Maximum-Likelihood Dual-Energy Tomographic Reconstruction

Jeffrey A. Fessler, Idris Elbakri

EECS Department

Predrag Sukovic, Neal H. Clinthorne

BME Department, Nuclear Medicine Division of Radiology

The University of Michigan

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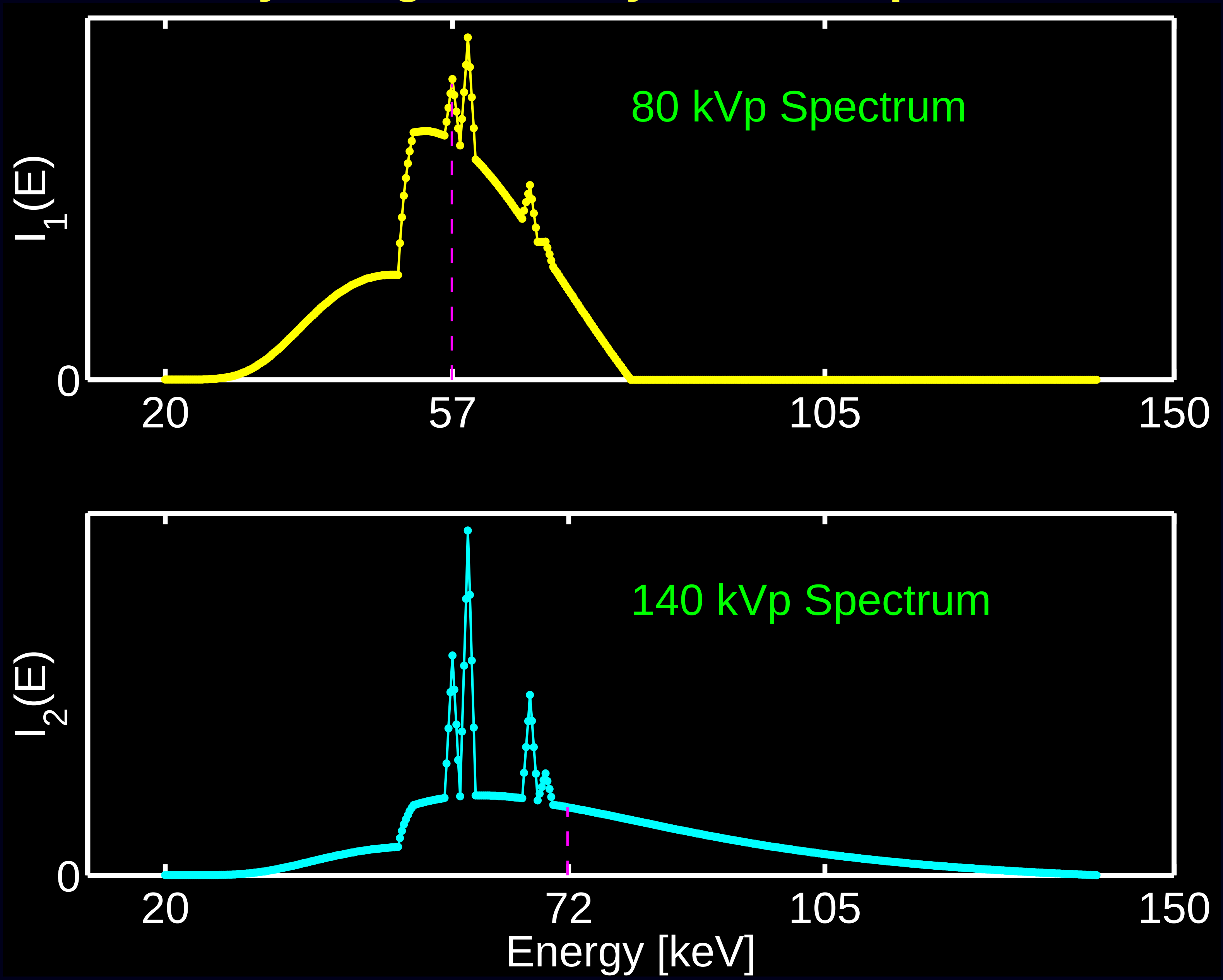
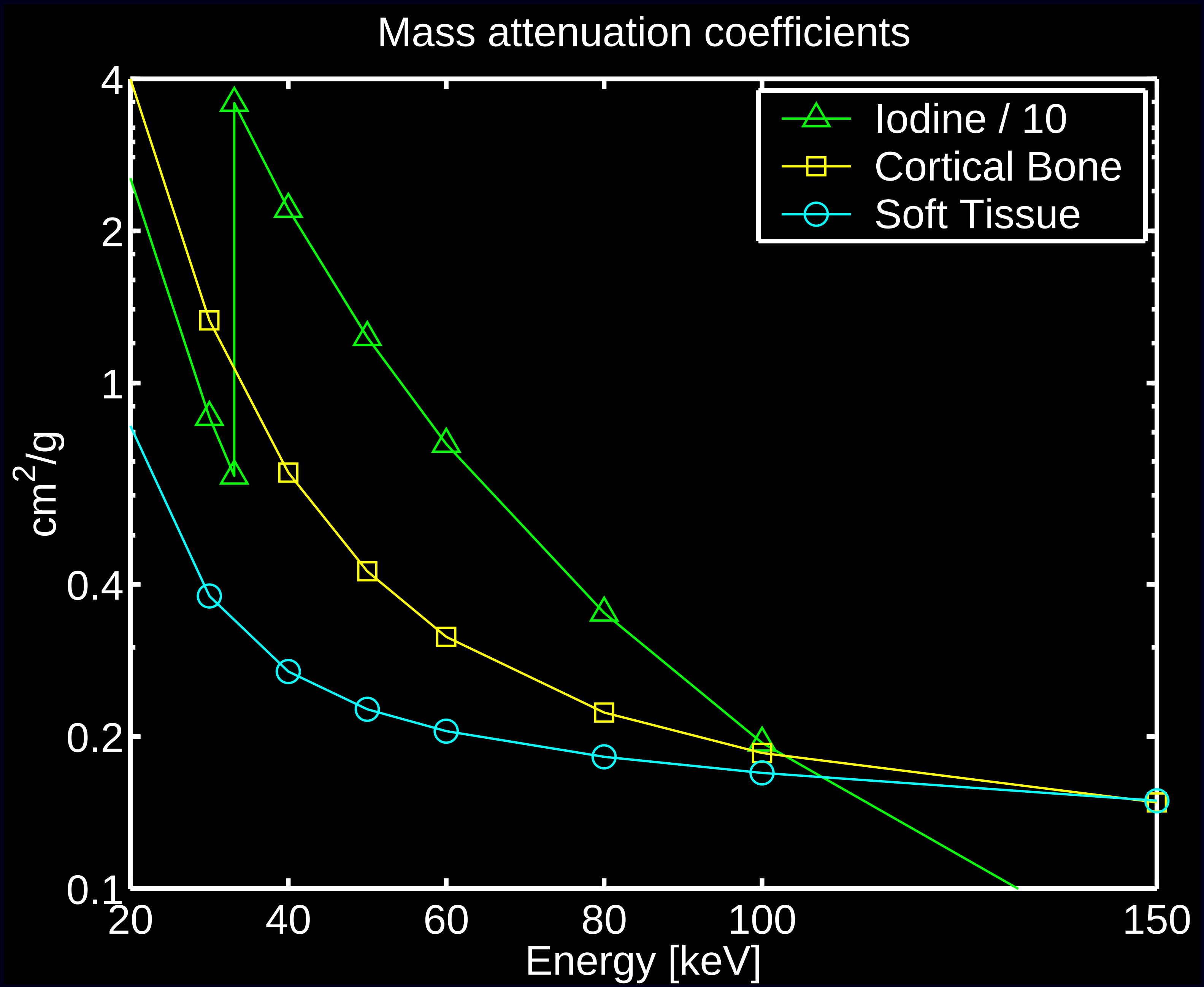
The Challenge for Quantitative X-ray CT: Polyenergetic X-ray Source Spectra



(Tungsten anode, 0.5 mm Be, 1mm Al, 0.5mm Cu, GSO scintillator)

The Opportunity for Dual-Energy CT: Energy-Dependent Attenuation



Applications of Dual-Energy CT

- Nondestructive evaluation
  - Security purposes like baggage inspection
  - Rock characterization for petrochemical industry
  - Soil sample analysis in agriculture
  - Radioactive waste drums
- Medical diagnosis (quantitative CT)
  - Bone mineral density measurements
  - Bone marrow composition
  - Adipose tissue volume determinations
  - Liver iron concentrations
  - Detection of contrast agents in spinal canal
  - Body composition
  - Carotid artery plaques
- SPECT & PET attenuation correction
  - Arms down
  - Iodine contrast
  - Dose / noise

