Low-Dose Dual-Energy CT for PET Attenuation Correction with Statistical Sinogram Restoration

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Outline

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  - CT-based attenuation correction for PET
- Conventional sinogram decomposition in DE-CT
- Statistically motivated sinogram restoration in DE-CT
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- Simulations
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PET/CT Background I

- For the i th ray, PET measurement is typically modeled as

Spatial distribution of radioisotope activity

Linear attenuation coefficient (LAC)

$$Y_i \sim \text{Poisson}\left\{\left(\int_{\mathcal{L}_i} \lambda(\vec{x})\ \mathrm{d}\ell\right) \exp\left(-\int_{\mathcal{L}_i} \mu(\vec{x},\mathcal{E})\ \mathrm{d}\ell\right)\Big|_{\mathcal{E}=511keV}\right\}.$$

Needed for PET image reconstruction

Attenuation

- Transmission scans are necessary for PET attenuation correction. For this purpose, the attenuation correction factor (ACF) is defined as follows:

$$\text{ACF}_i \triangleq \exp\left(\int_{\mathcal{L}_i} \mu(\vec{x},\mathcal{E})\ \mathrm{d}\ell\right)\Big|_{\mathcal{E}=511keV}.$$

Forward projection

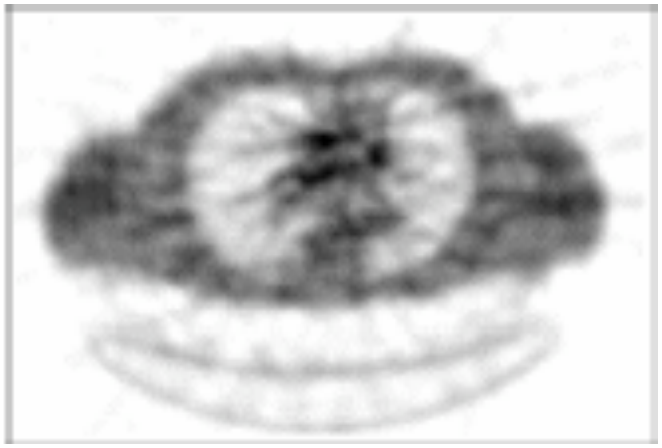
Evaluated at PET energy

- The ACF can be obtained from PET transmission scan or X-ray CT scan.

PET/CT Background II

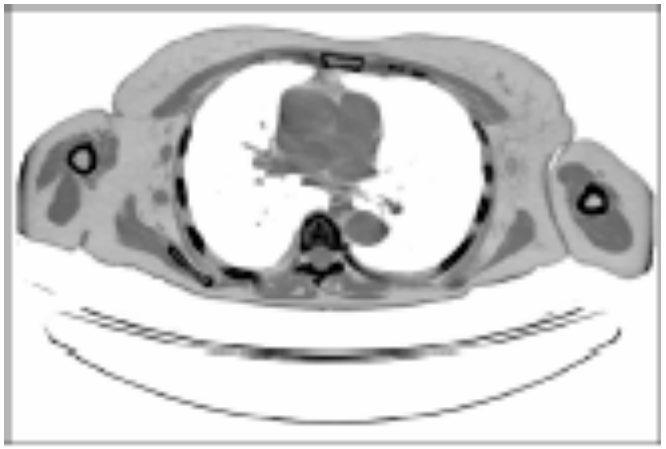
- Benefits and a challenge of CT-based attenuation correction (CTAC):

PET Transmission (511keV)



High noise
Long scan time
Emission contamination
Energy (511keV) matches PET

X-ray Transmission (~30-140keV)

