# Statistical reconstruction for low-dose CT


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- ...

# A picture is worth 1000 words

(and perhaps several 1000 seconds of computation?)

| Thin-slice FBP | ASIR | Statistical |
|---|---|---|
| Seconds | A bit longer | Much longer |

# Why statistical methods for CT?

- Accurate physical models
  - X-ray spectrum, beam-hardening, scatter, ...
    reduced artifacts? quantitative CT?
  - X-ray detector spatial response, focal spot size, ...
    improved spatial resolution?
  - detector spectral response (*e.g.*, photon-counting detectors)

- Nonstandard geometries
  - transaxial truncation (big patients)
  - long-object problem in helical CT
  - irregular sampling in "next-generation" geometries
  - coarse angular sampling in image-guidance applications
  - limited angular range (tomosynthesis)
  - "missing" data, *e.g.*, bad pixels in flat-panel systems

- Appropriate statistical models
  - weighting reduces influence of photon-starved rays
    (FBP treats all rays equally)
  - reducing image noise or dose

# and more...

- Object constraints
  - nonnegativity
  - object support
  - piecewise smoothness
  - object sparsity (*e.g.*, angiography)
  - sparsity in some basis
  - motion models
  - dynamic models
  - ...

**Disadvantages?**

- Computation time (super computer)
- Must reconstruct entire FOV
- Model complexity
- Software complexity
- Algorithm nonlinearities
  - Difficult to analyze resolution/noise properties (*cf.* FBP)
  - Tuning parameters
  - Challenging to characterize performance

# “Iterative” vs “Statistical”

- Traditional *successive substitutions* iterations
  - *e.g.*, Joseph and Spital (JCAT, 1978) bone correction
  - usually only one or two “iterations”
  - not statistical

- Algebraic reconstruction methods
  - Given sinogram data $\boldsymbol{y}$ and system model $\boldsymbol{A}$, reconstruct object $\boldsymbol{x}$ by “solving” $\boldsymbol{y} = \boldsymbol{A}\boldsymbol{x}$
  - ART, SIRT, SART, ...
  - iterative, but typically not statistical
  - Iterative filtered back-projection (FBP):

$$\boldsymbol{x}^{(n+1)} = \boldsymbol{x}^{(n)} + \underbrace{\alpha}_{\text{step size}} \mathrm{FBP}(\underbrace{\boldsymbol{y}}_{\text{data}} - \underbrace{\boldsymbol{A}\boldsymbol{x}^{(n)}}_{\text{forward project}})$$

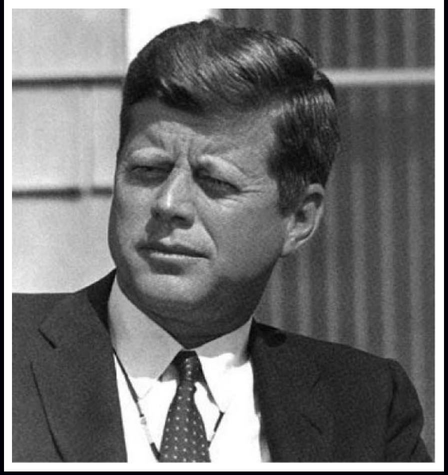
- Statistical reconstruction methods
  - Image domain
  - Sinogram domain
  - Fully statistical (both)
  - Hybrid methods (*e.g.*, AIR, SPIE 7961-18, Bruder *et al.*)

# "Statistical" methods: Image domain

- Denoising methods

$$\begin{array}{c}\text{sinogram}\\ \boldsymbol{y}\end{array} \rightarrow \boxed{\text{FBP}} \rightarrow \begin{array}{c}\text{noisy}\\ \text{reconstruction}\\ \tilde{\boldsymbol{x}}\end{array} \rightarrow \boxed{\begin{array}{c}\text{iterative}\\ \text{denoiser}\end{array}} \rightarrow \begin{array}{c}\text{final}\\ \text{image}\\ \hat{\boldsymbol{x}}\end{array}$$

  - Relatively fast, even if iterative
  - Remarkable advances in denoising methods in last decade



Zhu & Milanfar, T-IP, Dec. 2010, using "steering kernel regression" (SKR) method

Challenges:
  - Typically assume *white noise*
  - Streaks in low-dose FBP appear like edges (highly correlated noise)

- Image denoising methods “guided by data statistics”

sinogram $\boldsymbol{y}$ $\longrightarrow$ $\boxed{\text{FBP}}$ $\longrightarrow$ noisy reconstruction $\tilde{\boldsymbol{x}}$ $\longrightarrow$ $\boxed{\text{magical iterative denoiser}}$ $\longrightarrow$ final image $\hat{\boldsymbol{x}}$

$\uparrow$

sinogram statistics?

  - Image-domain methods are fast (thus practical)
  - ASIR? IRIS? ...
  - The technical details are often a mystery...

Challenges:

  - FBP often does not use all data efficiently (*e.g.*, Parker weighting)
  - Low-dose CT statistics most naturally expressed in sinogram domain

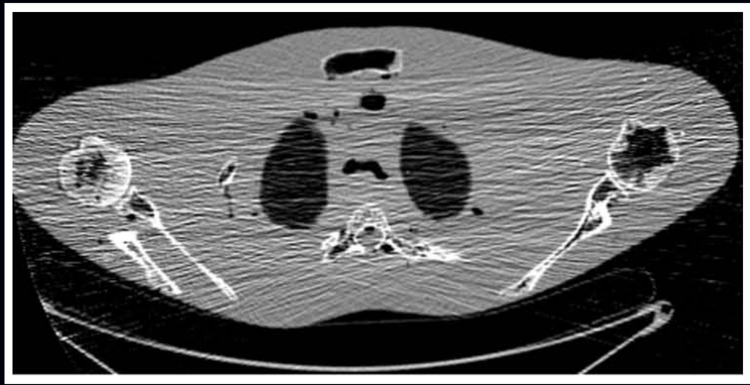
# “Statistical” methods: Sinogram domain

- Sinogram restoration methods

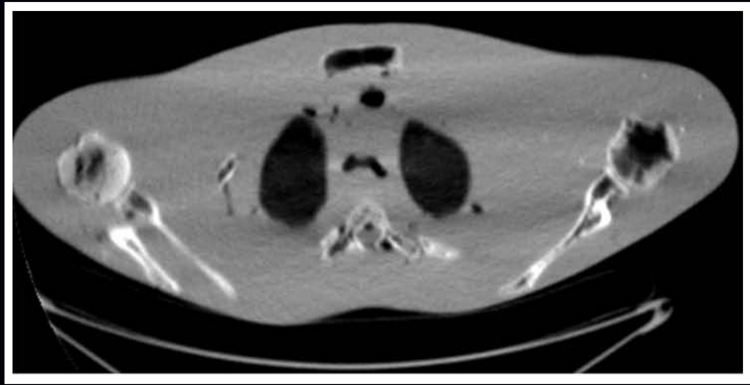
noisy sinogram $\boldsymbol{y}$ $\rightarrow$ $\boxed{\text{adaptive or iterative denoiser}}$ $\rightarrow$ cleaned sinogram $\hat{\boldsymbol{y}}$ $\rightarrow$ $\boxed{\text{FBP}}$ $\rightarrow$ final image $\hat{\boldsymbol{x}}$

  - Adaptive: J. Hsieh, Med. Phys., 1998; Kachelrieß, Med. Phys., 2001, ...
  - Iterative: P. La Riviere, IEEE T-MI, 2000, 2005, 2006, 2008
  - Relatively fast even if iterative

Challenges:
  - Limited denoising without resolution loss
  - Difficult to “preserve edges” in sinograms



FBP, 10 mA



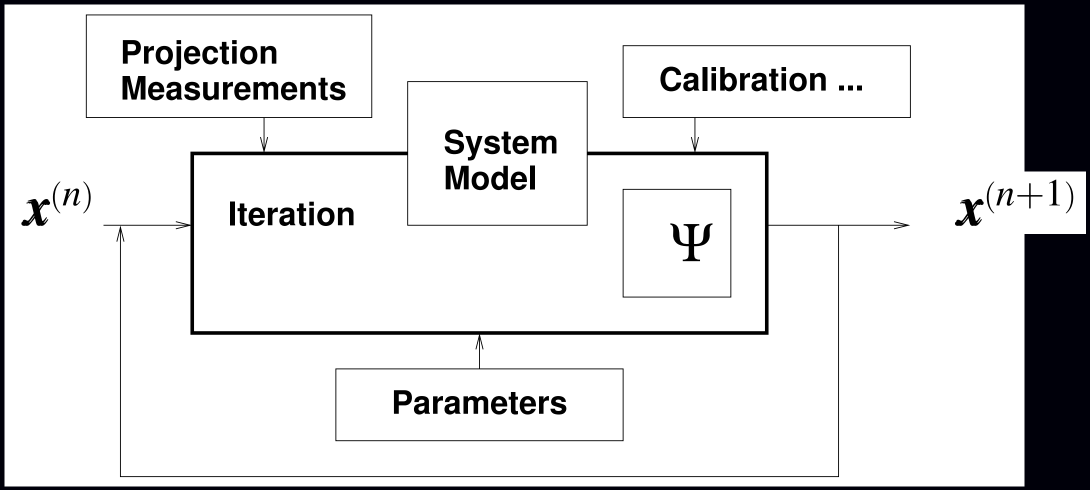
FBP from denoised sinogram

Wang *et al.*, T-MI, Oct. 2006, using PWLS-GS on sinogram

# (True? Fully? Slow?) Statistical reconstruction

- Object model
- Physics/system model
- Statistical model
- Cost function (log-likelihood + regularization)
- Iterative algorithm for minimization

"Find the image $\hat{x}$ that best fits the sinogram data $y$ according to the physics model, the statistical model and prior information about the object"




- Repeatedly revisiting the sinogram data can use statistics fully
- Repeatedly updating the image can exploit object properties
- ∴ greatest potential dose reduction, but repetition is expensive...

# History: Statistical reconstruction for PET

- Iterative method for emission tomography (Kuhl, 1963)
- Weighted least squares for 3D SPECT (Goitein, NIM, 1972)
- Richardson/Lucy iteration for image restoration (1972, 1974)
- Poisson likelihood (emission) (Rockmore and Macovski, TNS, 1976)
- Expectation-maximization (EM) algorithm (Shepp and Vardi, TMI, 1982)
- Regularized (aka Bayesian) Poisson emission reconstruction (Geman and McClure, ASA, 1985)
- Ordered-subsets EM (OSEM) algorithm (Hudson and Larkin, TMI, 1994)
- Commercial release of OSEM for PET scanners circa 1997

Today, most commercial PET systems include *unregularized* OSEM.

