appear like edges (highly correlated noise)

- Image denoising methods “guided by data statistics”



  - Image-domain methods are fast (thus very practical)
  - ASIR? IRIS? ...
  - The technical details are often a mystery...

Challenges:
  - FBP often does not use all data efficiently (*e.g.*, Parker weighting)
  - Low-dose CT statistics most naturally expressed in sinogram domain

Several studies of potential dose reduction with image-domain methods:
  - Hara *et al.*, AJR 193(3) 2009, 32-65 % dose reduction
  - Silva *et al.*, AJR 194(1) 2010, 50-65 % dose reduction
  - Leipsic *et al.*, AJR 195(3) 2010, 27 % dose reduction
  - ...

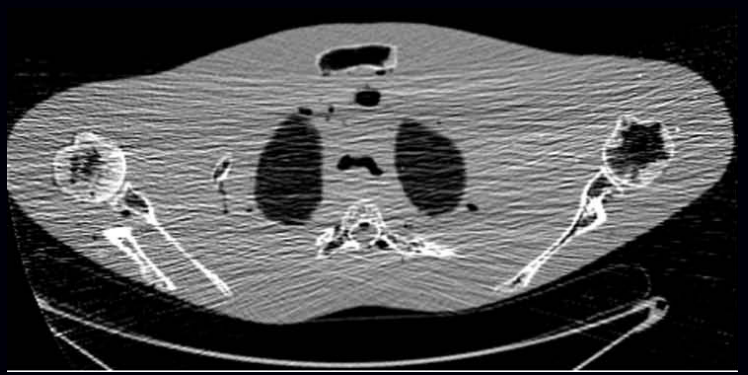
"Statistical" methods: Sinogram domain

- Sinogram restoration methods

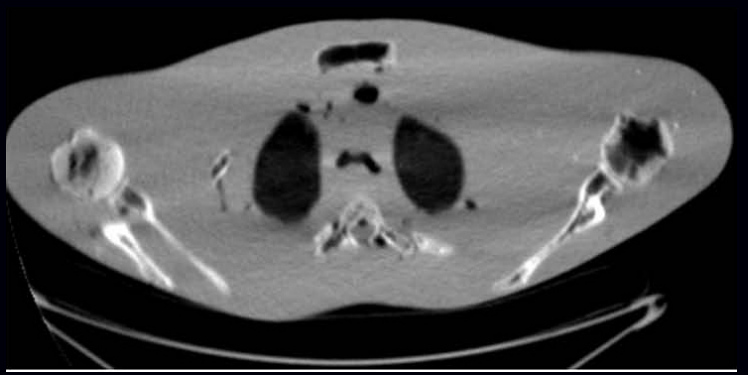
$$\underset{\boldsymbol{y}}{\text{noisy sinogram}} \rightarrow \boxed{\text{adaptive or iterative denoiser}} \rightarrow \underset{\hat{\boldsymbol{y}}}{\text{cleaned sinogram}} \rightarrow \boxed{\text{FBP}} \rightarrow \underset{\hat{\boldsymbol{x}}}{\text{final image}}$$

  - Adaptive: J. Hsieh, Med. Phys., 1998; Kachelrieß, Med. Phys., 2001, ...
  - Iterative: P. La Riviere, IEEE T-MI, 2000, 2005, 2006, 2008, ...
  - Relatively fast even if iterative

  Challenges:
  - Limited denoising without resolution loss $\Longrightarrow$ modest dose reduction?
  - Difficult to "preserve edges" in sinograms



