Recent and Current Topics

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Statistical Models for Randoms-Precorrected PET

(Mehmet Yavuz)
IEEE T-MI, Aug. 1999

Randoms-subtracted measurements (mean $\neq$ variance):

$$Y_i \sim \text{Poisson}\{\bar{y}_i + r_i\} - \text{Poisson}\{r_i\}$$

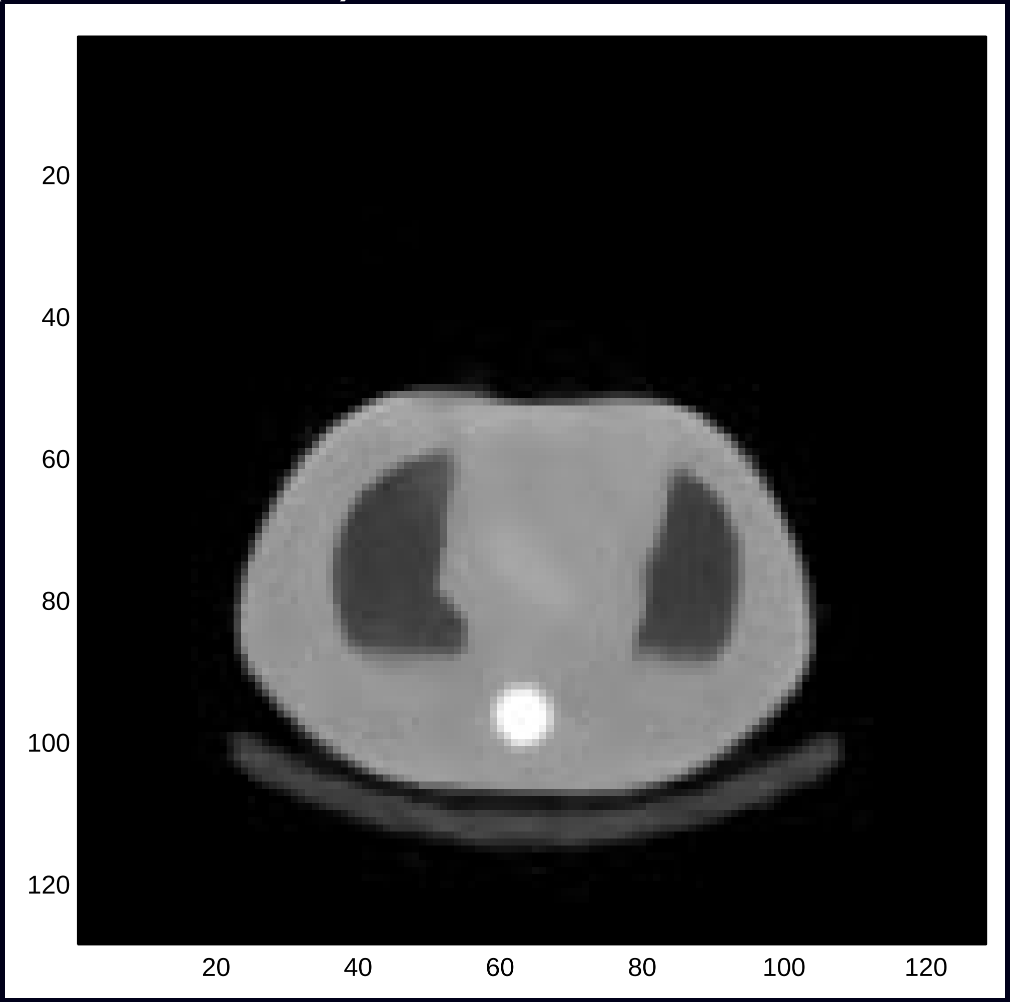
Exact log-likelihood impractical.
Practical approximate: statistical models:

- Data-weighted LS approach: $[Y_i]_+ \overset{?}{\sim} N(\bar{y}_i, Y_i + 2r_i)$
- Ordinary Poisson model: $[Y_i]_+ \overset{?}{\sim} \text{Poisson}\{\bar{y}_i\}$
- Shifted-Poisson model: $Y_i + 2r_i \overset{?}{\sim} \text{Poisson}\{\bar{y}_i + 2r_i\}$

Penalized-likelihood reconstruction easy for all three models.

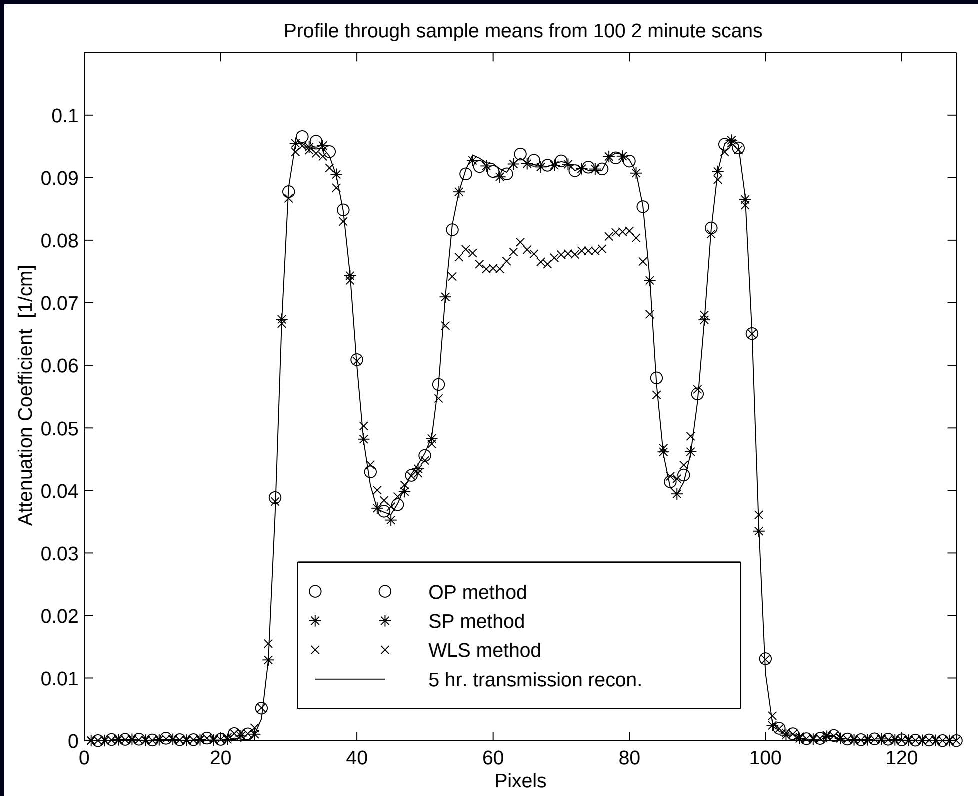
PET Transmission Scans of Thorax Phantom