15 years between key EM paper (1982) and commercial adoption (1997)
(25 years if you count the R/L paper in 1972 which is the same as EM)

## Key factors in PET

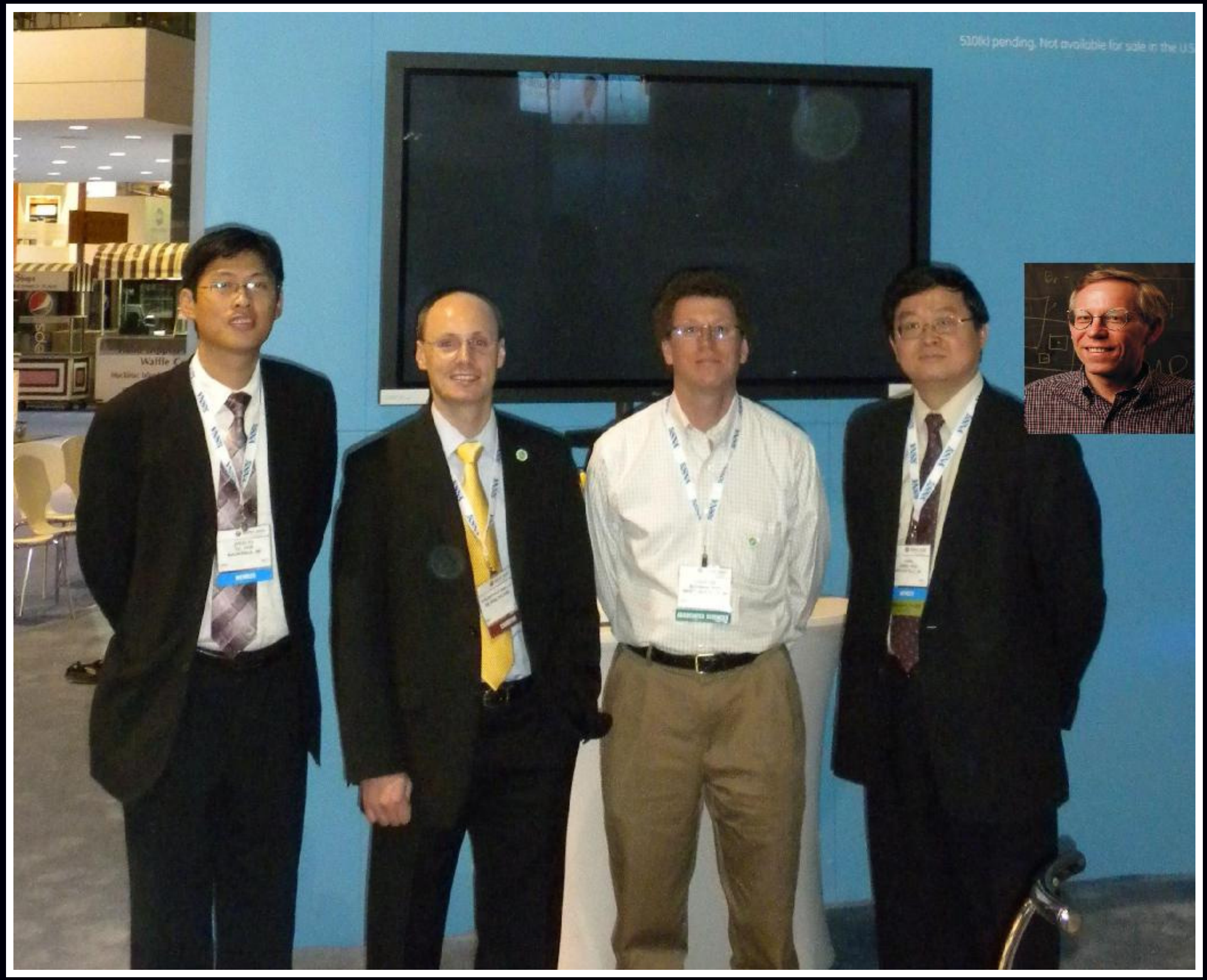
- OS algorithm accelerated convergence by order of magnitude
- Computers got faster (but problem size grew too)
- Key clinical validation papers?
- Key numerical observer studies?
- Nuclear medicine physicians grew accustomed to appearance of images reconstructed using statistical methods

## History: Statistical reconstruction for CT*

- Iterative method for X-ray CT (Hounsfield, 1968)
- ART for tomography (Gordon, Bender, Herman, JTB, 1970)
- ...
- Roughness regularized LS for tomography (Kashyap & Mittal, 1975)
- Poisson likelihood (transmission) (Rockmore and Macovski, TNS, 1977)
- EM algorithm for Poisson transmission (Lange and Carson, JCAT, 1984)
- Iterative coordinate descent (ICD) (Sauer and Bouman, T-SP, 1993)
- Ordered-subsets algorithms
  (Manglos *et al.*, PMB 1995)
  (Kamphuis & Beekman, T-MI, 1998)
  (Erdoğan & Fessler, PMB, 1999)
- ...
- Commercial introduction for CT scanners circa 2010

(* numerous omissions)

# RSNA 2010



Zhou Yu, Jean-Baptiste Thibault, Charles Bouman, Jiang Hsieh, Ken Sauer

https://engineering.purdue.edu/BME/AboutUs/News/HomepageFeatures/ResultsofPurdueResearchUnveiledatRSNA

# Five Choices for Statistical Reconstruction

1. Object model
2. System physical model
3. Measurement statistical model
4. Cost function: data-mismatch and regularization
5. Algorithm / initialization

No perfect choices - one can critique all approaches!

Historically these choices are often left implicit in publications, but being explicit facilitates reproducibility

# Choice 1. Object Parameterization

Finite measurements: $\{y_i\}_{i=1}^{M}$. Continuous object: $f(\vec{r}) = \mu(\vec{r})$.

“All models are wrong but some models are useful.”

Linear *series expansion* approach. Represent $f(\vec{r})$ by $\boldsymbol{x} = (x_1, \ldots, x_N)$ where

$$f(\vec{r}) \approx \tilde{f}(\vec{r}) = \sum_{j=1}^{N} x_j b_j(\vec{r}) \leftarrow \text{“basis functions”}$$

Reconstruction problem becomes “discrete-discrete:” estimate $\boldsymbol{x}$ from $\boldsymbol{y}$

Numerous basis functions in literature. Two primary contenders:

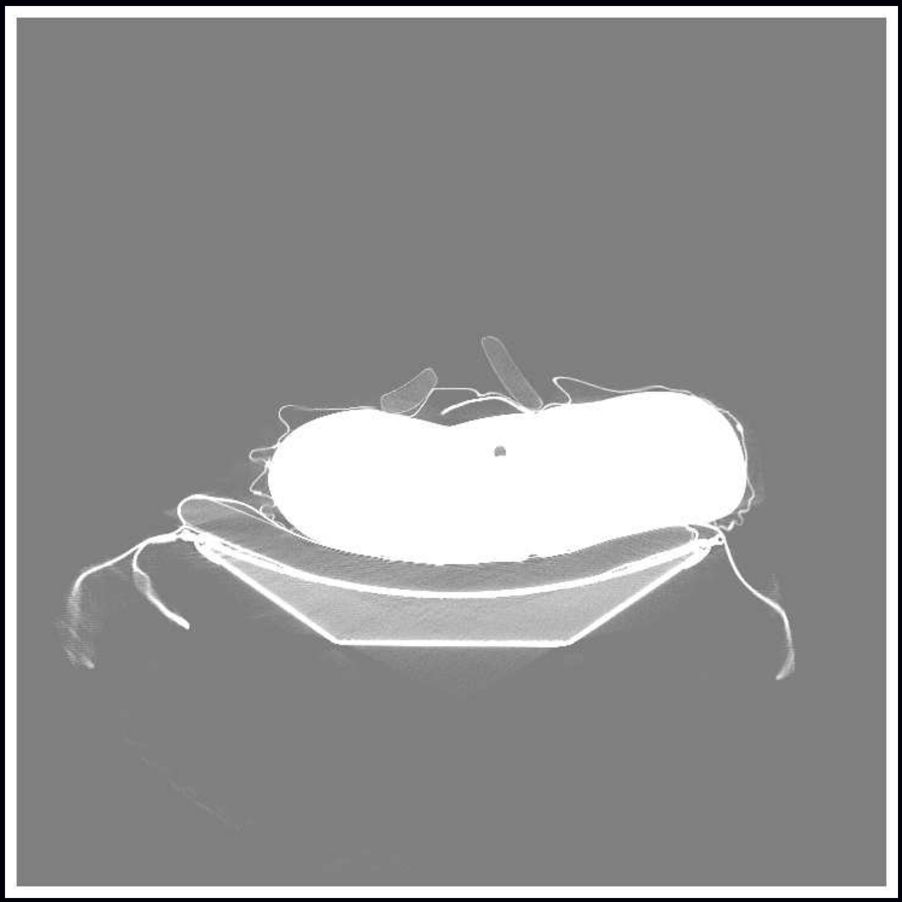
- voxels
- blobs (Kaiser-Bessel functions)
  + Blobs are approximately band-limited (reduced aliasing?)
  – Blobs have larger footprints, increasing computation.

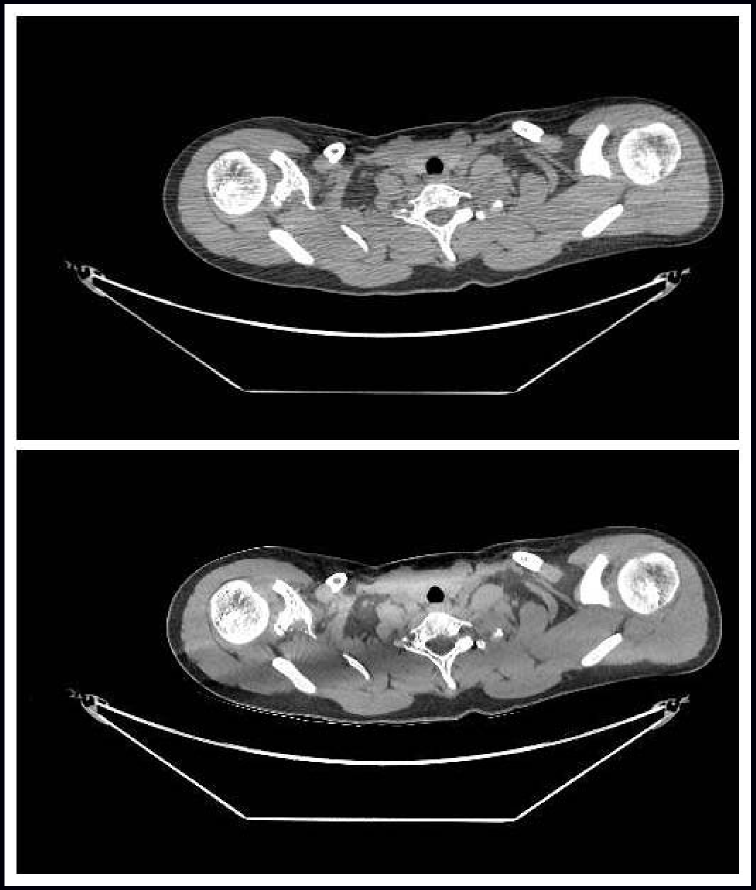
Open question: how small should the voxels be?