Conventional Dual-Energy CT Model

(Alvarez and Macovski, 1976)

Object Model

Component material basis functions for linear attenuation coefficient:

$$\mu(\vec{r}, E) = \sum_{l=1}^{L} \beta_l(E)\, \rho_l(\vec{r})$$

- $L = 2$ usually. (Compton/photoelectric or bone/water or iodine/tissue)
- $\beta_l(E)$: mass attenuation coefficient of lth material - known
- $\rho_l(\vec{r})$: spatial distribution of material *density* - unknown

Measurement Model

Implicit model for log-processed dual-energy sinogram measurements $\{y_{mi}\}$:

$$\hat{f}_{im} \triangleq -\log\left(\frac{y_{mi} - r_{mi}}{I_{mi}}\right) \approx f_{im}(\boldsymbol{s}_i) \quad (1)$$

$$f_{im}(\boldsymbol{s}_i) \triangleq -\log\left(\frac{1}{I_{mi}} \int I_{mi}(E) \exp\left(-\sum_{l=1}^{L} \beta_l(E) s_{il}\right) dE\right) \quad (2)$$

$$s_{il}(\rho) \triangleq \int_{L_{mi}} \rho_l(\vec{r})\, d\ell \qquad \text{“component line integrals”}, \quad (3)$$

where i indexes rays, and m indexes energy settings, and $\boldsymbol{s}_i \triangleq (s_{i1}, \ldots, s_{iL})$.
Total source intensity: $I_{mi} \triangleq \int I_{mi}(E)\, dE$.

Conventional Dual-Energy CT Approach

- Ignoring noise leads to a (typically 2×2) system of nonlinear equations:

$$\hat{\boldsymbol{f}}_i = \boldsymbol{f}_i(\boldsymbol{s}_i), \quad \text{where } \boldsymbol{f}_i \triangleq (f_{i1}, \ldots, f_{iM}).$$

(One such equation for each ray in the sinogram.)

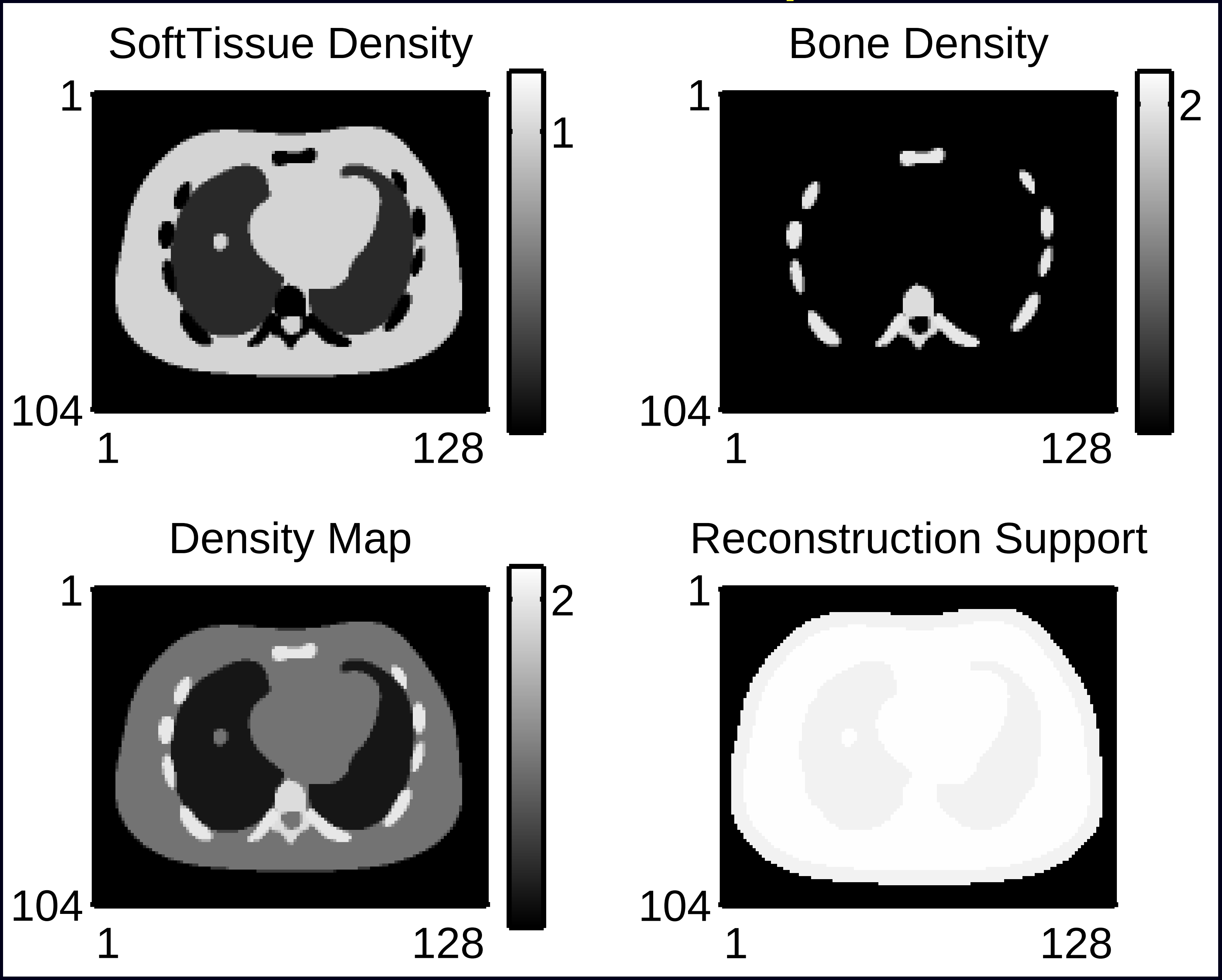
- Characterize nonlinear function $\boldsymbol{f}_i(\boldsymbol{s}_i)$ using polynomial approximations
- Solve nonlinear equations numerically for component line integrals:

$$\hat{\boldsymbol{s}}_i \triangleq \boldsymbol{f}_i^{-1}\left(\hat{\boldsymbol{f}}_i\right).$$

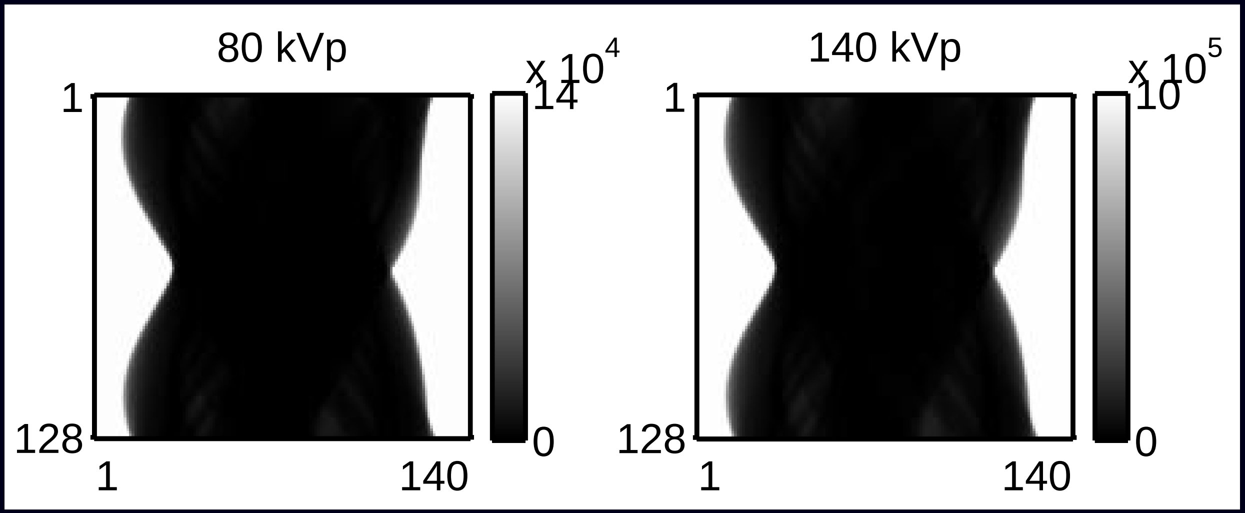
  - This sinogram preprocessing is a noise amplifying step!
  - Great care with implementation details required.

- Perform FBP reconstruction of lth component using sinogram $\{\hat{s}_{il}\}_{i=1}^{N_d}$.

