Statistical Reconstruction in CT

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Former MS students
- Kevin Brown, Philips
- ...

Why we are here

A picture is worth 1000 words
(and perhaps several 1000 seconds of computation?)

Thin-slice FBP	ASIR	Statistical
Seconds	A bit longer	Much longer*
		(* ask the vendors)

Why statistical methods for CT?

- Accurate physical models
  - X-ray spectrum, beam-hardening, scatter, ...
    reduced artifacts? quantitative CT?
  - X-ray detector spatial response, focal spot size, ...
    improved spatial resolution?
  - detector spectral response (*e.g.*, photon-counting detectors)

- Nonstandard geometries
  - transaxial truncation (big patients)
  - long-object problem in helical CT
  - irregular sampling in "next-generation" geometries
  - coarse angular sampling in image-guidance applications
  - limited angular range (tomosynthesis)
  - "missing" data, *e.g.*, bad pixels in flat-panel systems

- Appropriate statistical models
  - weighting reduces influence of photon-starved rays
    (FBP treats all rays equally)
  - reducing image noise or dose

and more...

- Object constraints
  - nonnegativity
  - object support
  - piecewise smoothness
  - object sparsity (*e.g.*, angiography)
  - sparsity in some basis
  - motion models
  - dynamic models
  - ...

Disadvantages?

- Computation time (Dr. Thrall says super computer)
- Must reconstruct entire FOV
- Model complexity
- Software complexity
- Algorithm nonlinearities
  - Difficult to analyze resolution/noise properties (*cf.* FBP)
  - Tuning parameters
  - Challenging to characterize performance

"Iterative" vs "Statistical"

- Traditional *successive substitutions* iterations
  - *e.g.*, Joseph and Spital (JCAT, 1978) bone correction
  - usually only one or two "iterations"
  - not statistical

- Algebraic reconstruction methods
  - Given sinogram data y and system model A, reconstruct object x by "solving" $y = Ax$
  - ART, SIRT, SART, ...
  - iterative, but typically not statistical
  - Iterative filtered back-projection (FBP):

$$x^{(n+1)} = x^{(n)} + \underbrace{\alpha}_{\substack{\text{step} \\ \text{size}}} \text{FBP}(\underbrace{y}_{\text{data}} - \underbrace{Ax^{(n)}}_{\substack{\text{forward} \\ \text{project}}})$$

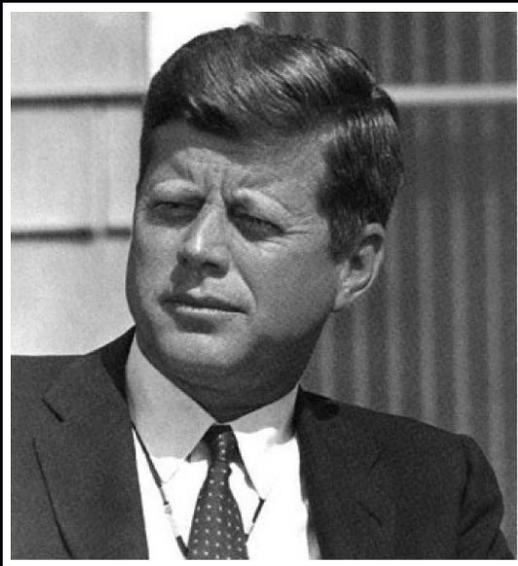- Statistical reconstruction methods
  - Image domain
  - Sinogram domain
  - Fully statistical (both)
  - Hybrids (*e.g.*, AIR, 7961-18, Bruder *et al.*)

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"Statistical" methods: Image domain
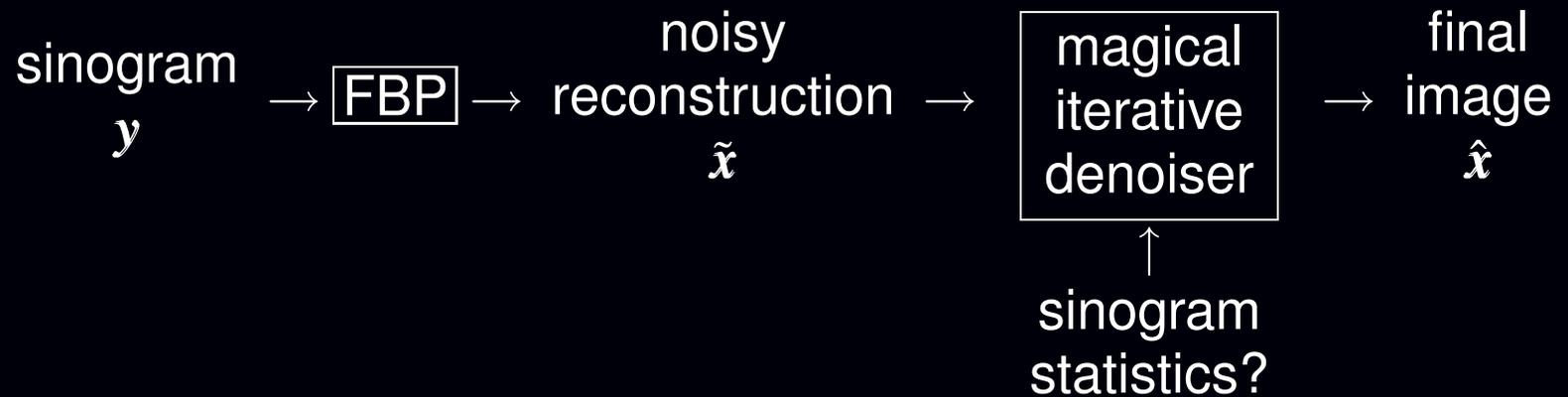
- Denoising methods

$$\text{sinogram } \boldsymbol{y} \;\rightarrow\; \boxed{\text{FBP}} \;\rightarrow\; \begin{array}{c}\text{noisy}\\ \text{reconstruction}\\ \tilde{x}\end{array} \;\rightarrow\; \boxed{\begin{array}{c}\text{iterative}\\ \text{denoiser}\end{array}} \;\rightarrow\; \begin{array}{c}\text{final}\\ \text{image}\\ \hat{x}\end{array}$$

  ○ Remarkable advances in denoising methods in last decade

Zhu & Milanfar, T-IP, Dec. 2010, using "steering kernel regression" (SKR) method
  ○ Typically assume *white noise*
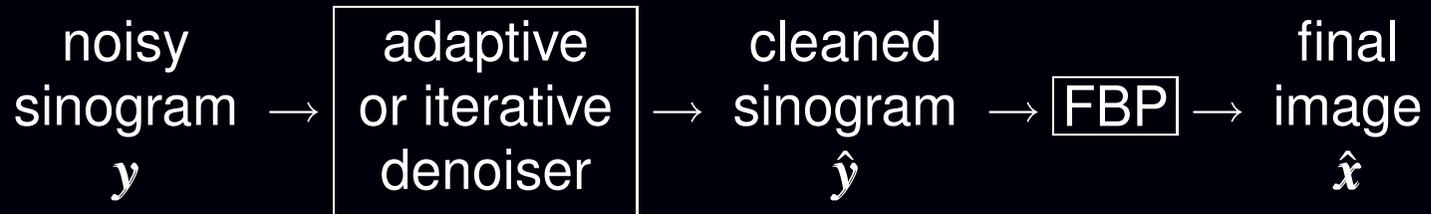 ○ Streaks in low-dose FBP appear like edges (highly correlated noise)

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- Denoising methods "guided by data statistics"

sinogram y $\rightarrow$ $\boxed{\text{FBP}}$ $\rightarrow$ noisy reconstruction $\tilde{x}$ $\rightarrow$ $\boxed{\begin{array}{c}\text{magical}\\\text{iterative}\\\text{denoiser}\end{array}}$ $\rightarrow$ final image $\hat{x}$

$\uparrow$

sinogram statistics?

- Image-domain methods are fast (thus practical)
- The technical details are a mystery...
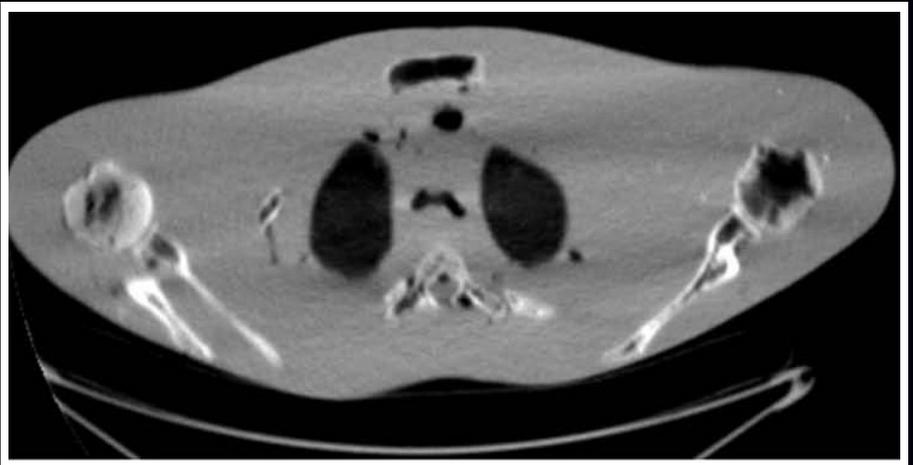- ASIR? IRIS? ...