One practical compromise: wide FOV coarse-grid reconstruction followed by fine-grid refinement over ROI, *e.g.*, Ziegler *et al.*, Med. Phys., Apr. 2008

# Global reconstruction: An inconvenient truth

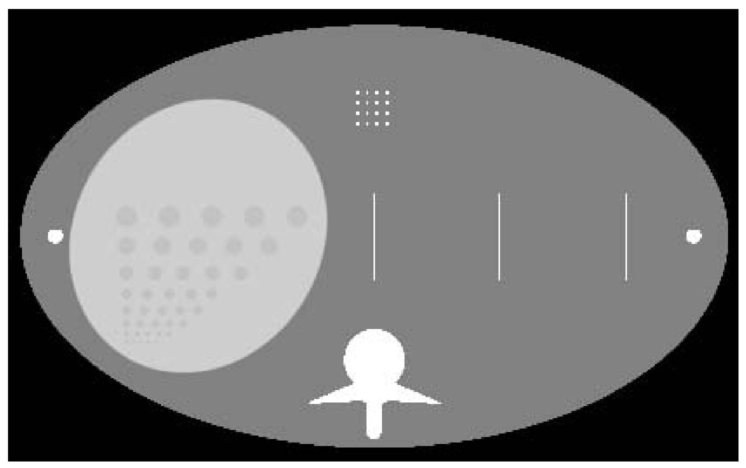
70-cm FOV reconstruction

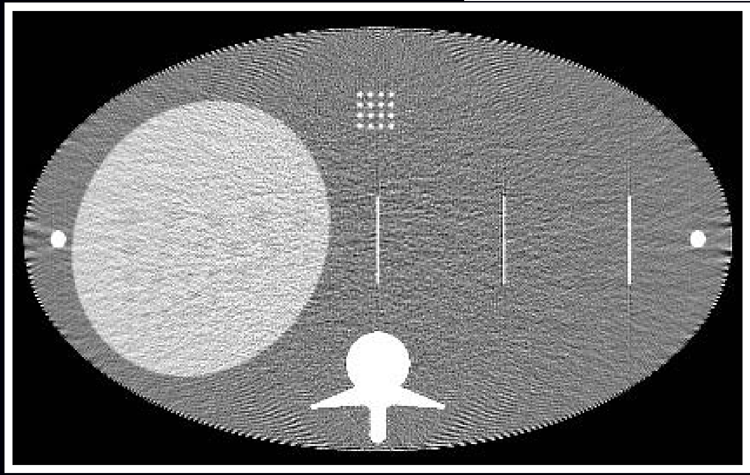




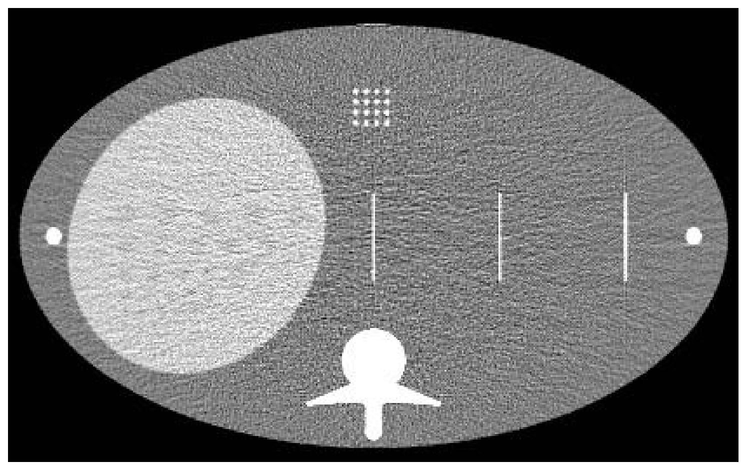
Thibault *et al.*, Fully3D, 2007

# Voxel size matters?





512 grid



1024 grid

Unregularized OS reconstructions. Zbijewski & Beekman, PMB, Jan. 2004

# Choice 2. System model / Physics model

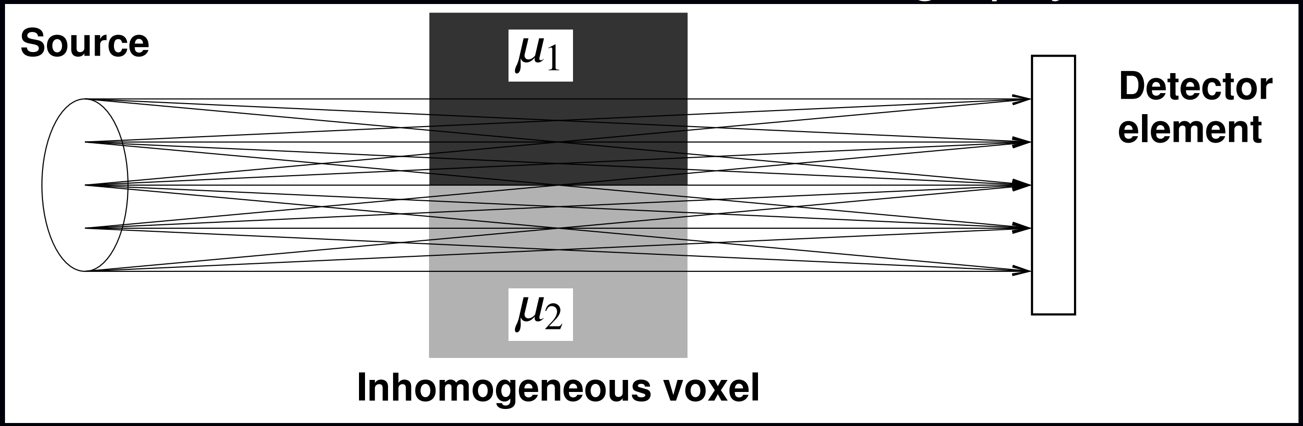
- scan geometry
- source intensity $I_0$
  - spatial variations (air scan)
  - intensity fluctuations
- resolution effects
  - finite detector size / detector spatial response
  - finite X-ray spot size / anode angulation Inhomogeneous
  - detector afterglow
- spectral effects
  - X-ray source spectrum
  - bowtie filters
  - detector spectra response
- scatter
- ...

Challenges / trade-offs

- computation time versus
- accuracy/artifacts/resolution/contrast versus
- dose?

# Exponential edge-gradient effect

Fundamental difference between emission tomography and CT:




Recorded intensity for $i$th ray: (Joseph and Spital, PMB, May 1981)

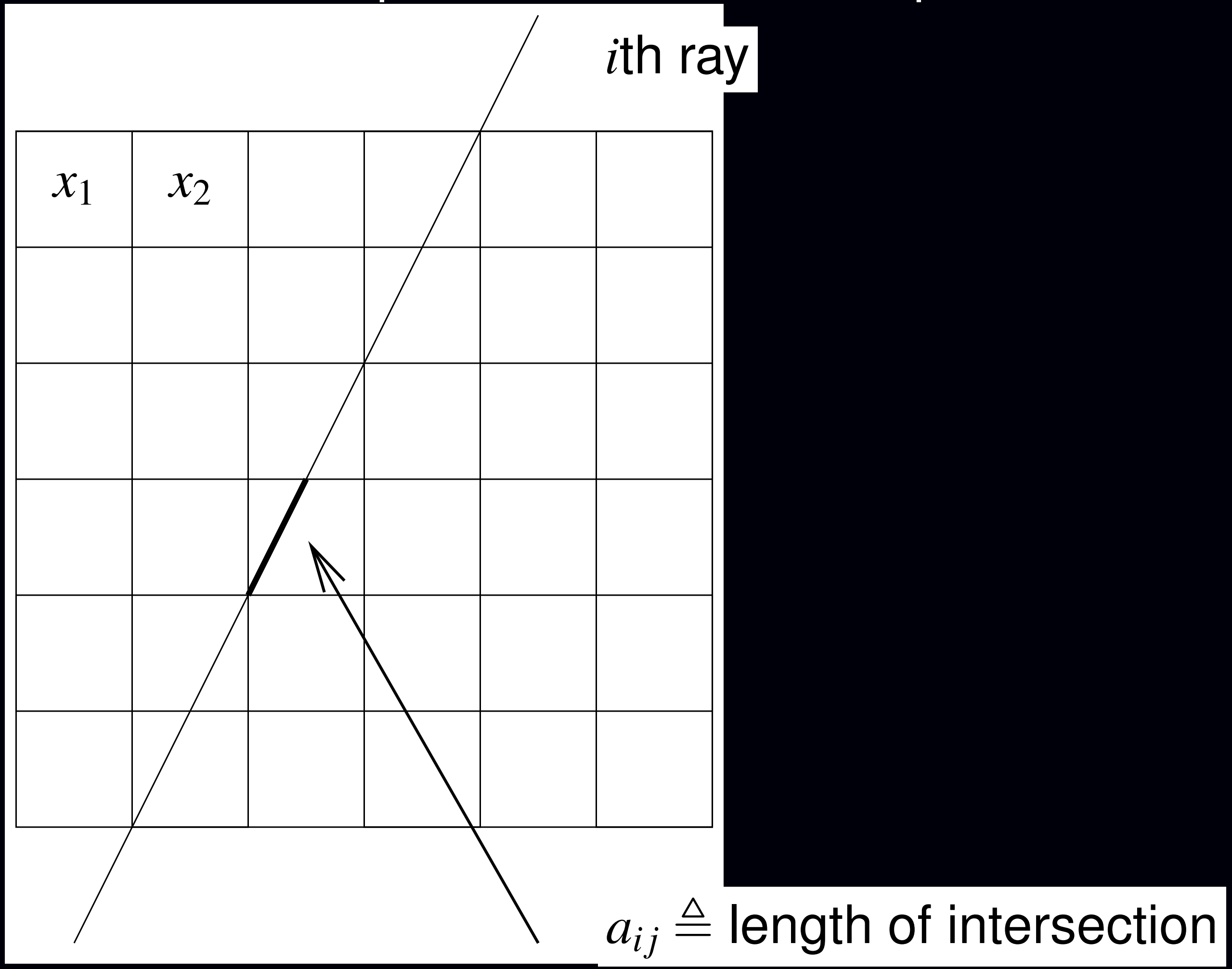
$$I_i = \int_{\text{source}} \int_{\text{detector}} I_0(\vec{p}_s, \vec{p}_d) \exp\left( - \int_{\mathcal{L}(\vec{p}_s,\vec{p}_d)} \mu(\vec{r})\, \mathrm{d}\ell \right) \mathrm{d}\vec{p}_d \, \mathrm{d}\vec{p}_s$$
$$\neq I_0 \exp\left( - \int_{\text{source}} \int_{\text{detector}} \int_{\mathcal{L}(\vec{p}_s,\vec{p}_d)} \mu(\vec{r})\, \mathrm{d}\ell \, \mathrm{d}\vec{p}_d \, \mathrm{d}\vec{p}_s \right).$$