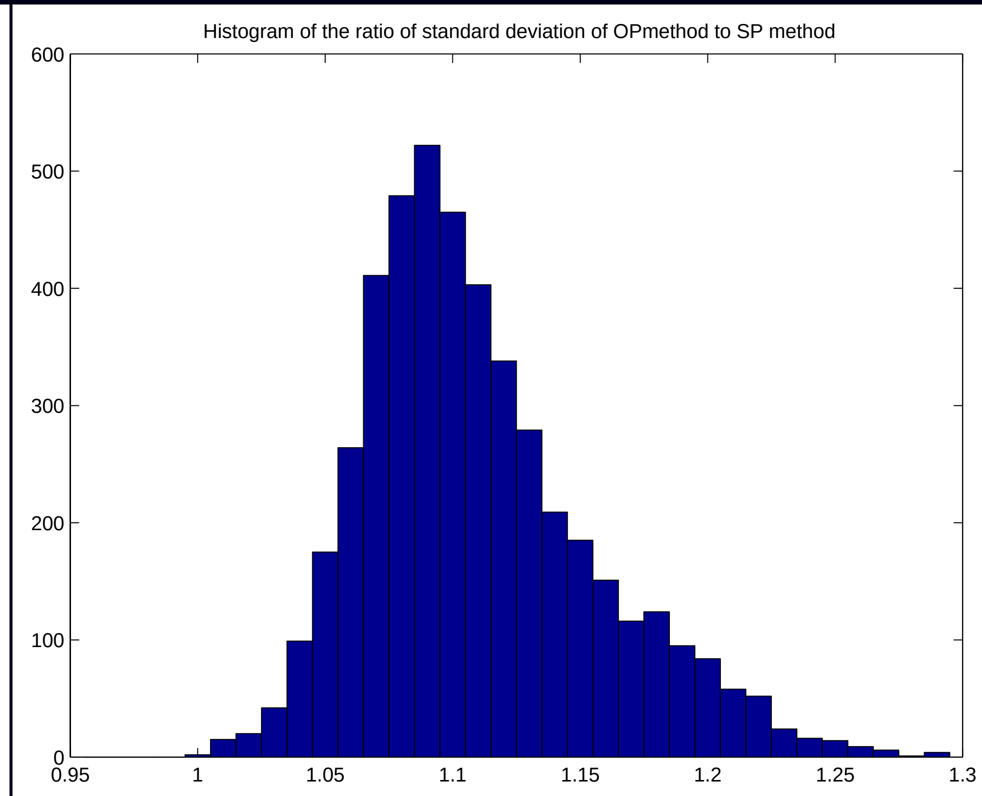
- One 5-hour scan (ECAT EXACT)



- 100 2-minute scans
  Each reconstructed by all three methods and by FBP.

Bias/Variance Comparison



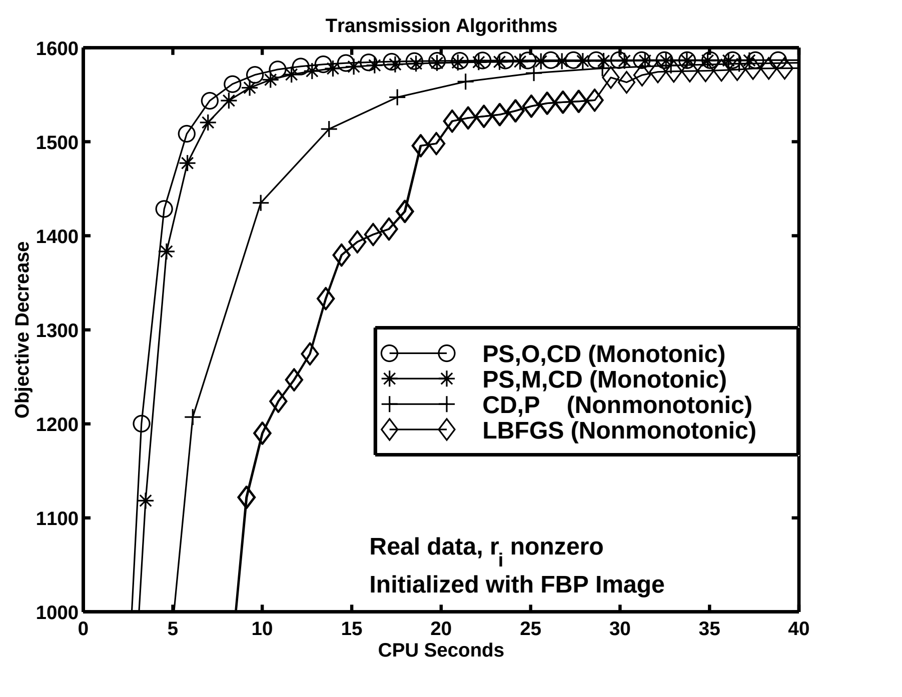


- Data-weighted LS (and FBP) are severely biased for low-count transmission scans.
- Ordinary Poisson model yields about 11% higher standard deviation than shifted-Poison.
- Computation time indistinguishable. Programming time almost indistinguishable.
- Robust to crude approximations for randoms (r_i's)
- Attenuation map noise somewhat amplified when propagated into emission images (observed empirically and verified analytically).
- Collecting separate prompts/randoms nevertheless preferable.

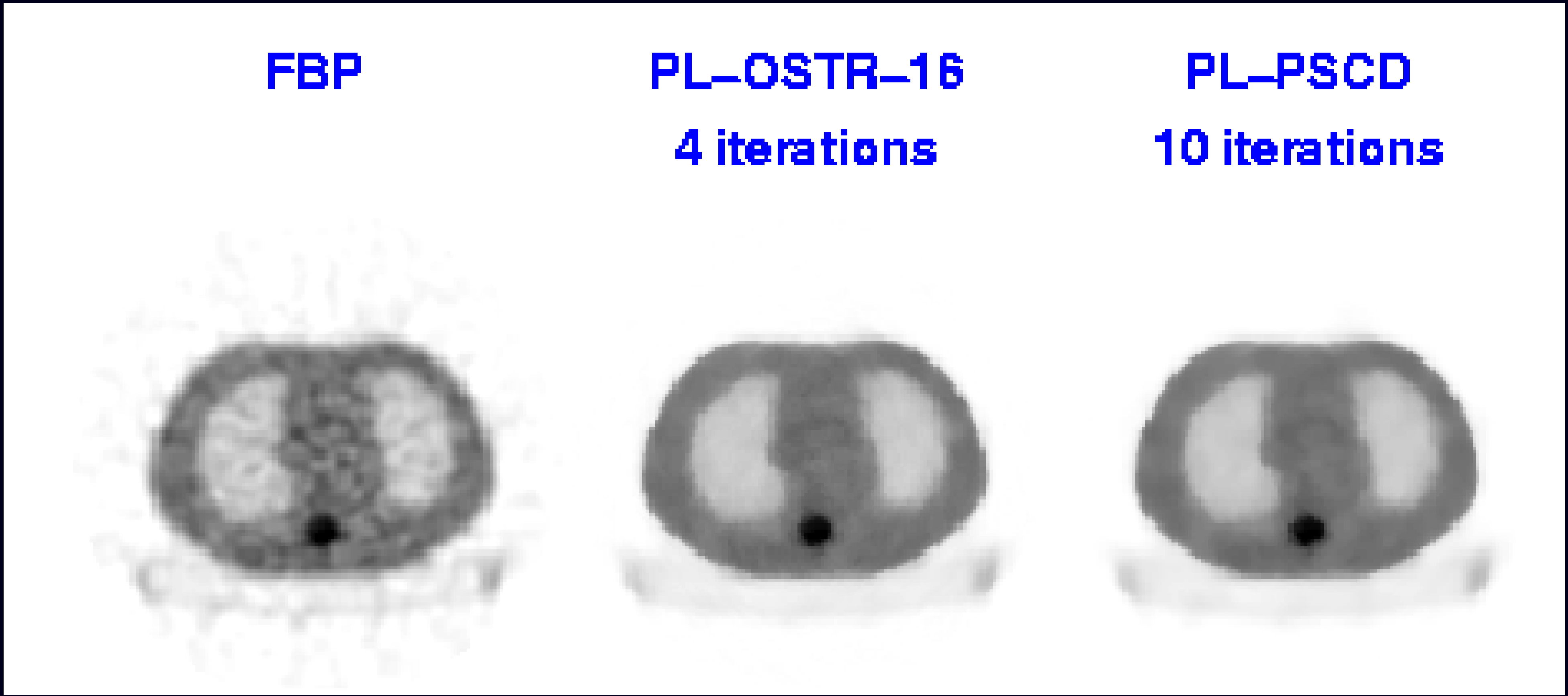
Paraboloidal Surrogate Iterations

(Hakan Erdogan)
IEEE T-MI, Sep. 1999, PMB Nov. 1999

- Replace complicated minimization problem with sequence of easier ones
- Fast: monotonic version (globally convergent for stricly convex cost functions)
- Faster: “usually monotonic” version (precomputed diagonal preconditioner)
- Fastest: ordered-subsets version (not monotonic)



