Regularized B1+ map estimation in MRI

Amanda Funai and Jeffrey A. Fessler

EECS Dept.
The University of Michigan

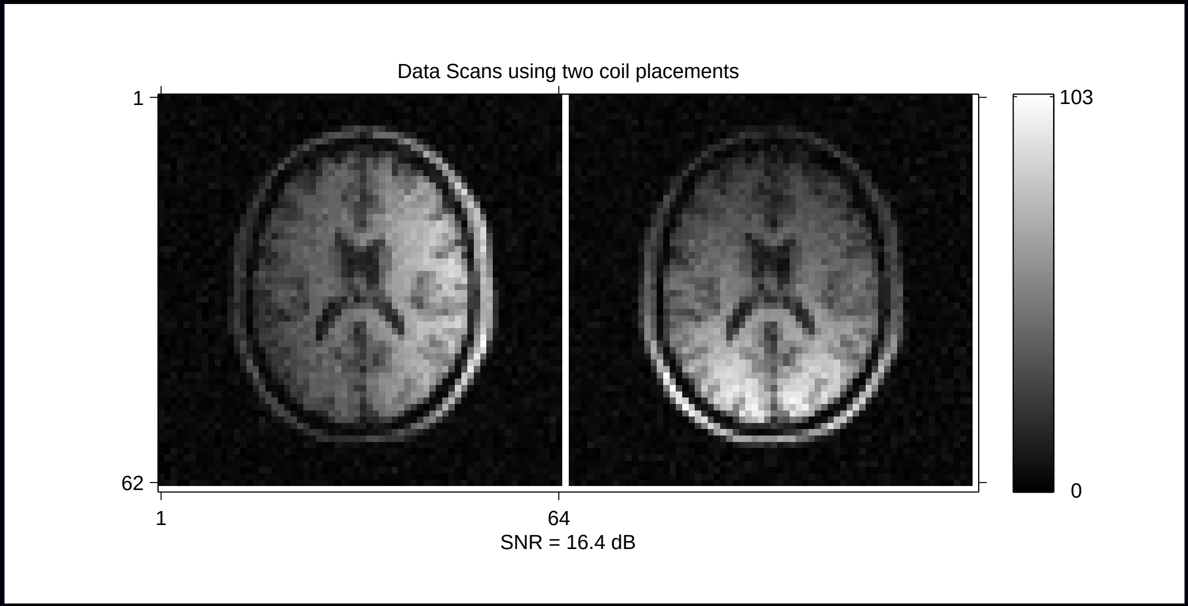
William Grissom and Douglas C. Noll

BME Dept.
The University of Michigan

ISBI
Apr. 13, 2007

Introduction

- RF transmit coils produce non-uniform field strengths
  $\Longrightarrow$ tip angles vary over the field of view
- Focus: parallel transmit excitation (using a coil array)
  (Katscher *et al.*, 2003, MRM)
- RF pulse design requires map of B1+ field strength
- At high fields (≥ 3T), use for pre-scan calibration
  (Cunningham *et al.*, 2006, MRM)



Conventional Measurement Model

Two scans, one with twice the RF amplitude

$$\begin{aligned} y_{j1} &= f_j \sin(\alpha_j) + \varepsilon_{1j} \\ y_{j2} &= f_j \sin(2\alpha_j) + \varepsilon_{2j}, \end{aligned} \qquad j = 1, \ldots, n_{\mathrm{p}}$$

- f_j: underlying object transverse magnetization of the jth voxel (times receive coil sensitivity map)
- n_{p}: number of voxels
- α_j: unknown tip angle of the jth voxel
- ε_j: (complex) gaussian noise

Estimating α_j equivalent to estimating B1+ field strength

Standard approach - Double Angle Formula

- Double Angle Formula:

$$\frac{\mathrm{E}[y_{j2}]}{\mathrm{E}[y_{j1}]} = \frac{\sin(2\alpha_j)}{\sin(\alpha_j)} = \frac{2\sin(\alpha_j)\cos(\alpha_j)}{\sin(\alpha_j)} = 2\cos(\alpha_j)\,.$$

- Method-of-moments estimator:

$$\hat{\alpha}_j = \arccos\left(\frac{1}{2}\frac{y_{j2}}{y_{j1}}\right).$$

- Possible problems
  - ignores noise
  - performs poorly in areas with low spin density
  - suffers from 2π ambiguities if α_j is too large
  - unstable where α_j too small
  - provides no phase information
  - does not generalize to more tip angles