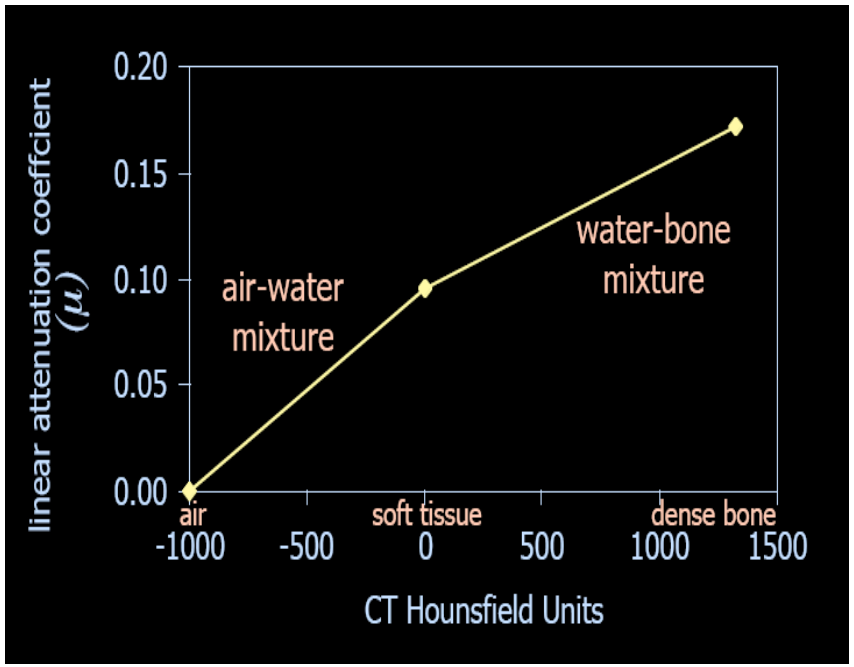


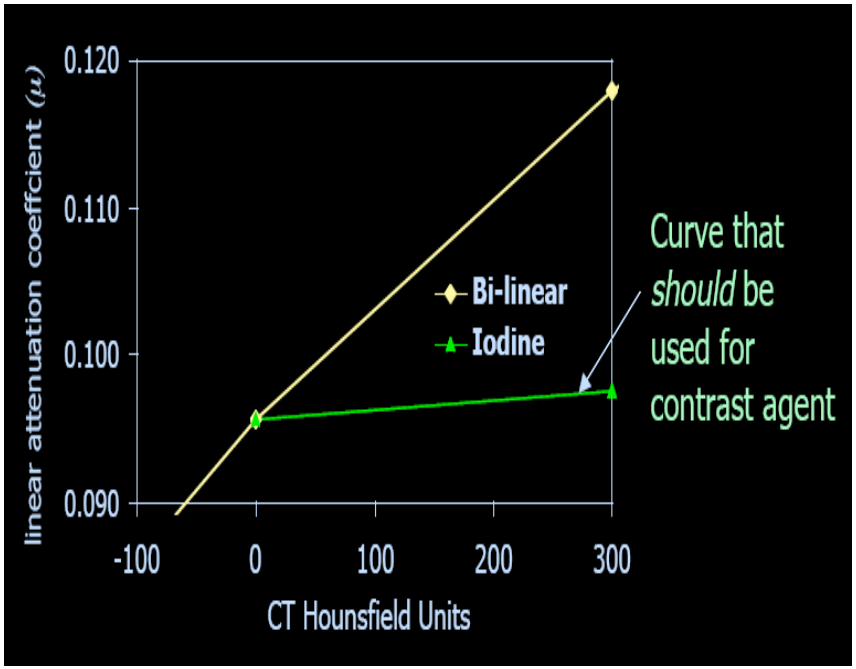
Low noise
Short scan time
No emission contamination
Energies do not match PET

- Challenge: We need to transform LACs in the range of CT energies (~30–140 keV) to LACs at the PET energy (511keV). However, there is no exact way for this transform.

Conventional CTAC

- Conventional method for CTAC is bilinear scaling (with a single-kVp source spectrum) [Blankespoor *et al.*, IEEE TNS, ’94].
- Drawback: ambiguity between bone and non-bone materials with high atomic numbers, e.g., iodine contrast agent.





➡ This may cause biases in ACFs and errors can propagate from ACFs to PET images [Kinahan *et al.,* TCRT, ’06].

Proposed Approaches

- We propose two **statistically** motivated approaches for **DE-CT sinogram** restoration, PWLS and PL methods.
- Why DE-CT instead of bilinear scaling? [Kinahan *et al.,* TCRT, '06]

  To avoid the ambiguity between bone and iodine contrast agent
- Why sinogram domain instead of image domain?

  To compute ACF, we do not have to compute LACs directly.