Simulation Example

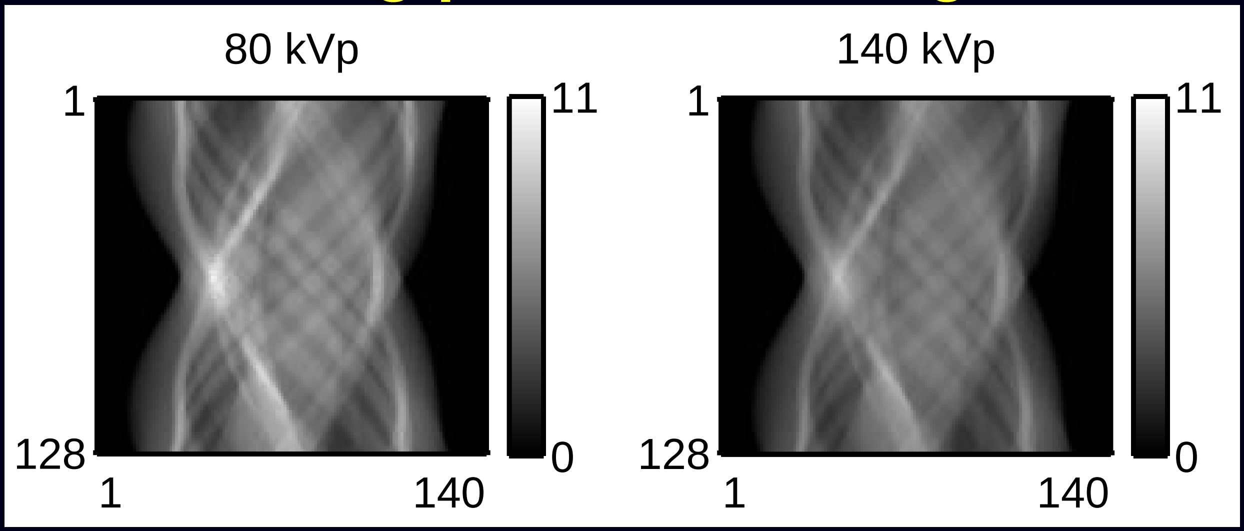


Raw Dual-Energy CT Sinograms



$\{y_{mi}\}$

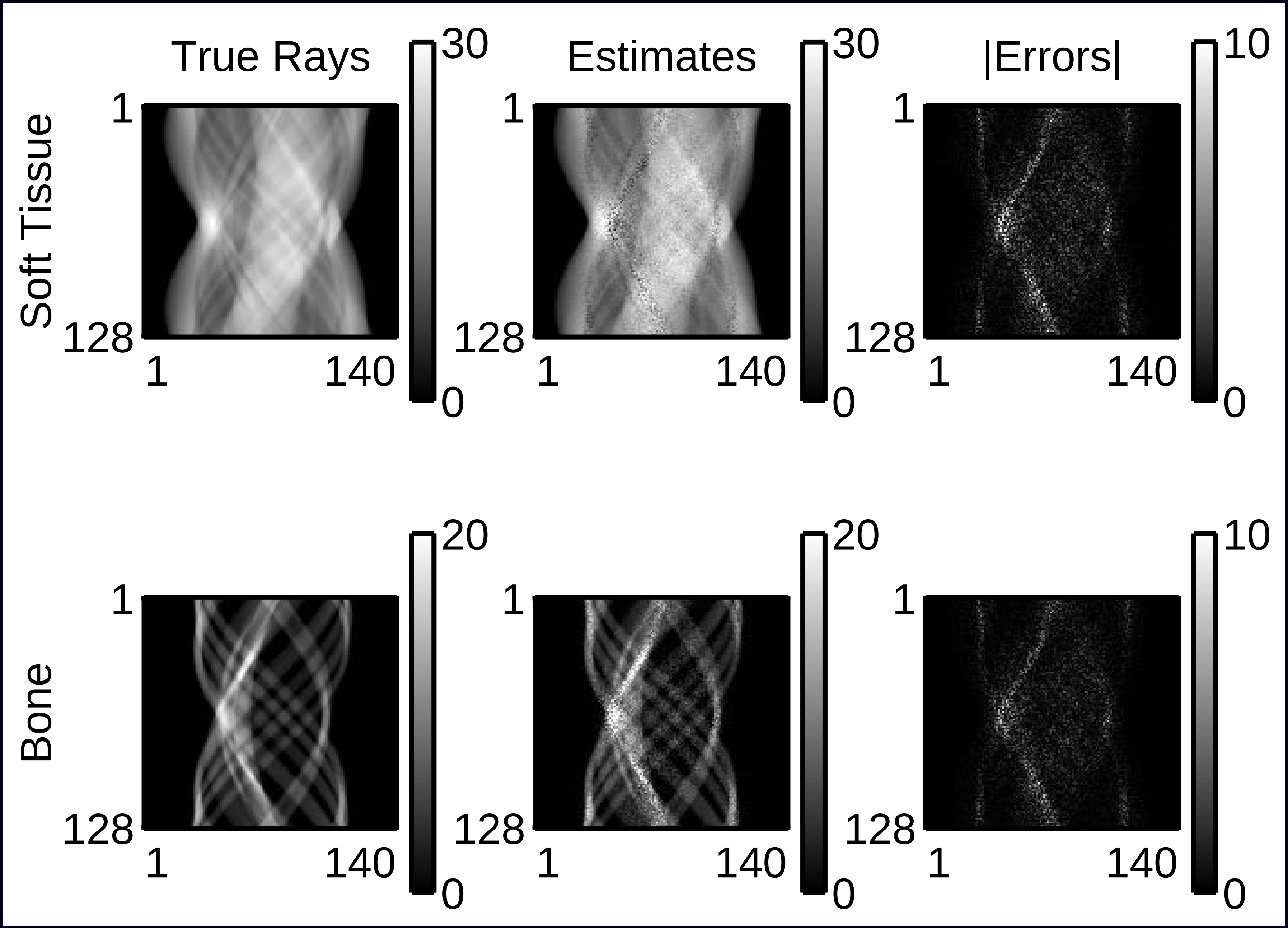
Log-processed Sinograms



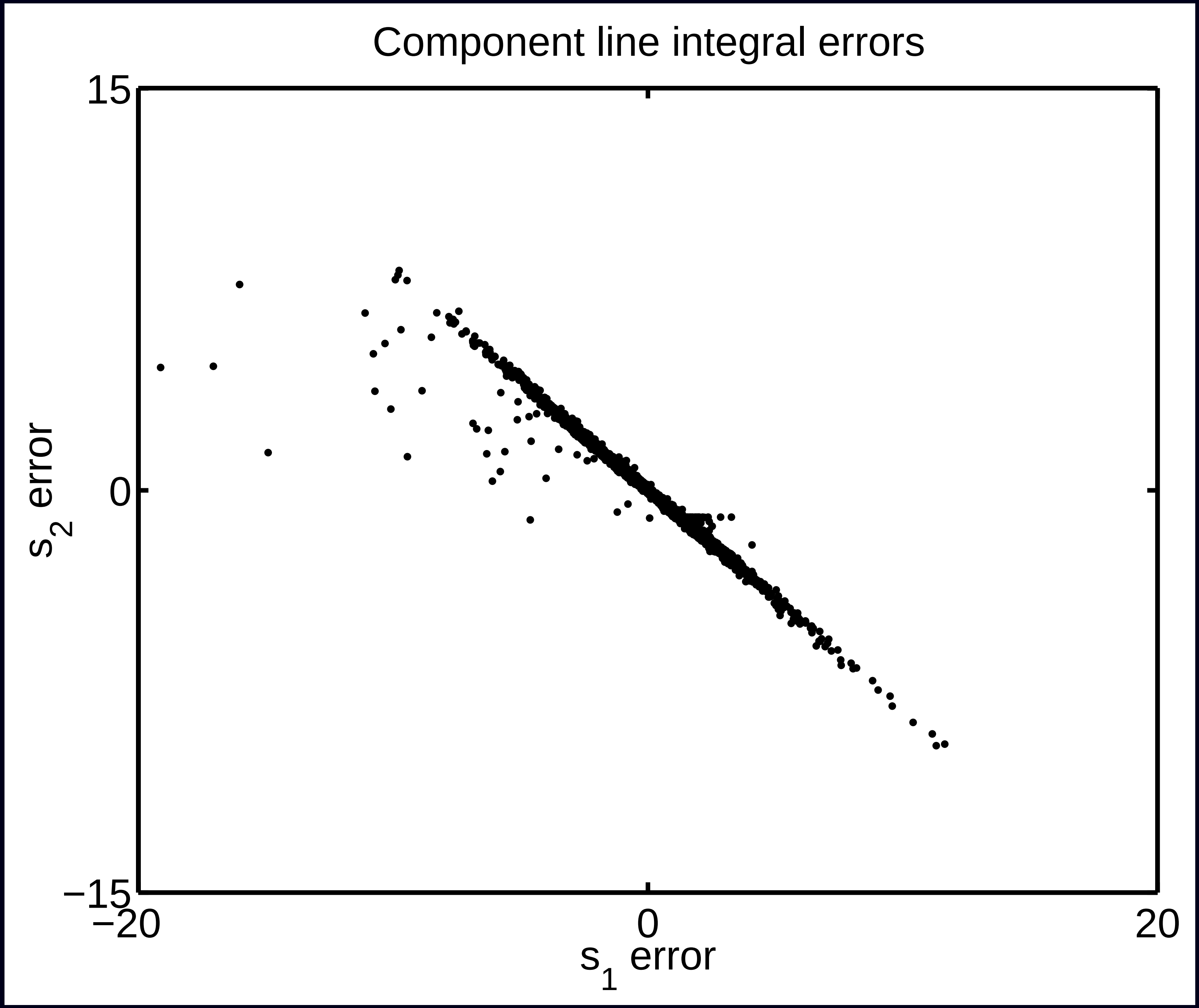
$\{\hat{f}_{im}\}$

- Poisson measurements based on 10^6 incident photons/ray for 140 kVp case.
- No electronic readout noise simulated.