FBP, 10 mA



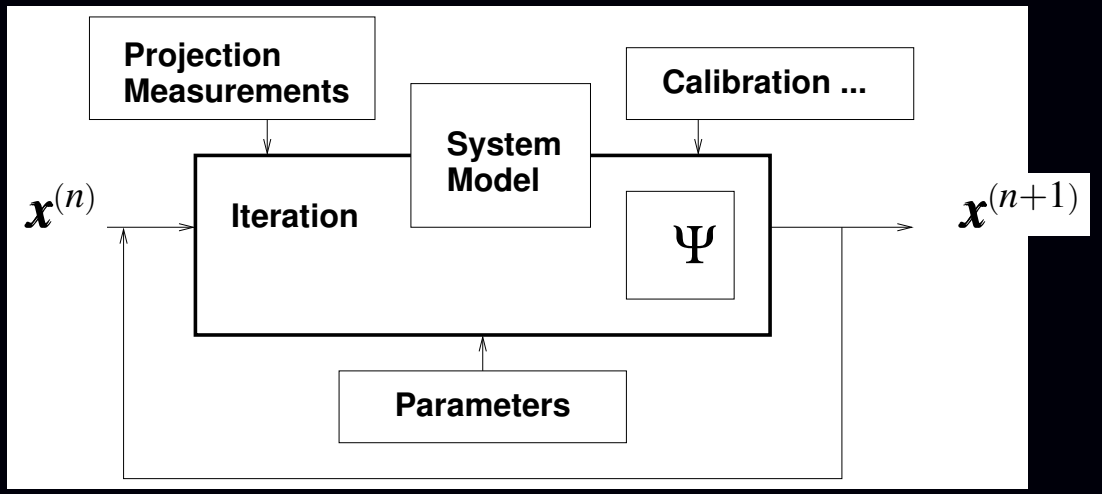
FBP from denoised sinogram

Wang *et al.*, T-MI, Oct. 2006, using PWLS-GS on sinogram

"Fully" (?) statistical image reconstruction

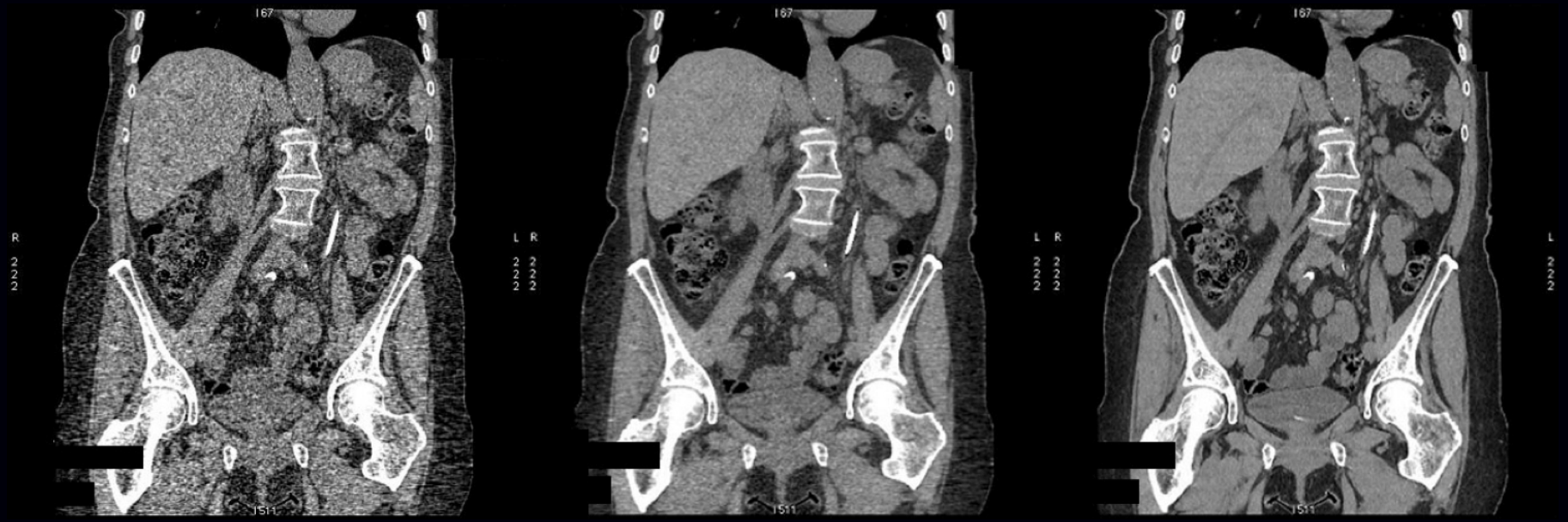
- Object model
- Physics/system model
- Statistical model
- Cost function (log-likelihood + regularization)
- Iterative algorithm for minimization

"Find the image $\hat{x}$ that best fits the sinogram data y according to the physics model, the statistical model and prior information about the object"



- Repeatedly revisiting the sinogram data can use measurement statistics fully
- Repeatedly updating the image can exploit object properties
- ∴ greatest potential dose reduction, but repetition is expensive...

Example comparison



Thin-slice FBP

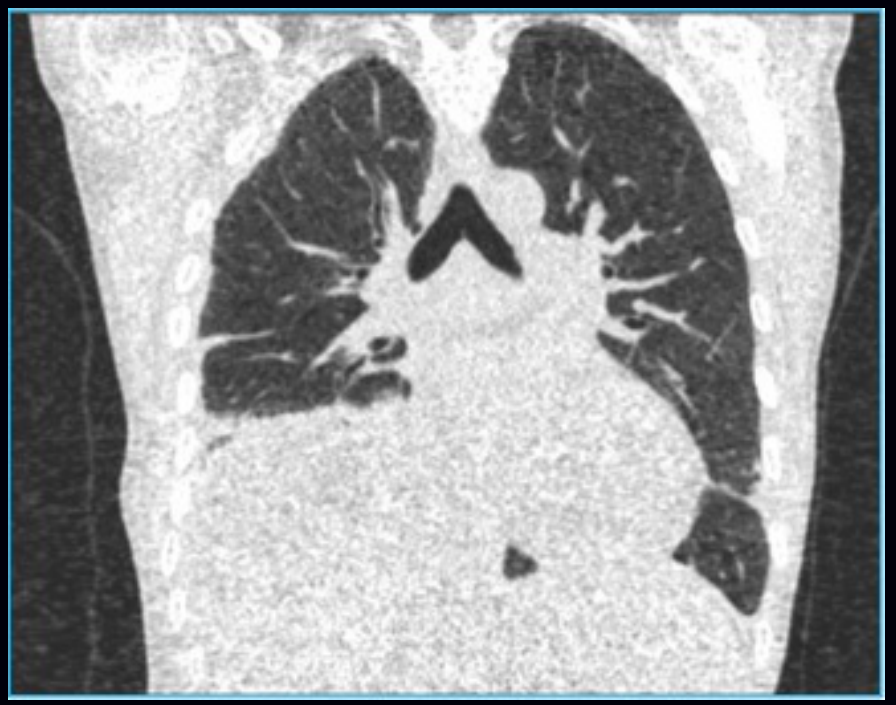
ASIR

Statistical

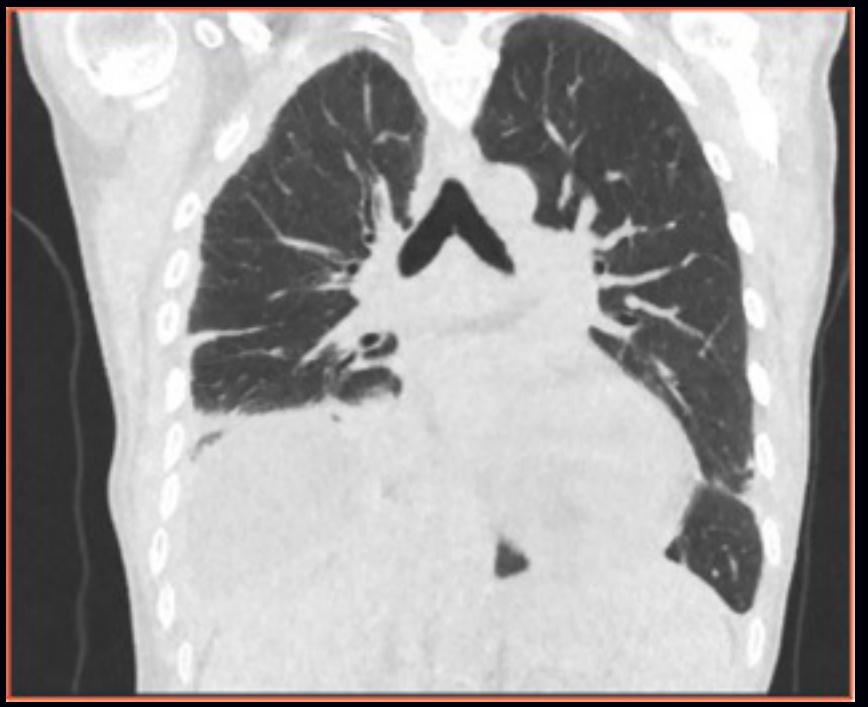
(Same sinogram, so all at same dose)

MBIR example: Routine chest CT

Helical chest CT study with dose = 0.09 mSv.
Typical CXR effective dose is about 0.06 mSv. Source: Health Physics Society.
http://www.hps.org/publicinformation/ate/q2372.html



FBP



MBIR

Veo (MBIR) images courtesy of Jiang Hsieh, GE Healthcare

Five Choices for Statistical Image Reconstruction

1. Object model
2. System physical model
3. Measurement statistical model
4. Cost function: data-mismatch and regularization
5. Algorithm / initialization

No perfect choices - one can critique all approaches!

Historically these choices are often left implicit in publications,
but being explicit facilitates reproducibility.

Next: overview of each choice, focusing on how they may affect dose.

Choice 1. Object Parameterization

Finite measurements: $\{y_i\}_{i=1}^{M}$. Continuous object: $f(\vec{r}) = \mu(\vec{r})$.

"All models are wrong but some models are useful."