PET Transmission Scan



- When monotonicity is desired, use paraboloidal surrogates coordinate descent
- Otherwise, use ordered-subsets separable paraboloidal surrogates
- Benefits of penalized-likelihood attenuation map reconstruction
  - Proper transmission scan statistical model (no logarithm!)
  - Better attenuation maps than FBP (or OSEM)
  - Preferable even if segmentation used

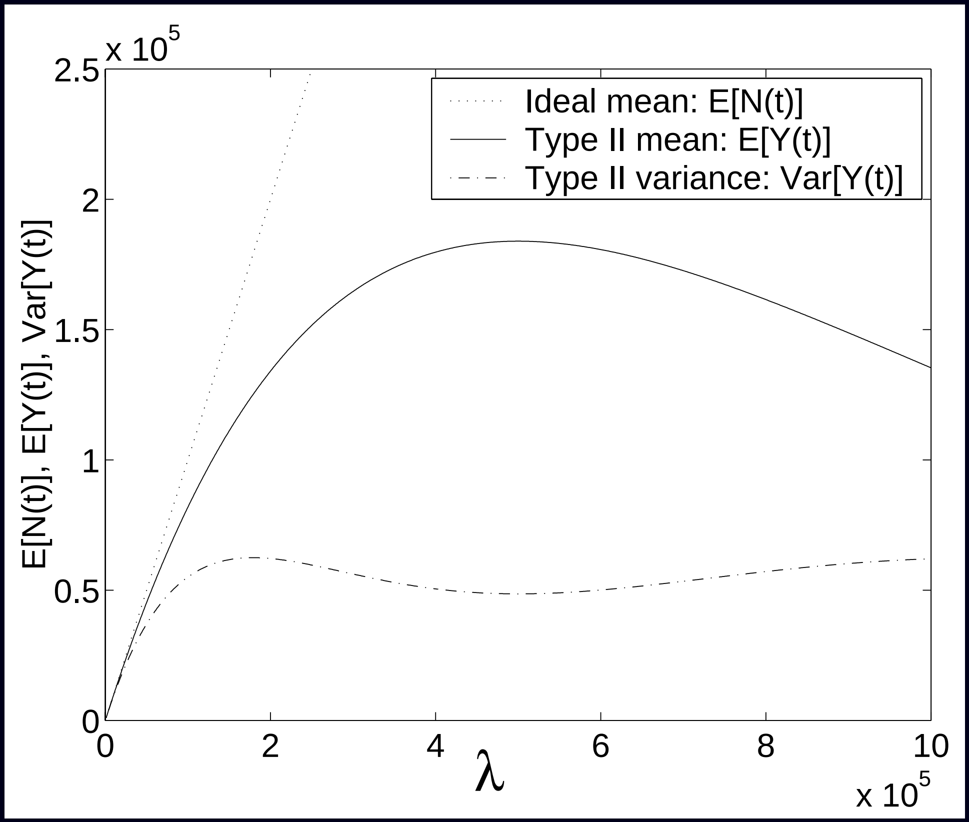
Paralyzable Deadtime Statistics

(Dan Yu)
PMB Jul. 2000, NIM 2002

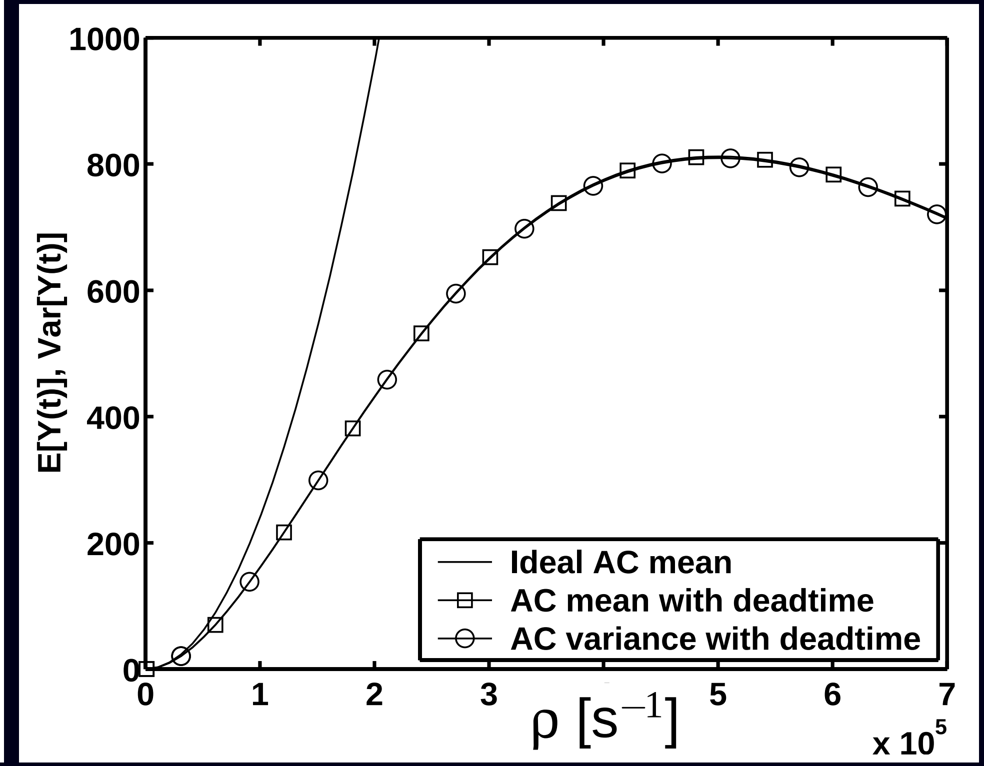
- Deadtime introduces statistical dependencies, $\therefore$ non-Poisson measurements
- Nevertheless, mean $\approx$ variance for
  - singles counting with “block” detectors (*e.g.*, Anger camera)
  - coincidence counting
- Effect of deadtime should be included in system matrix (not precorrected)

Paralyzable Deadtime Analysis

Singles Counting



Coincidence Counting



Resolution Properties and Regularization

(J. Webster Stayman)
TMI June 2000

Local impulse response:

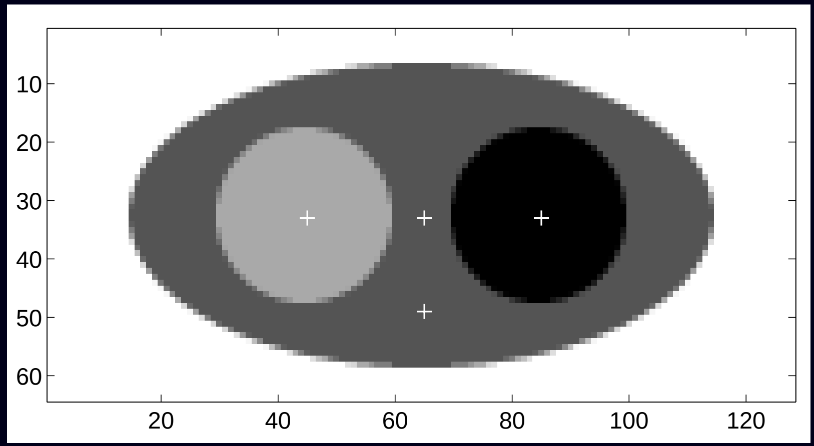
$$\boldsymbol{l}_j \approx [\boldsymbol{G}'\boldsymbol{D}\boldsymbol{G} + \beta \boldsymbol{R}]^{-1} \boldsymbol{G}'\boldsymbol{D}\boldsymbol{G}\boldsymbol{e}_j,$$