"Statistical" methods: Sinogram domain

- Sinogram restoration methods

<div align="center">

noisy | adaptive | cleaned   final

sinogram $\rightarrow$ | or iterative | $\rightarrow$ sinogram $\rightarrow$ [FBP] $\rightarrow$ image

y | denoiser | $\hat{y}$ $\hat{x}$

</div>

- ○ Adaptive: J. Hsieh, Med. Phys., 1998; Kachelrieß, Med. Phys., 2001, ...
- ○ Iterative: P. La Riviere, IEEE T-MI, 2000, 2005, 2006, 2008
- ○ fast, but limited denoising without resolution loss (local, no edges)


FBP, 10 mA   FBP from denoised sinogram

Wang *et al.*, T-MI, Oct. 2006, using PWLS-GS on sinogram

(True? Fully? Slow?) Statistical reconstruction

- Object model
- Physics/system model
- Statistical model
- Cost function (log-likelihood + regularization)
- Iterative algorithm for minimization

"Find the image $\hat{x}$ that best fits the sinogram data y according to the physics model, the statistical model and prior information about the object"
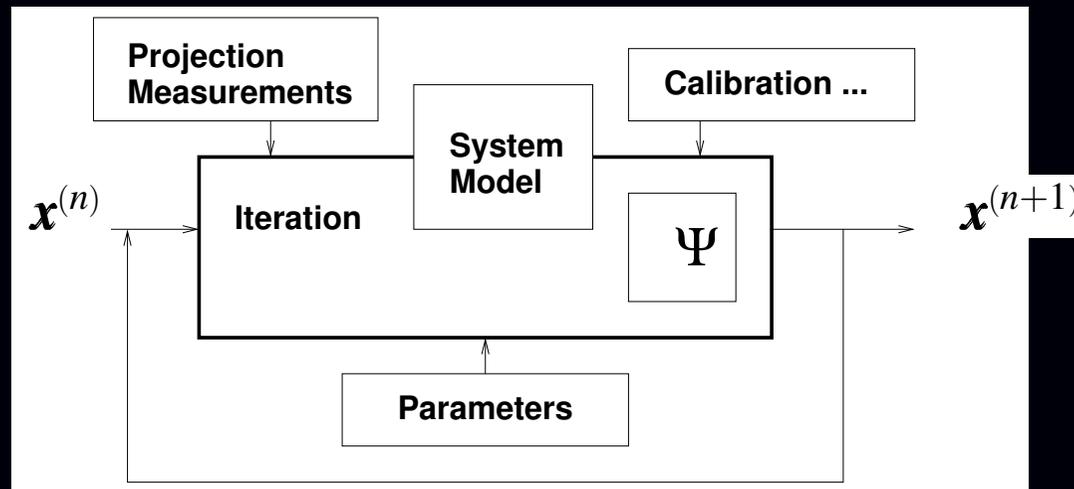
- Repeatedly revisiting the sinogram data can use statistics fully
- Repeatedly updating the image can exploit object properties
- But repetition is expensive...

History: Statistical reconstruction for PET

- Iterative method for emission tomography   (Kuhl, 1963)
- Weighted least squares for 3D SPECT   (Goitein, NIM, 1972)
- Richardson/Lucy iteration for image restoration   (1972, 1974)
- Poisson likelihood (emission)   (Rockmore and Macovski, TNS, 1976)
- Expectation-maximization (EM) algorithm (Shepp and Vardi, TMI, 1982)
- Regularized (aka Bayesian) Poisson emission reconstruction
                                              (Geman and McClure, ASA, 1985)
- Ordered-subsets EM algorithm   (Hudson and Larkin, TMI, 1994)
- Commercial introduction of OSEM for PET scanners   circa 1997

Today, most commercial PET systems include *unregularized* OSEM.

15 years between key EM paper (1982) and commercial adoption (1997)
(25 years if you count the R/L paper in 1972 which is the same as EM)

History: Statistical reconstruction for CT*

- Iterative method for X-ray CT   (Hounsfield, 1968)
- ART for tomography   (Gordon, Bender, Herman, JTB, 1970)
- ...
- Roughness regularized LS for tomography   (Kashyap & Mittal, 1975)
- Poisson likelihood (transmission) (Rockmore and Macovski, TNS, 1977)
- EM algorithm for Poisson transmission (Lange and Carson, JCAT, 1984)
- Iterative coordinate descent (ICD)   (Sauer and Bouman, T-SP, 1993)
- Ordered-subsets algorithms

$$\text{(Manglos } et\ al.\text{, PMB 1995)}$$
$$\text{(Kamphuis \& Beekman, T-MI, 1998)}$$
$$\text{(Erdoğan \& Fessler, PMB, 1999)}$$

- ...
- Commercial introduction for CT scanners   circa 2010

(* numerous omissions)

RSNA 2010
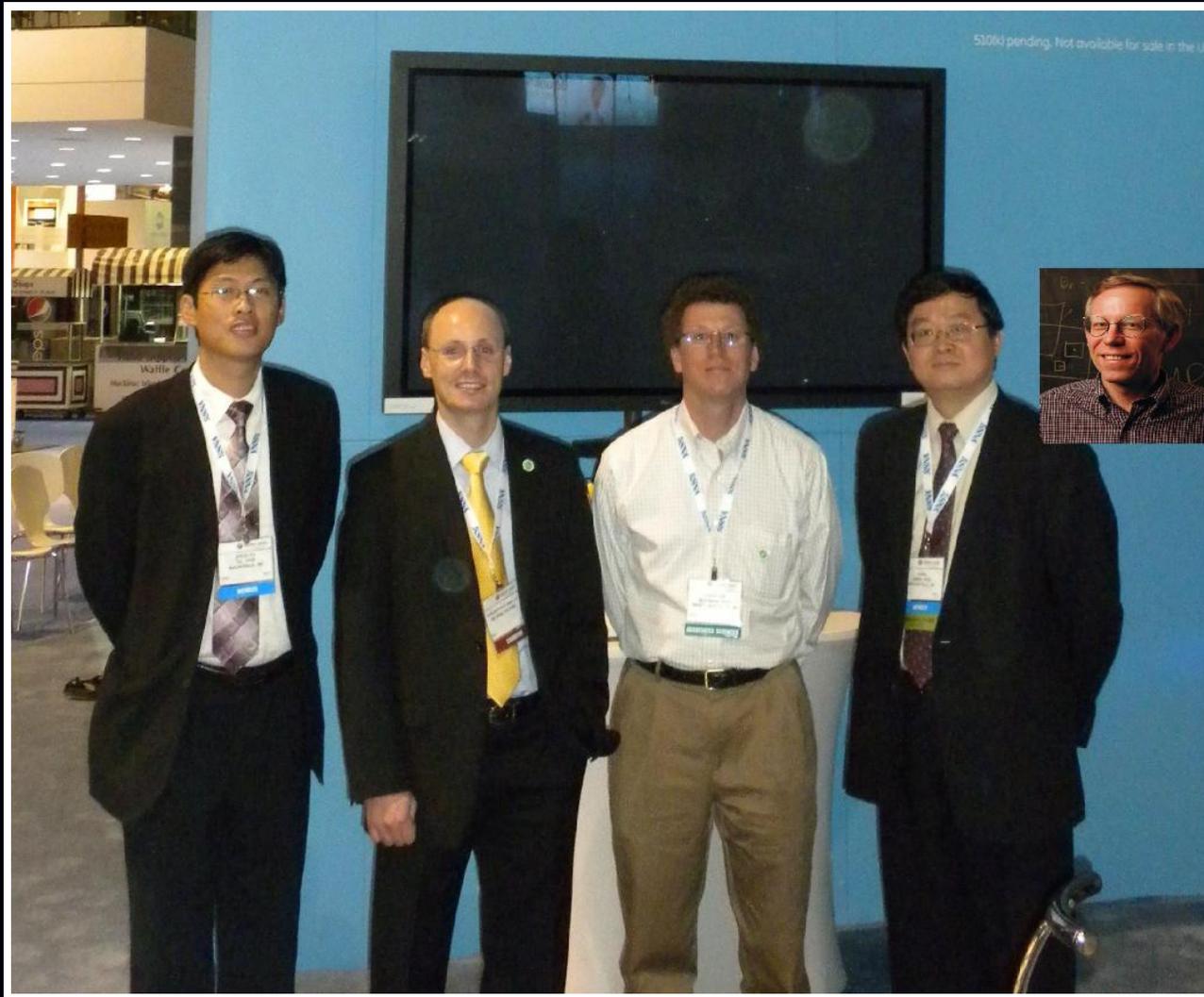
Zhou Yu, Jean-Baptiste Thibault, Charles Bouman, Jiang Hsieh, Ken Sauer

Five Choices for Statistical Reconstruction

1. Object model

2. System physical model

3. Measurement statistical model

4. Cost function: data-mismatch and regularization

5. Algorithm / initialization

No perfect choices - one can critique all approaches!