Linear *series expansion* approach. Represent $f(\vec{r})$ by $\boldsymbol{x} = (x_1, \ldots, x_N)$ where

$$f(\vec{r}) \approx \tilde{f}(\vec{r}) = \sum_{j=1}^{N} x_j b_j(\vec{r}) \quad \leftarrow \quad \text{"basis functions"}$$

Reconstruction problem becomes "discrete-discrete:" estimate $\boldsymbol{x}$ from $\boldsymbol{y}$

Numerous basis functions in literature. Two primary contenders:

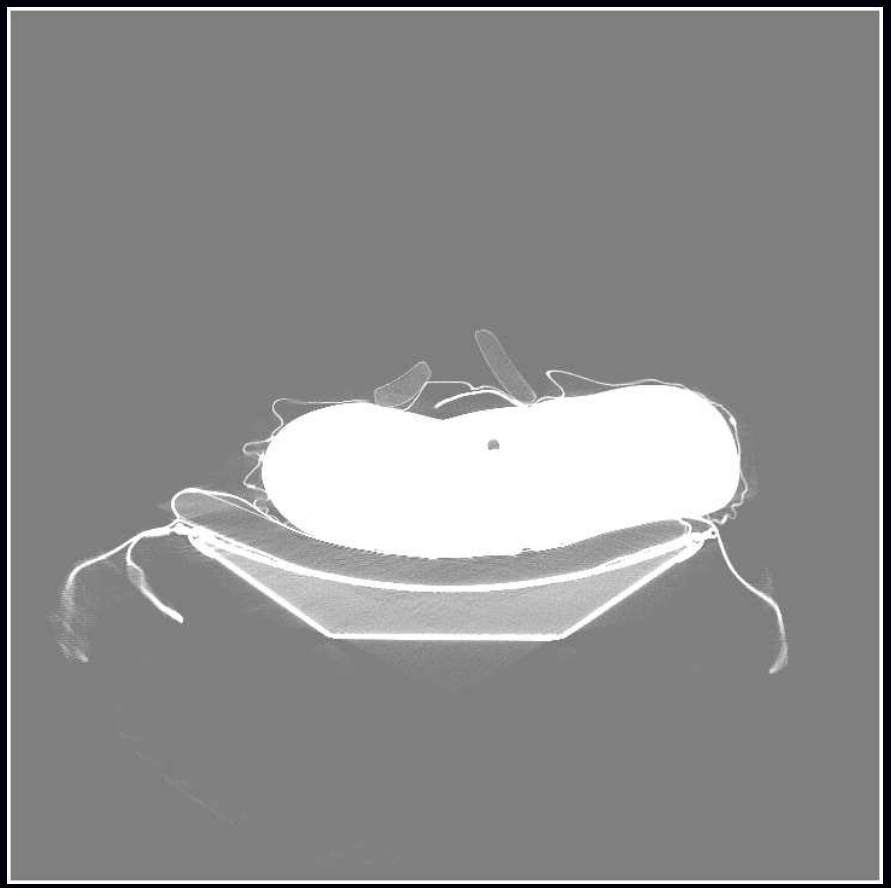
- voxels
- blobs (Kaiser-Bessel functions)
  - \+ Blobs are approximately band-limited (reduced aliasing?)
  - – Blobs have larger footprints, increasing computation.

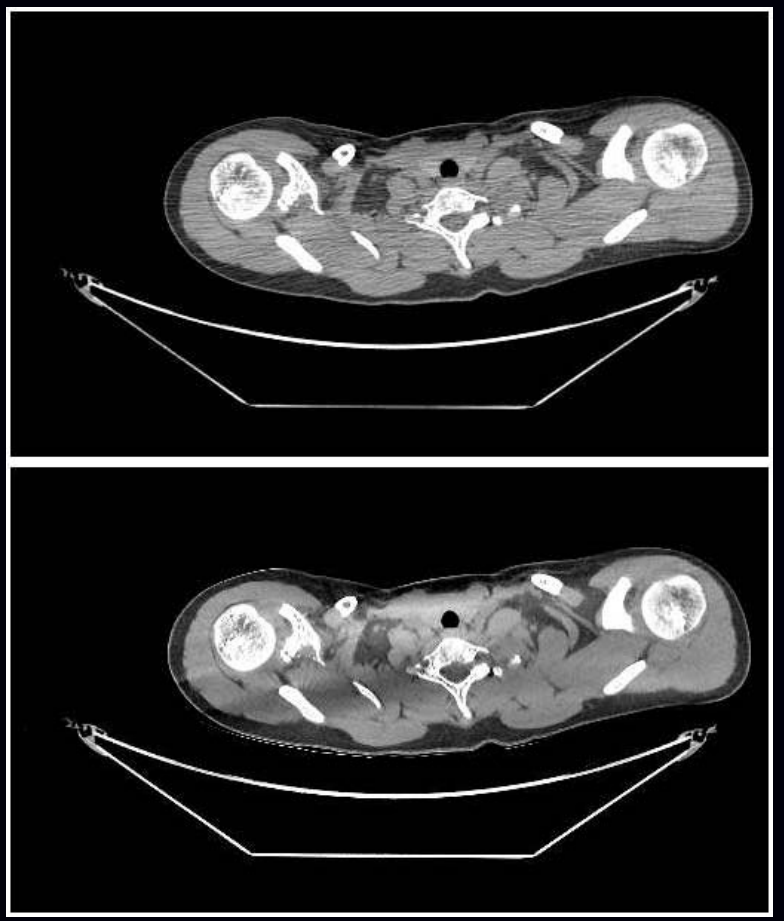
Open question: how small should the voxels be?

One practical compromise: wide FOV coarse-grid reconstruction followed by fine-grid refinement over ROI, *e.g.*, Ziegler *et al.*, Med. Phys., Apr. 2008