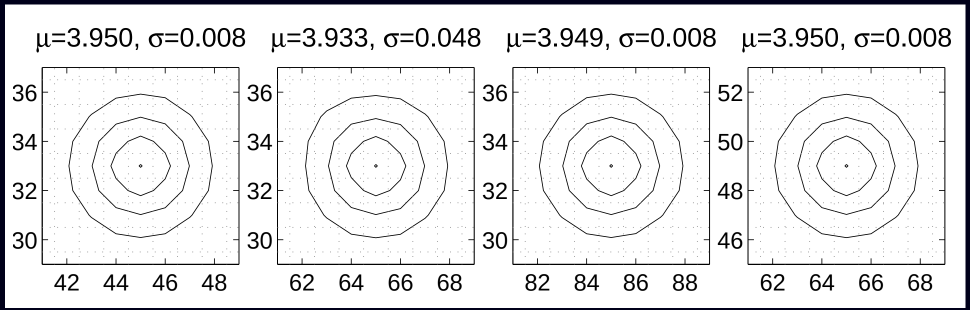
where diagonal $\boldsymbol{D}$ depends on statistical model and data.

For shift-invariant PET systems:

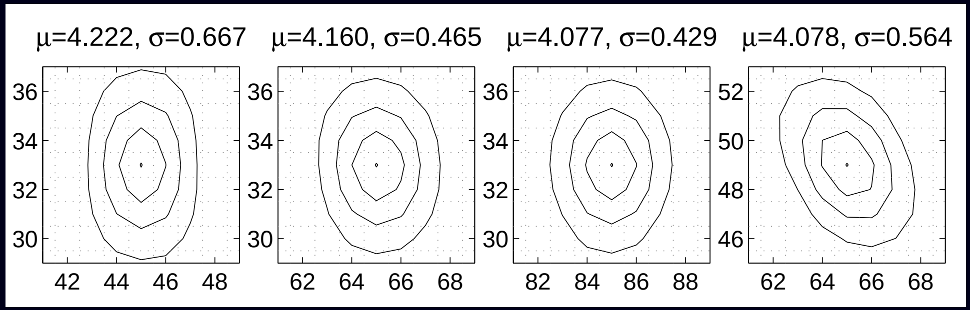
- FBP yields uniform spatial resolution (but higher noise)
- Conventional penalized-likelihood (or PWLS) yields non-uniform & anisotropic spatial resolution
- Modified (data-dependent) regularization ($\boldsymbol{R}$) yields nearly uniform & isotropic resolution

Example PSF Profiles (2D shift-invariant PET)

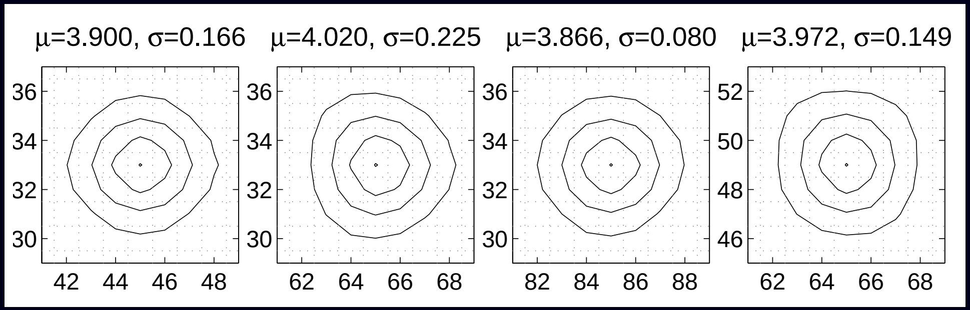




FBP Reconstruction

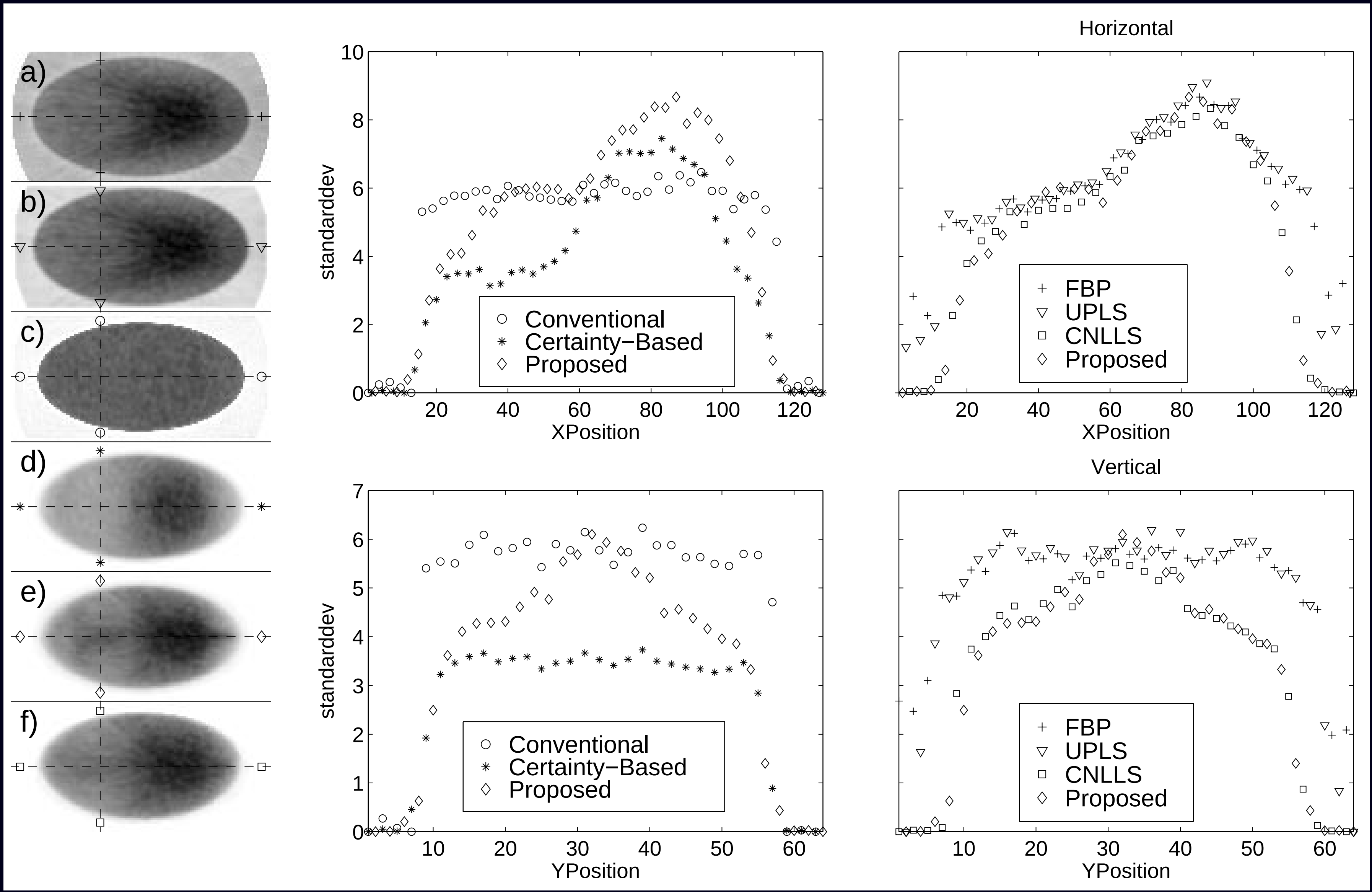


Conventional Regularization



Modified Regularization

Effects on Noise



a) Filtered backprojection (+),
b) Penalized unweighted least-squares PULS (▽),
c) PLE with conventional regularization (○),
d) PLE with certainty-based penalty (∗),
e) PLE with proposed penalty (◊), and
f) PLE with CNLLS penalty (□).