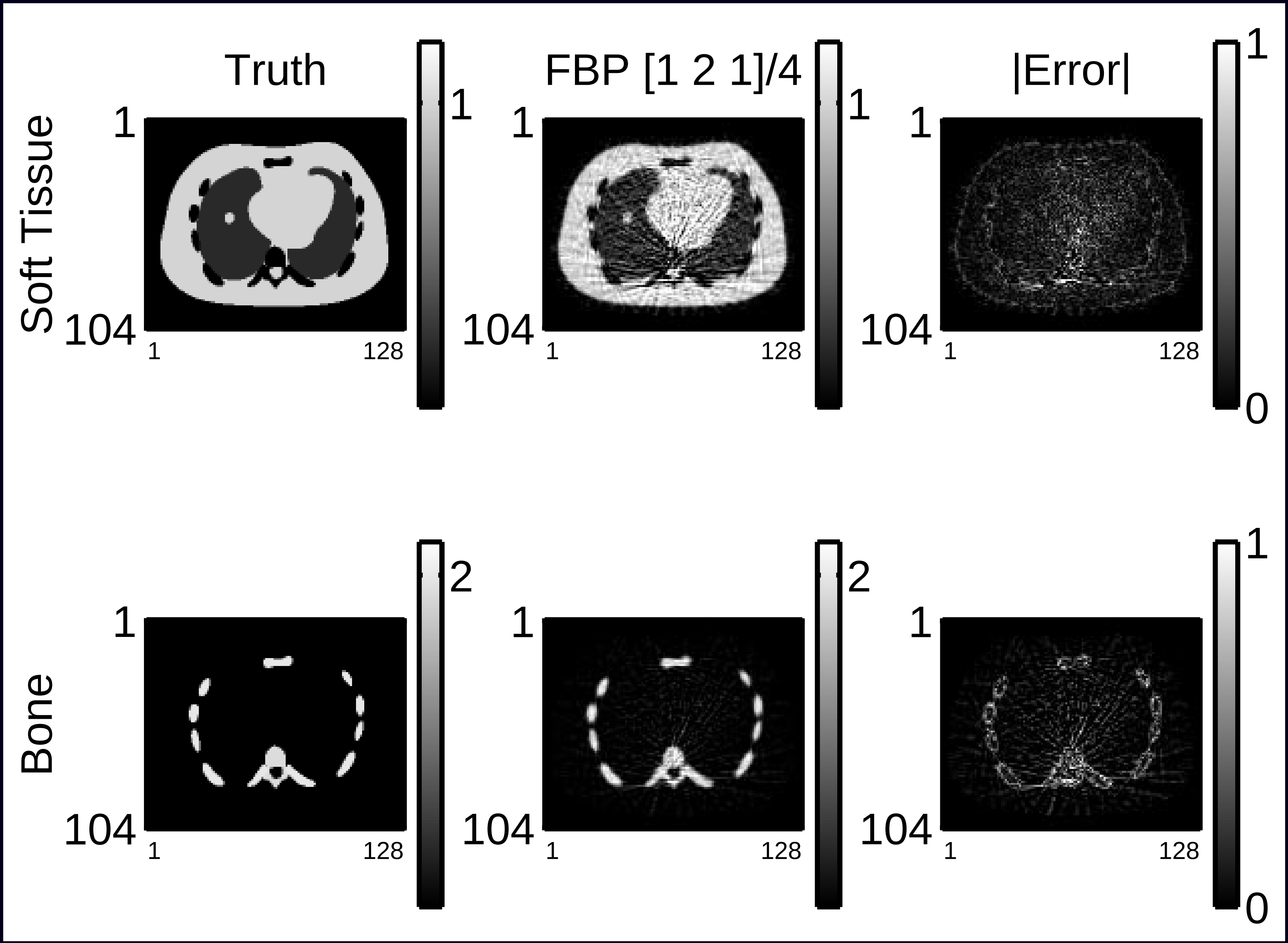
Estimated Component Line Integrals $\{\hat{s}_i\}$ (g/cm^2)



Correlation of Component Line Integral Errors $\{\hat{s}_i - s_i\}$

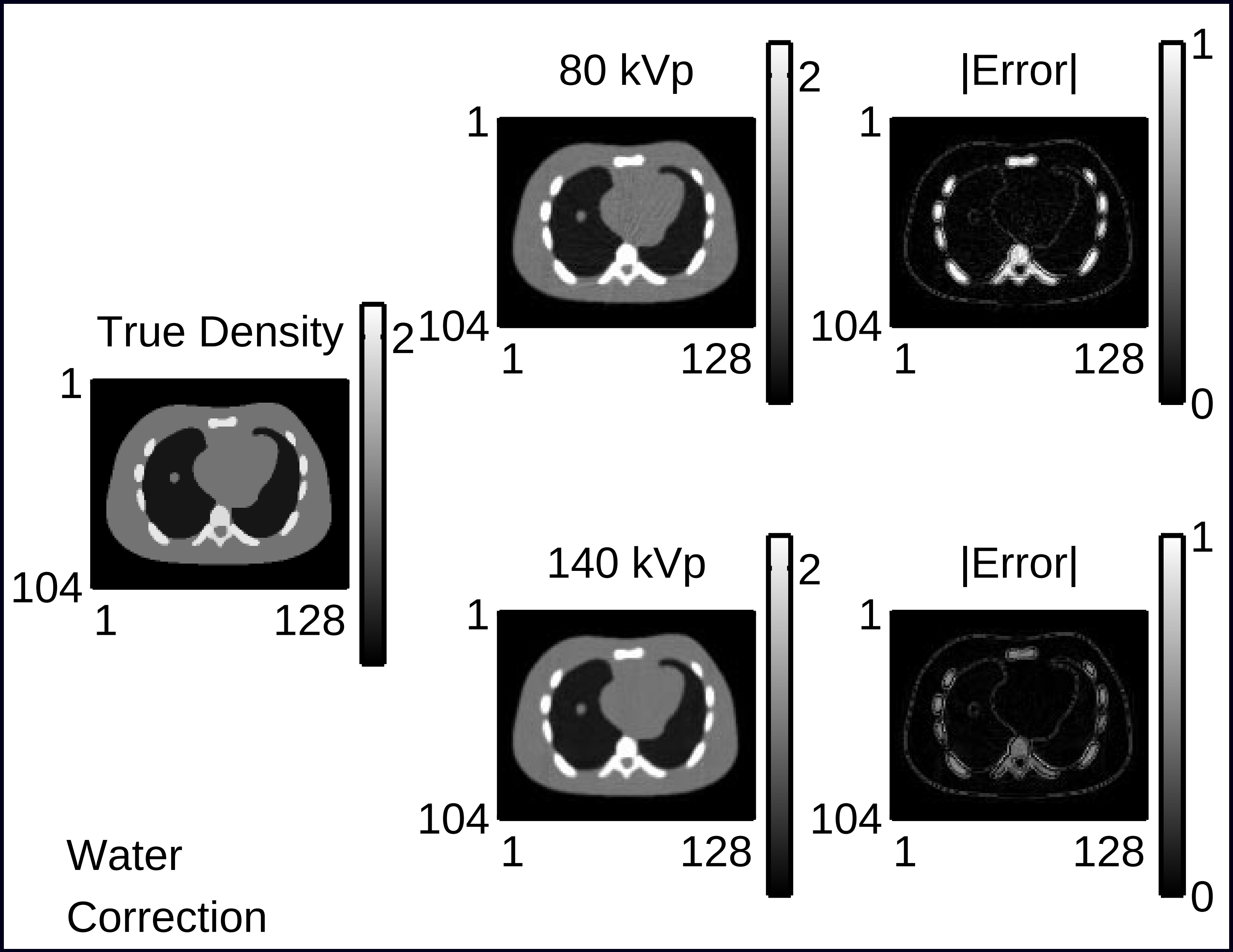


FBP Reconstruction



FBP treats all rays equally, ignoring information in nonlinearity and in noise statistics.

