Improved FMRI Time-series Registration Using Probability Density Priors

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Time-series Rigid Registration

FMRI time-series studies are used to detect brain activation.

Most subjects display varying rates of head motion over the data acquisition duration. This movement hampers statistical analysis used to detect brain-activation.

Freire et al. showed that registration based on $L2$ metrics is affected by brain activation and yields biased motion estimates.

Two strategies commonly used for time-series rigid motion estimation are: volume-to-volume (VV) and slice-to-volume (SV) registration.

VV Registration: A single rigid transform is estimated and applied to all slices in an fMRI volume, i.e., piece-wise constant motion.

SV Registration: An independent and distinct rigid transform is estimated for each fMRI slice, i.e., can handle more elaborate motion trajectories.

Mutual Information-Based Time-series Registration

REFERENCE IMAGE: VV-fMRI volume; SV - fMRI slice $s = 1, 2 \ldots S$ with intensities $\{u_i^s\}_{i=1}^M$ at coordinates $\{x_i^s\}_{i=1}^M$.

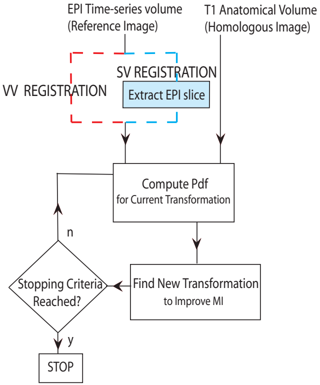
HOMOLOGOUS IMAGE: High resolution anatomical T1 volume with intensities $\{v_j\}_{j=1}^N$ at coordinates $\{y_j\}_{j=1}^N$.

SIMILARITY METRIC: MI between the ref. and hom. images

$$\begin{aligned}\hat{\Psi}_{\mathrm{MI}}(\theta_s) &= \hat{H}_u + \hat{H}_v(\theta_s) - \hat{H}_{uv}(\theta_s) \\ &= \sum_{l=1}^{L}\sum_{k=1}^{K} \hat{P}_{uv}(g_k, h_l; \theta_s) \log\left(\frac{\hat{P}_{uv}(g_k, h_l; \theta_s)}{\hat{P}_u(g_k)\hat{P}_v(h_l; \theta_s)}\right)\end{aligned}$$

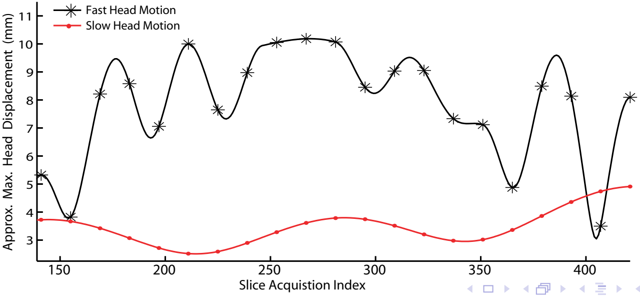
Rigid motion estimated using gradient descent (GD).
SV Registration: Rigid transform for slice s given by T_{θ_s}.
VV Registration: Rigid transform $T_{\theta_s} = T_\theta, \forall s$.

Transformed hom. image coordinates $\{y_i^{\theta_s} = T_{\theta_s}(x_i^s)\}_{i=1}^M$



Time-series Simulation

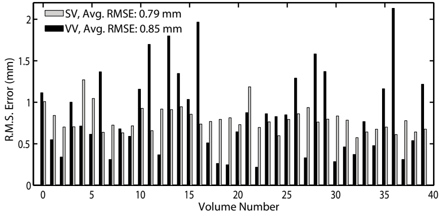
- Two time-series simulated using a $180 \times 216 \times 180$ T2 ICBM volume with 1 mm^3 voxels.
- Time-series parameters: TR = 3000 ms/volume i.e. ≈ 0.214 sec/slice, interleaved slice acquisition, 40 volumes with 14 slices per volume, EPI voxel dimensions: $2 \times 2 \times 6$ mm^3.
- Motion was smooth without being periodic. Range of rotational motion: $\pm 5^\circ$ and $\pm 2^\circ$.
- Avg. speed at a point on the circumference of the head (radius ≈ 87.5 mm) : Fast motion 1.35 mm/sec and Slow motion 0.14 mm/sec.
- EPI voxels at a given slice acquisition time point were simulated by averaging the corresponding neighborhood of $2 \times 2 \times 6$ voxels in the T2 volume.
- Gaussian noise $N(0, 49)$ and Rayleigh noise $\sigma = 7$ was added to voxels with non-zero and no signal intensities resp. Each slice was blurred with a 5×5 Gaussian kernel.