Improved Signal Model

- transmit separately from K coils and receive from a common coil
- apply a sequence of L tip angles with known RF amplitudes a_l
- $K \times L$ reconstructed images (assumes ideal rect profile):

$$y_{jkl} = f_j e^{\imath \varphi_{jk}} \sin(a_l x_{jk}) + \varepsilon_{jkl}$$

- Variables:
  - f_j : underlying object transverse magnetization (real)
  - φ_{jk} : phase of the kth coil
  - x_{jk} : unknown “B1+ map”
  - ε_{jkl} : zero-mean complex gaussian noise
  - j : voxel index, k : coil index, l : tip sequence index

Goal: estimate each B1+ map $\boldsymbol{x}$ and each B1+ phase map $\boldsymbol{\phi}$ from the images y_{jkl}.

The unknown object magnetization f_j is a nuisance parameter.

Signal Model Comments

- Units of x_{jk} are arbitrary
  i.e. a_l in gauss $\Longrightarrow$ x_{jk} radian/gauss
- f_j real $\Longrightarrow$ model is identifiable,
  ambiguity only in sign of f_j
- Similar model considered in (Kerr *et al.*, 2006, MRM)
  - a_l restricted to powers of two
  - cost function:

$$\sum_l \left(|y_{jkl}| - |f_j| \sin(|a_l x_{jk}|)\right)^2 .$$

  - not a complex gaussian statistical model
  - general purpose minimization model from Matlab
  - used value of tip index at each voxel where tip closest to $\pi/2$
- Our model allows for arbitrary a_l and uses all data at each voxel.

Regularized Iterative Estimator

- Goal : estimate $\boldsymbol{x}$, $\boldsymbol{\phi}$, and $\boldsymbol{f}$ by minimizing Ψ
- Cost function:

$$\Psi(\boldsymbol{x}, \boldsymbol{\phi}, \boldsymbol{f}) = \sum_{k=1}^{K} \sum_{j=1}^{n_\mathrm{p}} \sum_{l=1}^{L} \frac{1}{2} \left| y_{jkl} - f_j \, e^{\imath \varphi_{jk}} \sin(a_l x_{jk}) \right|^2 + \beta_1 \, \mathsf{R}(\boldsymbol{x}_k) + \beta_2 \, \mathsf{R}(\boldsymbol{\phi}_k)$$

- Maps are smooth
  $\Longrightarrow$ regularize $\boldsymbol{x} = (\boldsymbol{x}_1, \ldots, \boldsymbol{x}_K)$ and $\boldsymbol{\phi} = (\boldsymbol{\phi}_1, \ldots, \boldsymbol{\phi}_K)$
- $\mathsf{R}(\boldsymbol{x}_k)$ and $\mathsf{R}(\boldsymbol{\phi}_k)$ - quadratic regularizing roughness penalty functions
- β_1 and β_2 - regularized parameters
- No analytical solution over all 3 sets of parameters so we use block alternating minimization

Optimization Transfer - Object

- f_j has analytic solution (given $\boldsymbol{\phi}$ and $\boldsymbol{x}$) :

$$f_j^{(n+1)} = \text{real}\left(\frac{\sum_{k=1}^{K} \sum_{l=1}^{L} \mathrm{e}^{-\imath \varphi_{jk}^{(n)}} \sin\left(a_l x_{jk}^{(n)}\right) y_{jkl}}{\sum_{k=1}^{K} \sum_{l=1}^{L} \left|\sin^2\left(a_l x_{jk}^{(n)}\right)\right|} \right).$$

Optimization Transfer - B1+ Magnitude Map

- $\boldsymbol{x}$ (given $\boldsymbol{\phi}$ and $\boldsymbol{f}$)
  upper bound for the curvature given by:

$$\frac{\partial^2}{\partial x^2}\psi \leq |f_j|^2 a_l^2$$

- Using Separable quadratic surrogates

$$\boldsymbol{x}_k^{(n+1)} = \boldsymbol{x}_k^{(n)} - \mathrm{diag}\left\{\frac{1}{b_j}\right\} \nabla_{\boldsymbol{x}_k} \Psi\left(\boldsymbol{x}^{(n)}, \boldsymbol{\phi}^{(n)}, \boldsymbol{f}^{(n+1)}\right),$$

  where

$$b_j \triangleq \sum_{l=1}^{L} a_l^2 \left| f_j^{(n+1)} \right|^2 + r\beta_1.$$

- r depends on choice of regularizer
  For second order differences with 8 nearest neighbors,
  $r = 4 \cdot 4 \cdot (2 + 2/\sqrt{(}2))$