Water-Corrected Reconstructions



FBP provides unfavorable bias-variance tradeoff...

Dual-Energy CT Reconstruction Methods

	Data	
Reconstruction Algorithm	Preprocessed	Unprocessed
FBP	Alvarez & Macovski, 1976 (many others since)	-
Algebraic	Kotzi *et al.*, 1992 Markham *et al.*, 1993	-
Statistical (Monoenergetic)		Clinthorne & Sukovic 2000
Statistical (Polyenergetic)	(here)	(here)

Penalized Weighted Least Squares Reconstruction for Polyenergetic Dual-Energy X-ray CT

- Preprocess by solving 2×2 equations in sinogram space: $\hat{\boldsymbol{s}}_i = \boldsymbol{f}_i^{-1}(\hat{\boldsymbol{f}}_i)$
- Using statistical model for measurement variances,
  apply error propagation to estimate 2×2 covariances for each ray pair:

$$\mathrm{Cov}\{\hat{\boldsymbol{s}}_i\} \approx \left[(\nabla' \boldsymbol{f}_i) \, \mathrm{Cov}\left\{ \hat{\boldsymbol{f}}_i \right\}^{-1} (\nabla \boldsymbol{f}_i) \right]^{-1}, \qquad \mathrm{Cov}\left\{ \hat{\boldsymbol{f}}_i \right\}^{-1} \approx \mathrm{diag}\{ \mathrm{Var}\{y_{mi}\}\}$$

- Penalized weighted least-squares (PWLS) estimator and cost function:

$$\hat{\boldsymbol{x}} \triangleq \arg \min_{\boldsymbol{x} \geq \boldsymbol{0}} \Psi(\boldsymbol{x})$$

$$\Psi(\boldsymbol{x}) \triangleq \sum_{i=1}^{N_d} \frac{1}{2} (\boldsymbol{y}_i - [\boldsymbol{A}\boldsymbol{x}]_i)' \, \mathrm{Cov}\{\hat{\boldsymbol{s}}_i\}^{-1} (\boldsymbol{y}_i - [\boldsymbol{A}\boldsymbol{x}]_i) + R(\boldsymbol{x})$$

  - x_{lj} : jth pixel in lth component image.
  - $[\boldsymbol{A}\boldsymbol{x}]_i$: discrete approximation to ith line integral

Regularization

Edge-preserving regularizing roughness penalty function:

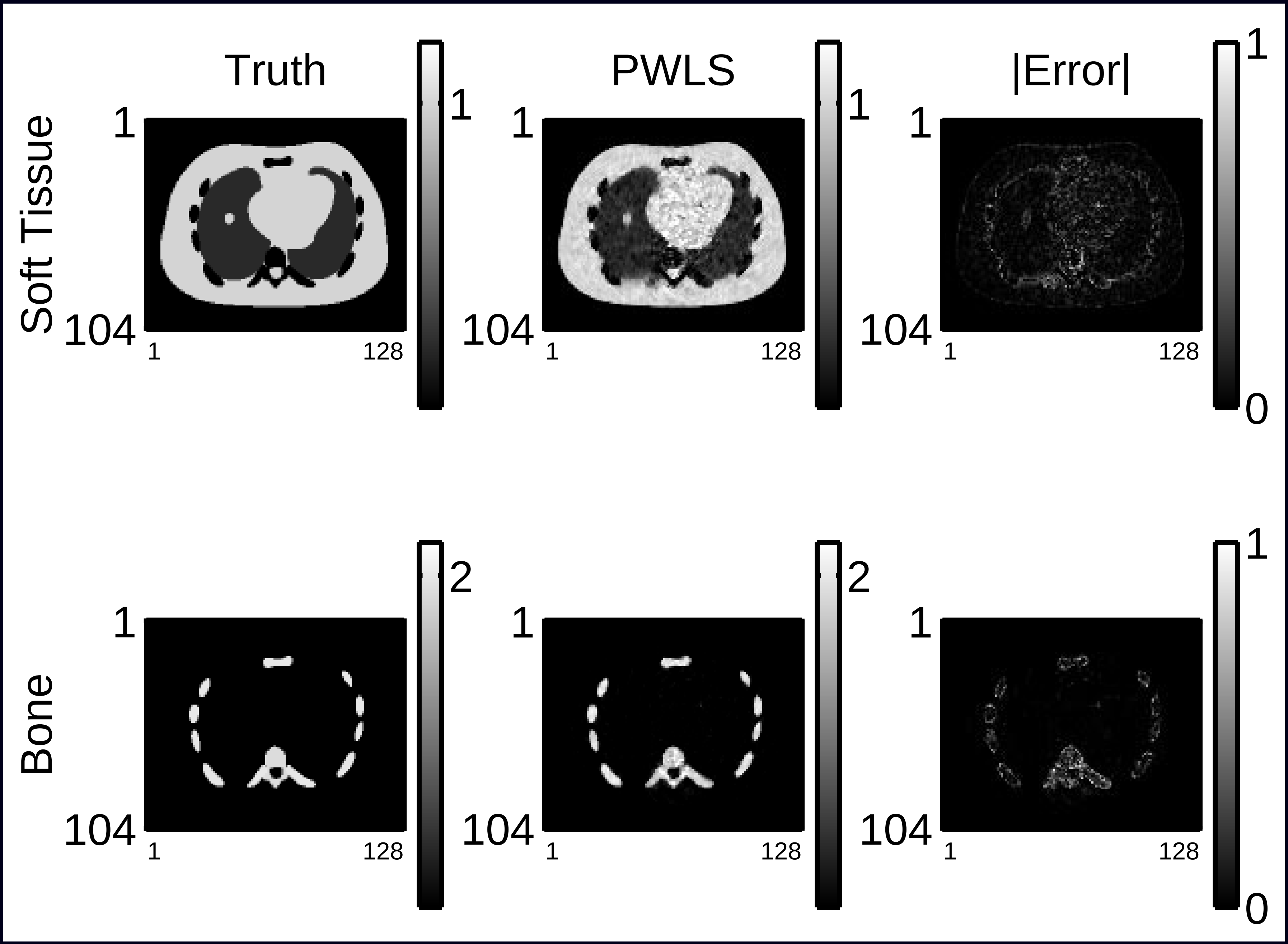
$$R(\boldsymbol{x}) = \beta \sum_{l=1}^{L} \sum_{j=1}^{p} \sum_{k=1}^{p} w_{jk} \psi(x_{lj} - x_{lk})$$

Hyperbola potential function:

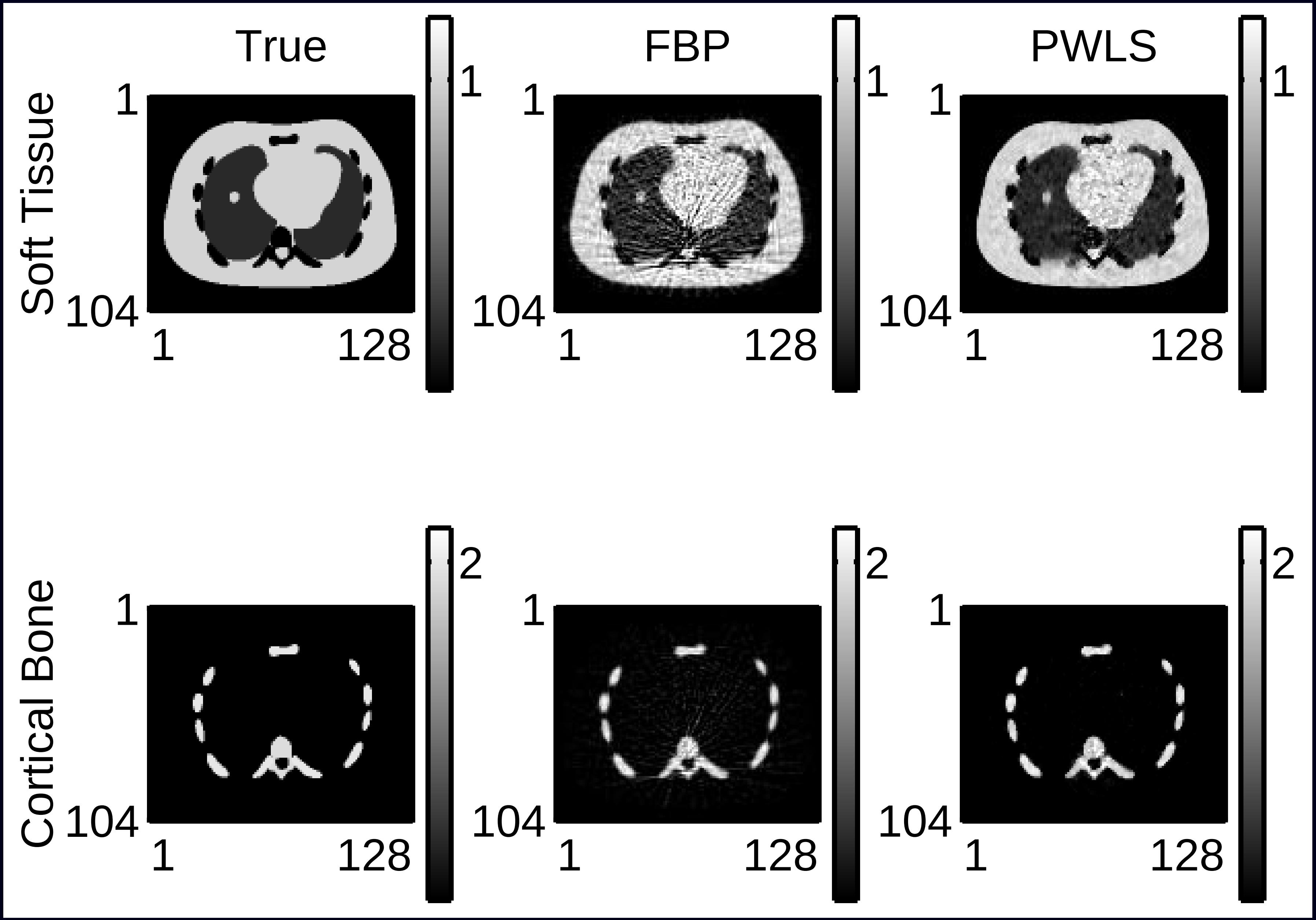
$$\psi(x_{lj} - x_{lk}) = \sqrt{1 + \left(\frac{x_{lj} - x_{lk}}{\delta} \right)^2}$$

- Separable between basis components.
- β controls resolution/noise tradeoff.
- δ controls degree of edge preservation
- Quadratic data-fit term + convex penalty $\Rightarrow$ simple iterative minimization. (Globally convergent, relaxed ordered-subsets PWLS algorithm.)

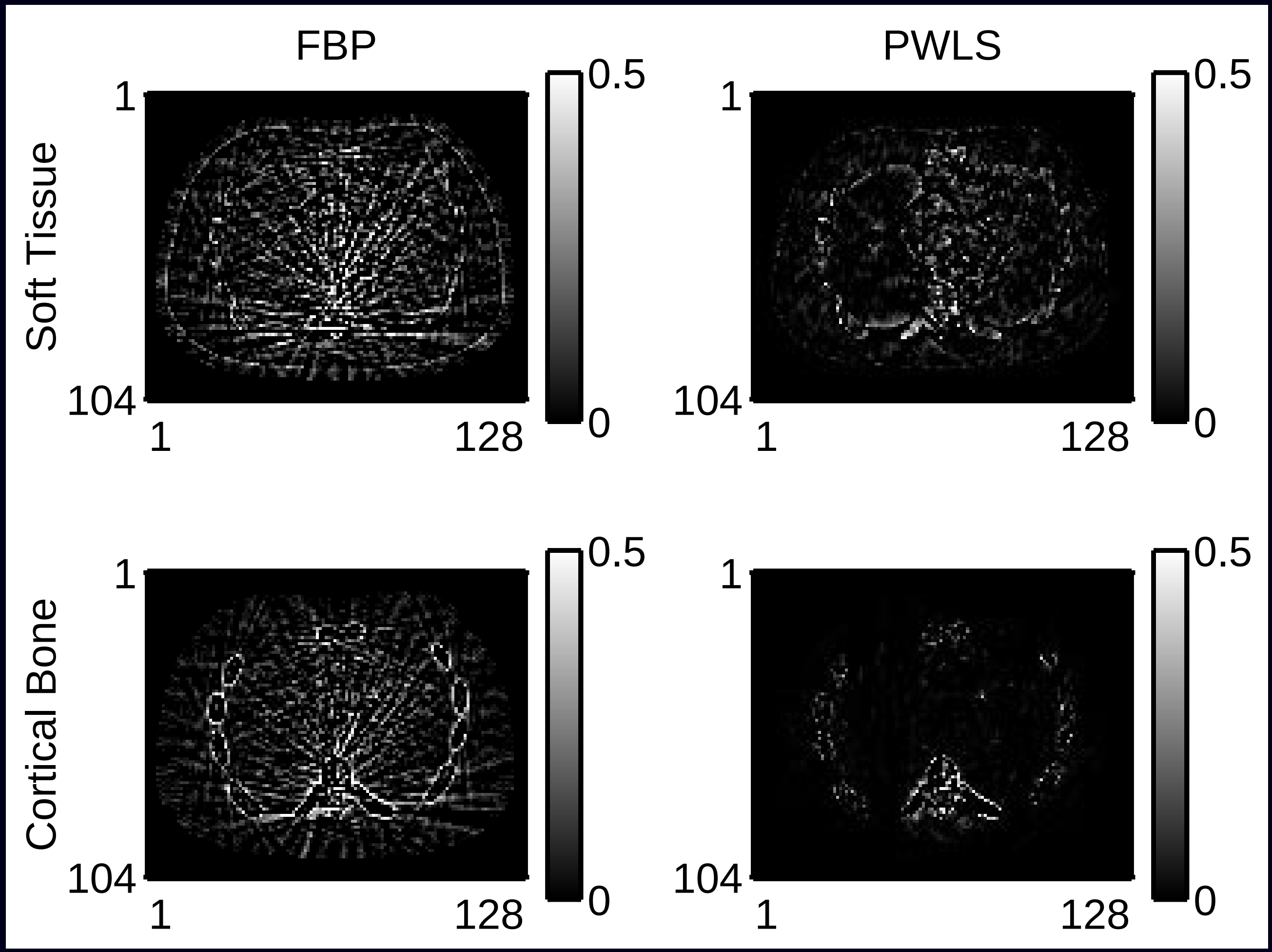
PWLS Simulation Results



FBP vs PWLS



FBP vs PWLS Errors



	NRMS Error	
Method	Soft Tissue	Cortical Bone
FBP	22.7%	29.6%
PWLS	13.6%	16.2%
PL	11.8%	15.8%

Maximum-Likelihood Dual-Energy CT Approach

Parameterized object model:

$$\mu(\vec{r}, E) = \sum_{l=1}^{L} \sum_{j=1}^{p} \beta_l(E)\, b_j(\vec{r}) x_{lj}$$

- $\beta_l(E)$: spectral basis
- $b_j(\vec{r})$: spatial basis (*e.g.*, voxels)

Polyenergetic model for sinogram measurements $\{y_{mi}\}$:

$$\begin{aligned}
\bar{y}_{mi}(\boldsymbol{x}) &= \int I_{mi}(E) \exp\left(-\sum_{l=1}^{L} \beta_l(E) s_{il}(\boldsymbol{x}) \right) dE + \text{scatter}_{mi} \\
s_{il}(\boldsymbol{x}) &\triangleq \sum_{j=1}^{p} a_{ij} x_{lj} \\
a_{ij} &\triangleq \int_{L_{mi}} b_j(\vec{r})\, d\ell \qquad \text{(system matrix / reprojector)}\ .
\end{aligned}$$

Statistical model:

- independent measurements $\{Y_{mi}\}$
- very mild regularity conditions on log-likelihood

Penalized-Likelihood DE CT Image Reconstruction

Choose a statistical model with marginal negative log-likelihood functions $\{\psi_{mi}\}$

Penalized-likelihood estimator:

$$\hat{\boldsymbol{x}} = \arg\min_{\boldsymbol{x}} \Psi(\boldsymbol{x})$$

Penalized-likelihood cost function:

$$\boxed{\Psi(\boldsymbol{x}) = \sum_{m=1}^{M} \sum_{i=1}^{N_d} \psi_{mi}(\bar{y}_{mi}(\boldsymbol{x})) + R(\boldsymbol{x})}$$