Global reconstruction: An inconvenient truth

70-cm FOV reconstruction

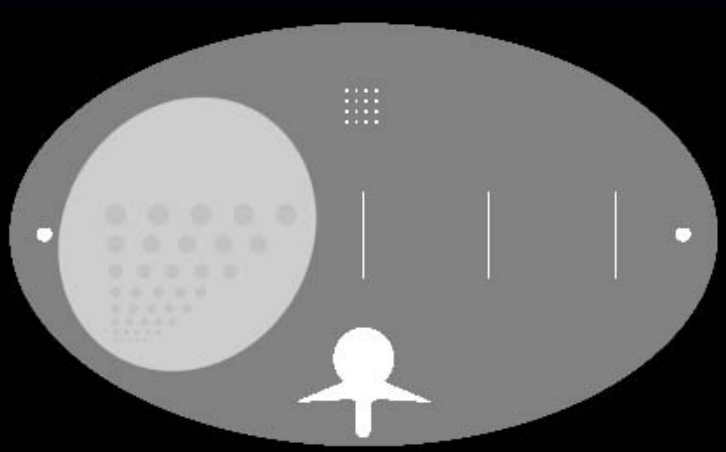




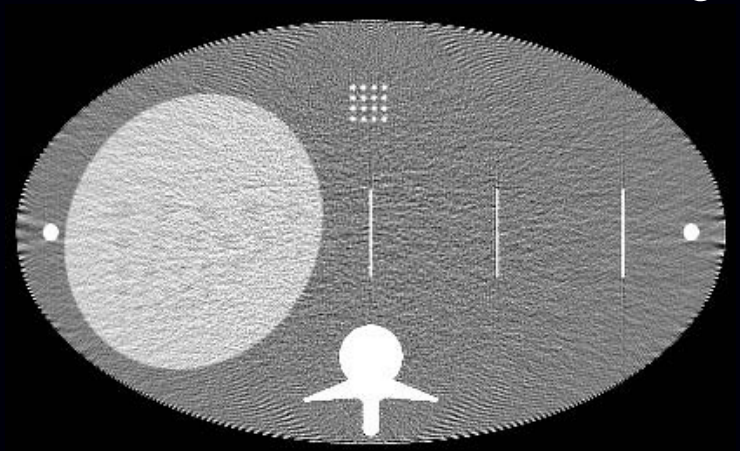
Thibault *et al.*, Fully3D, 2007

For a statistical approach to interior tomography, see Xu *et al.*, IEEE T-MI, May 2011.

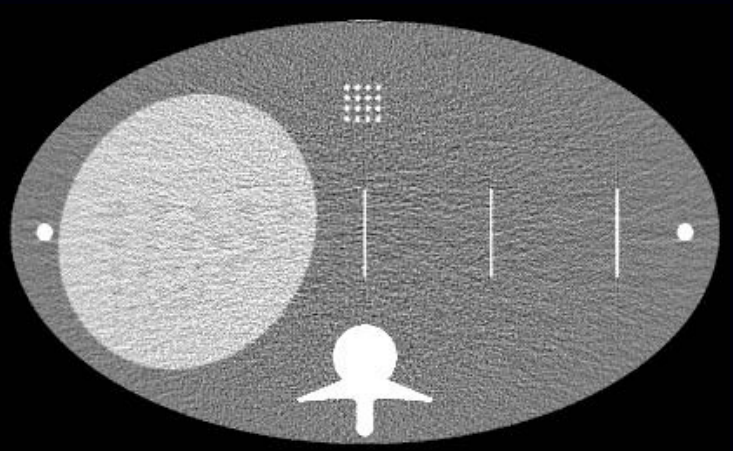
Voxel size matters?



digital phantom



512^2 grid



1024^2 grid

Unregularized OS reconstructions. Zbijewski & Beekman, PMB, Jan. 2004

Choice 2. System model / Physics model

- scan geometry
- source intensity I_0
  - spatial variations (air scan)
  - intensity fluctuations
- resolution effects
  - finite detector size / detector spatial response
  - finite X-ray spot size / anode angulation
  - detector afterglow / gantry rotation
- spectral effects
  - X-ray source spectrum
  - bowtie filters
  - detector spectra response
- scatter
- ...

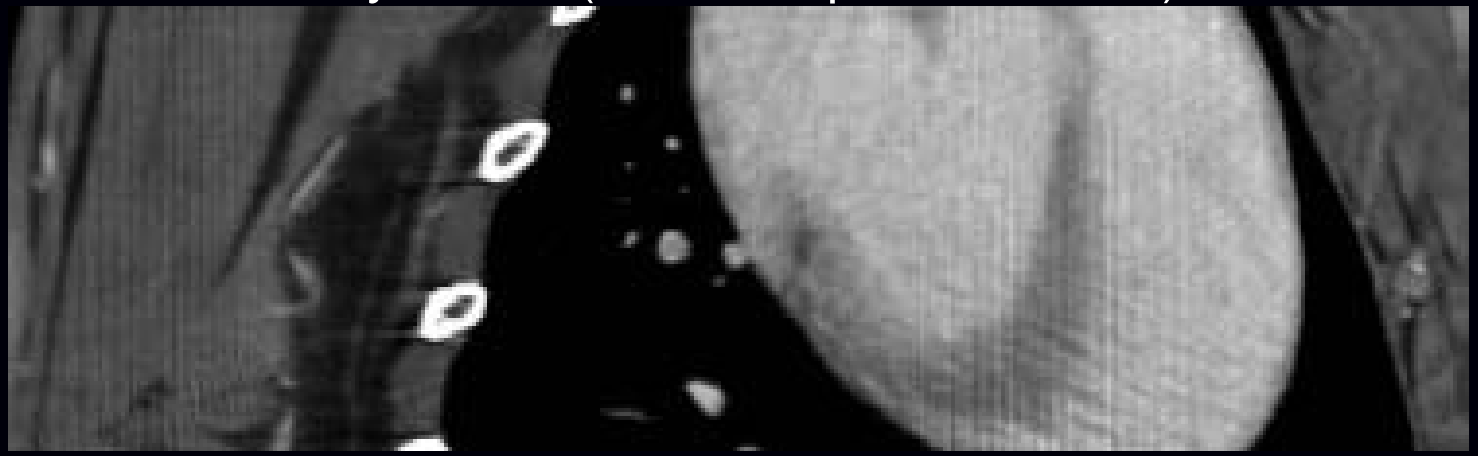
Challenges / trade-offs

- computation time
- accuracy/artifacts/resolution/contrast
- dose?

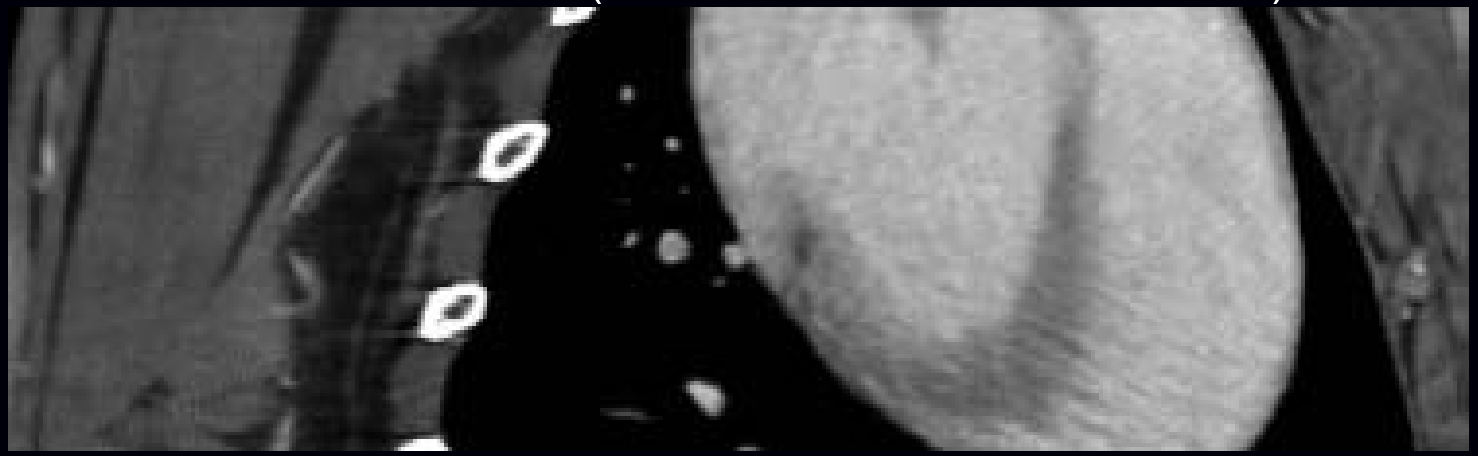
Detector size modeling matters

From (De Man and Basu, PMB, Jun. 2004) MLTR of rabbit heart

Ray-driven (idealized point detector)