Optimization Transfer - B1+ Phase Map

- $\boldsymbol{\phi}$ (given $\boldsymbol{f}$ and $\boldsymbol{x}$)
  upper bound for the curvature given by:

$$\frac{\partial^2}{\partial \varphi^2}\psi \leq |y_{jkl} f_j \sin(a_l x_{jk})|$$

- Using Separable quadratic surrogates,

$$\boldsymbol{\phi}_k^{(n+1)} = \boldsymbol{\phi}_k^{(n)} - \text{diag}\left\{\frac{1}{d_j}\right\} \nabla_{\boldsymbol{\phi}_k} \Psi\left(\boldsymbol{x}^{(n+1)}, \boldsymbol{\phi}^{(n)}, \boldsymbol{f}^{(n+1)}\right),$$

  where

$$d_j \triangleq \sum_{l=1}^{L} \left| y_{jkl} f_j^{(n+1)} \sin\left(a_l x_{jk}^{(n+1)}\right) \right| + r\beta_2.$$

- B1 map and phase map updates can be parallelized because no coupling between coil terms in cost function

Initialization

- Algorithm is non-convex - may descend to local minimum $\Longrightarrow$ Good initial estimates crucial
- $\boldsymbol{x}_k^{(0)}$ - use standard double angle method
- $\boldsymbol{f}^{(0)}$ - use analytic formula
- $\varphi_{jk}^{(0)}$ -
  Because we can write the cost function as

$$\sum_{k=1}^{K}\sum_{l=1}^{L}\frac{1}{2}\left|y_{jkl} - f_j \mathrm{e}^{\imath\varphi_{jk}}\sin(a_l x_{jk})\right|^2$$
$$\equiv -\sum_{k=1}^{K}\mathrm{real}\left(\left[f_j\sum_{l=1}^{L}\sin(a_l x_{jk})y_{jkl}\right]\mathrm{e}^{-\imath\varphi_{jk}}\right).$$