Usual "linear" approximation:

$$I_i \approx I_0 \exp\left( - \sum_{j=1}^{N} a_{ij} x_j \right), \qquad \underbrace{a_{ij} \triangleq \int_{\text{source}} \int_{\text{detector}} \int_{\mathcal{L}(\vec{p}_s,\vec{p}_d)} b_j(\vec{r})\, \mathrm{d}\ell \, \mathrm{d}\vec{p}_d \, \mathrm{d}\vec{p}_s}_{\text{elements of system matrix } \boldsymbol{A}}$$

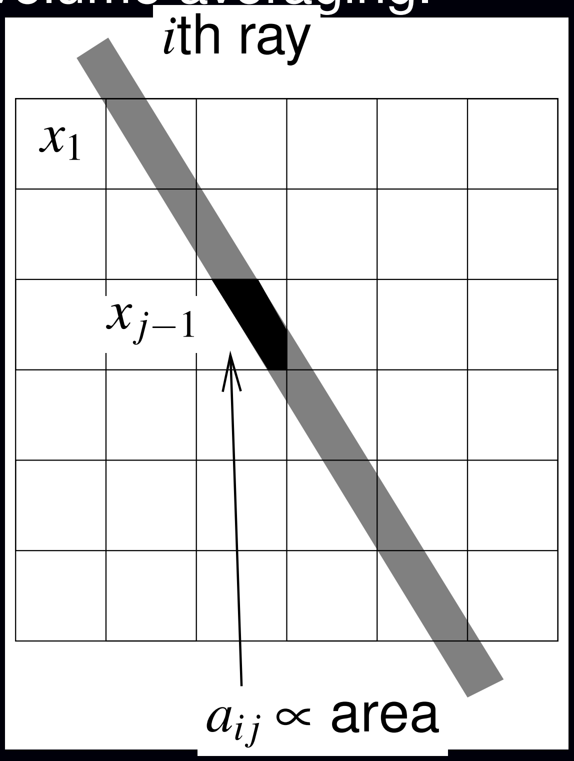
# "Line Length" System Model

Assumes (implicitly?) that source is a point and detector is a point.



# “Strip Area” System Model

Account for finite detector width.
Ignores nonlinear partial-volume averaging.



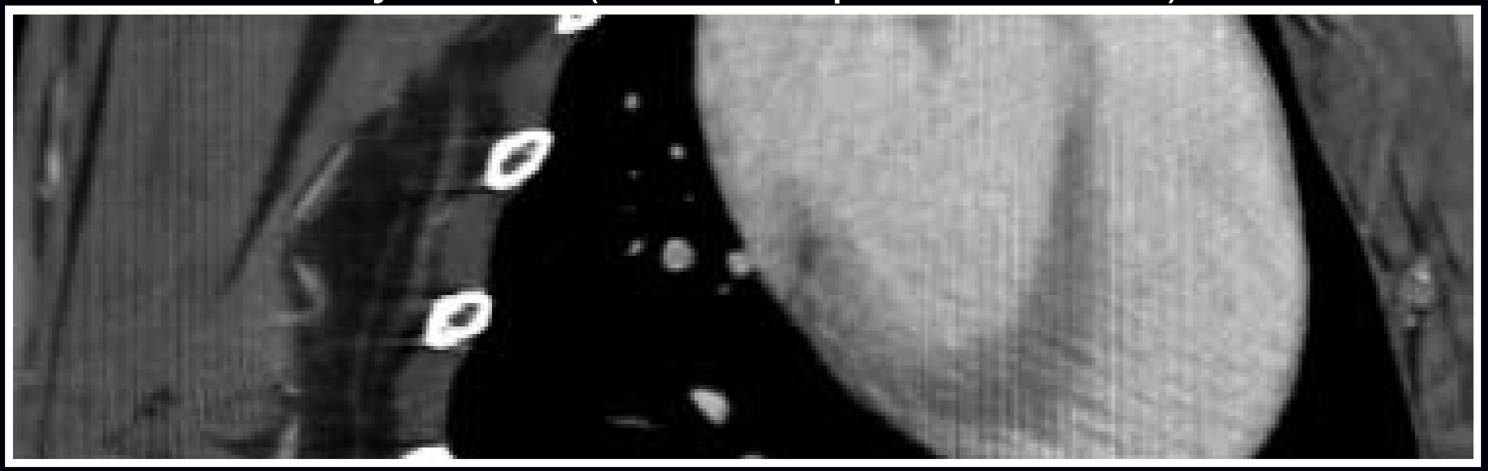

Practical (?) implementations in 3D include

- Distance-driven method (De Man and Basu, PMB, Jun. 2004)
- Separable-footprint method (Long *et al.*, T-MI, Nov. 2010)
- Further comparisons needed...

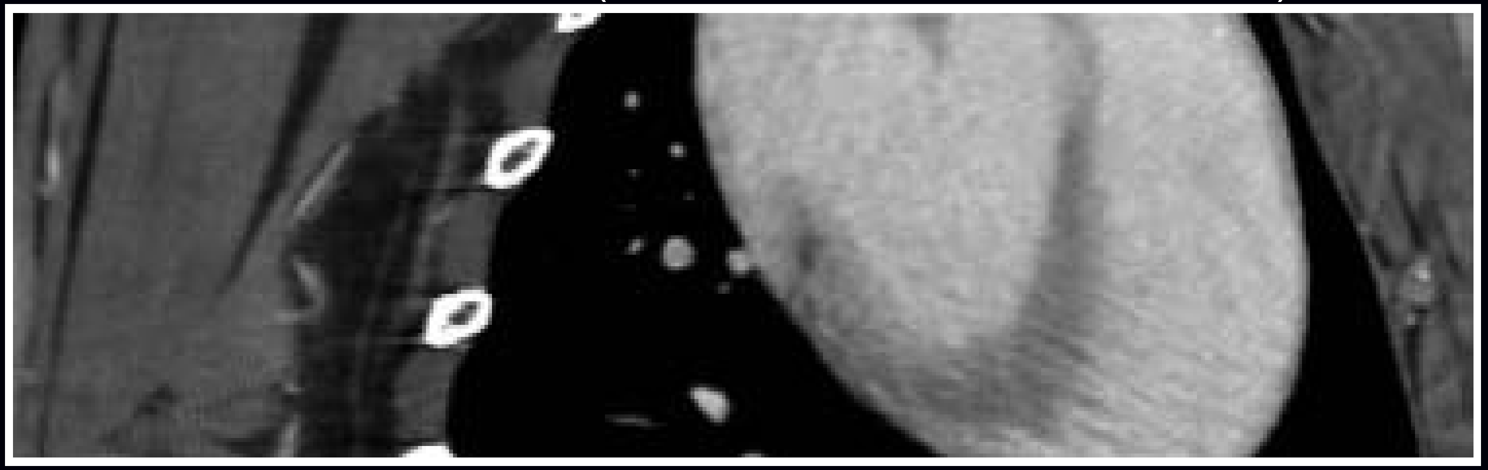
# Lines versus strips

From (De Man and Basu, PMB, Jun. 2004) MLTR of rabbit heart

Ray-driven (idealized point detector)



Distance-driven (models finite detector width)



# Forward- / Back-projector "Pairs"

Typically iterative algorithms require two key steps.

- forward projection (image domain to projection domain):

$$\bar{\boldsymbol{y}} = \boldsymbol{A}\boldsymbol{x}, \qquad \bar{y}_i = \sum_{j=1}^{N} a_{ij} x_j = [\boldsymbol{A}\boldsymbol{x}]_i$$

- backprojection (projection domain to image domain):

$$\boldsymbol{z} = \boldsymbol{A}'\boldsymbol{y}, \qquad z_j = \sum_{i=1}^{M} a_{ij} y_i$$

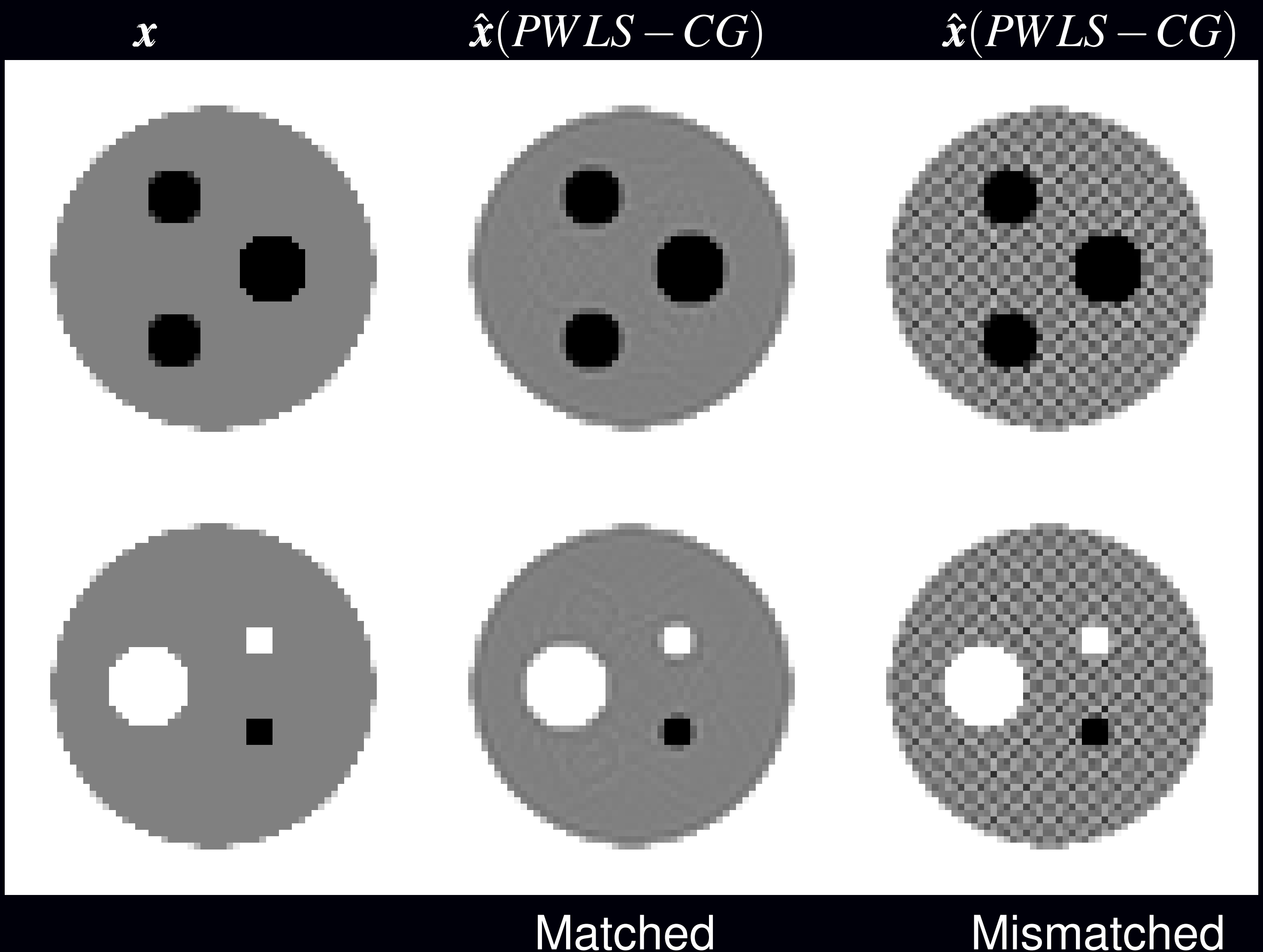
The term "forward/backprojection pair" often refers to some implicit choices for the object basis and the system model.