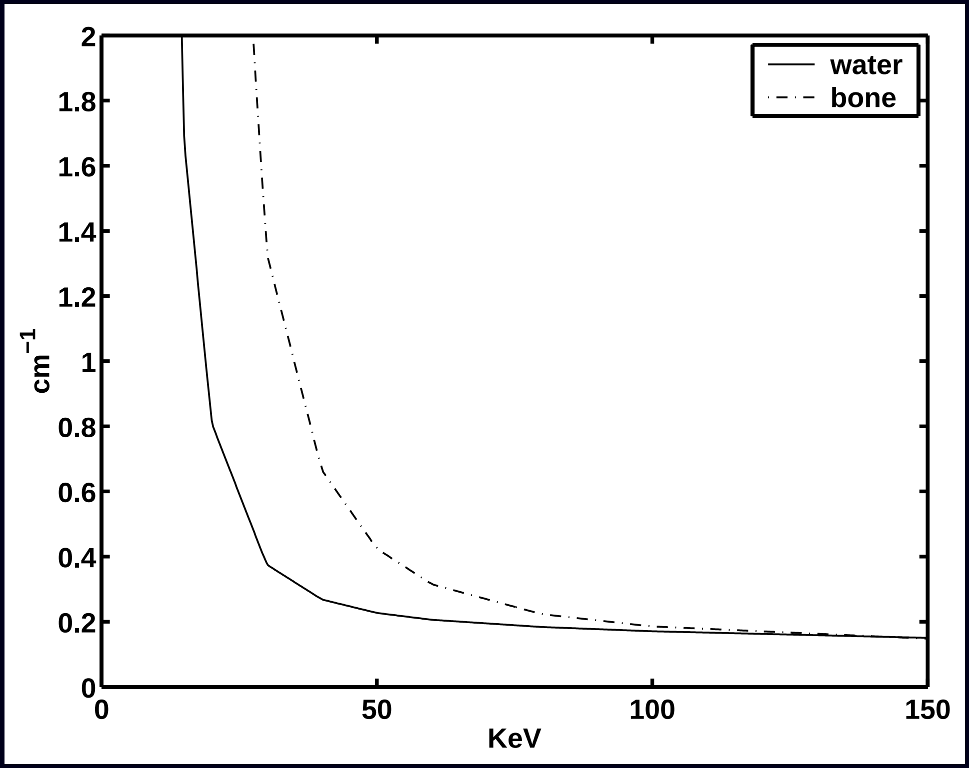
Current Regularization Work

- Highly shift-variant systems
  - 3D SPECT with depth-dependent detector response
  - Animal PET systems (large FOV relative to ring diameter)
- Faster design of regularizer
- Comparisons with post-filter maximum-likelihood estimates
- Hybrid penalized-likelihood / small post-filtering approaches
  - Retain convergence speed and stability of regularized methods over ML
  - Improve resolution uniformity by modest post-filtering