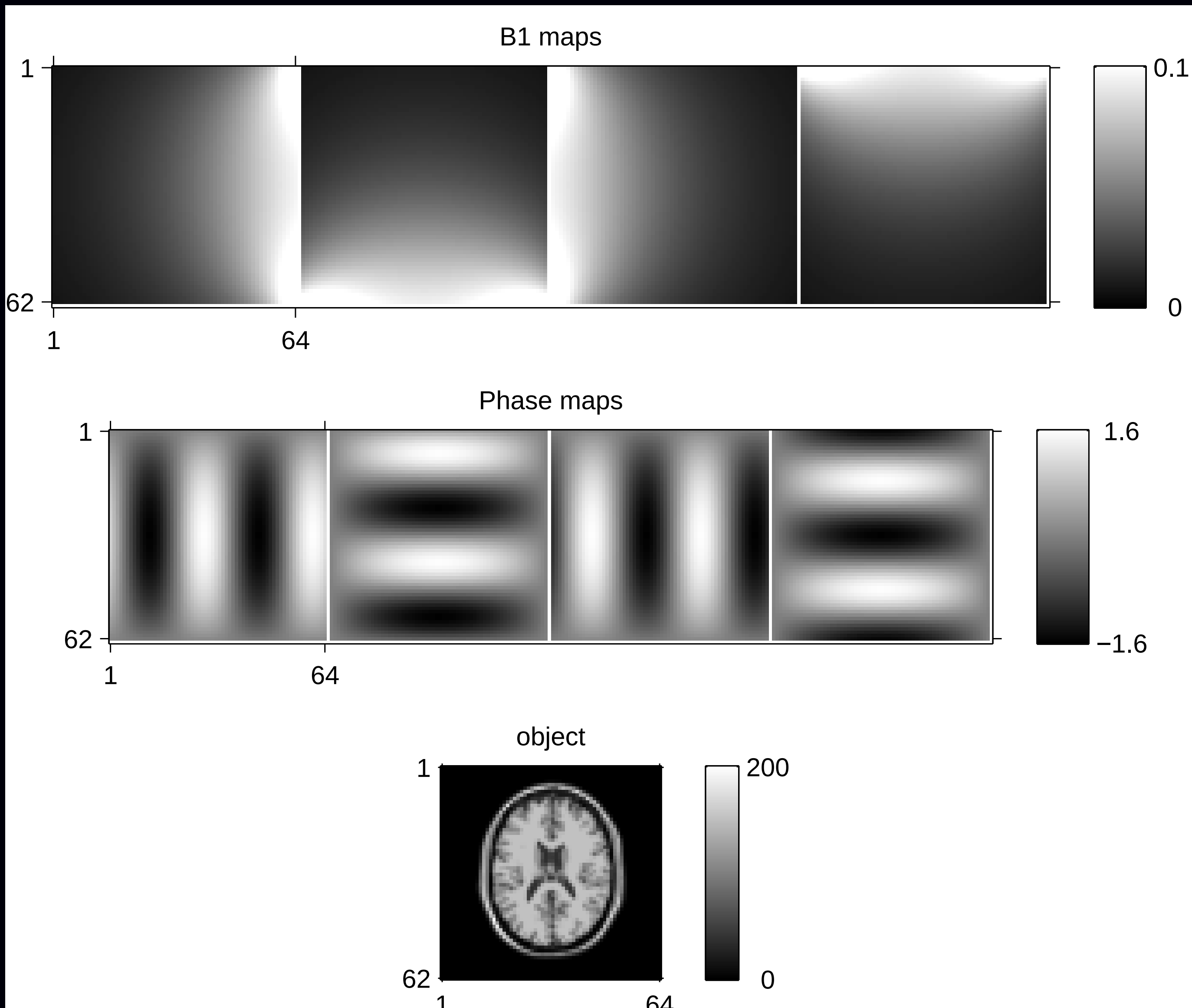
it suggests:

$$\varphi_{jk}^{(0)} = \angle\left(f_j\sum_{l=1}^{L}\sin(a_l x_{jk})y_{jkl}\right)$$