Example: for Poisson statistical model

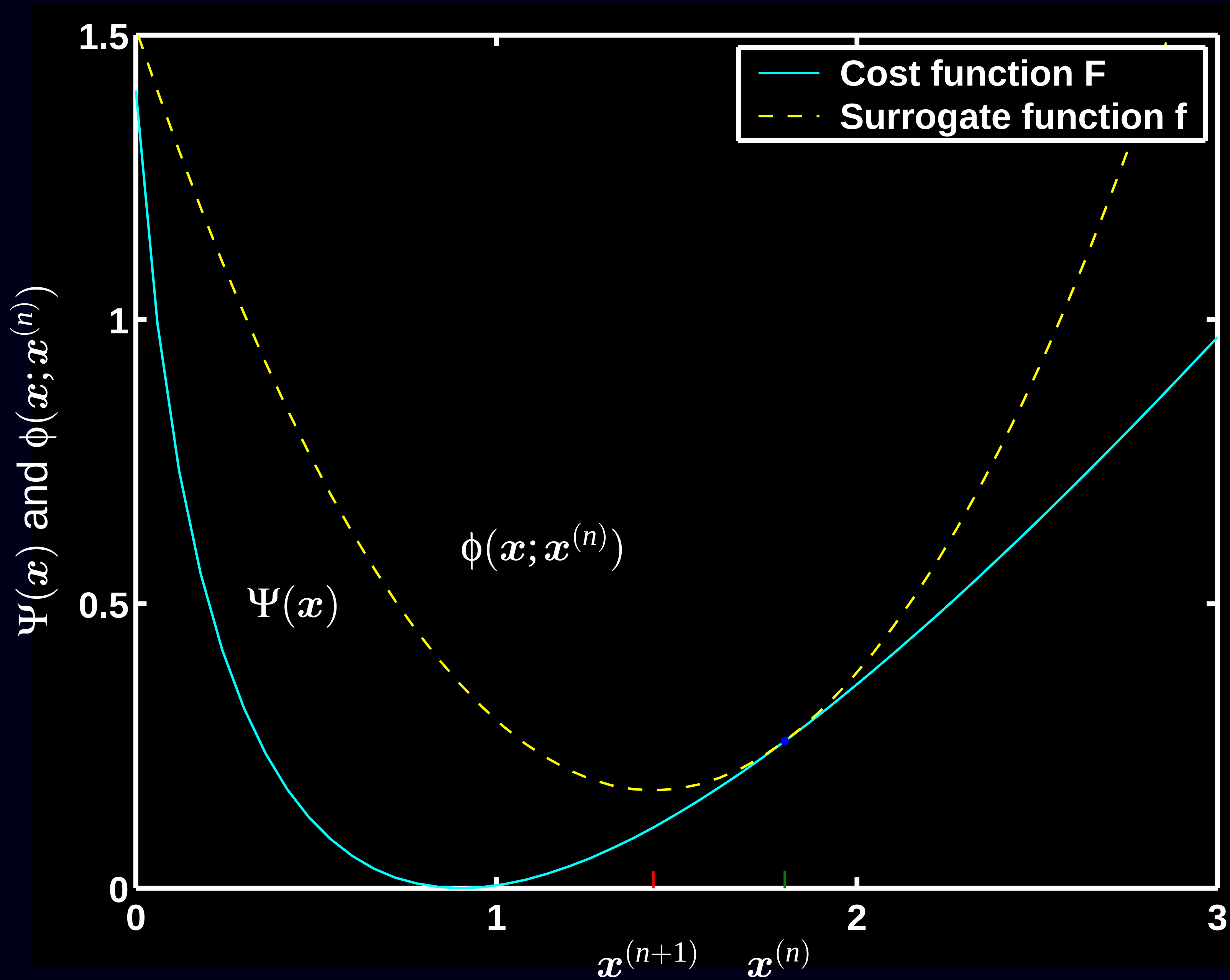
$$\psi_{mi}(y) = y - y_{mi} \log y.$$

Example: pairwise hyperbolic (edge preserving) roughness penalty:

$$R(\boldsymbol{x}) = \sum_{l=1}^{L} \sum_{j=1}^{p} \sum_{k=1}^{p} w_{jk} \sqrt{1 + \left(\frac{x_{lj} - x_{lk}}{\delta}\right)^2}$$

Challenging minimization problem! Requires iterative algorithms.

Optimization Transfer Illustrated



Separable Surrogate Algorithm

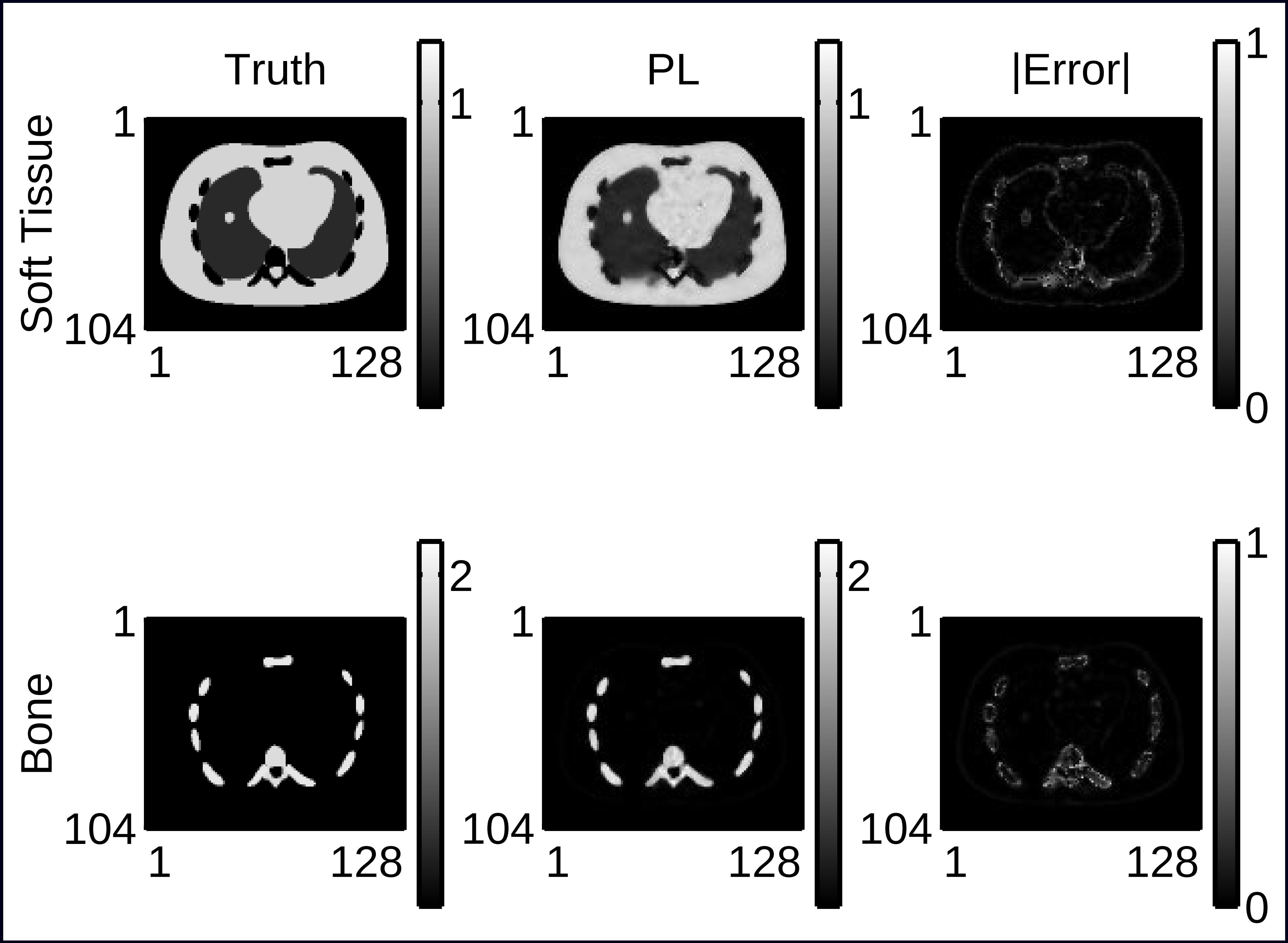
L e n g t h y derivation with repeated use of optimization transfer...