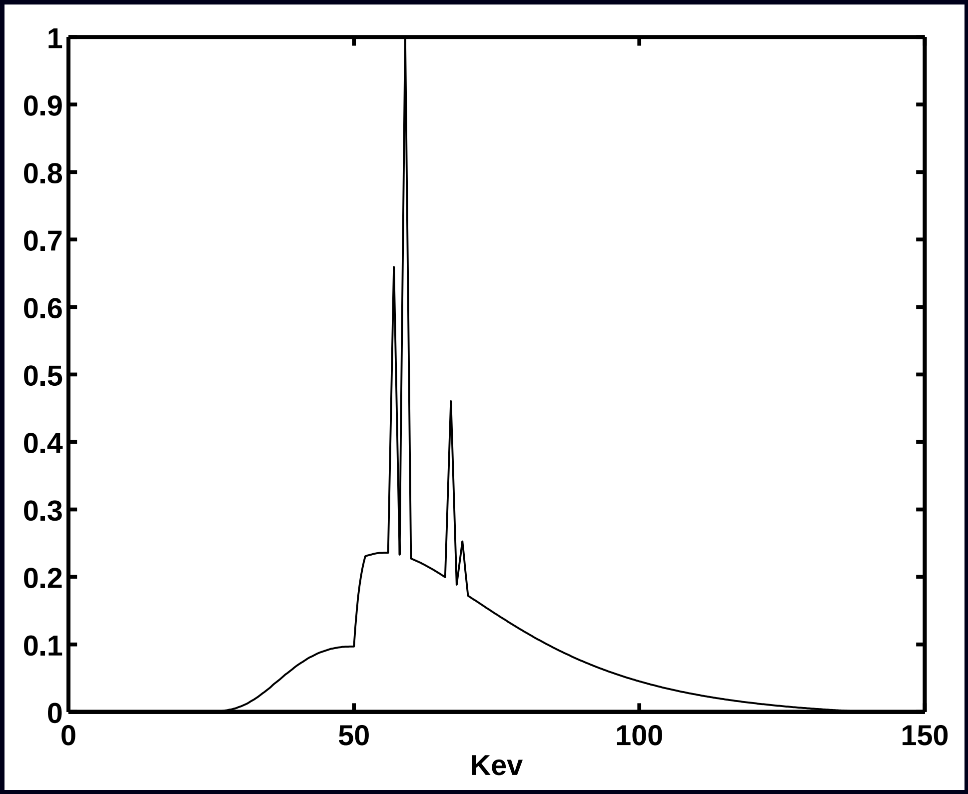
X-ray CT Image Reconstruction

(Idris Elbakri)
T-MI Feb. 2002

Energy-dependent attenuation

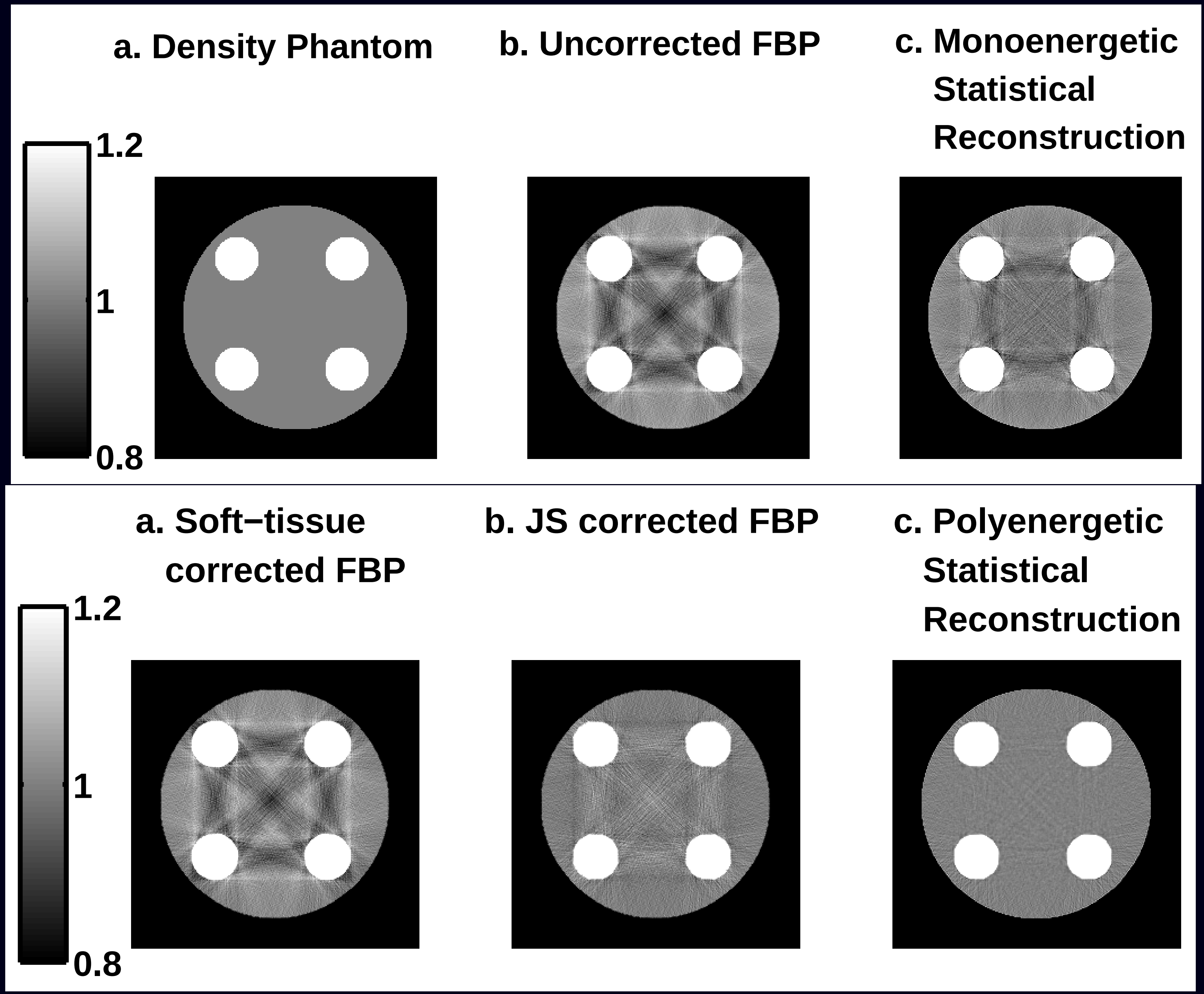


Polyenergetic source spectrum

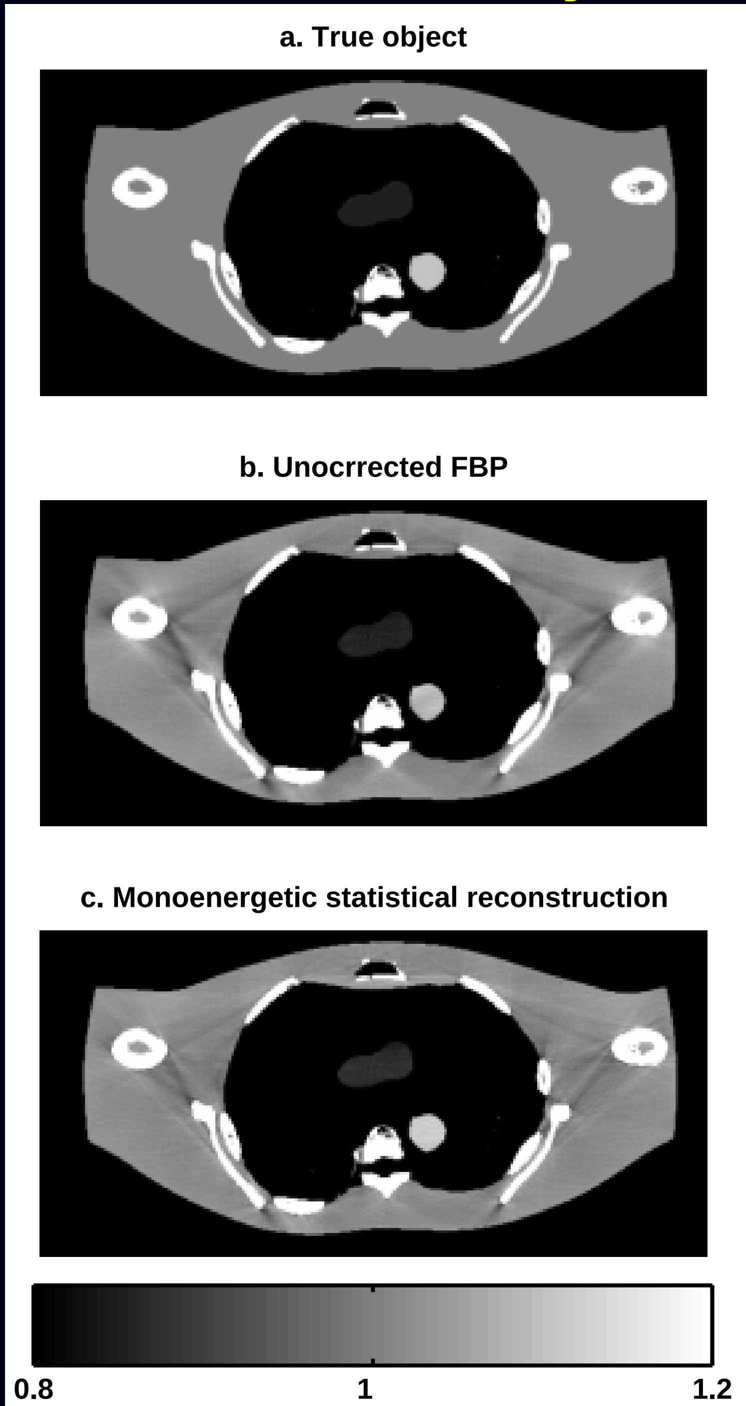


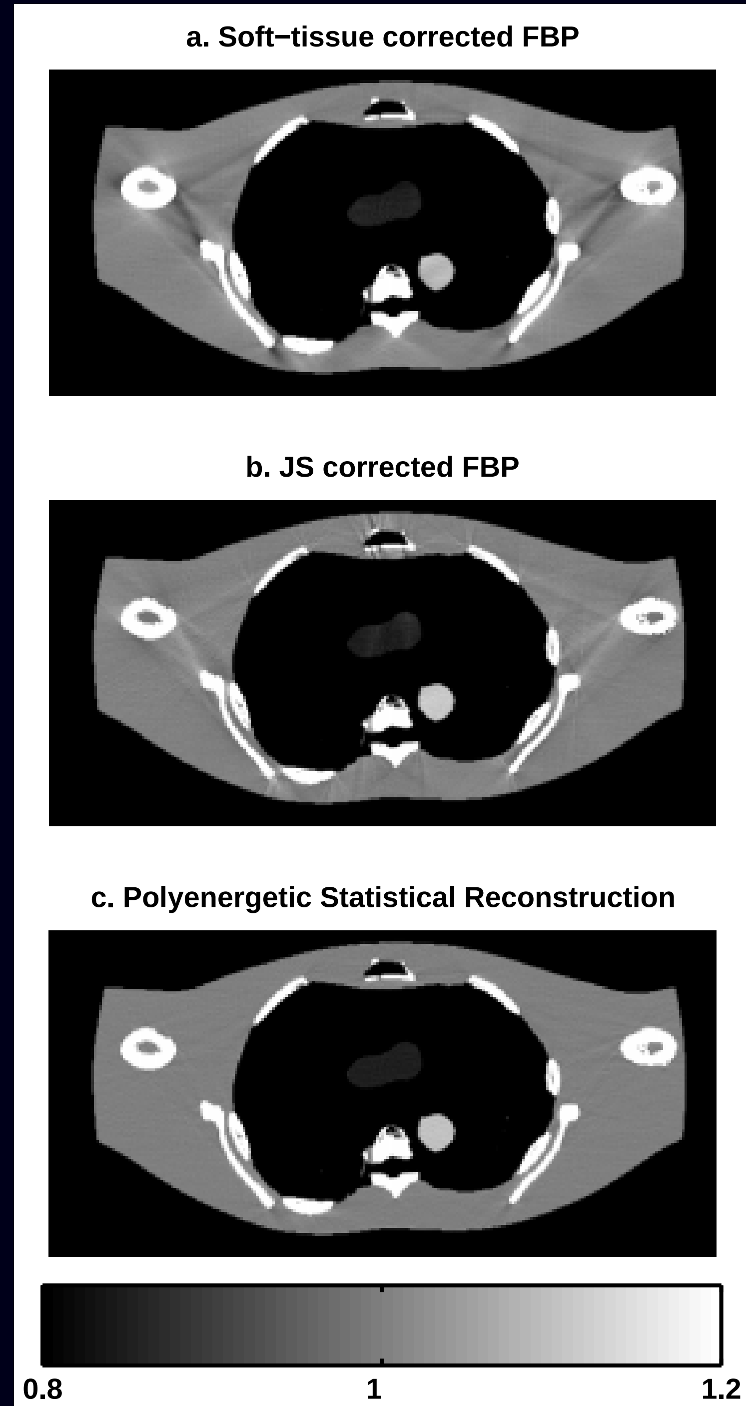
- Beam-hardening artifacts
- FBP is dose inefficient
- Additional motivation: PET attenuation correction in PET-CT