Distance-driven (models finite detector width)



Forward- / Back-projector "Pairs"

Typically iterative algorithms require two key steps.

- forward projection (image domain to projection domain):

$$\bar{\boldsymbol{y}} = \boldsymbol{A}\boldsymbol{x}, \qquad \bar{y}_i = \sum_{j=1}^{N} a_{ij} x_j = [\boldsymbol{A}\boldsymbol{x}]_i$$

- backprojection (projection domain to image domain):

$$\boldsymbol{z} = \boldsymbol{A}'\boldsymbol{y}, \qquad z_j = \sum_{i=1}^{M} a_{ij} y_i$$

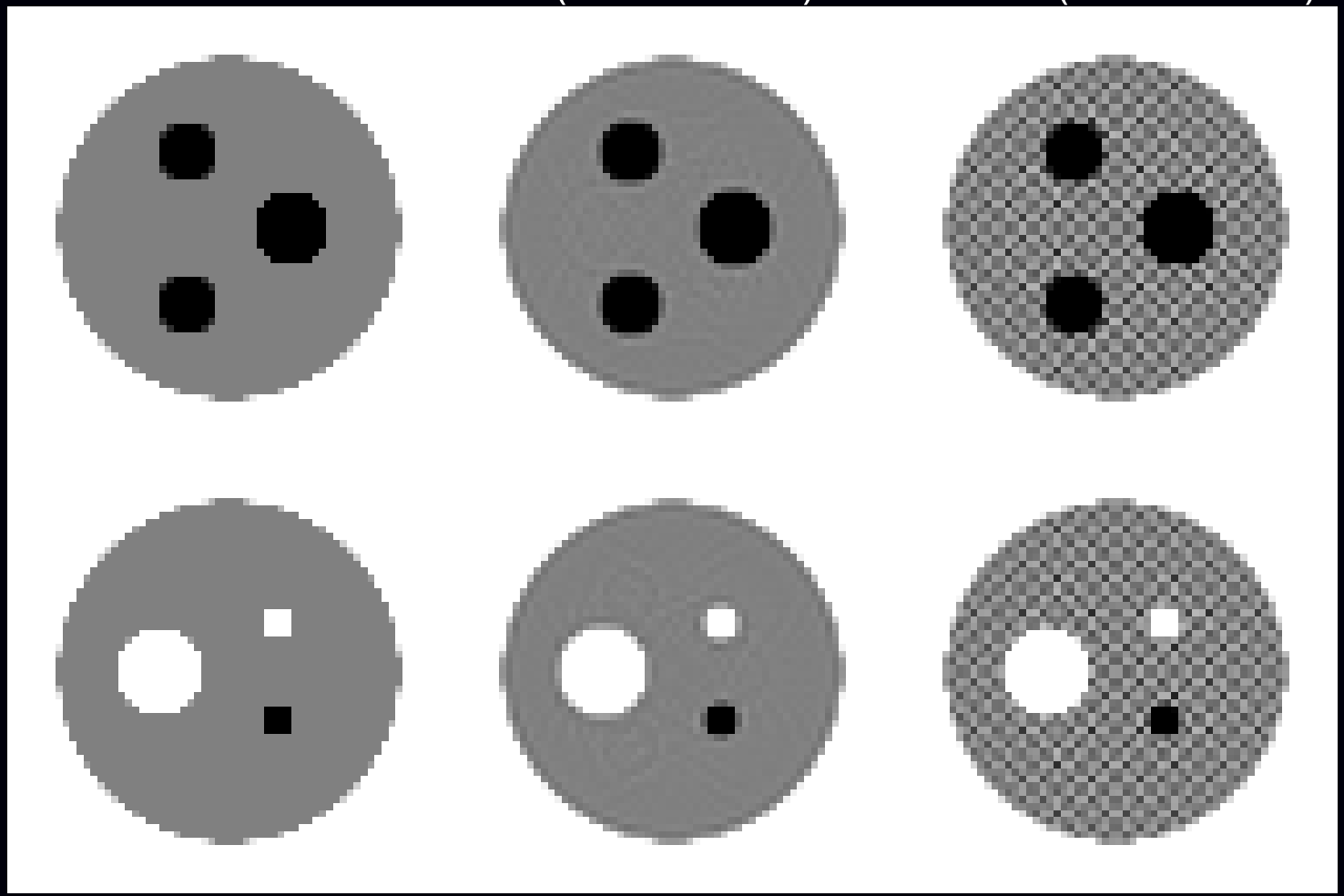
The term "forward/backprojection pair" often refers to some implicit choices for the object basis and the system model.

Sometimes $\boldsymbol{A}'\boldsymbol{y}$ is implemented as $\boldsymbol{B}\boldsymbol{y}$ for some "backprojector" $\boldsymbol{B} \neq \boldsymbol{A}'$. Especially in SPECT and sometimes in PET and CT.

Least-squares solutions (for example):

$$\hat{\boldsymbol{x}} = \arg\min_{\boldsymbol{x}} \|\boldsymbol{y} - \boldsymbol{A}\boldsymbol{x}\|^2 = \left[\boldsymbol{A}'\boldsymbol{A}\right]^{-1} \boldsymbol{A}'\boldsymbol{y} \neq \left[\boldsymbol{B}\boldsymbol{A}\right]^{-1} \boldsymbol{B}\boldsymbol{y}$$

Mismatched Backprojector $B \neq A'$



cf. SPECT/PET reconstruction – usually unregularized

Choice 3. Statistical Model

The physical model describes measurement mean,
e.g., for a monoenergetic X-ray source and ignoring scatter etc.:

$$\bar{I}_i = I_0 \, \mathrm{e}^{-\sum_{j=1}^{N} a_{ij} x_j} .$$

The raw noisy measurements $\{I_i\}$ are distributed around those means.
Statistical reconstruction methods require a model for that distribution.