Sometimes $\boldsymbol{A}'\boldsymbol{y}$ is implemented as $\boldsymbol{B}\boldsymbol{y}$ for some "backprojector" $\boldsymbol{B} \neq \boldsymbol{A}'$. Especially in SPECT and sometimes in PET.

Least-squares solutions (for example):

$$\hat{\boldsymbol{x}} = \arg\min_{\boldsymbol{x}} \|\boldsymbol{y} - \boldsymbol{A}\boldsymbol{x}\|^2 = \left[\boldsymbol{A}'\boldsymbol{A}\right]^{-1} \boldsymbol{A}'\boldsymbol{y} \neq [\boldsymbol{B}\boldsymbol{A}]^{-1} \boldsymbol{B}\boldsymbol{y}$$

# Mismatched Backprojector $B \neq A'$




*cf.* SPECT/PET reconstruction – usually unregularized

# Projector/back-projector bottleneck

Challenges

- Projector/backprojector algorithm design
  - Approximations (*e.g.*, transaxial/axial separability)
  - Symmetry
- Hardware / software implementation
  - GPU, CUDA, OpenCL, FPGA, SIMD, pthread, OpenMP, MPI, ...
- Further “wholistic” approaches?

  *e.g.*, Basu & De Man, “Branchless distance driven projection ...,” SPIE 2006
- ...

# Choice 3. Statistical Model

The physical model describes measurement mean,
*e.g.*, for a monoenergetic X-ray source and ignoring scatter etc.:

$$\bar{I}_i([\boldsymbol{A}\boldsymbol{x}]_i) = I_0 \, \mathrm{e}^{-\sum_{j=1}^{N} a_{ij} x_j}.$$

The raw noisy measurements $\{I_i\}$ are distributed around those means.
Statistical reconstruction methods require a model for that distribution.

Challenges / Trade offs: using more accurate statistical models

- *may* lead to less noisy images
- may incur additional computation
- may involve higher algorithm complexity.

CT measurement statistics are very complicated, more so at low doses

- incident photon flux variations (Poisson)
- X-ray photon absorption/scattering (Bernoulli)
- energy-dependent light production in scintillator (?)
- shot noise in photodiodes (Poisson?)
- electronic noise in readout electronics (Gaussian?)
  Whiting, SPIE 4682, 2002; Lasio *et al.*, PMB, Apr. 2007
- Inaccessibility of raw sinogram data

# To log() or not to log() – That is the question

Models for "raw" data $I_i$ (before logarithm)

- compound Poisson (complicated) Whiting, SPIE 4682, 2002; Elbakri & Fessler, SPIE 5032, 2003; Lasio *et al.*, PMB, Apr. 2007
- Poisson + Gaussian (photon variability and electronic readout noise):

$$I_i \sim \text{Poisson}\{\bar{I}_i\} + \text{N}\left(0, \sigma^2\right)$$

  Snyder *et al.*, JOSAA, May 1993 & Feb. 1995

- Shifted Poisson approximation (matches first two moments):

$$\tilde{I}_i \triangleq \left[I_i + \sigma^2\right]_+ \sim \text{Poisson}\left\{\bar{I}_i + \sigma^2\right\}$$

  Yavuz & Fessler, MIA, Dec. 1998

- Ordinary Poisson (ignore electronic noise):

$$I_i \sim \text{Poisson}\{\bar{I}_i\}$$

  Rockmore and Macovski, TNS, Jun. 1977; Lange and Carson, JCAT, Apr. 1984

- Photon-counting detectors would simplify statistical modeling

All are somewhat complicated by the nonlinearity of the physics: $\bar{I}_i = \mathrm{e}^{-[\boldsymbol{A}\boldsymbol{x}]_i}$

# After taking the log()

Taking the log leads to a linear model (ignoring beam hardening):

$$y_i \triangleq -\log\left(\frac{I_i}{I_0}\right) \approx [\boldsymbol{Ax}]_i + \varepsilon_i$$

Drawbacks:

- Undefined if $I_i \le 0$ (*e.g.*, due to electronic noise)
- It is *biased* (by Jensen's inequality): $\mathsf{E}[y_i] \ge -\log(\bar{I}_i/I_0) = [\boldsymbol{Ax}]_i$
- Exact distribution of noise $\varepsilon_i$ intractable

Practical approach: assume Gaussian noise model: $\varepsilon_i \sim \mathsf{N}\left(0, \sigma_i^2\right)$

Options for modeling noise variance $\sigma_i^2 = \mathsf{Var}\{\varepsilon_i\}$

- consider both Poisson and Gaussian noise effects: $\sigma_i^2 = \frac{I_i + \sigma^2}{I_i^2}$

  Thibault *et al.*, SPIE 6065, 2006
- consider just Poisson effect: $\sigma_i^2 = \frac{1}{I_i}$ (Sauer & Bouman, T-SP, Feb. 1993)
- pretend it is white noise: $\sigma_i^2 = \sigma_0^2$
- ignore noise altogether and "solve" $\boldsymbol{y} = \boldsymbol{Ax}$

Whether using pre-log data is better than post-log data is an open question.

# Choice 4. Cost Functions

Components:

- *Data-mismatch* term
- *Regularization* term (and regularization parameter $\beta$)
- Constraints (*e.g.*, nonnegativity)

Reconstruct image $\hat{\boldsymbol{x}}$ by minimizing a cost function:

$$\hat{\boldsymbol{x}} \triangleq \arg\min_{\boldsymbol{x} \geq \boldsymbol{0}} \Psi(\boldsymbol{x})$$

$$\Psi(\boldsymbol{x}) = \text{DataMismatch}(\boldsymbol{y}, \boldsymbol{A}\boldsymbol{x}) + \beta \, \text{Regularizer}(\boldsymbol{x})$$

Forcing too much “data fit” alone would give noisy images.

Equivalent to a Bayesian MAP (maximum *a posteriori*) estimator.

Distinguishes “statistical methods” from “algebraic methods” for “$\boldsymbol{y} = \boldsymbol{A}\boldsymbol{x}$.”

# Choice 4.1: Data-Mismatch Term

Standard choice is the negative log-likelihood of statistical model:

$$\text{DataMismatch} = -L(\boldsymbol{x};\boldsymbol{y}) = -\log \mathrm{p}(\boldsymbol{y}|\boldsymbol{x}) = \sum_{i=1}^{M} -\log \mathrm{p}(y_i|\boldsymbol{x}) \,.$$

- For pre-log data $\boldsymbol{I}$ with shifted Poisson model:

$$-L(\boldsymbol{x};\boldsymbol{I}) = \sum_{i=1}^{M} \left(\bar{I}_i + \sigma^2\right) - \left[I_i + \sigma^2\right]_+ \log\left(\bar{I}_i + \sigma^2\right), \qquad \bar{I}_i = I_0 \, \mathrm{e}^{-[\boldsymbol{A}\boldsymbol{x}]_i}$$

  This can be non-convex if $\sigma^2 > 0$;
  it is convex if we ignore electronic noise $\sigma^2 = 0$. Trade-off ...

- For post-log data $\boldsymbol{y}$ with Gaussian model:

$$-L(\boldsymbol{x};\boldsymbol{y}) = \sum_{i=1}^{M} w_i \frac{1}{2}(y_i - [\boldsymbol{A}\boldsymbol{x}]_i)^2 = \frac{1}{2}(\boldsymbol{y} - \boldsymbol{A}\boldsymbol{x})' \boldsymbol{W} (\boldsymbol{y} - \boldsymbol{A}\boldsymbol{x}), \qquad w_i = 1/\sigma_i^2$$

  This is a kind of (data-based) weighted least squares (WLS).
  It is always convex in $\boldsymbol{x}$. Quadratic functions are “easy” to minimize.

- ...

# Choice 4.2: Regularization

How to control noise due to ill-conditioning?