X-ray CT Disk Phantom Simulation



X-ray CT Thorax Simulation





Convergent Ordered-Subsets Algorithms

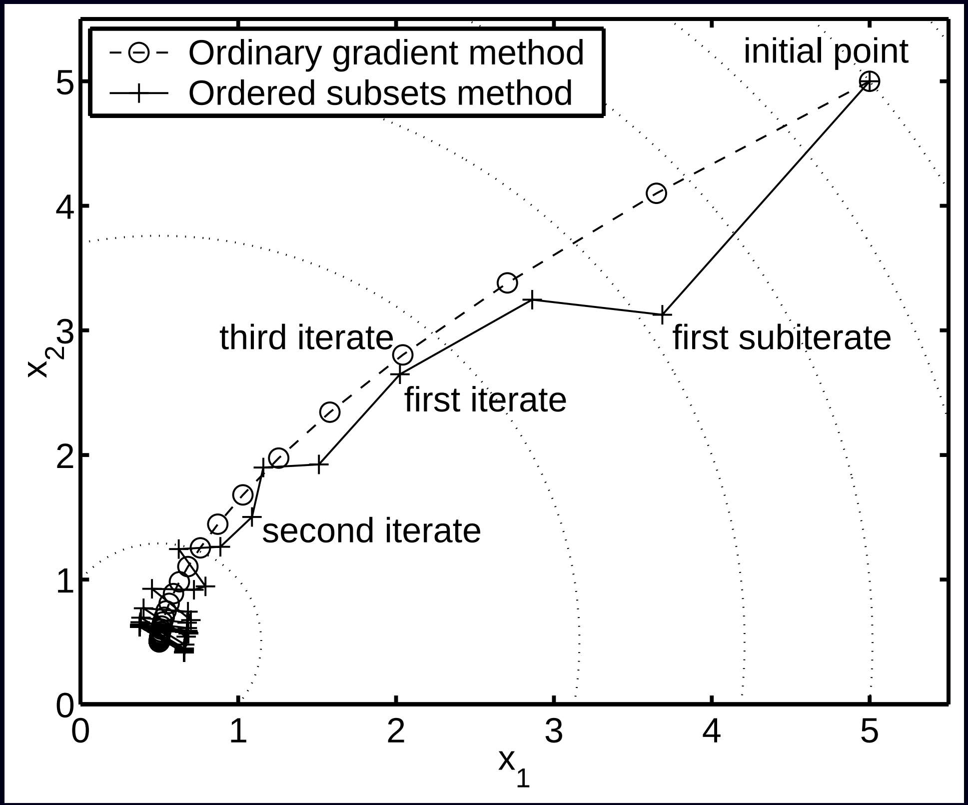
(Sangtae Ahn)
IEEE T-MI, Submitted 2002

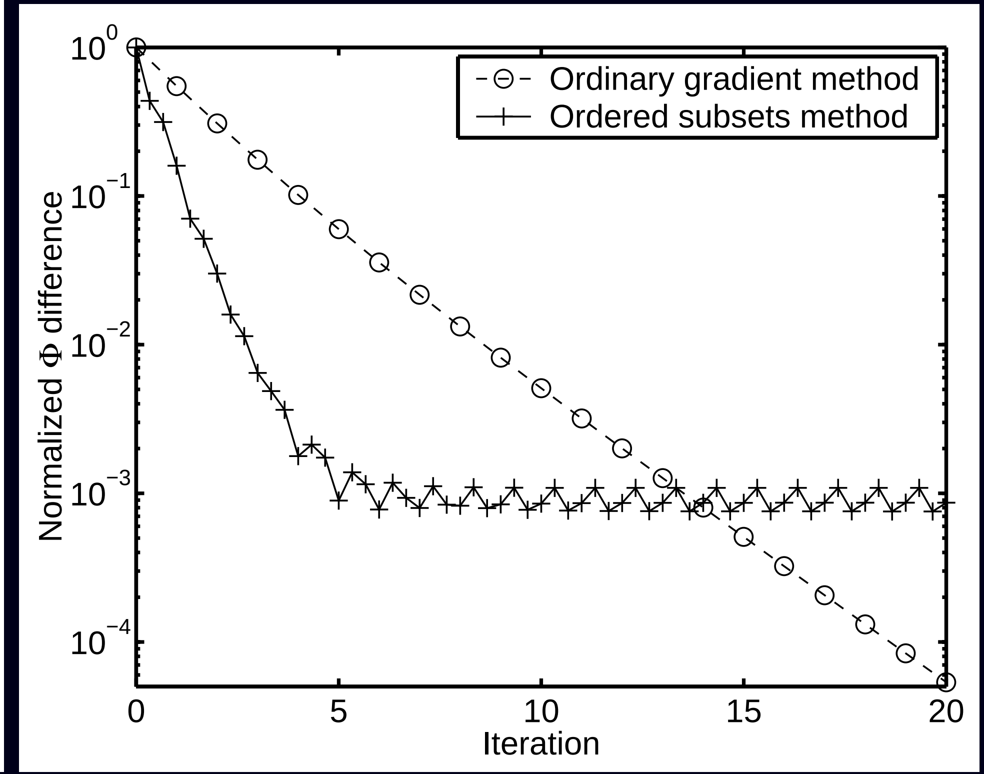
- Ordinary OS methods are not convergent
- Convergence is desired for regularized reconstruction methods
- Relaxation can guarantee convergence if
  - appropriate relaxation schedule is used: $\alpha_n \to 0$, $\sum \alpha_n = \infty$, and
  - appropriate (subiteration-independent!) diagonal scaling matrix is used.
- Two versions:
  - Multiplicative form, modified version of De Pierro's BSREM
  - Additive form, modified version of separable paraboloidal surrogates

$$\boldsymbol{x}^{(n+(m+1)/M)} = \boldsymbol{x}^{(n+m/M)} - \alpha_n \boldsymbol{D}\left(\boldsymbol{x}^{(n+m/M)}\right) \nabla\Psi_m\left(\boldsymbol{x}^{(n+m/M)}\right), \qquad m = 0, \ldots, M-1$$