Iteration:

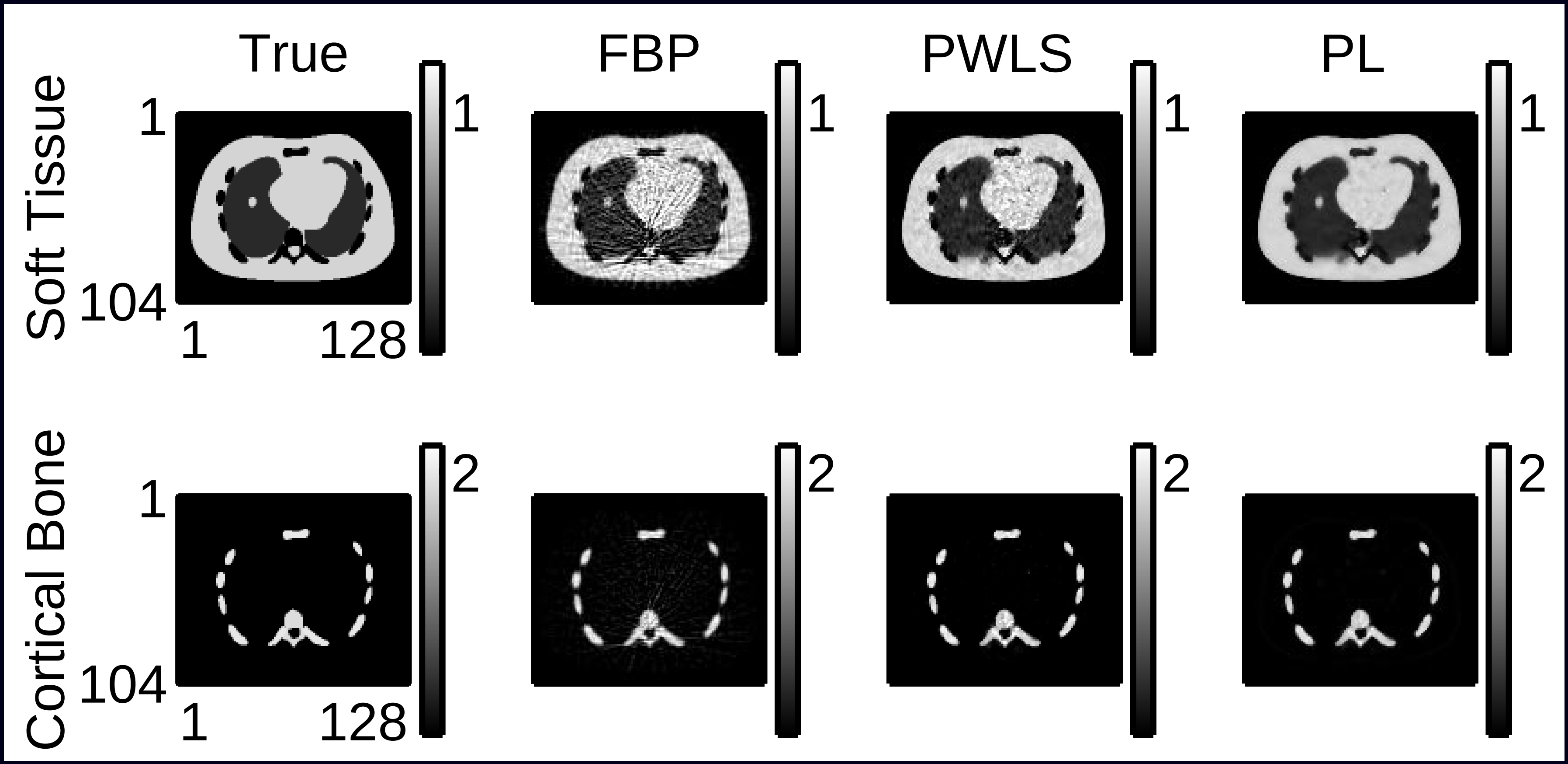
$$\boxed{\boldsymbol{x}^{(n+1)} = \left[\boldsymbol{x}^{(n)} - \boldsymbol{D}(\boldsymbol{x}^{(n)})\nabla\Psi\left(\boldsymbol{x}^{(n)}\right)\right]_+}$$

- $[\cdot]_+$ enforces the nonnegativity constraint.
- One choice of $\boldsymbol{D}$ guarantees monotonic decrease in $\Psi(\boldsymbol{x})$
  ($\Psi(\boldsymbol{x})$ is nonconvex so stronger convergence conditions are unlikely)
- An alternative choice of $\boldsymbol{D}$ is precomputable and usually is monotonic
- Ordered subsets variation is trivial:
  replace backprojections with partial backprojections

Penalized-Likelihood DE CT Simulation Results



DE CT Comparison



	NRMS Error	
Method	Soft Tissue	Cortical Bone
FBP	22.7%	29.6%
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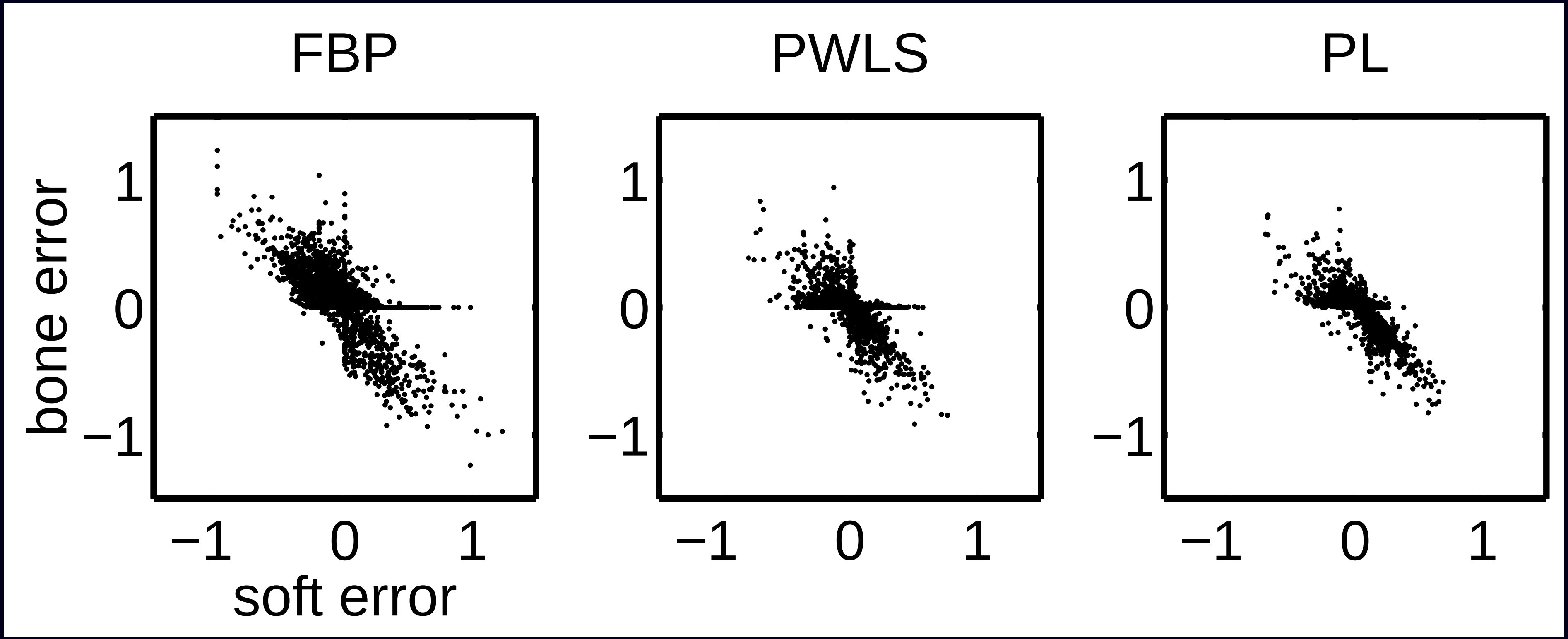
Correlated Errors



Summary

- Statistical image reconstruction methods for dual-energy X-ray CT
  - Penalized weighted least-squares approach (preprocessed sinograms)
  - Maximum-likelihood / penalized-likelihood algorithm (raw sinograms)
- First statistical approach based on fully polyenergetic source spectrum model.
  - Sukovic and Clinthorne (IEEE T-MI, 2000) used monoenergetic model
  - Prior polyenergetic dual-energy CT approaches have been non-statistical
- ML formulation/algorithm accommodates general statistical model
- Monotonic version of ML algorithm (but slow)
- Fast ordered-subsets ML algorithm
  Relaxation may not ensure convergence due to non-convexity?
- Reduced noise in idealized simulations
- Future work:
  - Identify suitable noise model / log-likelihood
  - Real CT data...
  - Explore applicability to attenuation correction for PET-CT systems