Challenges / Trade offs: using more accurate statistical models

- *may* lead to less noisy images
- may incur additional computation
- may involve higher algorithm complexity.

CT measurement statistics are very complicated, particularly at low doses.

- incident photon flux variations (Poisson)
- X-ray photon absorption/scattering (Bernoulli)
- energy-dependent light production in scintillator (?)
- shot noise in photodiodes (Poisson?)
- electronic noise in readout electronics (Gaussian?)
  Whiting, SPIE 4682, 2002; Lasio *et al.*, PMB, 2007; Massoumzadeh *et al.*, PMB 2009
- Inaccessibility of raw sinogram data

To log() or not to log() – That is the question

Models for "raw" data I_i (before logarithm)

- compound Poisson (complicated) Whiting, SPIE 4682, 2002; Elbakri & Fessler, SPIE 5032, 2003; Lasio *et al.*, PMB, Apr. 2007
- Poisson + Gaussian (photon variability and electronic readout noise):

$$I_i \sim \text{Poisson}\{\bar{I}_i\} + \text{N}\left(0, \sigma^2\right)$$

  Snyder *et al.*, JOSAA, May 1993 & Feb. 1995 .
- Shifted Poisson approximation (matches first two moments):

$$\tilde{I}_i \triangleq \left[I_i + \sigma^2\right]_+ \sim \text{Poisson}\left\{\bar{I}_i + \sigma^2\right\}$$

  Yavuz & Fessler, MIA, Dec. 1998
- Ordinary Poisson (ignore electronic noise):

$$I_i \sim \text{Poisson}\{\bar{I}_i\}$$

  Rockmore and Macovski, TNS, Jun. 1977; Lange and Carson, JCAT, Apr. 1984
- Photon-counting detectors would simplify statistical modeling

All are somewhat complicated by the nonlinearity of the physics: $\bar{I}_i = \mathrm{e}^{-[\boldsymbol{A}\boldsymbol{x}]_i}$

After taking the log()

Taking the log leads to a simpler linear model (ignoring beam hardening):

$$y_i \triangleq -\log\left(\frac{I_i}{I_0}\right) \approx [\boldsymbol{A}\boldsymbol{x}]_i + \varepsilon_i$$

Drawbacks:

- Undefined if $I_i \leq 0$ (due to electronic noise); bad for very low-dose scans
- It is *biased* (by Jensen's inequality): $\mathsf{E}[y_i] \geq -\log(\bar{I}_i/I_0) = [\boldsymbol{A}\boldsymbol{x}]_i$
- Exact distribution of log-domain noise ε_i is intractable.

Practical approach: assume Gaussian noise model: $\varepsilon_i \sim \mathsf{N}\left(0, \sigma_i^2\right)$

Options for modeling noise variance $\sigma_i^2 = \mathsf{Var}\{\varepsilon_i\}$

- consider both Poisson and Gaussian noise effects: $\sigma_i^2 = \frac{\bar{I}_i + \sigma^2}{\bar{I}_i^2}$
  (Thibault *et al.*, SPIE 6065, 2006)
- consider just Poisson effect: $\sigma_i^2 = \frac{1}{\bar{I}_i}$ (Sauer & Bouman, T-SP, Feb. 1993)
- pretend it is white noise: $\sigma_i^2 = \sigma_0^2$
- ignore noise altogether and "solve" $\boldsymbol{y} = \boldsymbol{A}\boldsymbol{x}$

Whether using pre-log data is better than post-log data is an open question, especially for very low-dose scans (*cf.* experience with PET rod scans).

Choice 4. Cost Functions

Components:

- 4.1 *Data-mismatch* term
- 4.2 *Regularization* term (and regularization parameter β)
- 4.3 *Constraints* (*e.g.*, nonnegativity: minor effect on noise / dose)

Reconstruct image $\hat{\boldsymbol{x}}$ by finding minimizer of a cost function:

$$\hat{\boldsymbol{x}} \triangleq \underbrace{\underset{\boldsymbol{x} \geq \boldsymbol{0}}{\arg\min}}_{\text{Constraints}} \Psi(\boldsymbol{x})$$

$$\Psi(\boldsymbol{x}) = \text{DataMismatch}(\boldsymbol{y}, \boldsymbol{A}\boldsymbol{x}) + \beta \, \text{Regularizer}(\boldsymbol{x})$$

Forcing too much "data fit" alone would give noisy images.

Equivalent to a Bayesian MAP (maximum *a posteriori*) estimator.

Distinguishes "statistical methods" from "algebraic methods" for "$\boldsymbol{y} = \boldsymbol{A}\boldsymbol{x}$."

Such optimization-based formulation appears to be key to low-dose CT.
Theoretically optimizes bias-variance trade-off (Eldar, IEEE T-SP, Jul. 2004).

Choice 4.1: Data-Mismatch Term

Standard choice is the negative log-likelihood of statistical model:

$$\text{DataMismatch} = -L(\boldsymbol{x};\boldsymbol{y}) = -\log \mathrm{p}(\boldsymbol{y}|\boldsymbol{x}) = \sum_{i=1}^{M} -\log \mathrm{p}(y_i|\boldsymbol{x})\,.$$

- For pre-log data $\boldsymbol{I}$ with shifted Poisson model:

$$-L(\boldsymbol{x};\boldsymbol{I}) = \sum_{i=1}^{M} \left(\bar{I}_i + \sigma^2\right) - \left[I_i + \sigma^2\right]_+ \log\left(\bar{I}_i + \sigma^2\right), \qquad \bar{I}_i = I_0\, \mathrm{e}^{-[\boldsymbol{A}\boldsymbol{x}]_i}$$

  This can be non-convex if $\sigma^2 > 0$;
  it is convex if we ignore electronic noise $\sigma^2 = 0$. Trade-off ...

- For post-log data $\boldsymbol{y}$ with Gaussian model:

$$-L(\boldsymbol{x};\boldsymbol{y}) = \sum_{i=1}^{M} w_i \frac{1}{2}(y_i - [\boldsymbol{A}\boldsymbol{x}]_i)^2 = \frac{1}{2}(\boldsymbol{y} - \boldsymbol{A}\boldsymbol{x})'\boldsymbol{W}(\boldsymbol{y} - \boldsymbol{A}\boldsymbol{x}), \qquad w_i = 1/\sigma_i^2$$

  This is a kind of (data-based) weighted least squares (WLS).
  It is always convex in $\boldsymbol{x}$. Quadratic functions are “easy” to minimize.

- ...
- Effect on dose is related to statistical model accuracy

Choice 4.2: Regularization

How to control noise due to ill-conditioning in tomography?
Perhaps the most important factor influencing "how low (dose) can we go"

Noise-control methods (used in clinical PET reconstruction today):

- Stop an unregularized algorithm before convergence
- Over-iterate an unregularized algorithm then post-filter

Other possible "simple" solutions:

- Modify the raw data (pre-filter / denoise)
- Filter between iterations
- ...