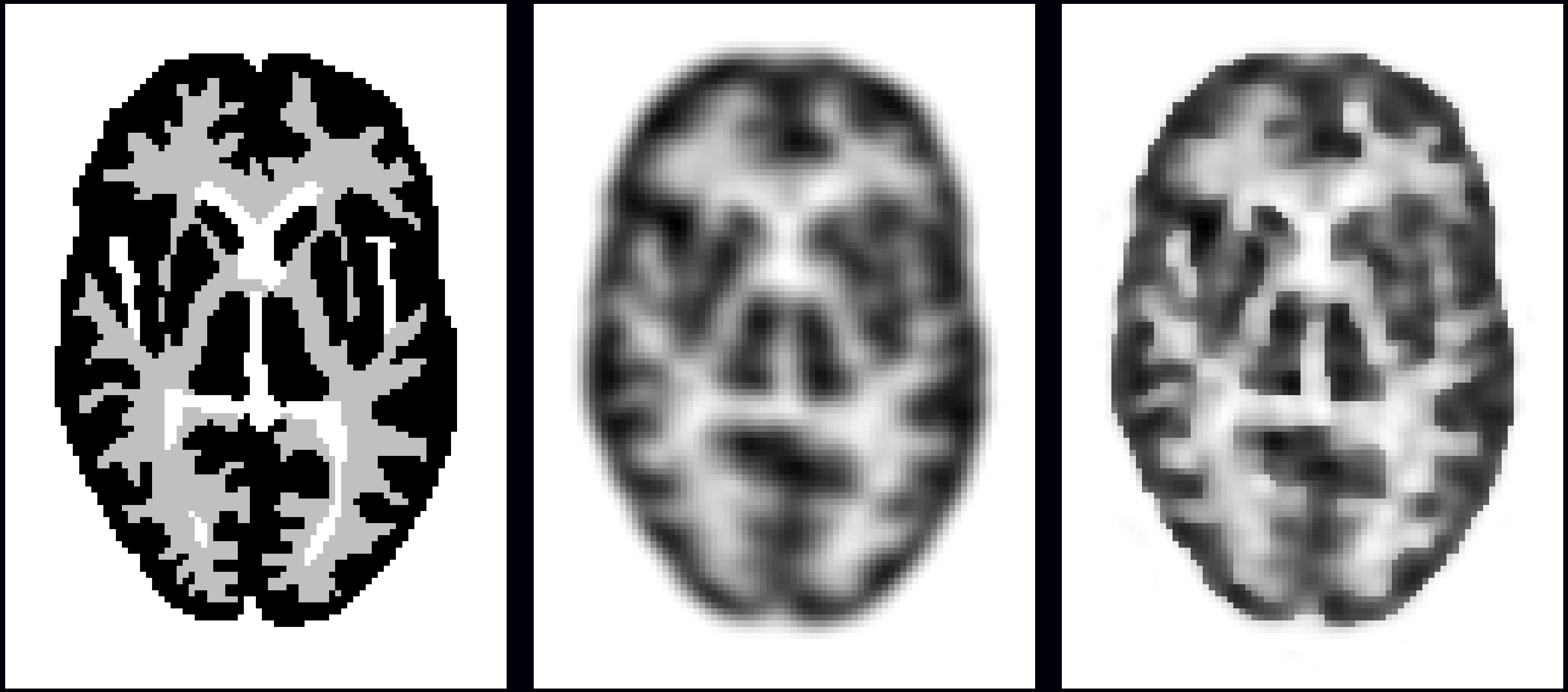
**Noise-control methods in clinical use in PET reconstruction today**:
- Stop an unregularized algorithm before convergence
- Over-iterate an unregularized algorithm then post-filter

**Other possible “simple” solutions**:
- Modify the raw data (pre-filter / denoise)
- Filter between iterations
- ...

**Appeal:**
- simple / familiar
- filter parameters have intuitive units (*e.g.*, FWHM), unlike a regularization parameter $\beta$
- Changing a post-filter does not require re-iterating, unlike changing a regularization parameter $\beta$

Dozens of papers on regularized methods for PET, but little clinical impact.
(USC MAP method is available in mouse scanners.)

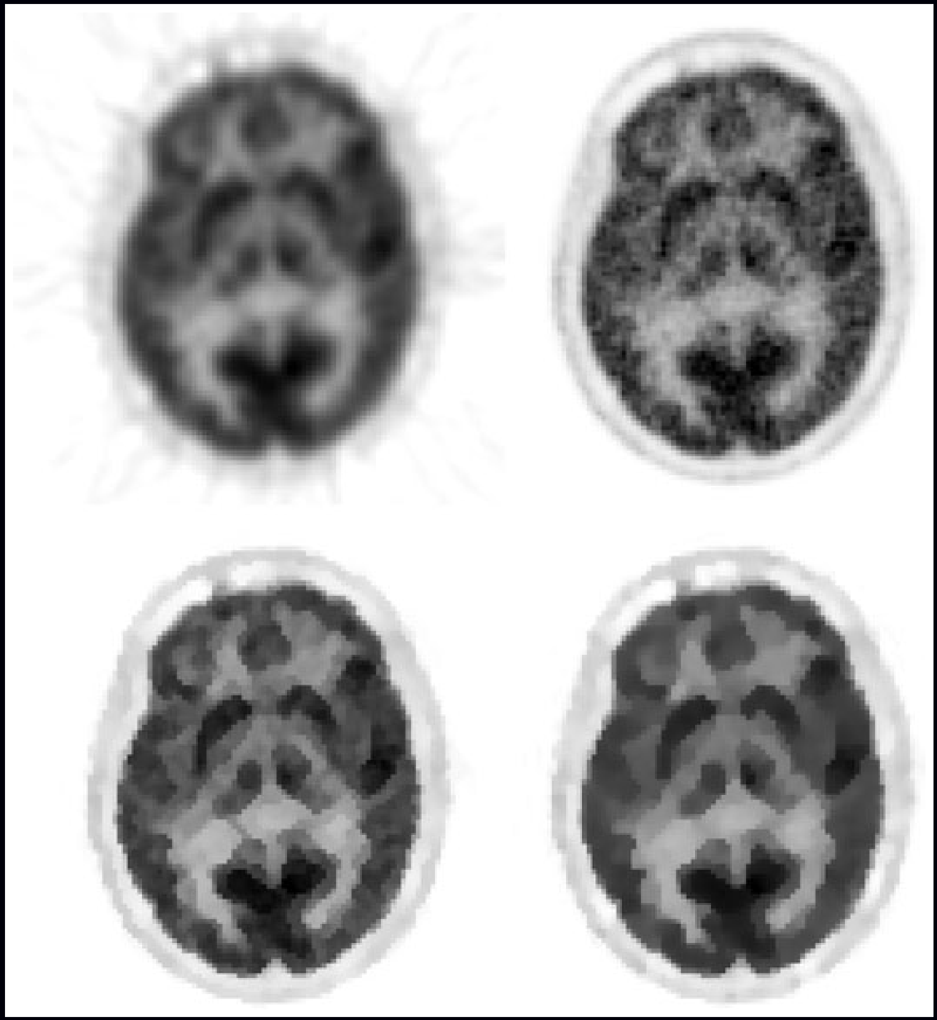
# Edge-Preserving Reconstruction: PET Example



Phantom | Quadratic Penalty | Huber Penalty

Quantification vs qualitative vs tasks...

# More “Edge Preserving” PET Regularization



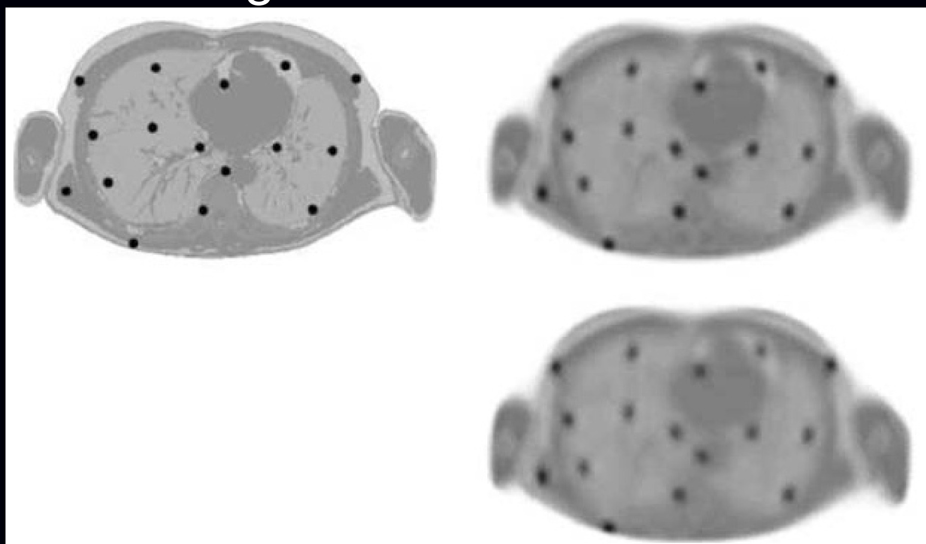
| FBP | ML-EM |
|---|---|
| Median-root prior | Huber regularizer |

Chlewicki *et al.*, PMB, Oct. 2004; “Noise reduction and convergence of Bayesian algorithms with blobs based on the Huber function and median root prior”

# Regularization in PET

Nuyts *et al.*, T-MI, Jan. 2009:
MAP method outperformed post-filtered ML for lesion detection in simulation

Noiseless images:



| Phantom | ML-EM filtered |
|---|---|
| | Regularized |

# Regularization options

Options for regularizer $\mathrm{R}(\boldsymbol{x})$ in increasing complexity:

- quadratic roughness
- convex, non-quadratic roughness
- non-convex roughness
- total variation
- convex sparsity
- non-convex sparsity

Challenges

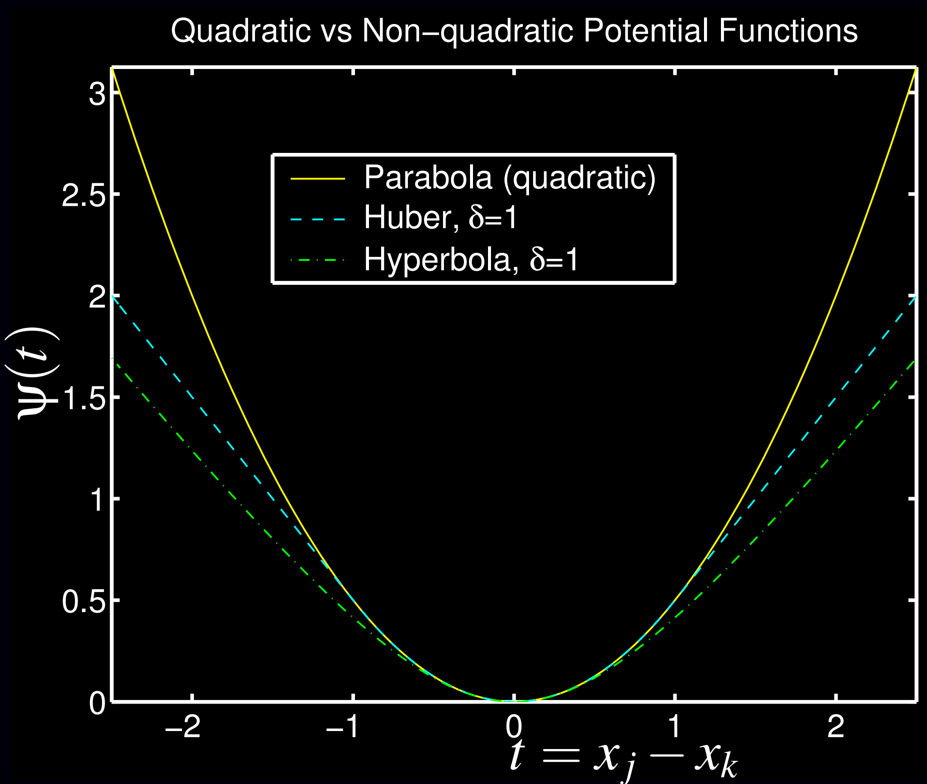
- Reducing noise without degrading spatial resolution
- Balancing regularization strength between and within slices
- Parameter selection
- Computational complexity (voxels have 26 neighbors in 3D)
- Preserving “familiar” noise texture
- Optimizing clinical task performance

Many open questions...