  (To avoid potential sources of errors and to reduce computational cost)
- Why statistical methods?

  For low radiation dose, statistical methods yield more accurate ACFs.

➡ Therefore DE-CT sinogram restoration is promising for better attenuation corrected PET images !!

Measurement Model in DE-CT

- For the mth source spectrum and ith ray, sinogram measurement is modeled as a random variable whose mean is

$$\mathbf{E}[y_{mi}] = \bar{y}_{mi} = \int I_m(\mathcal{E}) \exp\left(- \int_{\mathcal{L}_i} \mu(\vec{x}, \mathcal{E}) \; \mathrm{d}\ell \right) \mathrm{d}\mathcal{E} \; + \; r_{mi},$$

Sinogram measurement

Polychromatic source spectrum

Known additive contributions

where $m = 1, 2$.

- LAC can be decomposed with component material basis functions,

$$\mu(\vec{x}, \mathcal{E}) = \sum_{l=1}^{2} \beta_l(\mathcal{E}) \rho_l(\vec{x})$$

Spatial distribution of the lth material density

Mass attenuation coefficient

- A simplification gives

$$\bar{y}_{mi} = I_m e^{-f_{im}(\boldsymbol{s}_i)} + r_{mi},$$

$$f_{im}(\boldsymbol{s}_i) \triangleq -\log\left(\int \frac{I_m(\mathcal{E})}{I_m} \, e^{-\sum_{l=1}^{L_0} \beta_l(\mathcal{E}) s_{li}} \, \mathrm{d}\mathcal{E} \right), \qquad s_{li} \triangleq \int_{\mathcal{L}_i} \rho_l(\vec{x}) \; \mathrm{d}\ell, \qquad I_m \triangleq \int I_m(\mathcal{E}) \; \mathrm{d}\mathcal{E}.$$

Conventional Sinogram Decomposition

- By Ignoring measurement noise and inverting the simplified expression for $\bar{y}_{mi}$, we have the following estimate of f_{im} :

Sinogram measurement

$$\widehat{f}_{im} \triangleq -\log\left\{\text{smooth}\left(\frac{y_{mi} - r_{mi}}{I_m}\right)\right\}.$$

Smoothing in the radial direction

Thus, we have a system of nonlinear equations

$$\widehat{\boldsymbol{f}}_i = \boldsymbol{f}_i\left(\boldsymbol{s}_i\right), \qquad i = 1, \ldots, N_d$$

where, e.g., $\boldsymbol{f}_i \triangleq (f_{i1}, f_{i2})$ and $\boldsymbol{s}_i \triangleq (s_{1i}, s_{2i})$.

- Solving nonlinear equations numerically produces the estimates of component sinograms,

$$\widehat{\boldsymbol{s}}_i \triangleq \boldsymbol{f}_i^{-1}\left(\widehat{\boldsymbol{f}}_i\right),$$

- This conventional sinogram decomposition involves noise amplifying step and yields very noisy restored component sinograms and reconstructed images with streaks after performing FBP.

Penalized Weighted Least Squares (PWLS) I

- To obtain better component sinogram estimates, we use a statistically motivated method. We jointly fit the bone and soft tissue sinograms to the low and high energy log-scans.

$$\widehat{\mathbf{s}}_{PWLS} \triangleq \underset{\mathbf{s}\in\mathbf{R}^{L_0\times N_d}\geq 0}{\arg\min} \Phi(\mathbf{s}),$$

PWLS cost function

Roughness penalty function

$$\Phi(\mathbf{s}) \triangleq \sum_{i=1}^{N_d} \frac{1}{2}\left(\widehat{\boldsymbol{f}}_i - \boldsymbol{f}_i(\boldsymbol{s}_i)\right)^T \boldsymbol{D}_i\left(\widehat{\boldsymbol{f}}_i - \boldsymbol{f}_i(\boldsymbol{s}_i)\right) + R(\mathbf{s}),$$

where the sinogram matrix is defined as $\mathbf{s} \triangleq \left(\boldsymbol{s}_1, \ldots, \boldsymbol{s}_{N_d}\right)$. # of total rays