Historically these choices are often left implicit in publications, but being explicit facilitates reproducibility

Choice 1. Object Parameterization

Finite measurements: $\{y_i\}_{i=1}^{M}$. Continuous object: $f(\vec{r}) = \mu(\vec{r})$.

"All models are wrong but some models are useful."

Linear *series expansion* approach. Represent $f(\vec{r})$ by $\boldsymbol{x} = (x_1, \ldots, x_N)$ where

$$f(\vec{r}) \approx \tilde{f}(\vec{r}) = \sum_{j=1}^{N} x_j b_j(\vec{r}) \quad \leftarrow \quad \text{"basis functions"}$$

Reconstruction problem becomes "discrete-discrete:" estimate $\boldsymbol{x}$ from $\boldsymbol{y}$

Numerous basis functions in literature. Two primary contenders:
- voxels
- blobs (Kaiser-Bessel functions)
  - + Blobs are approximately band-limited (reduced aliasing?)
  - − Blobs have larger footprints, increasing computation.

Open question: how small should the voxels be?

One practical compromise: wide FOV coarse-grid reconstruction followed by fine-grid refinement over ROI, *e.g.*, Ziegler *et al.*, Med. Phys., Apr. 2008

# Choice 2. System model / Physics model

- scan geometry
- source intensity I_0
  - spatial variations (air scan)
  - intensity fluctuations
- resolution effects
  - finite detector size / detector spatial response
  - finite X-ray spot size / anode angulation Inhomogeneous
  - detector afterglow
- spectral effects
  - X-ray source spectrum
  - bowtie filters
  - detector spectra response
- scatter
- ...

Trade-off
- computation time
- accuracy/artifacts/resolution/contrast

Exponential edge-gradient effect

Fundamental difference between emission tomography and CT:
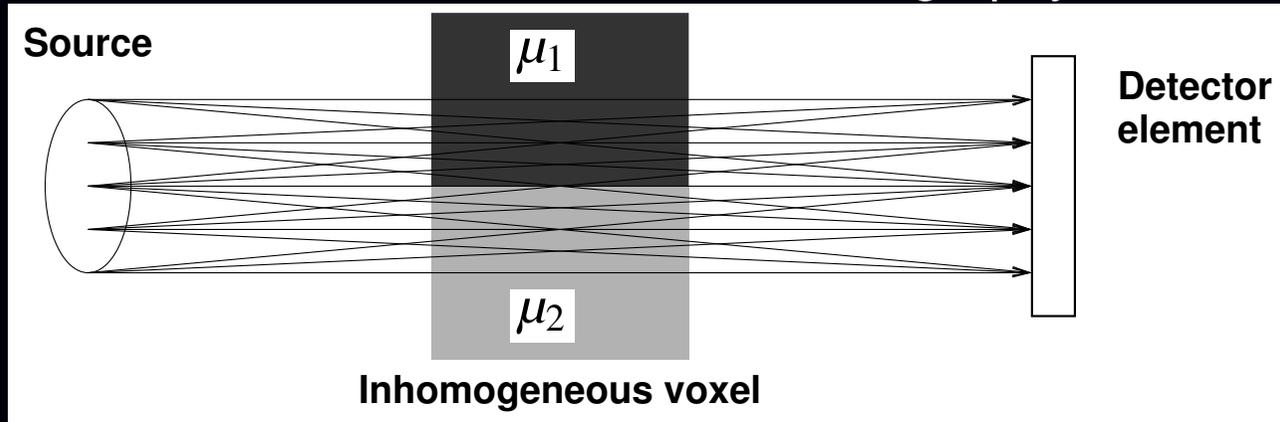
Recorded intensity for ith ray: (Joseph and Spital, PMB, May 1981)

$$I_i \; = \; \int_{\text{source}} \int_{\text{detector}} I_0(\vec{p}_s, \vec{p}_d) \exp\left(- \int_{\mathcal{L}(\vec{p}_s, \vec{p}_d)} \mu(\vec{r}) \, \mathrm{d}\ell \right) \mathrm{d}\vec{p}_d \, \mathrm{d}\vec{p}_s$$
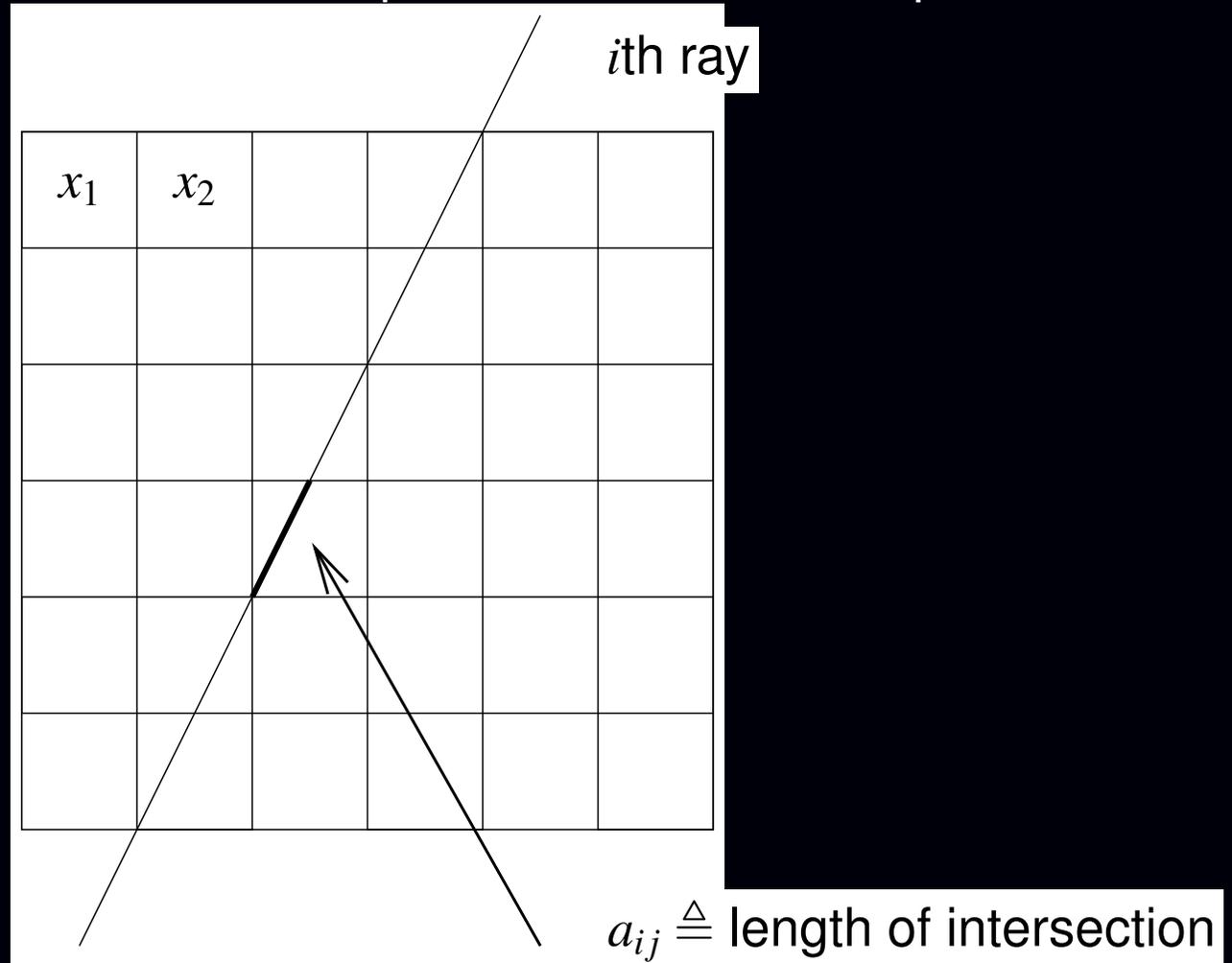
$$\neq \; I_0 \exp\left(- \int_{\text{source}} \int_{\text{detector}} \int_{\mathcal{L}(\vec{p}_s, \vec{p}_d)} \mu(\vec{r}) \, \mathrm{d}\ell \, \mathrm{d}\vec{p}_d \, \mathrm{d}\vec{p}_s \right).$$

Usual "linear" approximation:

$$I_i \approx I_0 \exp\left(- \sum_{j=1}^{N} a_{ij} x_j \right), \qquad a_{ij} \triangleq \underbrace{\int_{\text{source}} \int_{\text{detector}} \int_{\mathcal{L}(\vec{p}_s, \vec{p}_d)} b_j(\vec{r}) \, \mathrm{d}\ell \, \mathrm{d}\vec{p}_d \, \mathrm{d}\vec{p}_s}_{\text{elements of system matrix } A}$$

"Line Length" System Model

Assumes (implicitly?) that source is a point and detector is a point.