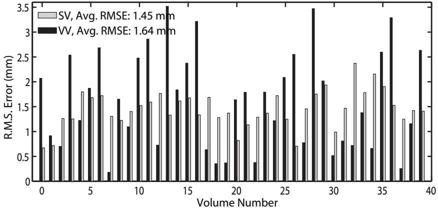
VV Versus SV Registration

Fast Motion

CENTER SLICES

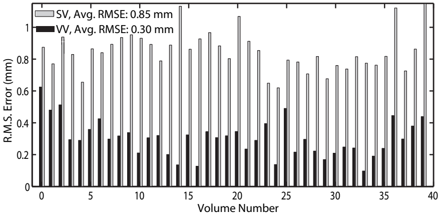


END SLICES



Slow Motion

CENTER SLICES



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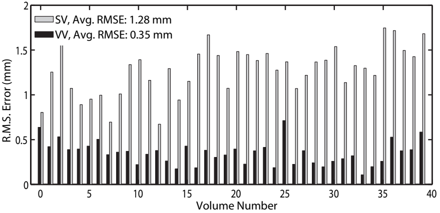


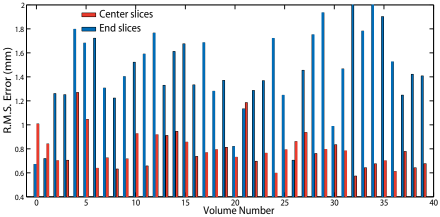
SV Registration

1. Better suited than VV to estimate fast head motion.
2. Can handle elaborate motion trajectories. Each slice has 6 independent degrees of freedom.
3. MI estimates based on few histogram counts; may be noisy.
4. Motion estimates for low complexity slices unreliable.

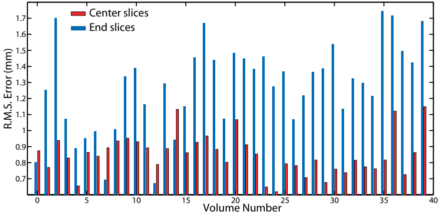
VV Registration

1. Gives better estimates than SV for slow head motion.
2. All estimated motion trajectories are piece-wise constant. One rigid transform per volume.
3. MI estimates based on many histogram counts; more reliable.
4. Motion estimates for each volume typically reliable.

Effect of Image Complexity on SV Registration Accuracy



(e) Fast Motion

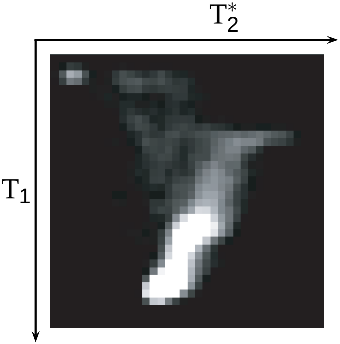


(f) Slow Motion

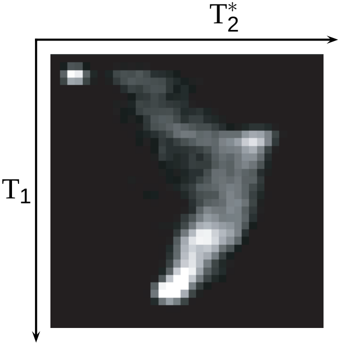
Strategies to Improve SV Registration

- Encourage smooth motion trajectories, i.e., use a regularization term.
- Use a pdf estimate that retains as much information about voxel intensities from the higher resolution anatomical dataset as possible.
- Incorporate an informative prior on the nature of the joint pdf or histogram obtained from fMRI time-series data (previously) registered with a T1 anatomical volume.