- The weight matrix D_i (2 x 2 in DECT) are determined based on an approximate variance of $\widehat{f}_{im}$. For Poisson distributed measurements and small r_{mi} [Fessler, IEEE TIP, '96],

$$\text{var}\left(\widehat{f}_{im}\right) \approx \frac{\text{var}(y_{mi})}{(\bar{y}_{mi} - r_{mi})^2} \approx \frac{1}{y_{mi}}.$$

From this, we define the weight matrix for each ray as follows:

$$\boldsymbol{D}_i \triangleq \text{diag}_{m=1}^{M_0}\{y_{mi}\}.$$

Penalized Weighted Least Squares (PWLS) II

- The roughness penalty function is defined as

Regularization parameter

First order difference in the radial direction only

$$R(\mathbf{s}) \triangleq \sum_{l=1}^{2} \gamma_l \sum_{i=2}^{N_d} \frac{1}{2}(s_{li} - s_{li-1})^2,$$

where the regularization parameters (γ_1 and γ_2) control resolution/noise tradeoff.

- We use the optimization transfer principle to perform PWLS minimization. Using a sequence of separable quadratic surrogates, we arrive at the following equation for update:

$$s_{li}^{(n+1)} = \left[s_{li}^{(n)} - \frac{1}{h_{li}^{(n)}} \frac{\partial \Phi\left(s_{li}^{(n)}\right)}{\partial s_{li}} \right]_+,$$

Due to the non-negativity constraint on sinogram matrix

where we precompute the curvature $h_{li}^{(n)}$ that monotonically decreases the PWLS cost function.

Penalized Likelihood (PL) Approach

- PWLS uses the logarithmic transform to obtain $\widehat{f}_{im}$, so it is suboptimal in terms of noise. To improve ACFs, we propose a PL approach that is fully based on a statistical model.
- Assuming Poisson distributed raw sinogram measurements leads to the PL cost function:

$$\Psi(\mathrm{s}) \triangleq \sum_{m=1}^{M_0} \sum_{i=1}^{N_d} \bar{y}_{mi}(\boldsymbol{s}_i) - y_{mi} \log \bar{y}_{mi}(\boldsymbol{s}_i) \; + \; R(\mathrm{s}),$$

Negative Poisson log-likelihood

- With the same penalty function as in PWLS, we minimize the PL cost function.

$$\widehat{\mathrm{s}}_{PL} = \underset{\mathrm{s} \in \mathbf{R}^{L_0 \times N_d} \geq 0}{\arg\min} \; \Psi(\mathrm{s})$$

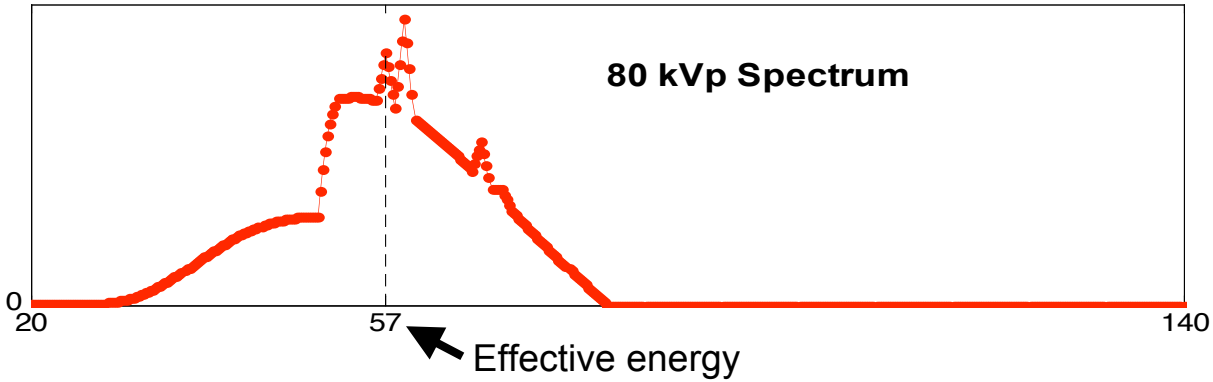
- Applying the optimization transfer principle yields

$$s_{li}^{(n+1)} = \left[s_{li}^{(n)} - \frac{1}{d_{li}^{(n)}} \frac{\partial \Psi \left(s_{li}^{(n)} \right)}{\partial s_{li}} \right]_+ ,$$

where we precompute the curvature $d_{li}^{(n)}$ that monotonically decreases the PL cost function.