ith ray

x_1 x_2

$a_{ij} \triangleq$ length of intersection

"Strip Area" System Model

Account for finite detector width.
Ignores nonlinear partial-volume averaging.
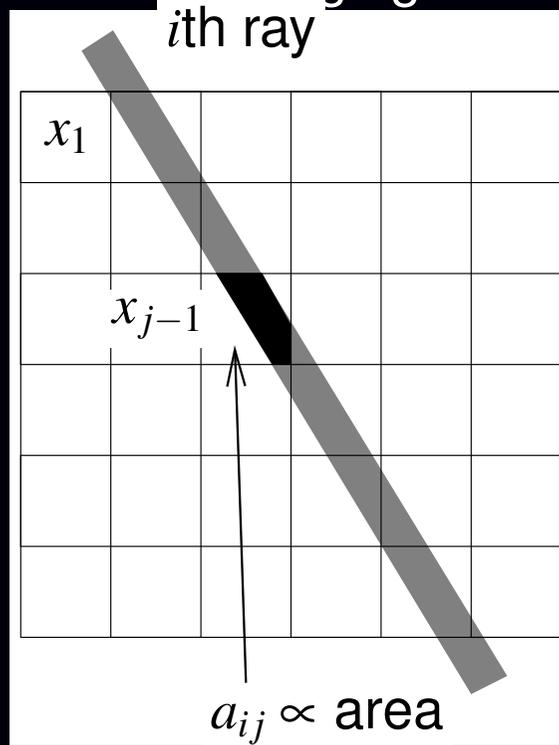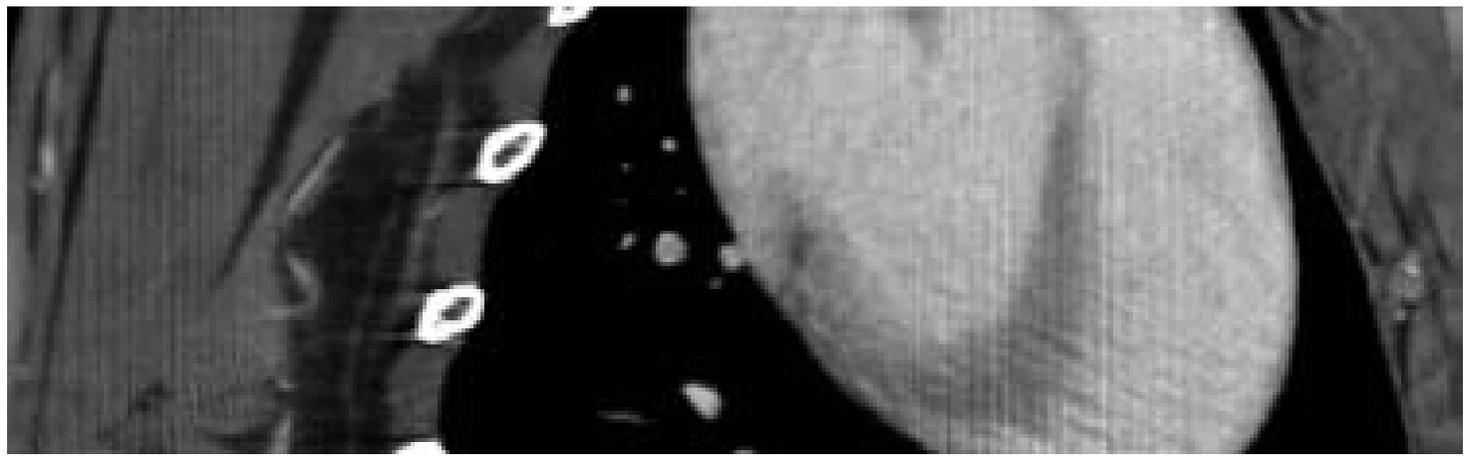

Practical (?) implementations in 3D include
- Distance-driven method (De Man and Basu, PMB, Jun. 2004)
- Separable-footprint method (Long *et al.*, T-MI, Nov. 2010)
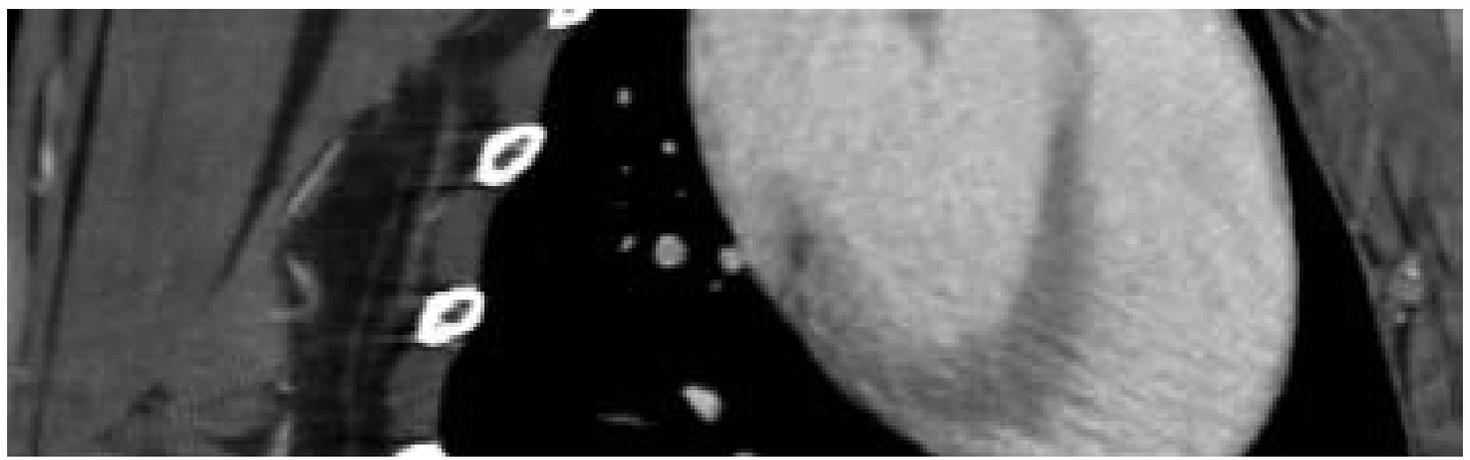- Further comparisons needed...

Lines versus strips

From (De Man and Basu, PMB, Jun. 2004)   MLTR of rabbit heart

Ray-driven

Distance-driven

Forward- / Back-projector "Pairs"

Typically iterative algorithms require two key steps.
- forward projection (image domain to projection domain):

$$\bar{y} = Ax, \qquad \bar{y}_i = \sum_{j=1}^{N} a_{ij}x_j = [Ax]_i$$

- backprojection (projection domain to image domain):
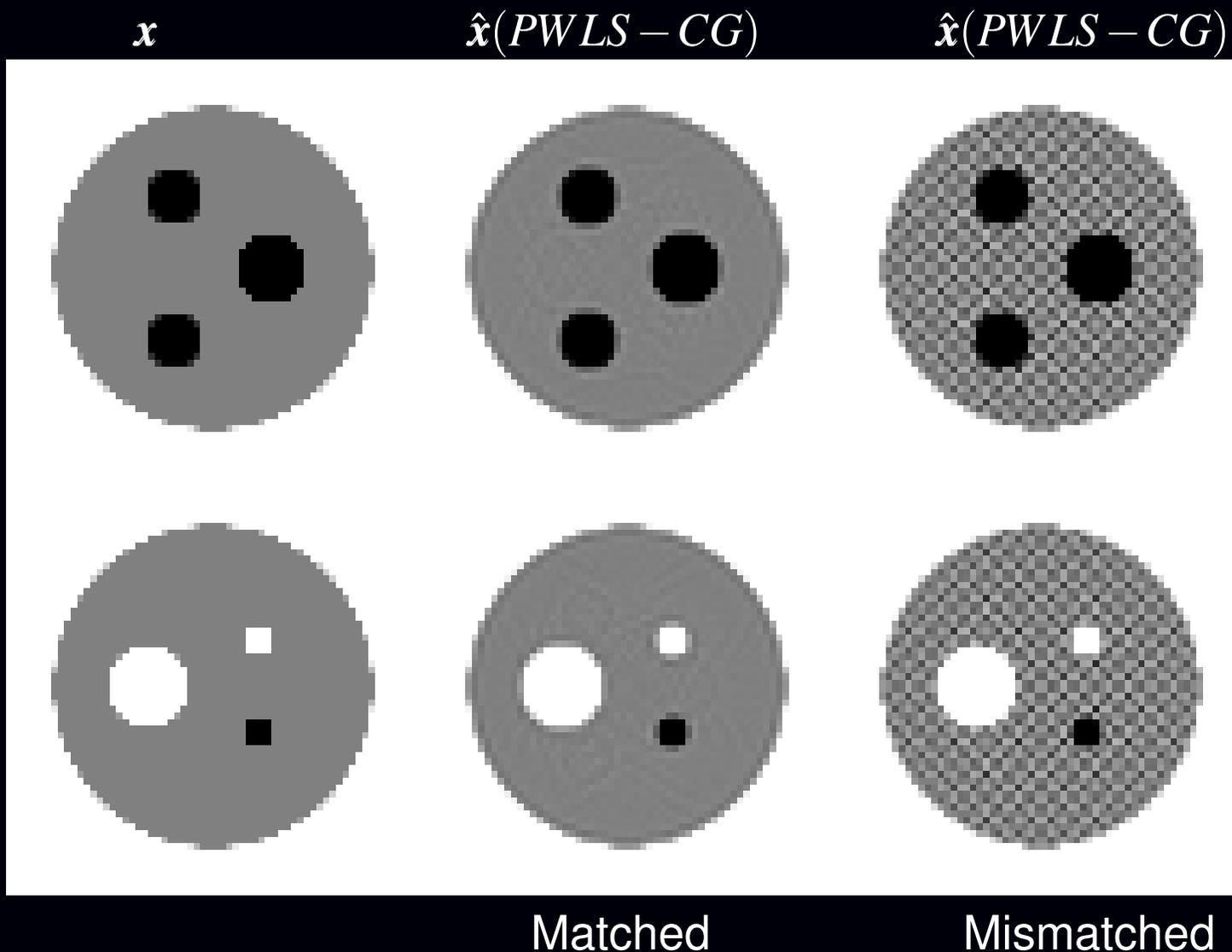
$$z = A'y, \qquad z_j = \sum_{i=1}^{M} a_{ij}y_i$$

The term "forward/backprojection pair" often refers to some implicit choices for the object basis and the system model.