Joint PDF Estimates at Registration: Center-slice v/s End-slice



Center-slice



End-slice

Maximum Likelihood PDF Estimation Using Joint Histograms

For discrete histograms, integer bin counts $\{d_{kl}^{\theta}\}_{k=1,l=1}^{KL}$ can be treated as Multinomial r. v. with parameters $\{P_{uv}(f_k, g_l) = P_{uv}^{kl}\}_{k=1,l=1}^{KL}$ and M trials;

$$P(\{d_{kl}^{\theta}\}_{k=1,l=1}^{K,L}) = M! \prod_{k=1,l=1}^{K,L} \frac{(P_{uv}^{kl})^{d_{kl}^{\theta}}}{d_{kl}^{\theta}!}.$$

The usual pmf estimates given by

$$P_{uv}^{\mathrm{ML}}(f_k, g_l; \theta) = \frac{d_{kl}^{\theta}}{\sum_{i,j} d_{ij}^{\theta}} = \frac{d_{kl}^{\theta}}{M},$$

are maximum likelihood (ML) estimates of the parameters of this Multinomial distribution. For low complexity images i.e. sparse histograms, these estimates may become unreliable.

MAP PDF Estimates: Including Prior Information in Joint Histograms

To facilitate maximum a posteriori (MAP) pdf estimation, a Dirichlet prior with parameters $\{\alpha_{kl}\}_{k=1,l=1}^{K,L}$, $\alpha_{kl} > 0 \; \forall k,l$, given by

$$P\left(\{P_{uv}^{kl}\}_{k=1,l=1}^{K,L}; \{\alpha_{kl}\}_{k=1,l=1}^{K,L}\right) = \Gamma\left(\sum_{k,l}\alpha_{kl}\right)\prod_{k=1,l=1}^{K,L}\frac{(P_{uv}^{kl})^{\alpha_{kl}-1}}{\Gamma(\alpha_{kl})},$$

can be used. [1] The Dirichlet distribution is a conjugate prior on the Multinomial distribution, resulting in MAP pdf estimates:

$$P_{uv}^{\mathrm{MAP}}(g_k,h_l;\theta) = \frac{d_{kl}^{\theta}+\alpha_{kl}-1}{\sum_{i,j}(d_{ij}^{\theta}+\alpha_{ij}-1)},$$

where the parameters $\{\alpha_{kl}\}_{k=1,l=1}^{K,L}$ represent prior histogram bin counts. We use $\alpha_{kl} \geq 1$, $\forall k,l$ to ensure that $P_{uv}^{\mathrm{MAP}}(g_k,h_l;\theta) \geq 0, \forall k,l$. The MAP pdf estimate can be re-written as

$$P_{uv}^{\mathrm{MAP}}(g_k,h_l;\theta) = (1-\beta)\hat{P}(g_k,h_l;\theta) + \beta P^*(g_k,h_l),$$

where, $\hat{P}(g_k,h_l;\theta) = \frac{d_{kl}^{\theta}}{M}$, $P^*(g_k,h_l) = \frac{\alpha_{kl}-1}{N^*}$, $N^* = \sum_{i,j}(\alpha_{ij}-1)$ and $\beta = \frac{N^*}{N^*+M}$.

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[1] Zollei et al.

SV Registration With PDF Priors (SV-JP)

Replace the joint pdf $\hat{P}_{uv}(g_k, h_l; \theta_S)$ in $\hat{\Psi}_{\mathrm{MI}}$ by

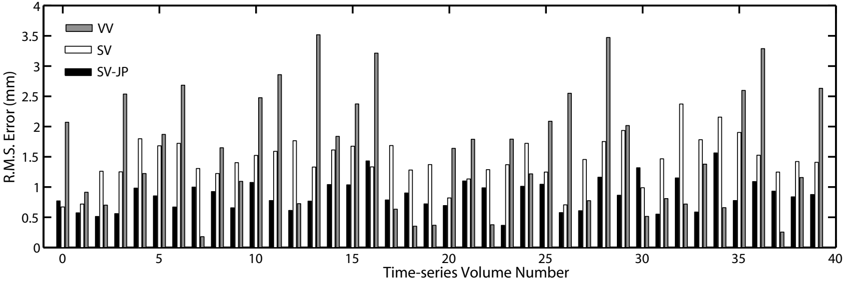
$$\tilde{P}_{uv}(g_k, h_l; \theta_S, \beta) = (1-\beta)\hat{P}_{uv}(g_k, h_l; \theta_S) + \beta P^*_{uv}(g_k, h_l);$$

where,
$\hat{P}_{uv}(g_k, h_l; \theta_S)$ depends only on intensity counts from the to-be registered slice S and changes with θ_S.
$P^*_{uv}(g_k, h_l)$ is based only on intensity counts from previously registered center slices and remains fixed.
$\beta \in [0, 1)$ is a user defined constant.