Simulations I

- We simulate two incident source spectra with 80kVp and 140kVp:



To simulate low radiation doses, we use 5 x 10^4 photons per ray for the 140kVp spectrum. The total number of rays is 140 (radius) x 128 (angle).

Simulations II

- NRMS errors obtained from the conventional sinogram decomposition with post smoothing in the radial direction, PWLS decomposition, and PL restoration

	Sinogram restoration method $(\gamma_1 = \gamma_2 = 2^{-5})$		
NRMS error	Conventional decomp	PWLS decomp	PL restoration
Sinogram of soft tissue	21%	13%	12%
Sinogram of bone	56%	34%	30%
Image of soft tissue	54%	33%	31%
Image of bone	64%	42%	41%
ACFs	22%	9%	8%
PET image	33%	19%	18%

ACF is defined as

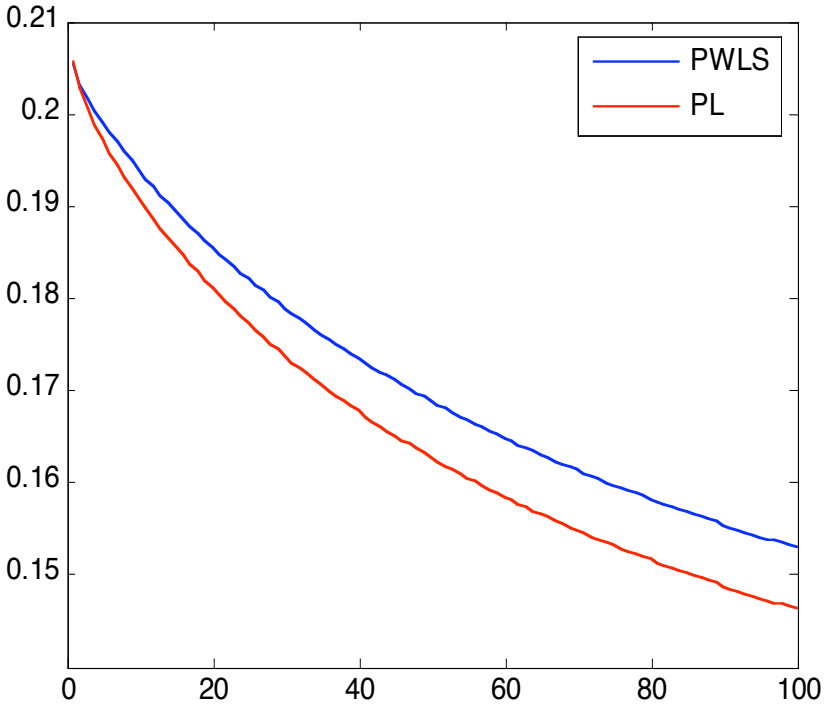
$$\text{ACF}_i \triangleq \exp\left(\sum_{l=1}^{2} \beta_l(\mathcal{E})\hat{s}_{li}\right)\Bigg|_{\mathcal{E}=511 \text{ keV}}$$