Sometimes $A'y$ is implemented as By for some "backprojector" $B \neq A'$. Especially in SPECT and sometimes in PET.

Least-squares solutions (for example):

$$\hat{x} = \arg\min_{x} \|y - Ax\|^2 = [A'A]^{-1}A'y \neq [BA]^{-1}By$$

Mismatched Backprojector $B \neq A'$

x	$\hat{x}(PWLS-CG)$	$\hat{x}(PWLS-CG)$

| Matched | Mismatched |

cf. SPECT/PET reconstruction – usually unregularized

Acceleration

- Projector/backprojector algorithm
  - Approximations (*e.g.*, transaxial/axial separability)
  - Symmetry

- Hardware / software
  - GPU, CUDA, OpenCL, FPGA, SIMD, pthread, OpenMP, MPI, ...

- ...

Choice 3. Statistical Model

The physical model describes measurement mean,
e.g., for a monoenergetic X-ray source and ignoring scatter etc.:

$$\bar{I}_i([\boldsymbol{Ax}]_i) = I_0\, e^{-\sum_{j=1}^{N} a_{ij}x_j}.$$

The raw noisy measurements $\{I_i\}$ are distributed around those means.
Statistical reconstruction methods require a model for that distribution.

Trade offs: using more accurate statistical models
- *may* lead to less noisy images
- may incur additional computation
- may involve higher algorithm complexity.

CT measurement statistics are very complicated (*cf.* PET)
- incident photon flux variations (Poisson)
- X-ray photon absorption/scattering (Bernoulli)
- energy-dependent light production in scintillator (?)
- shot noise in photodiodes (Poisson?)
- electronic noise in readout electronics (Gaussian?)

Whiting, SPIE 4682, 2002; Lasio *et al.*, PMB, Apr. 2007

To log() or not to log() – That is the question

Models for "raw" data I_i (before logarithm)

- compound Poisson (complicated) — Whiting, SPIE 4682, 2002; Elbakri & Fessler, SPIE 5032, 2003; Lasio *et al.*, PMB, Apr. 2007

- Poisson + Gaussian (photon variability and electronic readout noise):
$$I_i \sim \text{Poisson}\{\bar{I}_i\} + \text{N}(0, \sigma^2)$$

  Snyder *et al.*, JOSAA, May 1993 & Feb. 1995

- Shifted Poisson approximation (matches first two moments):
$$\tilde{I}_i \triangleq \left[I_i + \sigma^2\right]_+ \sim \text{Poisson}\{\bar{I}_i + \sigma^2\}$$

  Yavuz & Fessler, MIA, Dec. 1998

- Ordinary Poisson (ignore electronic noise):
$$I_i \sim \text{Poisson}\{\bar{I}_i\}$$

  Rockmore and Macovski, TNS, Jun. 1977; Lange and Carson, JCAT, Apr. 1984

All are somewhat complicated by the nonlinearity of the physics: $\bar{I}_i = e^{-[\boldsymbol{Ax}]_i}$

After taking the log()

Taking the log leads to a linear model (ignoring beam hardening):

$$y_i \triangleq -\log\left(\frac{I_i}{I_0}\right) \approx [\boldsymbol{A}\boldsymbol{x}]_i + \varepsilon_i$$

Drawbacks:
- Undefined if $I_i \leq 0$ (*e.g.*, due to electronic noise)
- It is *biased* (by Jensen's inequality): $\mathsf{E}[y_i] \geq -\log(\bar{I}_i/I_0) = [\boldsymbol{A}\boldsymbol{x}]_i$
- Exact distribution of noise ε_i intractable

Practical approach: assume Gaussian noise model: $\varepsilon_i \sim \mathsf{N}\left(0, \sigma_i^2\right)$

Options for modeling noise variance $\sigma_i^2 = \mathsf{Var}\{\varepsilon_i\}$
- consider both Poisson and Gaussian noise effects: $\sigma_i^2 = \frac{I_i + \sigma^2}{I_i^2}$
  Thibault *et al.*, SPIE 6065, 2006
- consider just Poisson effect: $\sigma_i^2 = \frac{1}{I_i}$ (Sauer & Bouman, T-SP, Feb. 1993)
- pretend it is white noise: $\sigma_i^2 = \sigma_0^2$
- ignore noise altogether and "solve" $\boldsymbol{y} = \boldsymbol{A}\boldsymbol{x}$

Whether using pre-log data is better than post-log data is an open question.

Choice 4. Cost Functions

Components:
- *Data-mismatch* term
- *Regularization* term (and regularization parameter β)
- Constraints (*e.g.*, nonnegativity)

Reconstruct image $\hat{\boldsymbol{x}}$ by minimizing a cost function:

$$\hat{\boldsymbol{x}} \triangleq \arg\min_{\boldsymbol{x} \geq \boldsymbol{0}} \Psi(\boldsymbol{x})$$

$$\Psi(\boldsymbol{x}) = \text{DataMismatch}(\boldsymbol{y}, \boldsymbol{A}\boldsymbol{x}) + \beta\,\text{Regularizer}(\boldsymbol{x})$$

Forcing too much "data fit" alone would give noisy images.

Equivalent to a Bayesian MAP (maximum *a posteriori*) estimator.

Distinguishes "statistical methods" from "algebraic methods" for "$\boldsymbol{y} = \boldsymbol{A}\boldsymbol{x}$."

Choice 4.1: Data-Mismatch Term

Standard choice is the negative log-likelihood of statistical model:

$$\text{DataMismatch} = -L(\boldsymbol{x};\boldsymbol{y}) = -\log \mathsf{p}(\boldsymbol{y}|\boldsymbol{x}) = \sum_{i=1}^{M} -\log \mathsf{p}(y_i|\boldsymbol{x}).$$

- For pre-log data $\boldsymbol{I}$ with shifted Poisson model:

$$-L(\boldsymbol{x};\boldsymbol{I}) = \sum_{i=1}^{M} \left(\bar{I}_i + \sigma^2\right) - \left[I_i + \sigma^2\right]_+ \log\left(\bar{I}_i + \sigma^2\right), \qquad \bar{I}_i = I_0 \, \mathrm{e}^{-[\boldsymbol{A}\boldsymbol{x}]_i}$$

This can be non-convex if $\sigma^2 > 0$;
it is convex if we ignore electronic noise $\sigma^2 = 0$. Trade-off ...

- For post-log data $\boldsymbol{y}$ with Gaussian model:

$$-L(\boldsymbol{x};\boldsymbol{y}) = \sum_{i=1}^{M} w_i \frac{1}{2}(y_i - [\boldsymbol{A}\boldsymbol{x}]_i)^2 = \frac{1}{2}(\boldsymbol{y} - \boldsymbol{A}\boldsymbol{x})'\boldsymbol{W}(\boldsymbol{y} - \boldsymbol{A}\boldsymbol{x}), \qquad w_i = 1/\sigma_i^2$$

This is a kind of (data-based) weighted least squares (WLS).
It is always convex in $\boldsymbol{x}$. Quadratic functions are "easy" to minimize.

- ...

Choice 4.2: Regularization

How to control noise due to ill-conditioning?

Noise-control methods in clinical use in PET reconstruction today:
- Stop an unregularized algorithm before convergence
- Over-iterate an unregularized algorithm then post-filter

Other possible "simple" solutions:
- Modify the raw data (pre-filter / denoise)
- Filter between iterations
- ...

