Statistical Methods for Image Reconstruction

Jeffrey A. Fessler

EECS Department
The University of Michigan

EMBS Summer School

June 2002
Image Reconstruction Methods
(Simplified View)

Analytical (FBP)

Iterative (OSEM?)
Image Reconstruction Methods / Algorithms

ANALYTICAL
- FBP
- BPF
- Gridding
  ...

ITERATIVE

Algebraic
- (y = Ax)
  - ART
  - MART
  - SMART
  ...

Statistical

(Weighted)
- Least Squares
  - CG
  - CD
  - ISRA
  ...

Likelihood
- (e.g., Poisson)
  - EM (etc.)
  - OSEM
  - SAGE
  - CG
  - Int. Point
  - GCA
  - PSCD
  - FSCD
  ...

(Weighted) Likelihood
(e.g., Poisson)
Outline

Part 0: Introduction / Overview

Part 1: From Physics to Statistics (Emission tomography)
- Assumptions underlying Poisson statistical model
- Emission reconstruction problem statement

Part 2: Four of Five Choices for Statistical Image Reconstruction
- Object parameterization
- System physical modeling
- Statistical modeling of measurements
- Cost functions and regularization

Part 3: Fifth Choice: Iterative algorithms
- Classical optimization methods
- Considerations: nonnegativity, convergence rate, ...
- Optimization transfer: EM etc.
- Ordered subsets / block iterative / incremental gradient methods

Part 4: Performance Analysis
- Spatial resolution properties
- Noise properties
- Detection performance

Part 5: Miscellaneous topics (?)
- ...
History

- Iterative method for X-ray CT (Hounsfield, 1968)
- ART for tomography (Gordon, Bender, Herman, JTB, 1970)
- Richardson/Lucy iteration for image restoration (1972, 1974)
- Weighted least squares for 3D SPECT (Goitein, NIM, 1972)
- Proposals to use Poisson likelihood for emission and transmission tomography
  - Emission: (Rockmore and Macovski, TNS, 1976)
  - Transmission: (Rockmore and Macovski, TNS, 1977)
- First expectation-maximization (EM) algorithms for Poisson model
  - Emission: (Shepp and Vardi, TMI, 1982)
  - Transmission: (Lange and Carson, JCAT, 1984)
- First regularized (aka Bayesian) Poisson emission reconstruction
  - Geman and McClure, ASA, 1985
- Ordered-subsets EM algorithm
  - Hudson and Larkin, TMI, 1994
- Commercial introduction of OSEM for PET scanners
  - circa 1997
Why Statistical Methods?

- Object constraints (e.g., nonnegativity, object support)
- Accurate physical models (less bias $\Rightarrow$ improved quantitative accuracy)
  improved spatial resolution?
  (e.g., nonuniform attenuation in SPECT)
- Appropriate statistical models (less variance $\Rightarrow$ lower image noise)
  (FBP treats all rays equally)
- Side information (e.g., MRI or CT boundaries)
- Nonstandard geometries (“missing” data)

Disadvantages?

- Computation time
- Model complexity
- Software complexity

Analytical methods (a different short course!)

- Idealized mathematical model
  - Usually geometry only, greatly over-simplified physics
  - Continuum measurements
- No statistical model
- Easier analysis of properties (due to linearity)
  e.g., Huesman (1984) FBP ROI variance for kinetic fitting
What about Moore’s Law?
Benefit Example: Statistical Models

<table>
<thead>
<tr>
<th>Method</th>
<th>Soft Tissue</th>
<th>Cortical Bone</th>
</tr>
</thead>
<tbody>
<tr>
<td>FBP</td>
<td>22.7%</td>
<td>29.6%</td>
</tr>
<tr>
<td>PWLS</td>
<td>13.6%</td>
<td>16.2%</td>
</tr>
<tr>
<td>PL</td>
<td>11.8%</td>
<td>15.8%</td>
</tr>
</tbody>
</table>
Benefit Example: Physical Models

a. True object

b. Unocorrected FBP

c. Monoenergetic statistical reconstruction

a. Soft-tissue corrected FBP

b. JS corrected FBP

c. Polyenergetic Statistical Reconstruction
Benefit Example: Nonstandard Geometries
Truncated Fan-Beam SPECT Transmission Scan

Truncated FBP
Truncated PWLS
Untruncated FBP
One Final Advertisement: Iterative MR Reconstruction

Spin Echo

Iterative NUFFT with min-max

Conjugate Phase

Field Map in Hz

SPHERE

Uncorrected