# Roughness Penalty Functions

$$\mathsf{R}(\boldsymbol{x}) = \sum_{j=1}^{N} \frac{1}{2} \sum_{k \in \mathcal{N}_j} \psi(x_j - x_k)$$

$\mathcal{N}_j \triangleq$ *neighborhood* of $j$th pixel (*e.g.*, left, right, up, down)
$\psi$ called the *potential function*



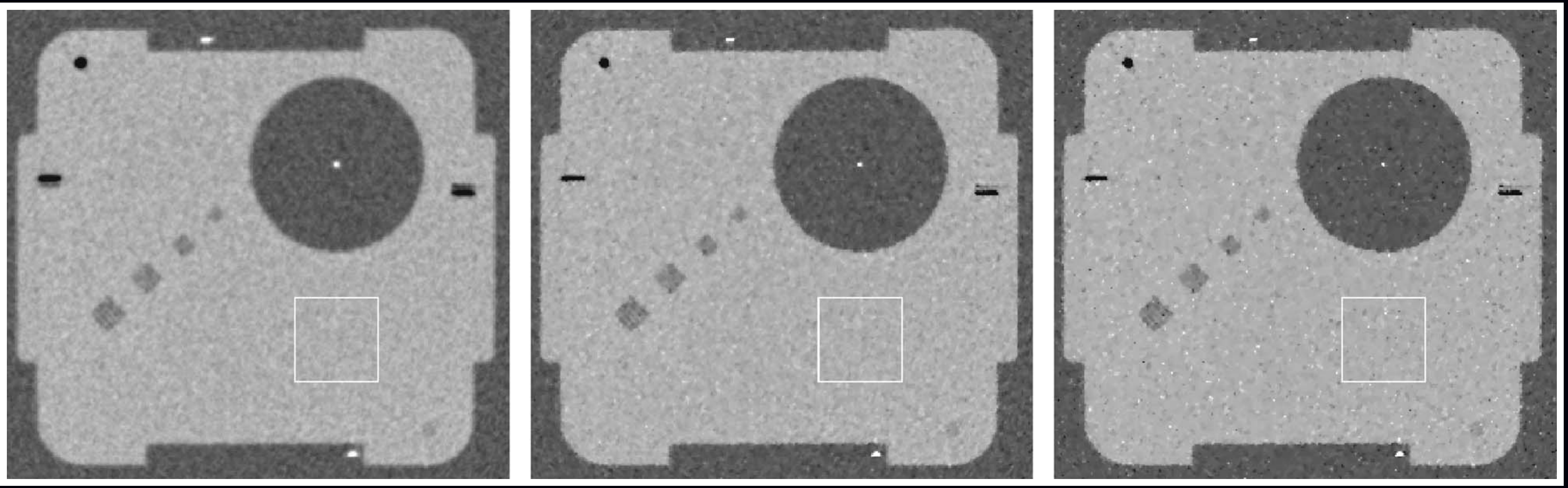

quadratic: $\psi(t) = t^2$
hyperbola: $\psi(t) = \sqrt{1 + (t/\delta)^2}$
(edge preservation)

# Regularization parameters: Dramatic effects

Thibault *et al.*, Med. Phys., Nov. 2007

"$q$ generalized gaussian" potential function with tuning parameters: $\beta, \delta, p, q$:

$$\beta\,\psi(t) = \beta \frac{\frac{1}{2}|t|^p}{1 + |t/\delta|^{p-q}}$$



| $p = q = 2$ | $p = 2, q = 1.2, \delta = 10$ HU | $p = q = 1.1$ |
|---|---|---|

| | | | |
|---|---|---|---|
| noise: | 11.1 | 10.9 | 10.8 |
| (#lp/cm): | 4.2 | 7.2 | 8.2 |

# Summary thus far

1. Object parameterization
2. System physical model
3. Measurement statistical model
4. Cost function: data-mismatch / regularization / constraints

$$\boxed{\textbf{Reconstruction Method} \triangleq \textbf{Models + Cost Function + Algorithm}}$$

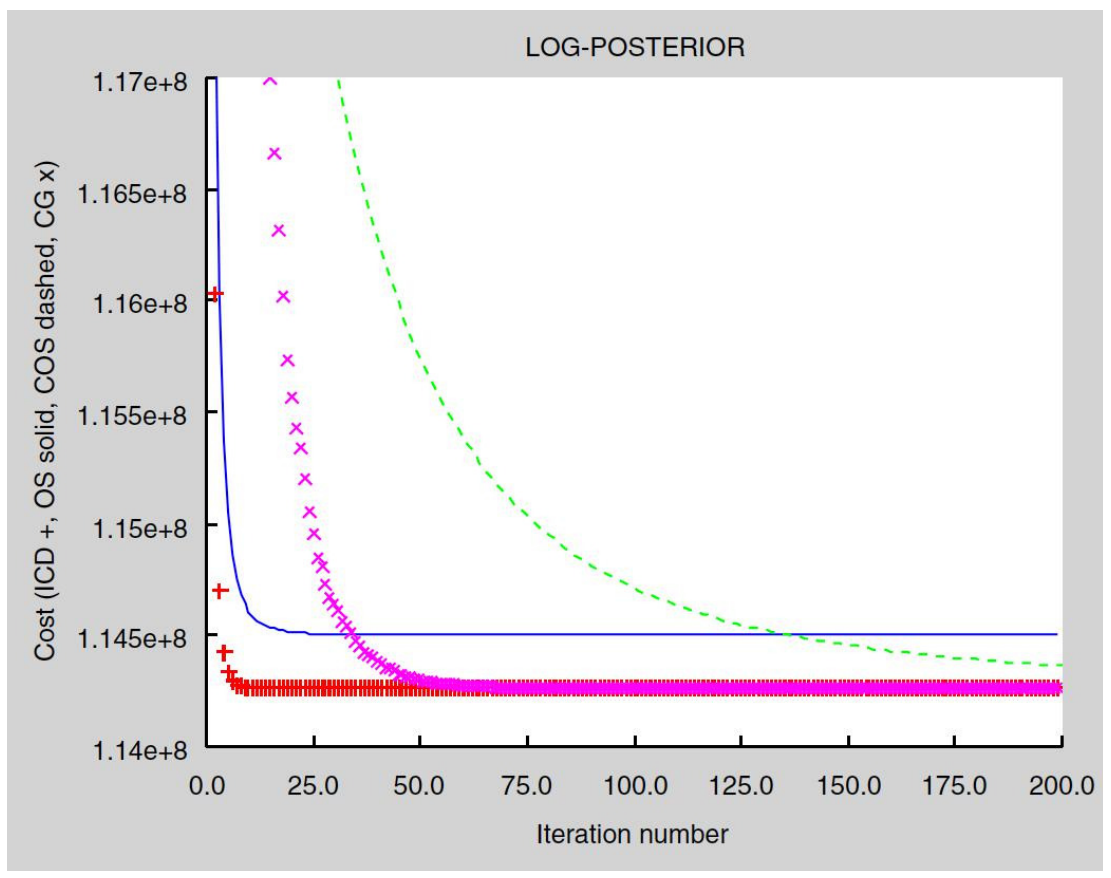
5. Minimization algorithms:

$$\hat{\boldsymbol{x}} = \arg\min_{\boldsymbol{x}} \Psi(\boldsymbol{x})$$

# Choice 5: Minimization algorithms

- Conjugate gradients
  - Converges slowly for CT
  - Difficult to precondition due to weighting and regularization
  - Difficult to enforce nonnegativity constraint
  - Very easily parallelized
- Ordered subsets
  - Initially converges faster than CG if many subsets used
  - Does not converge without relaxation etc., but those slow it down
  - Computes regularizer gradient $\nabla \mathsf{R}(\boldsymbol{x})$ for every subset - expensive?
  - Easily enforces nonnegativity constraint
  - Easily parallelized
- Coordinate descent (Sauer and Bouman, T-SP, 1993)
  - Converges high spatial frequencies rapidly, but low frequencies slowly
  - Easily enforces nonnegativity constraint
  - Challenging to parallelize
- Block coordinate descent (Benson *et al.*, NSS/MIC, 2010)
  - Spatial frequency convergence properties depend...
  - Easily enforces nonnegativity constraint
  - More opportunity to parallelize than CD

# Convergence rates

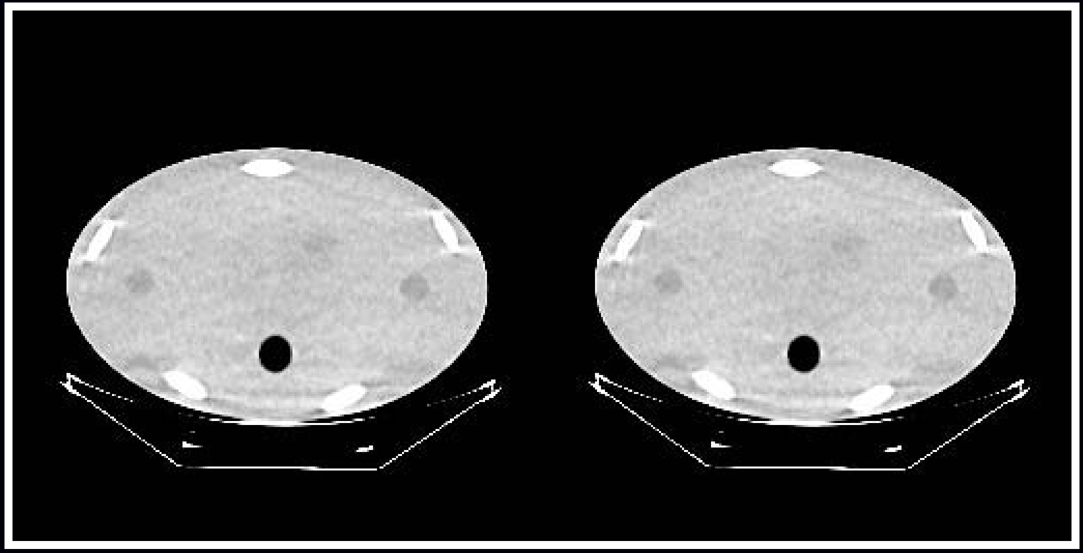


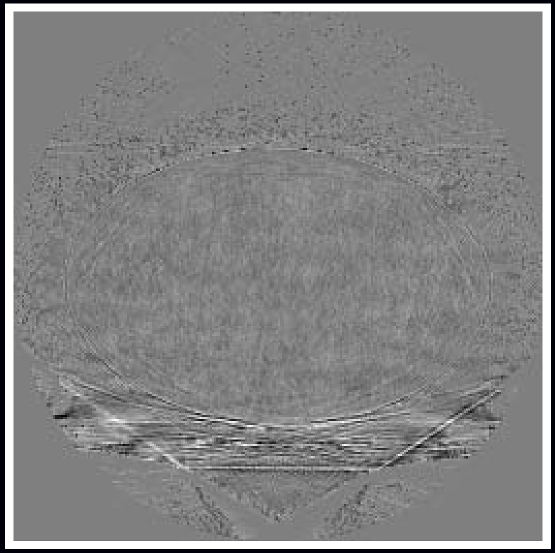

(De Man *et al.*, NSS/MIC 2005)

In terms of iterations: CD $<$ OS $<$ CG $<$ Convergent OS
In terms of compute time? (it depends...)