Appeal:
- simple / familiar
- filter parameters have intuitive units (*e.g.*, FWHM),
  unlike a regularization parameter β
- Changing a post-filter does not require re-iterating,
  unlike changing a regularization parameter β

Dozens of papers on regularized methods for PET, but little clinical impact.
(USC MAP method is available in mouse scanners.)
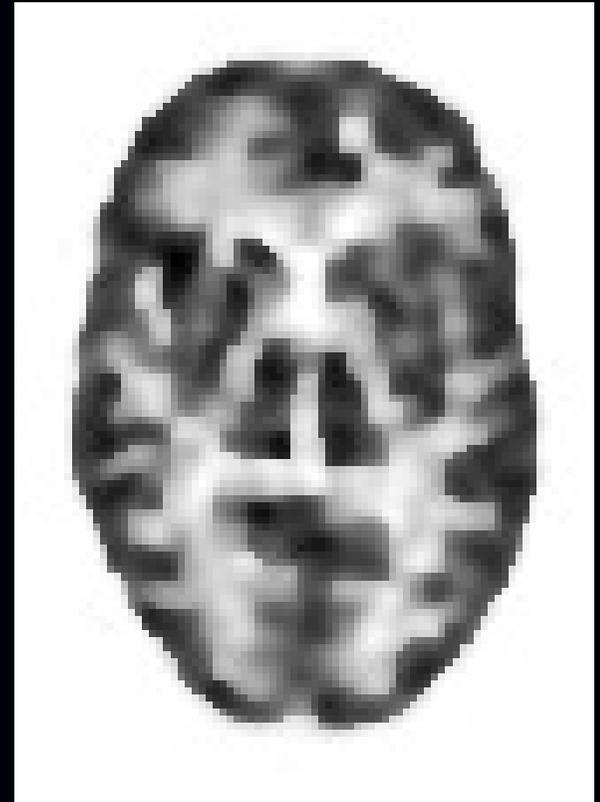
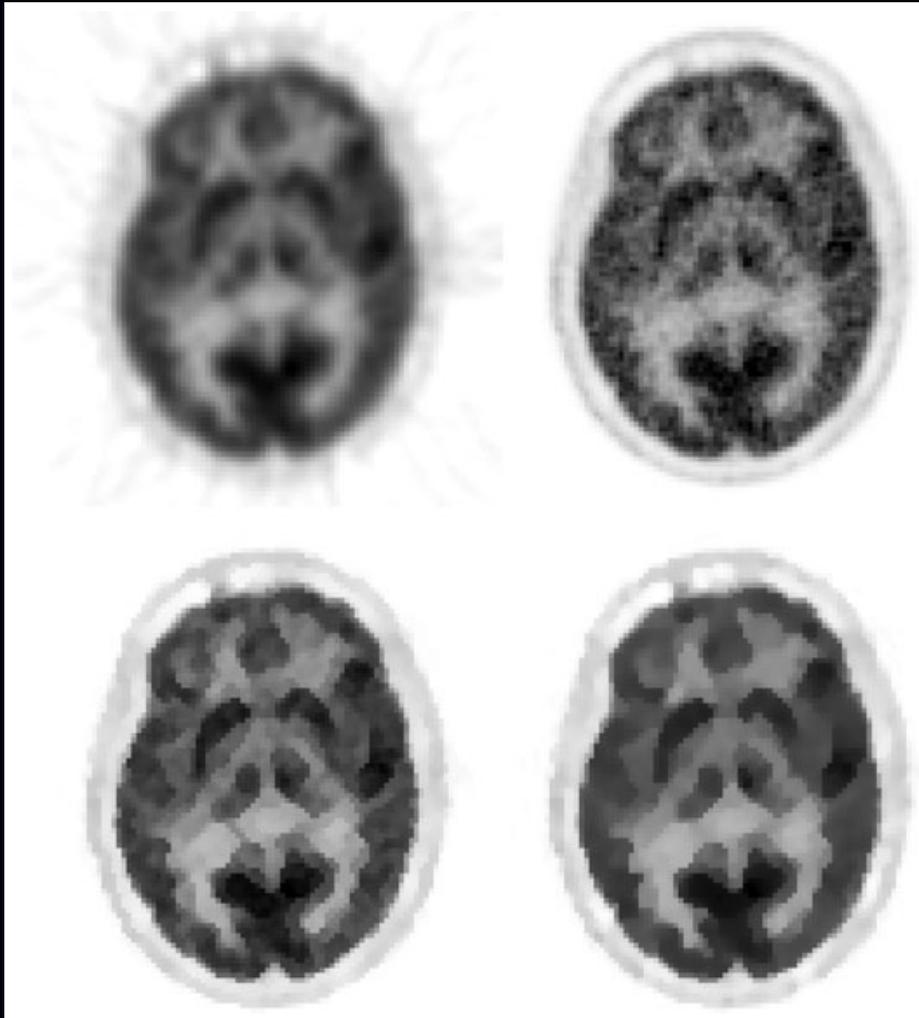# Edge-Preserving Reconstruction: PET Example
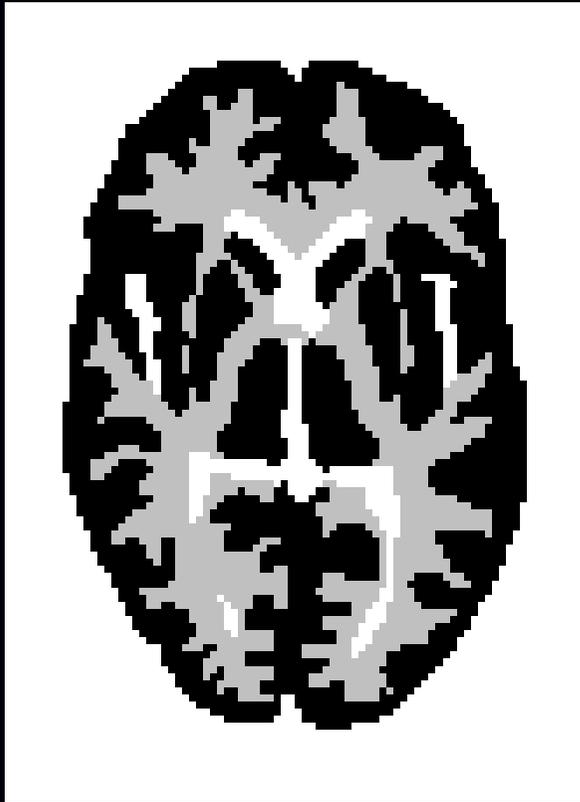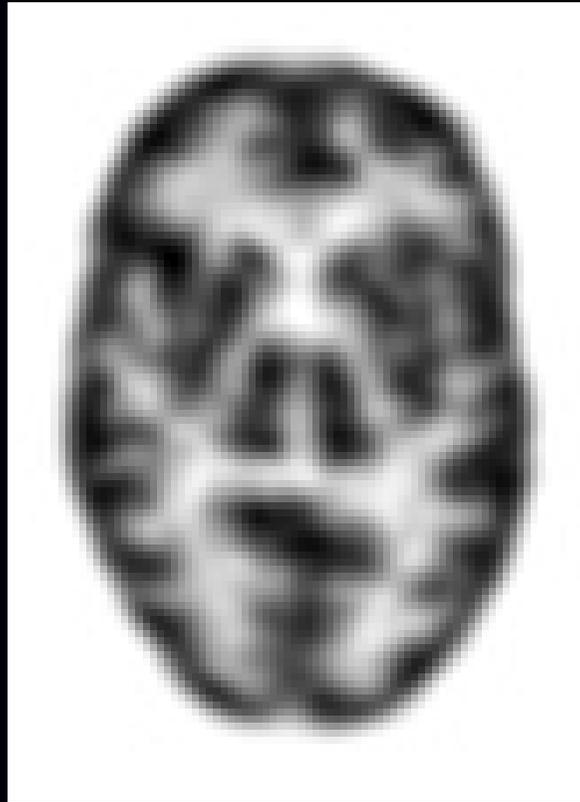
Phantom   Quadratic Penalty   Huber Penalty

Quantification vs qualitative vs tasks...

More "Edge Preserving" PET Regularization

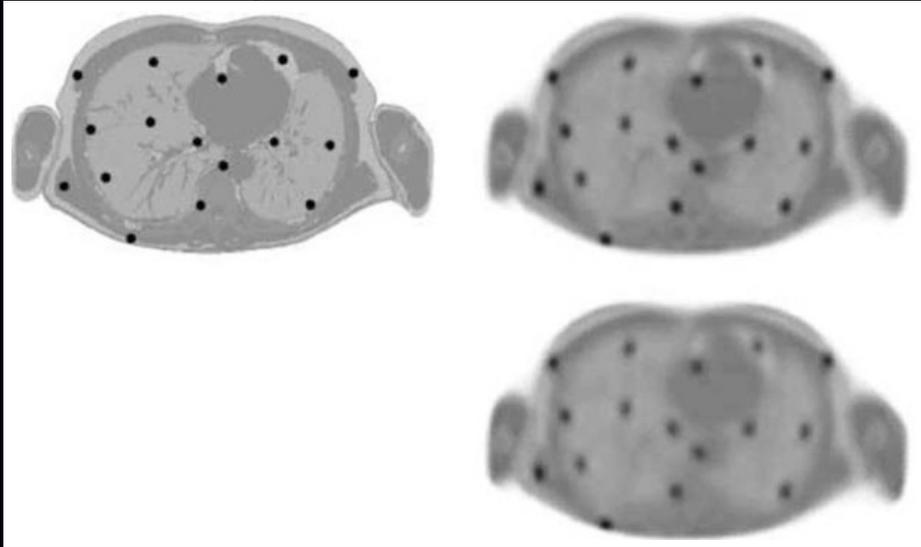| FBP	ML-EM
Median-root prior	Huber regularizer