- P^*_{uv} generated by averaging all 40 joint pdf estimates, obtained from each center-slice after SV registration, over time.
- $\beta = \frac{N^*}{\hat{N}+N^*}$, where N^*: avg. no. of intensity counts in a time-series center slice and $\hat{N}$: no. of intensity counts in slice S.

SV Versus SV-JP Registration

Fast Motion



	Avg Speed (mm/sec)	Avg. RMS Error (Std. Error) (mm)		
		VV	SV	SV-JP
Slow motion	0.14	0.35 (0.13)	1.28 (0.27)	0.90 (0.26)
Fast motion	1.35	1.64 (0.98)	1.45 (0.37)	0.87 (0.26)

Comparison of average RMS error values of motion estimates for times-series end-slices using VV, SV and SV-JP registration.

Effective Similarity Metric in SV-JP

SV-JP registration rigid motion estimate for slice s, $\tilde{\theta}_s$ is given by:

$$\begin{aligned}\tilde{\theta}_s &= \arg\max_{\theta_s} \tilde{\Psi}_{\mathrm{MI}}(\theta_s, \beta) = \arg\max_{\theta_s} \tilde{H}_v(\theta_s, \beta) - \tilde{H}_{uv}(\theta_s, \beta) \\ &= \arg\max_{\theta_s} \sum_{l=1}^{L} \sum_{k=1}^{K} \tilde{P}_{uv}(g_k, h_l; \theta_s, \beta) \log \left(\frac{\tilde{P}_{uv}(g_k, h_l; \theta_s, \beta)}{\tilde{P}_v(h_l; \theta_s, \beta)} \right).\end{aligned}$$

The entropy terms can be written as:

$$\tilde{H}_{uv}(\theta_s, \beta) = (1-\beta)\left(D_{\mathrm{KL}}\left(\hat{P}_{uv}(\theta_s) \| \tilde{P}_{uv}(\theta_s, \beta)\right) + \hat{H}_{uv}(\theta_s) \right) + \beta \left(D_{\mathrm{KL}}\left(P^*_{uv} \| \tilde{P}_{uv}(\theta_s, \beta)\right) + H^*_{uv} \right),$$

where, the Kullback-Leibler (KL) divergence D_{KL} is

$$D_{\mathrm{KL}}\left(\hat{P}_{uv}(\theta_s) \| \tilde{P}_{uv}(\theta_s, \beta)\right) = \sum_{l=1}^{L} \sum_{k=1}^{K} \hat{P}_{uv}(g_k, h_l; \theta_s) \log \frac{\hat{P}_{uv}(g_k, h_l; \theta_s)}{\tilde{P}_{uv}(g_k, h_l; \theta_s, \beta)}.$$

Dropping all terms that do not depend on θ_s, the estimate $\tilde{\theta}_s$ is given by

$$\begin{aligned}\tilde{\theta}_s &= \arg\max_{\theta_s} \Big\{ (1-\beta)\hat{\Phi}_{\mathrm{MI}}(\theta_s) - \beta \sum_{l=1}^{L} P^*_v(h_l) D_{\mathrm{KL}}\left(P^*_{u|v}(.|h_l) \| \tilde{P}_{u|v}(.|h_l; \theta_s, \beta)\right) \\ &\quad -(1-\beta) \sum_{l=1}^{L} \hat{P}_v(h_l; \theta_s) D_{\mathrm{KL}}\left(\hat{P}_{u|v}(.|h_l; \theta_s) \| \tilde{P}_{u|v}(.|h_l; \theta_s, \beta)\right) \Big\};\end{aligned}$$

where, $\hat{\Phi}_{\mathrm{MI}}(\theta_s) \triangleq \hat{H}_v(\theta_s) - \hat{H}_{uv}(\theta_s)$.

Discussion and Future Work

- A full validation of SV-JP using real time-series from a variety of fMRI stimulus studies (e.g. motor tasks, verbal tasks) will be valuable.
- For slow motion VV registration was most accurate, while SV-JP registration was the most accurate for faster head motion. Hence, a scheme that uses prior knowledge of head motion speed to use some combination of VV and SV-JP registration may show improved accuracy.
- Incorporating motion priors that reflect expected correlation in motion at adjacent slice-acquisition time-points may improve SV/SV-JP registration.
- β is a tuning parameter of the SV-JP method and studying its effect on the accuracy of SV-JP registration for different head motion speeds may be useful.