Appeal:

- simple / familiar
- filter parameters have intuitive units (*e.g.*, FWHM), unlike a regularization parameter β
- Changing a post-filter does not require re-iterating, unlike changing a regularization parameter β

Regularization options

Options for regularizer $\mathsf{R}(\boldsymbol{x})$ in increasing complexity:

- quadratic roughness
- convex, non-quadratic roughness
- non-convex roughness
- total variation
- convex sparsity
- non-convex sparsity

Challenges

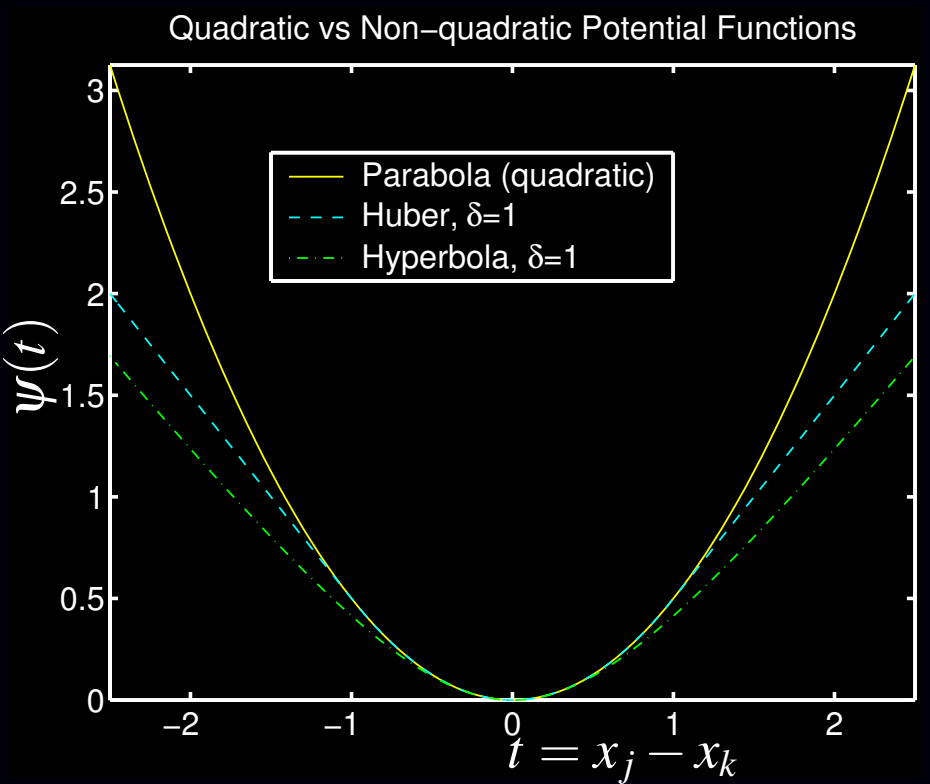
- Reducing noise without degrading spatial resolution
- Balancing regularization strength between and within slices
- Parameter selection
- Computational complexity (voxels have 26 immediate neighbors in 3D)
- Preserving “familiar” noise texture
- Optimizing clinical task performance

Many open questions...
Which regularization method can enable the lowest-dose scans?

Roughness Penalty Functions

$$\mathrm{R}(\boldsymbol{x}) = \sum_{j=1}^{N} \frac{1}{2} \sum_{k \in \mathcal{N}_j} \psi(x_j - x_k)$$

- $\mathcal{N}_j \triangleq$ *neighborhood* of jth pixel
  (*e.g.*, left, right, up, down in 2D; 26 neighbors in 3D)
- ψ called the *potential function*



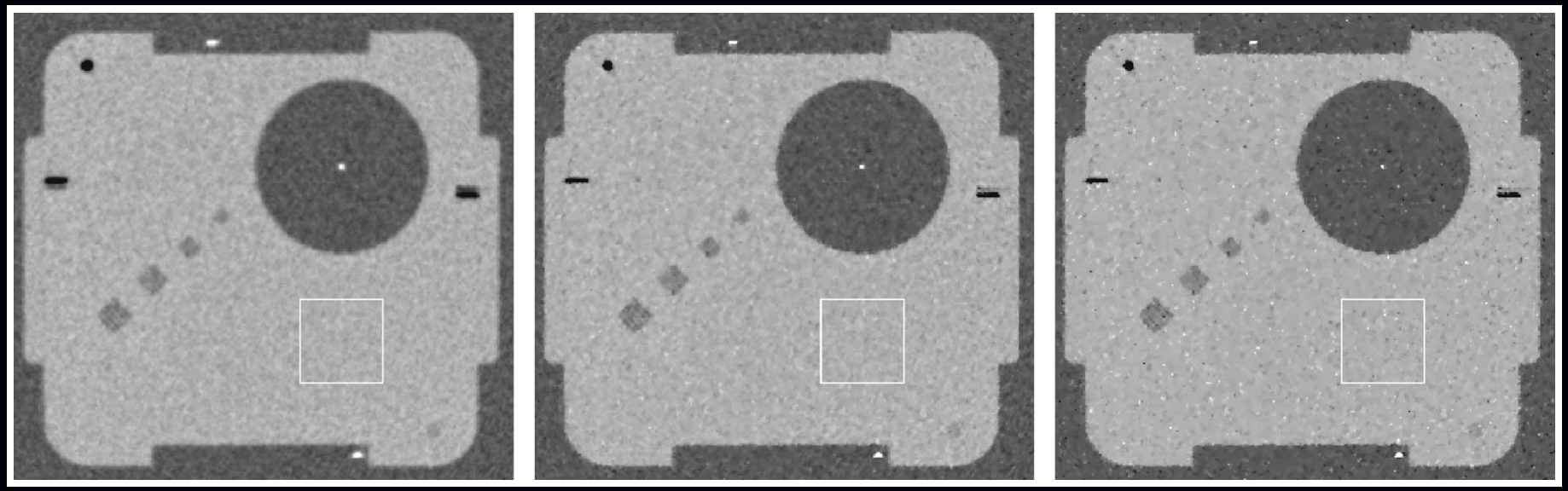
quadratic: $\psi(t) = t^2$
hyperbola: $\psi(t) = \sqrt{1 + (t/\delta)^2}$
(edge preservation)

Regularization parameters: Dramatic effects on IQ

"q generalized gaussian" potential function with tuning parameters: β, δ, p, q:

$$\beta\,\psi(t) = \beta \frac{\frac{1}{2}|t|^p}{1 + |t/\delta|^{p-q}}.$$

(Thibault *et al.*, Med. Phys., Nov. 2007)