Chlewicki *et al.*, PMB, Oct. 2004; "Noise reduction and convergence of Bayesian algorithms with blobs based on the Huber function and median root prior"

Regularization in PET

Nuyts *et al.*, T-MI, Jan. 2009:
MAP method outperformed post-filtered ML for lesion detection in simulation

Noiseless images:

Phantom	ML-EM filtered
	Regularized

Regularization options

Options for $R(\boldsymbol{x})$

In increasing complexity:

- quadratic roughness
- convex, non-quadratic roughness
- non-convex roughness
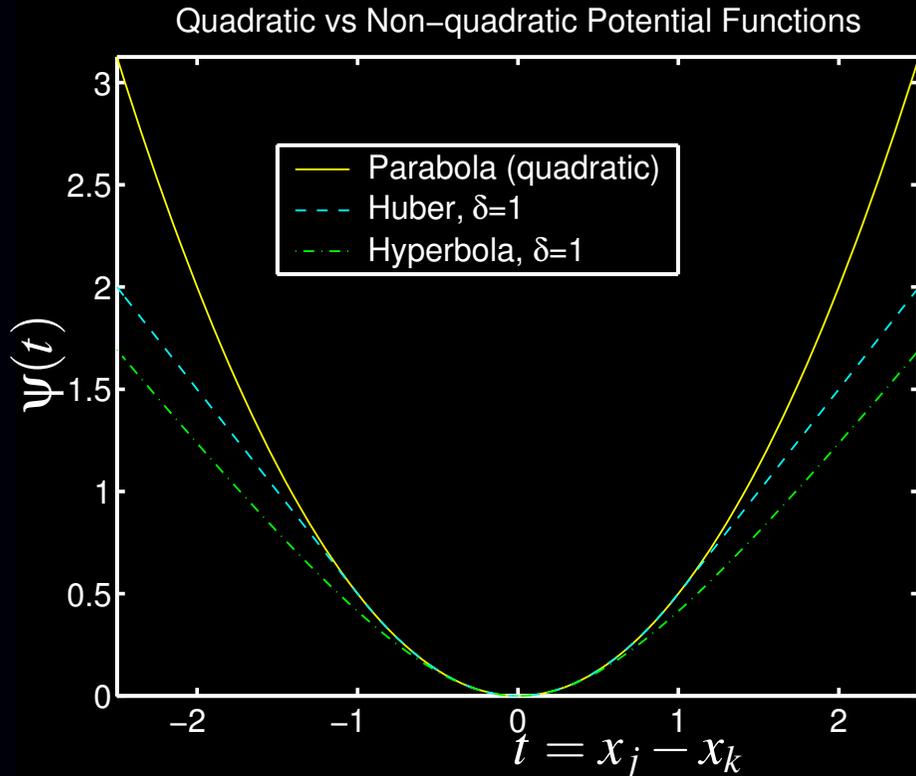- total variation
- convex sparsity
- non-convex sparsity

Goal: reduce noise without degrading spatial resolution

Many open questions...

Roughness Penalty Functions

$$R(\boldsymbol{x}) = \sum_{j=1}^{N} \frac{1}{2} \sum_{k \in \mathcal{N}_j} \psi(x_j - x_k)$$

$\mathcal{N}_j \triangleq$ *neighborhood* of jth pixel (*e.g.*, left, right, up, down)
ψ called the *potential function*

Quadratic vs Non−quadratic Potential Functions

Parabola (quadratic)
Huber, δ=1
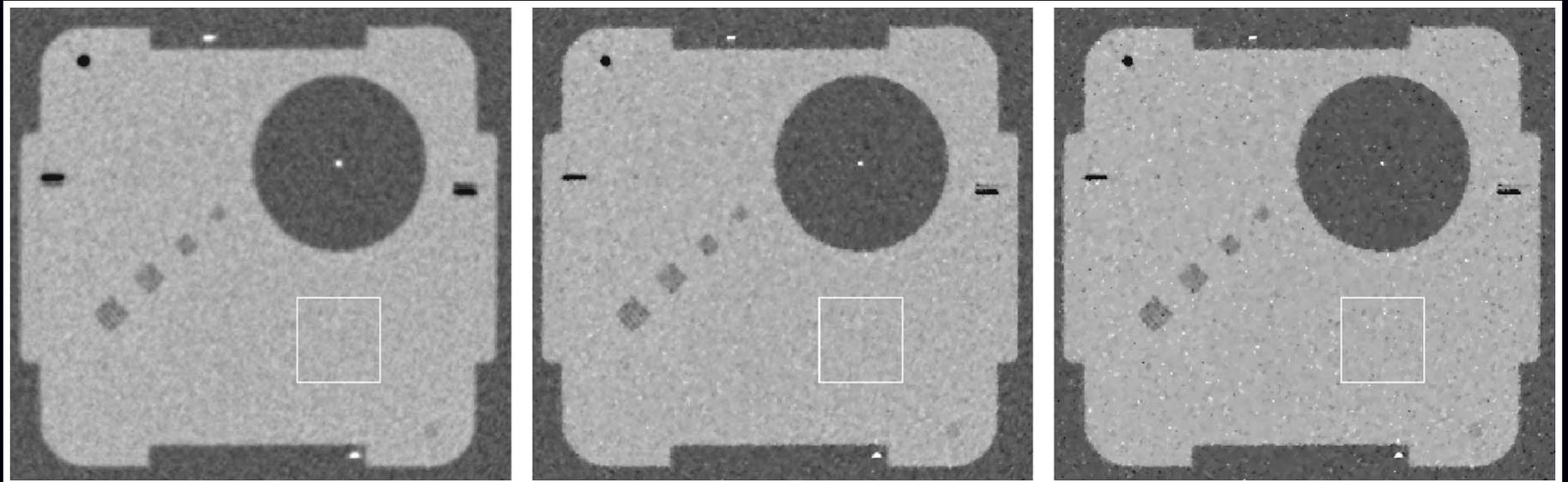Hyperbola, δ=1

$\psi(t)$

$t = x_j - x_k$

quadratic: $\psi(t) = t^2$
hyperbola: $\psi(t) = \sqrt{1 + (t/\delta)^2}$
(edge preservation)

Regularization parameters: Dramatic effects

Thibault *et al.*, Med. Phys., Nov. 2007

"q generalized gaussian" potential function with tuning parameters: β, δ, p, q:

$$\beta\,\psi(t) = \beta\,\frac{\frac{1}{2}\,|t|^p}{1 + |t/\delta|^{p-q}}$$

	$p = q = 2$	$p = 2, q = 1.2, \delta = 10$ HU	$p = q = 1.1$
noise:	11.1	10.9	10.8
(#lp/cm):	4.2	7.2	8.2

Summary thus far

1. Object parameterization

2. System physical model

3. Measurement statistical model

4. Cost function: data-mismatch / regularization / constraints

$$\boxed{\textbf{Reconstruction Method} \triangleq \textbf{Models + Cost Function + Algorithm}}$$

5. Minimization algorithms:

$$\hat{\boldsymbol{x}} = \arg\min_{\boldsymbol{x}} \Psi(\boldsymbol{x})$$

Choice 5: Minimization algorithms

- Conjugate gradients
  - Converges slowly for CT
  - Difficult to precondition due to weighting and regularization
  - Difficult to enforce nonnegativity constraint
  - Very easily parallelized
- Ordered subsets
  - Initially converges faster than CG if many subsets used
  - Does not converge without relaxation etc., but those slow it down
  - Computes regularizer gradient $\nabla R(x)$ for every subset - expensive?
  - Easily enforces nonnegativity constraint
  - Easily parallelized
- Coordinate descent   (Sauer and Bouman, T-SP, 1993)
  - Converges high spatial frequencies rapidly, but low frequencies slowly
  - Easily enforces nonnegativity constraint
  - Challenging to parallelize
- Block coordinate descent   (Benson et al., NSS/MIC, 2010)
  - Spatial frequency convergence properties depend...
  - Easily enforces nonnegativity constraint
  - More opportunity to parallelize than CD
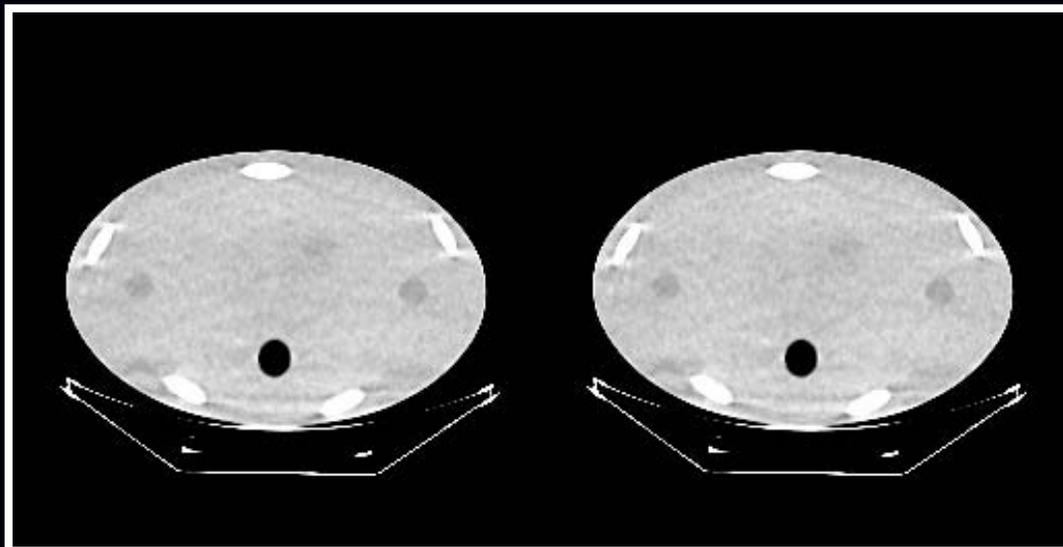
Convergence rates