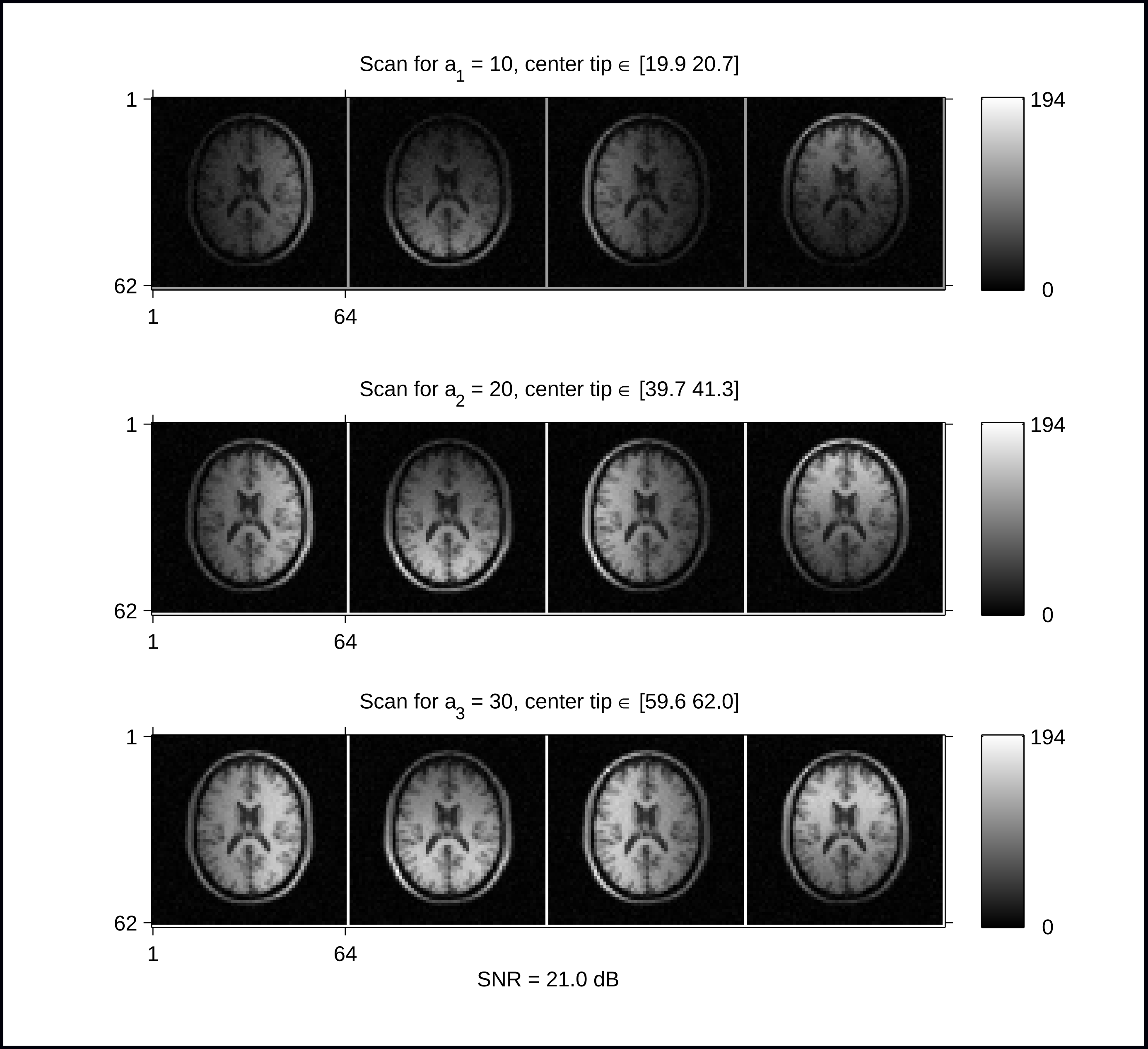
Simulations

- Parameters:
  - K = 4 coils
  - L = 3 different tip angles a_l = [10 20 30]
  - 100 iterations
  - Computation time - 17 sec total (in MATLAB)
  - SNR of about 21 dB as calculated by $10\log_{10}(\|\boldsymbol{y}\| / \|\boldsymbol{y} - \mathrm{E}[\boldsymbol{y}]\|)$