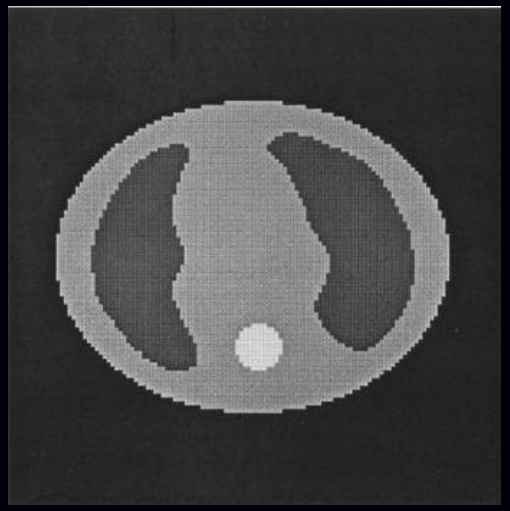
$p = q = 2$ $p = 2, q = 1.2, \delta = 10$ HU $p = q = 1.1$

noise:	11.1	10.9	10.8
(#lp/cm):	4.2	7.2	8.2

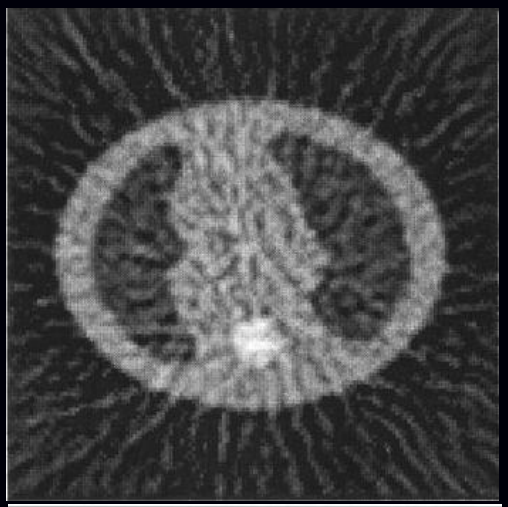
Which parameter choice enables the lowest dose scan?

Piecewise constant phantoms

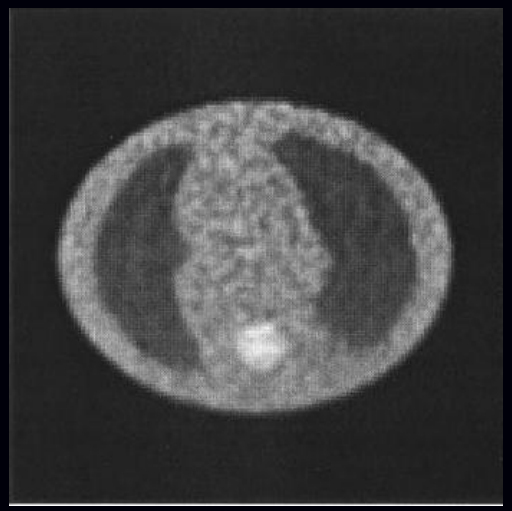
Phantom:



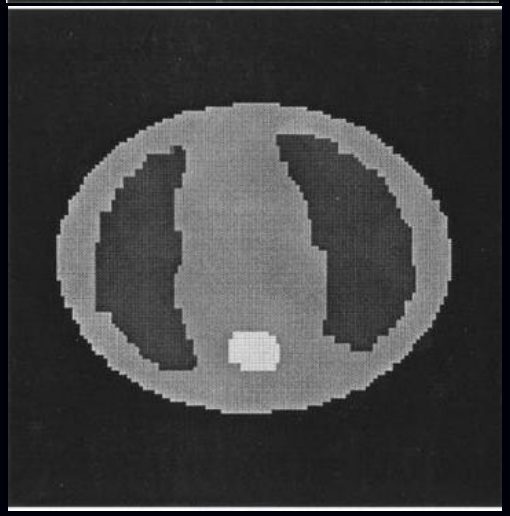
FBP:



MLEM:



MAP:



Lee *et al.*, IEEE T-NS, 2002, 300K counts
non-convex “broken parabola” potential function and deterministic annealing

Summary of statistical image reconstruction choices

1. Object parameterization
2. System physical model
3. Measurement statistical model
4. Cost function: data-mismatch / regularization / constraints

Reconstruction Method $\triangleq$ Models + Cost Function + Algorithm

5. Minimization algorithms: $\hat{\boldsymbol{x}} = \arg\min_{\boldsymbol{x}} \Psi(\boldsymbol{x})$

- The minimization algorithm has *no affect on image quality* / dose *if* we iterate "until convergence."
- For convergent algorithms, IQ determined entirely by cost function Ψ including the models within it.
- What happens we stop iterating "before convergence?" Affects dose? (Depends on algorithm and initial image...)
- Finding fast algorithms for arg min is key to ubiquitous low-dose CT. *e.g.*, alternating direction method of multipliers (ADMM) (Ramani & Fessler, IEEE T-MI, Mar. 2012)

Summary: Factors that affect dose reduction

In conjectured order from most important to least:

1. Regularizer
2. Statistical model / data-mismatch term
3. System model
4. Object model / non-negativity constraint
5. Iterative minimization algorithm

Other considerations that affect dose reduction

- 2D versus 3D reconstruction
  - 8 immediate neighbors in 2D versus 26 neighbors in 3D
  - transaxial versus axial relationships
- axial versus helical scans
- angular sampling: fine versus sparse
- dual energy / spectral CT versus single kVp

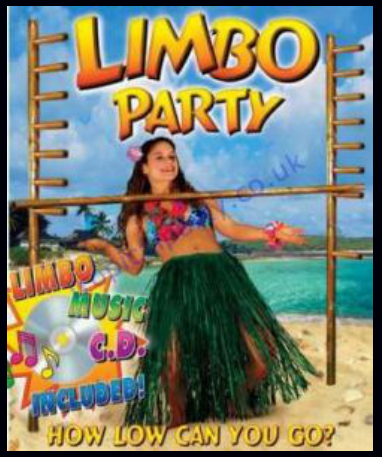
Recent dose figures for statistical image reconstruction

Yadava *et al.*, AAPM 2010, AAPM 2011
"average-probability-of-detection and average-detection-score of full dose FBP and 1/4th dose Veo [MBIR] are comparable."

Cohen *et al.*, AAPM 2011
[Phantom] "measurement differences between full dose FBP and 1/4th dose Veo were found to be statistically insignificant, indicating similar image quality."

Katsura *et al.*, Eur. Radiol. Aug. 2012 (hot off the press)
Compared FBP, ASIR, and "MBIR" (with no almost description);
concluded MBIR can work with "80% less radiation."

How low (dose) can you go?



http://laidoffinnyc.wordpress.com/2009/03/04/

- Depends on where you start
- Depends on many reconstruction method choices
- all of which affect IQ
  (which is harder to assess for nonlinear reconstruction methods)
- See next symposium...
- Conjecture: sub-mSv exams are achievable

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