LOG-POSTERIOR
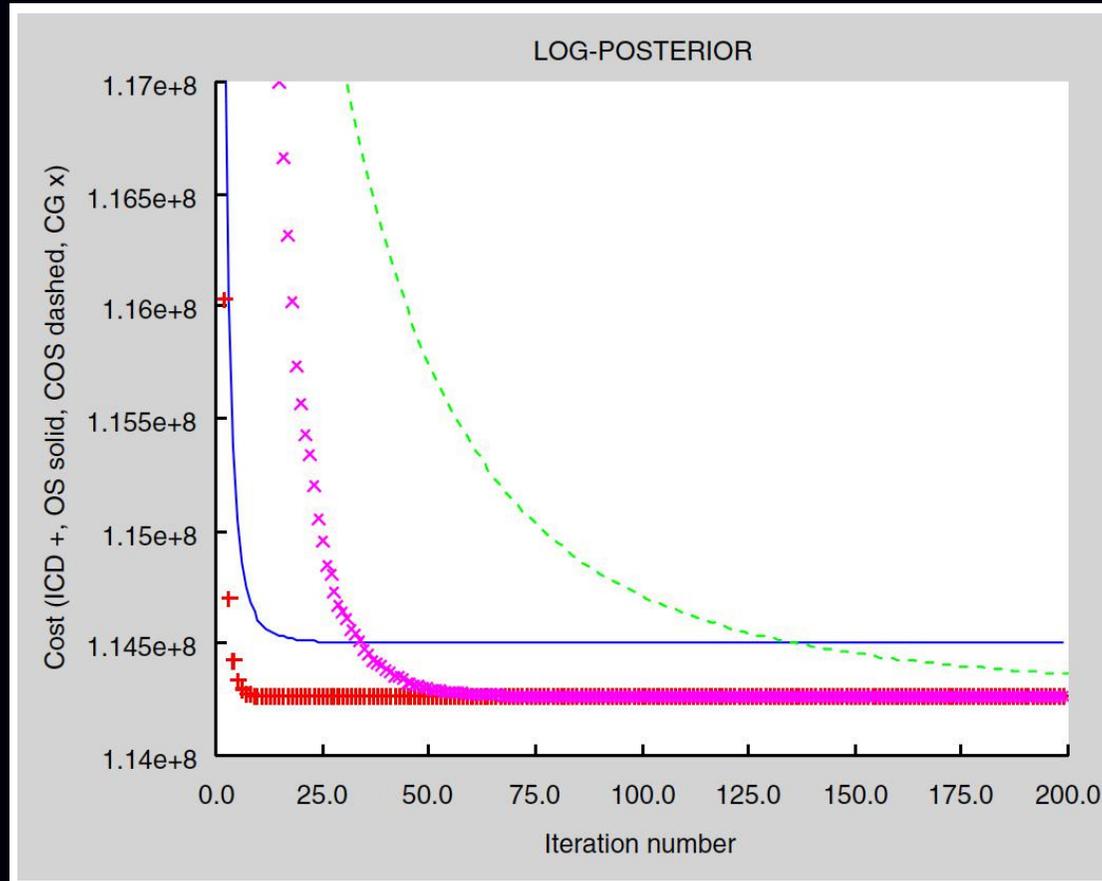
(De Man *et al.*, NSS/MIC 2005)

In terms of iterations: CD $<$ OS $<$ CG $<$ Convergent OS
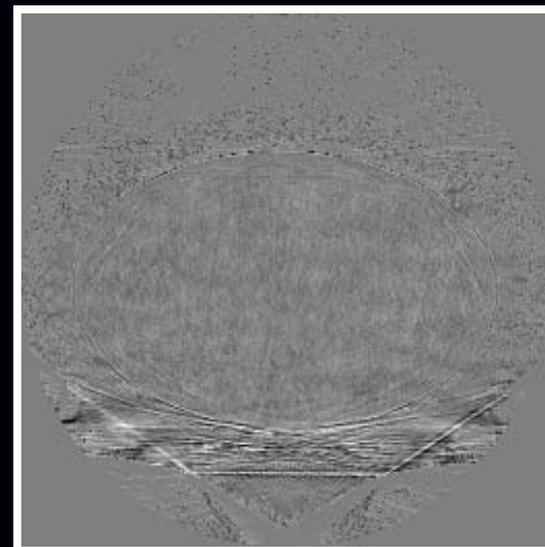In terms of compute time? (it depends...)

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Ordered subsets convergence

Theoretically OS does not converge, but it may get "close enough," even with regularization.

CD
200 iter

OS
41 subsets
200 iter

difference
0 ± 10HU

display: 930 HU $\pm$ 58 HU
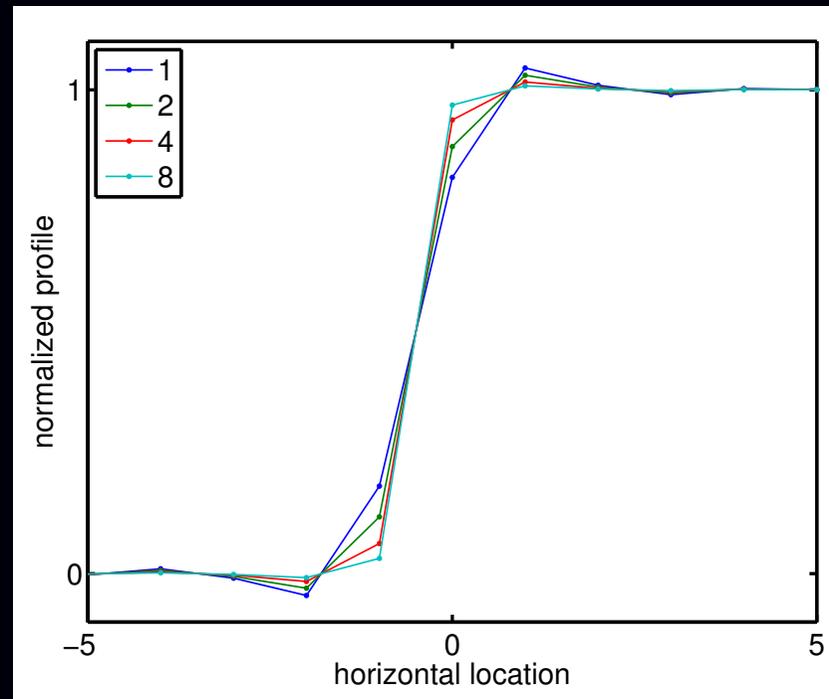
(De Man *et al.*, NSS/MIC 2005)

Ongoing saga...

(SPIE, ISBI, Fully 3D, ...)

Example

(movie)

82-subset OS with two different (but similar) edge-preserving regularizers. One frame per every 10th iteration.

Resolution characterization challenge

Orig:

Noiseless blurry image:

Restored image using PWLS $\delta = 1$:

Shape of edge response depends on contrast for edge-preserving regularizers.

Assessing image quality

- Several talks in Session 4
- Poster 7961-115, Rongping Zeng, Kyle J. Myers
  Task-based comparative study on iterative image reconstruction methods for limited-angle x-ray tomography
- Poster 7961-113, Pascal Theriault Lauzier, Jie Tang, Guang-Hong Chen
  Quantitative evaluation method of noise texture for iteratively reconstructed x-ray CT images
- ...

Very important to be specific about which statistical reconstruction method is being evaluated because results may vary significantly for different choices of models, parameters, stopping rules, ...

- What does "MTF" mean for nonlinear, shift-variant systems?
- "Less dose reduction for larger patients"
- "Resolution degrades as dose decreases"

Some open problems

- Modeling
  - Statistical modeling for very low-dose CT
  - Resolution effects
  - Spectral CT
  - Object motion
- Parameter selection / performance characterization
  - Performance prediction for nonquadratic regularization
  - Effect of nonquadratic regularization on detection tasks
  - Choice of regularization parameters for nonquadratic regularization
- Algorithms
  - optimization algorithm design
  - software/hardware implementation
  - Moore's law alone will not suffice
    (dual energy, dual source, motion, dynamic, smaller voxels ...)
- Clinical evaluation
- ...