True maps

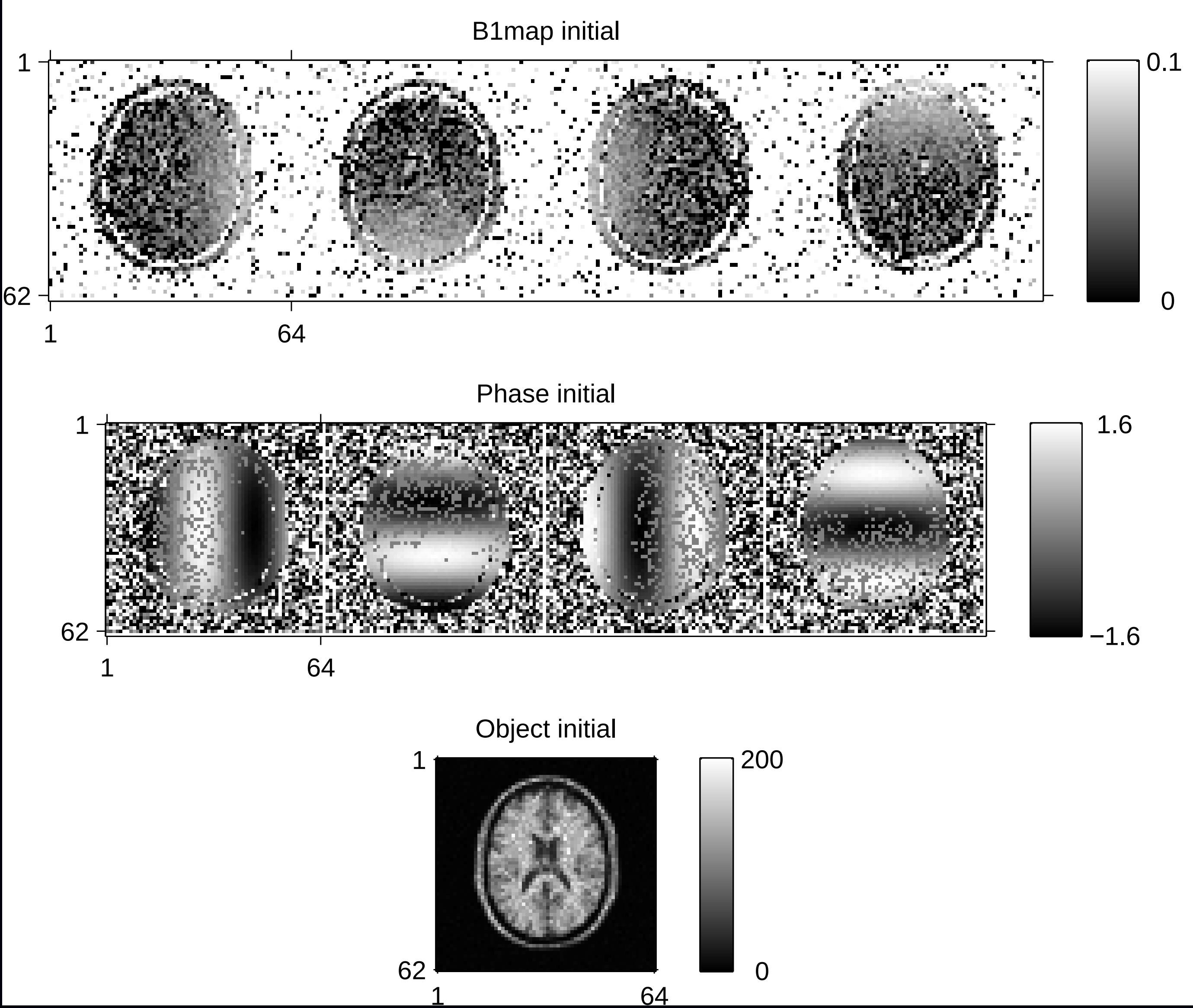

B1 maps
1
62
1
64
0.1
0
Phase maps
1
62
1
64
1.6
−1.6
object
1
62
1
64
200
0

Noisy Data

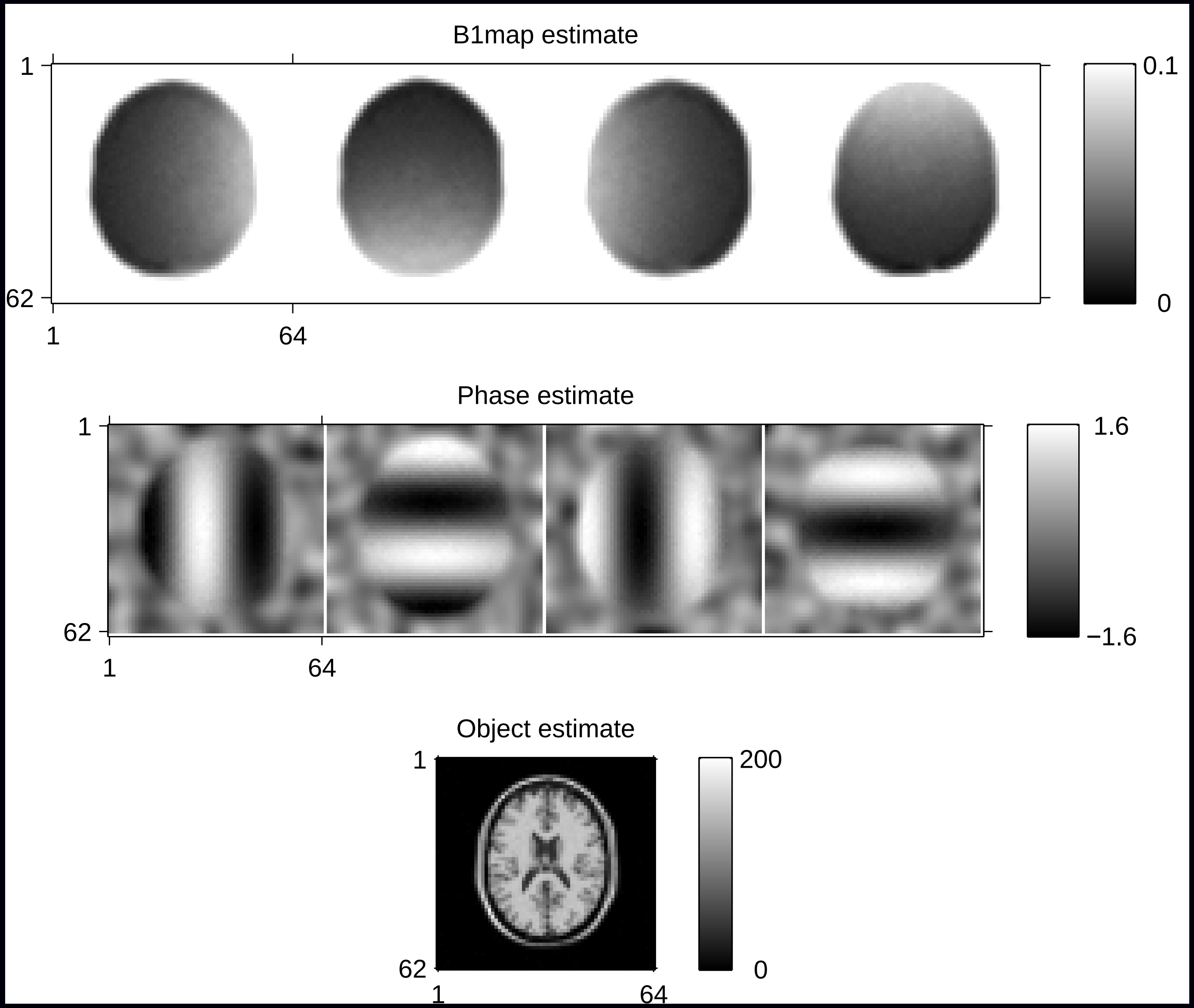


Scan for a_1 = 10, center tip ∈ [19.9 20.7]
1
62
1
64
194
0
Scan for a_2 = 20, center tip ∈ [39.7 41.3]
1
62
1
64
194
0
Scan for a_3 = 30, center tip ∈ [59.6 62.0]
1
62
1
64
194
0
SNR = 21.0 dB