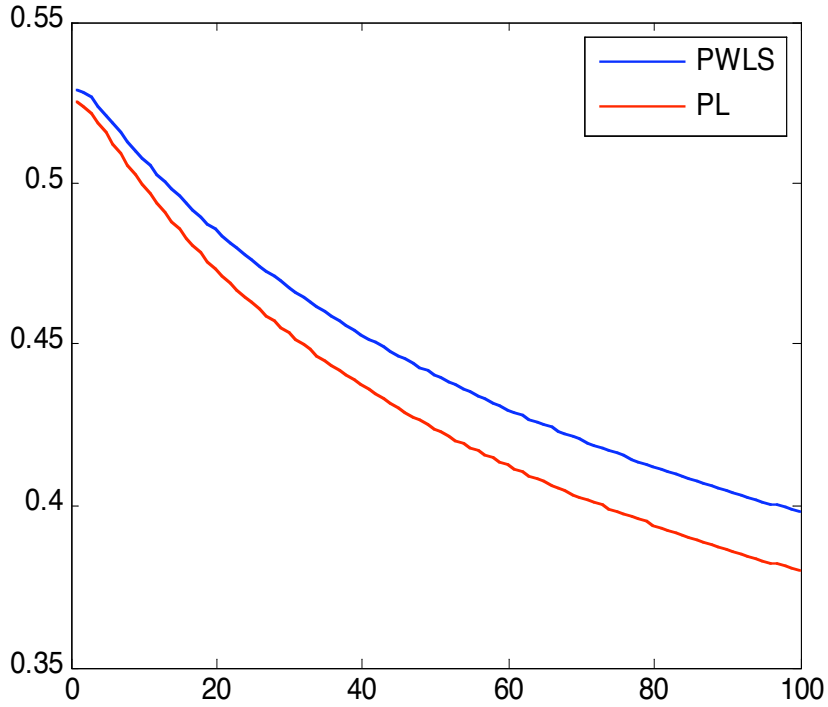
Restored component sinogram

PET image is reconstructed as follows:

$$\hat{\lambda} = \text{FBP}\left\{\text{ACF}_i \left(\int_{\mathcal{L}_i} \lambda(\vec{x}) \, d\ell\right) \exp\left(-\int_{\mathcal{L}_i} \mu(\vec{x}, \mathcal{E}) \, d\ell\right)\Bigg|_{\mathcal{E}=511keV}\right\}.$$

PWLS vs PL

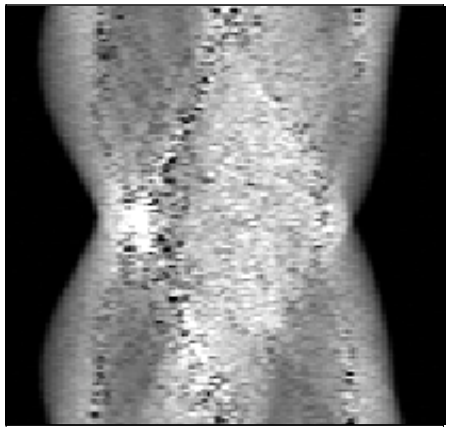




➡ For a given iteration number, PL provides lower NRMS error than PWLS.