# Ordered subsets convergence

Theoretically OS does not converge, but it may get "close enough," even with regularization.





| CD | OS | difference |
|---|---|---|
| 200 iter | 41 subsets<br>200 iter | $0 \pm 10$HU |

display: 930 HU $\pm$ 58 HU

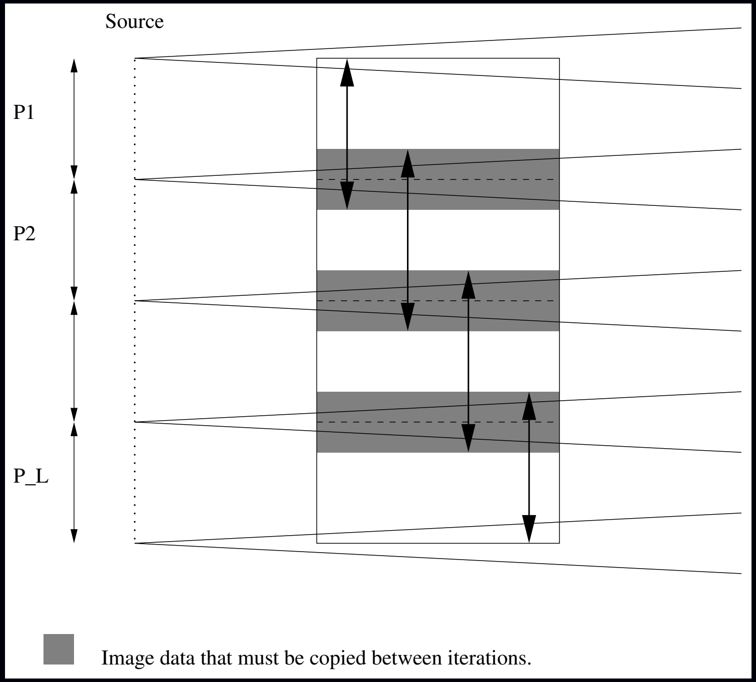
(De Man *et al.*, NSS/MIC 2005)

Ongoing saga... (SPIE, ISBI, Fully 3D, ...)

# Optimization algorithms

Challenges:

- theoretical convergence (to establish gold standards)
- practical: near convergence in few iterations
- highly parallelizable
- efficient use of hardware: memory bandwidth, cache, ...
- predictable stopping rules
- partitioning of helical CT data across multiple compute nodes

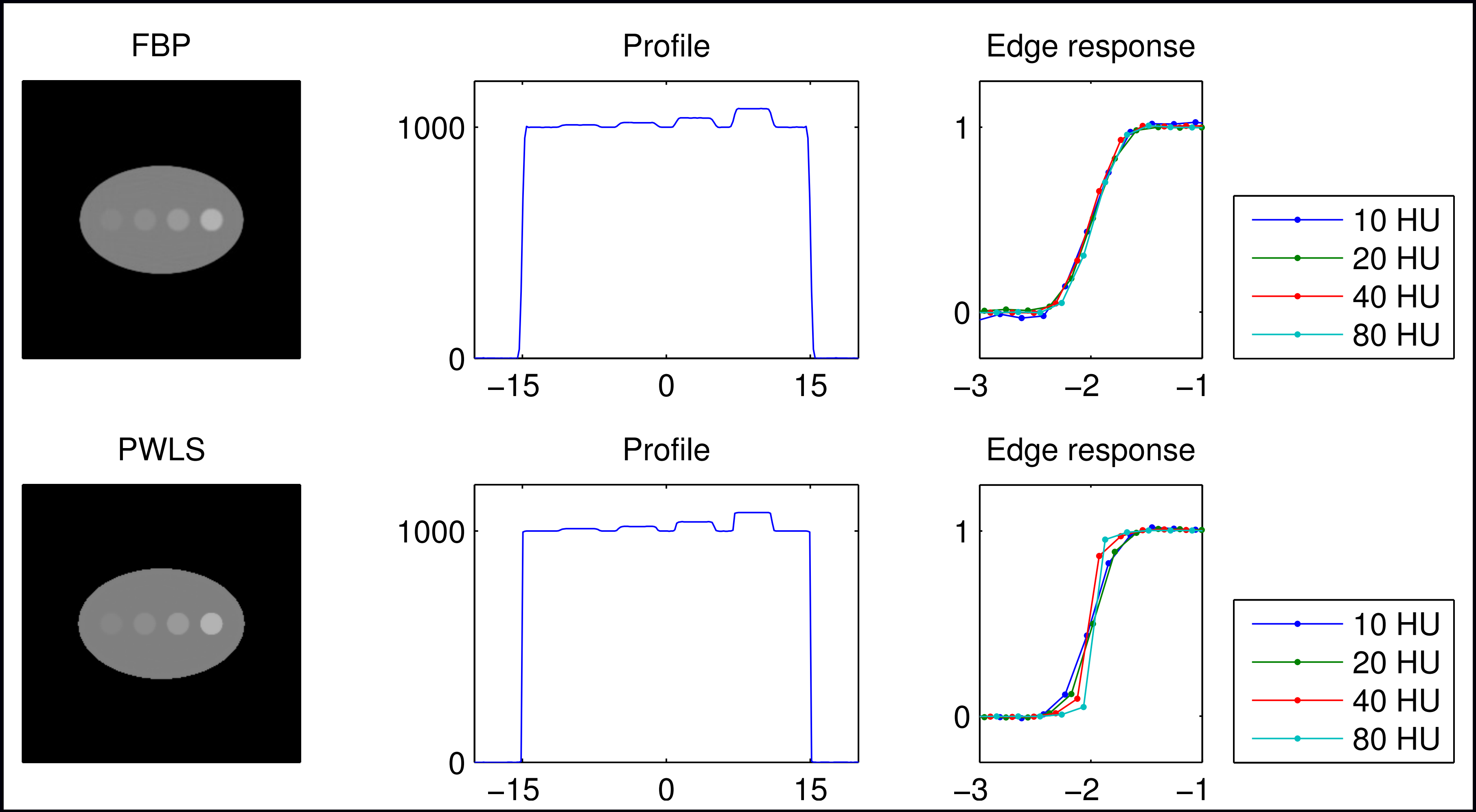


# Example

(movie in pdf)

82-subset OS with two different (but similar) edge-preserving regularizers.
One frame per every 10th iteration.

# Resolution characterization: 2D CT




Challenge:

*Shape of edge response depends on contrast for edge-preserving regularization.*

# Assessing image quality

Challenges:

- Resolution (PSF, edge response, MTF)
- Noise (predictions)
- Task-based performance measures
  Known-location versus unknown-location tasks
- ...

"How low can the dose go" – quite challenging to answer

# Some open problems

- Modeling
  - Statistical modeling for very low-dose CT
  - Resolution effects
  - Spectral CT
  - Object motion
- Parameter selection / performance characterization
  - Performance prediction for nonquadratic regularization
  - Effect of nonquadratic regularization on detection tasks
  - Choice of regularization parameters for nonquadratic regularization
- Algorithms
  - optimization algorithm design
  - software/hardware implementation
  - Moore's law alone will not suffice
    (dual energy, dual source, motion, dynamic, smaller voxels ...)
- Clinical evaluation
- ...

# Current CT research in my group

## Recent work

- Y. Long, J. A. Fessler, and J. M. Balter. 3D forward and back-projection for X-ray CT using separable footprints. *IEEE Trans. Med. Imag.*, 29(11):1839–50, Nov. 2010.
- Y. Lu, H-P. Chan, J. A. Fessler, L. Hadjiiski, J. Wei, M. Goodsitt, A. Schmitz, B. E. H. Claus, and F. W. Wheeler. Adaptive diffusion regularization for enhancement of microcalcifications in digital breast tomosynthesis (DBT) reconstruction. In *Proc. SPIE 7961*, 2011.
- Y. Lu, H-P. Chan, J. A. Fessler, L. Hadjiiski, J. Wei, M. Goodsitt, A. Schmitz, B. E. H. Claus, and F. W. Wheeler. Adaptive diffusion regularization for enhancement of microcalcifications in digital breast tomosynthesis (DBT) reconstruction. In *Proc. SPIE 7961*, 2011.
- W. Huh and J. A. Fessler. Iterative image reconstruction for dual-energy x-ray CT using regularized material sinogram estimates. ISBI 2011.
- D. Kim and J. A. Fessler. Accelerated ordered-subsets algorithm based on separable quadratic surrogates for regularized image reconstruction in X-ray CT. ISBI 2011.
- J-K. Kim, Z. Zhang, and J. A. Fessler. Hardware acceleration of iterative image reconstruction for X-ray computed tomography. ICASSP, 2011.

## Forthcoming work

- J. A. Fessler and D. Kim. Axial block coordinate descent (ABCD) algorithm for X-ray CT image reconstruction. *Fully 3D*, 2011.
- S. Ramani and J. A. Fessler. Convergent iterative CT reconstruction with sparsity-based regularization. *Fully 3D*, 2011.
- M. Wu and J. A. Fessler. GPU acceleration of 3D forward and backward projection using separable footprints for X-ray CT image reconstruction. *Fully 3D (workshop)*, 2011.

## Work in progress

- Spectral CT from a single sinogram using bow-tie filter
- Motion-compensated cardiac CT reconstruction
- Noise predictions for iterative CT reconstruction
- Application to lung CT (NIH R01 with GE GRC)
  - lung nodule quantification
  - airway quantification
  - observer studies?