Initializations



Final Estimate

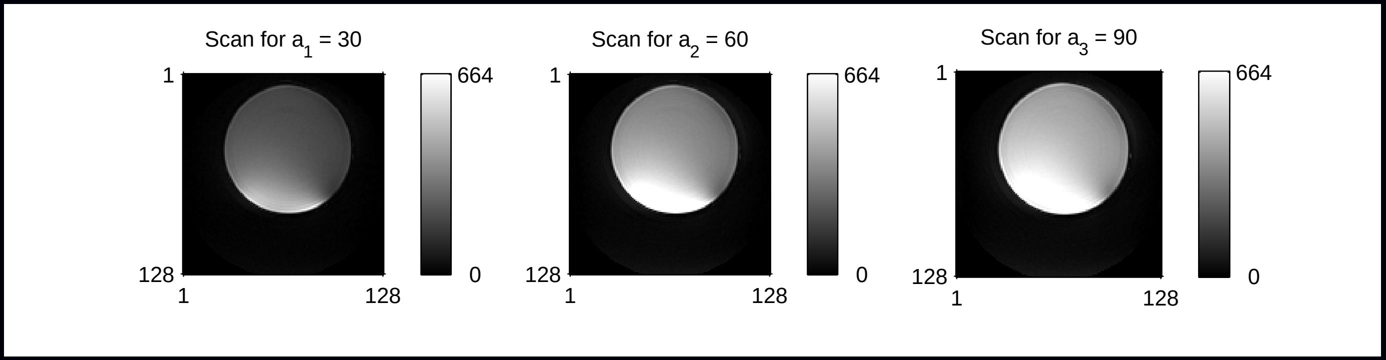


Simulation conclusions

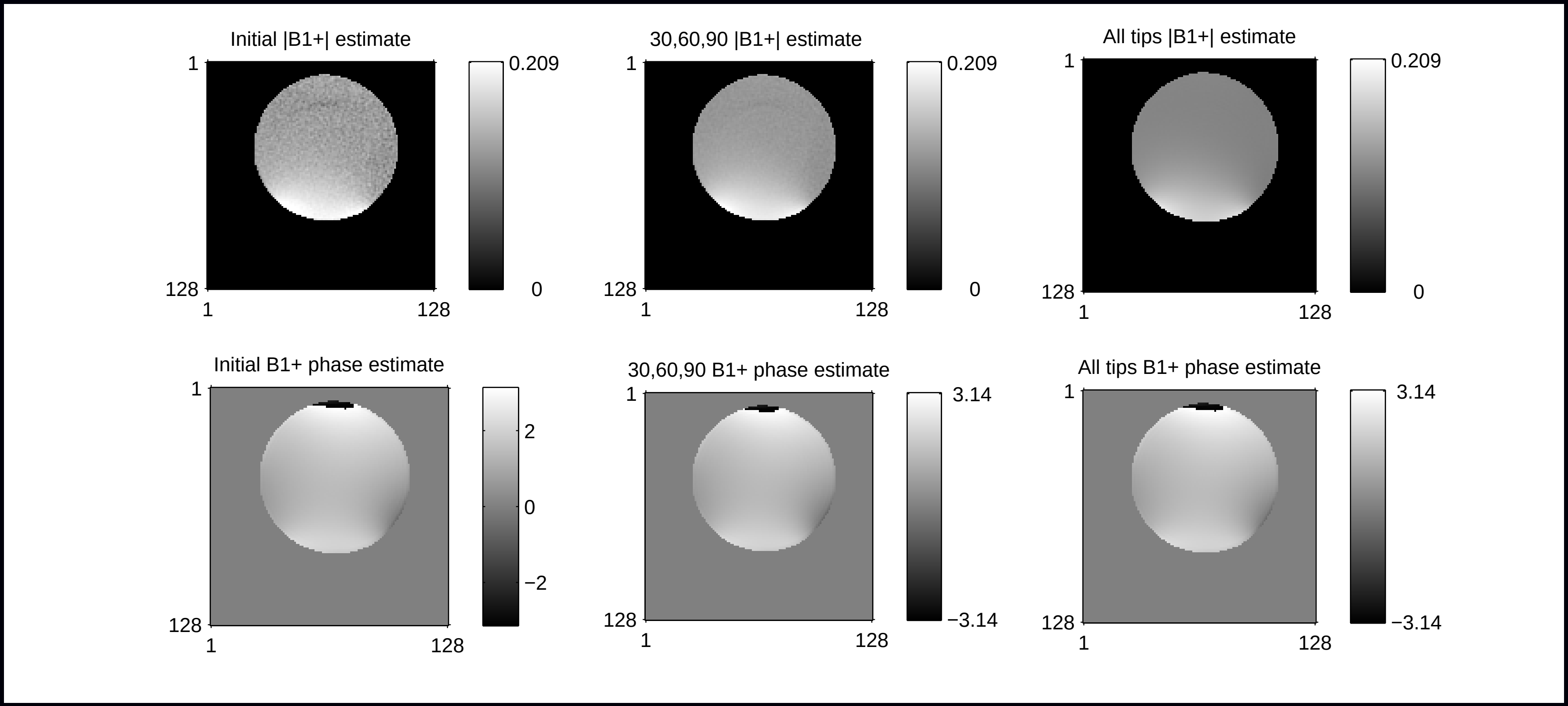
- Masked NRMSE (calculated where $f_j > .1 * \max(f_j)$)
  - B1+ magnitude map (double angle formula) - 72%
  - B1+ magnitude map (regularized iterative) - 12%
  - B1+ phase map (initial) - 52%
  - B1+ phase map (regularized iterative) - 3%
  - Object (initial) - 15%
  - Object (iterative) - 3%
- Reduces the NRMSE by over a factor of 5 compared to the double angle formula!

MRI dataset

- Phantom with coils positioned to create a B1+ map with a large magnitude difference
- One coil used for transmit
- TR ≈ 8 sec
- 18 different nominal tip angles from 10° to 180°
- Estimated with all data and with just $30^\circ, 60^\circ$, and 90°



MRI estimates



Using all tips as truth,

$\left|B_1^+\right|$ NRMSE (conventional) - 19.5% (masked)

$\left|B_1^+\right|$ NRMSE (30°, 60°, 90°) - 14.0% (masked)

B_1^+ phase NRMSE under 5% (masked) for both estimates

Conclusion

- Large improvement (RMSE and smoothness) over double angle formula
- Also estimates the phase for each coil
- Areas for future research
  - Investigate in conjuction with slice selection effects
  - Further explore the spatial resolution/CRB
  - Modify model to include T1 effects
  - Modify model to jointly estimate mosfet nonlinearity
  - Use for parallel excitation RF pulse design