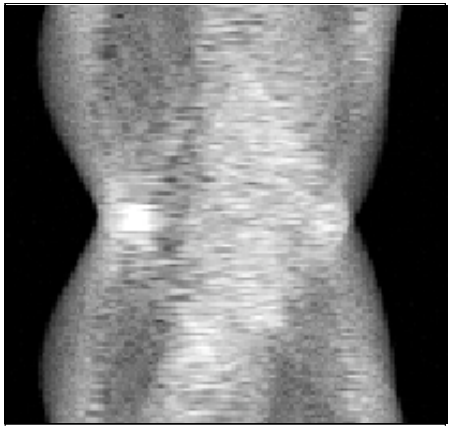
Restored Component Sinograms

Soft Tissue

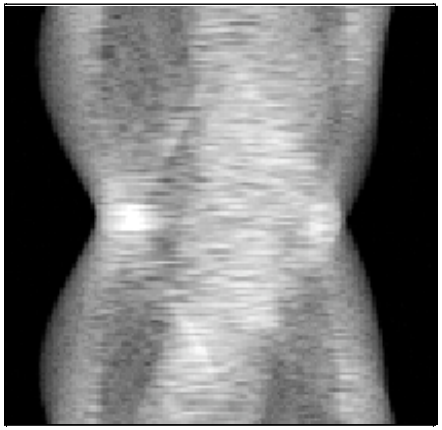


NRMS error: 21%

Post-Smoothed



NRMS error: 13%



NRMS error: 12%

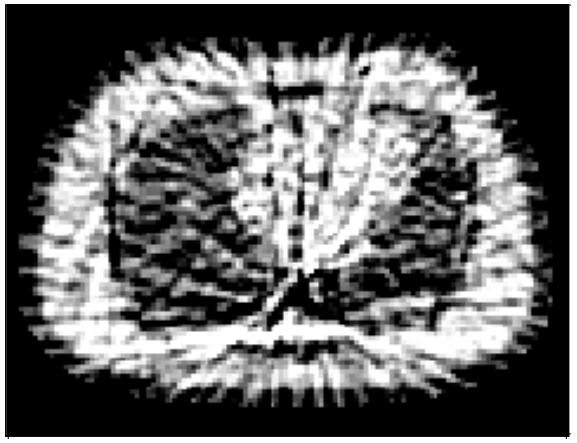
Bone

NRMS error: 56%

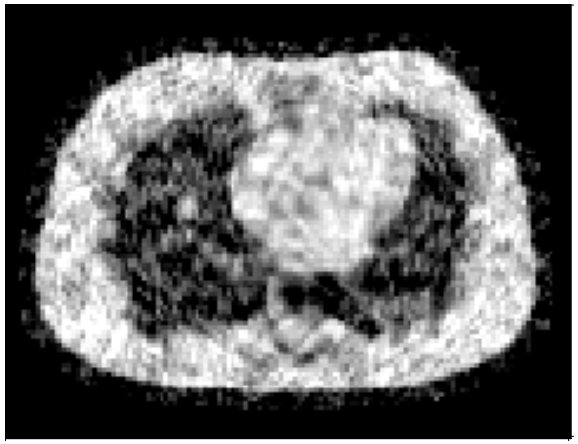
NRMS error: 34%

NRMS error: 30%

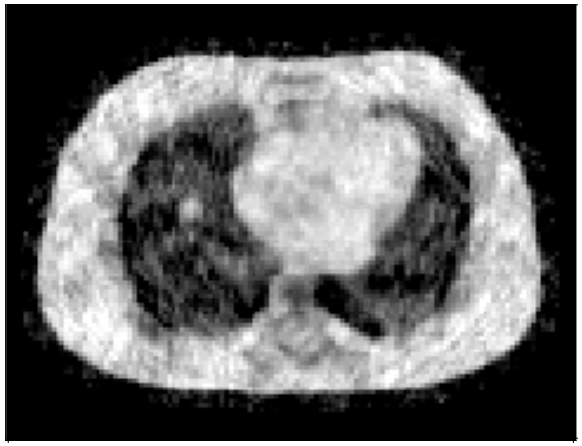
Reconstructed Component CT Images I



NRMS error: 54%

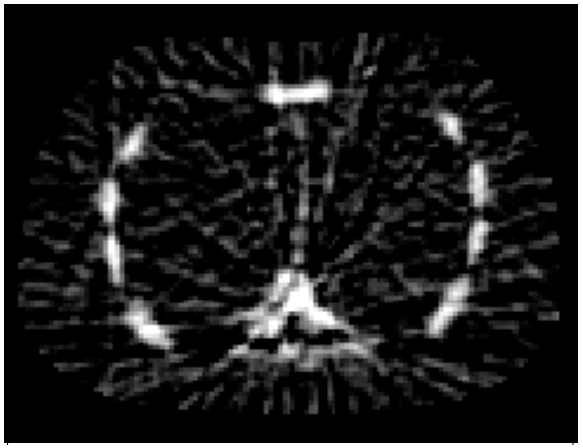


NRMS error: 33%

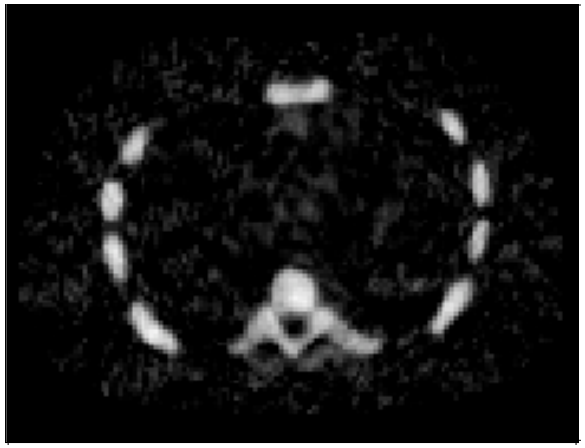


NRMS error: 31%

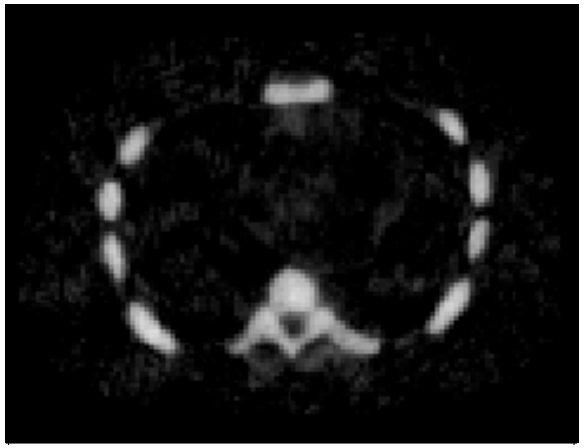
Reconstructed Component CT Images II



NRMS error: 64%

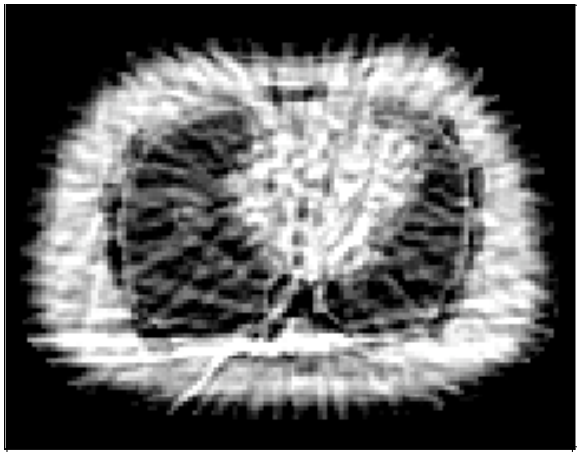


NRMS error: 42%

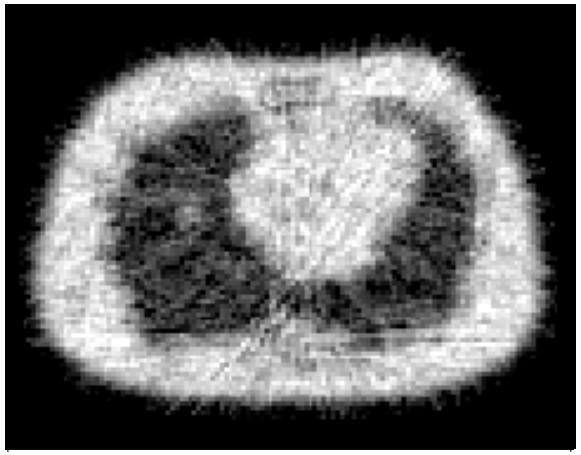


NRMS error: 41%

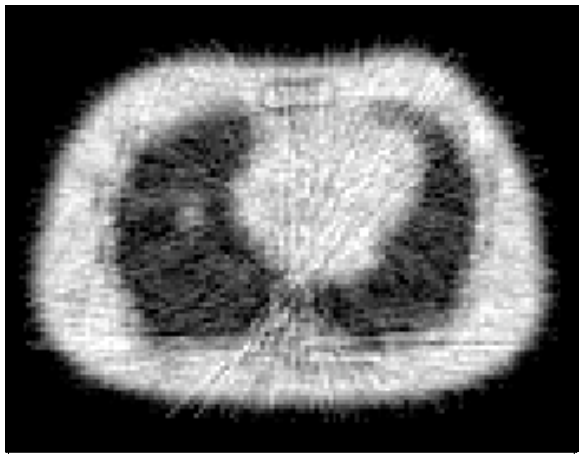
Reconstructed PET Images with CTAC



NRMS error: 33%



NRMS error: 19%



NRMS error: 18%

Conclusions and Future Works

- For low-dose DE-CT, two statistically motivated sinogram restoration methods were proposed for attenuation correction of PET images.
- The proposed PWLS and PL methods provided lower NRMS errors than the conventional sinogram decomposition in the sinogram domain, in the image domain, and in terms of ACFs. The PL approach had the lowest NRMS errors.
- Future works will include

  - experiments with real data.

  - analysis for approximately uniform spatial resolution in sinograms.

  - comparison with bilinear scaling using iodine contrast agents.

Backup Slides