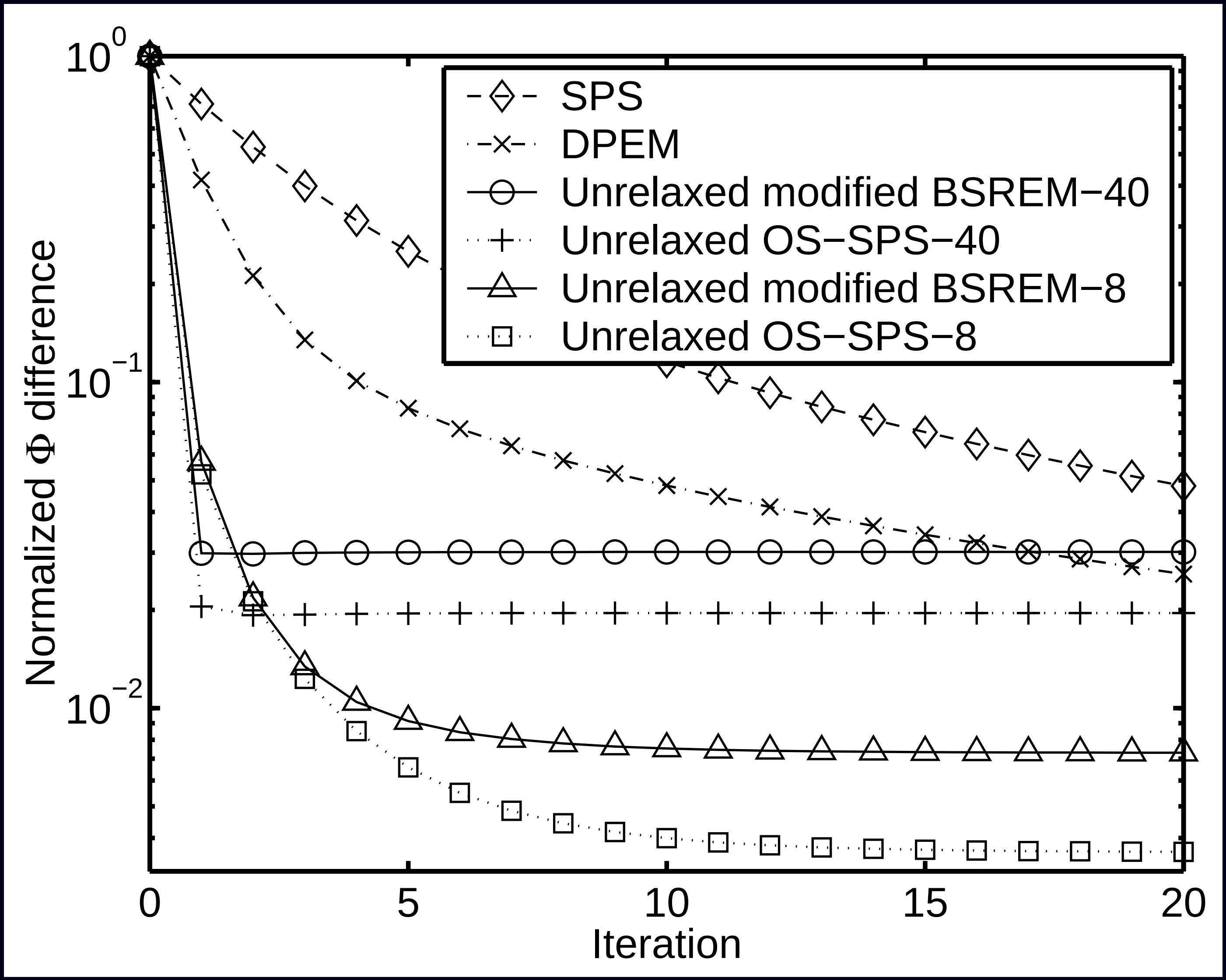
where $\Psi(\boldsymbol{x}) = \sum_{m=0}^{M-1} \Psi_m(\boldsymbol{x})$ decomposes cost-function into subsets.

OS Non-Convergence



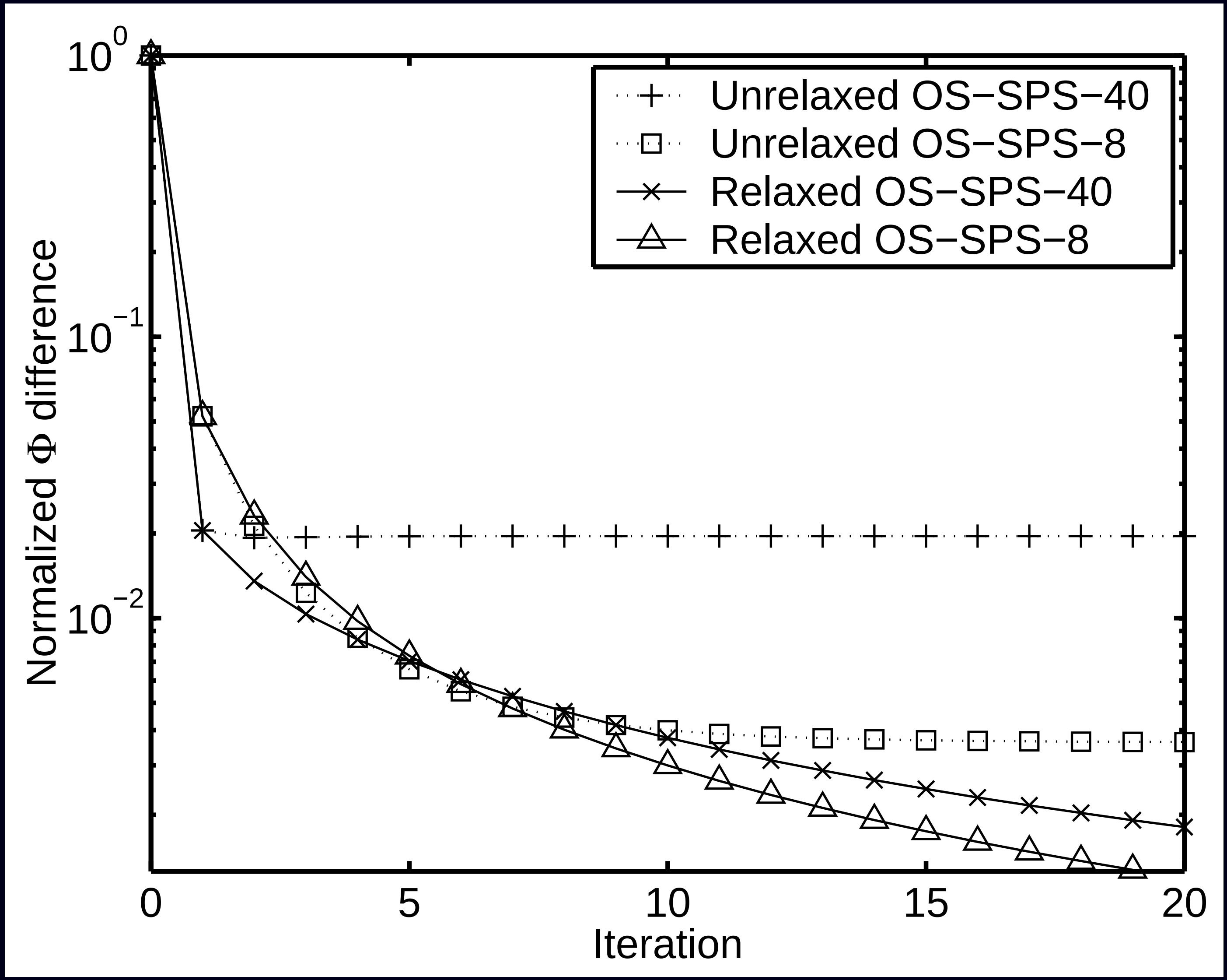


Non-relaxed OS Example



- More subsets $\Rightarrow$ faster initial convergence but faster stagnation
- 2D SPECT system with attenuation and depth-dependent detector response

Relaxed OS Results



MRI Reconstruction with Field Inhomogeneity

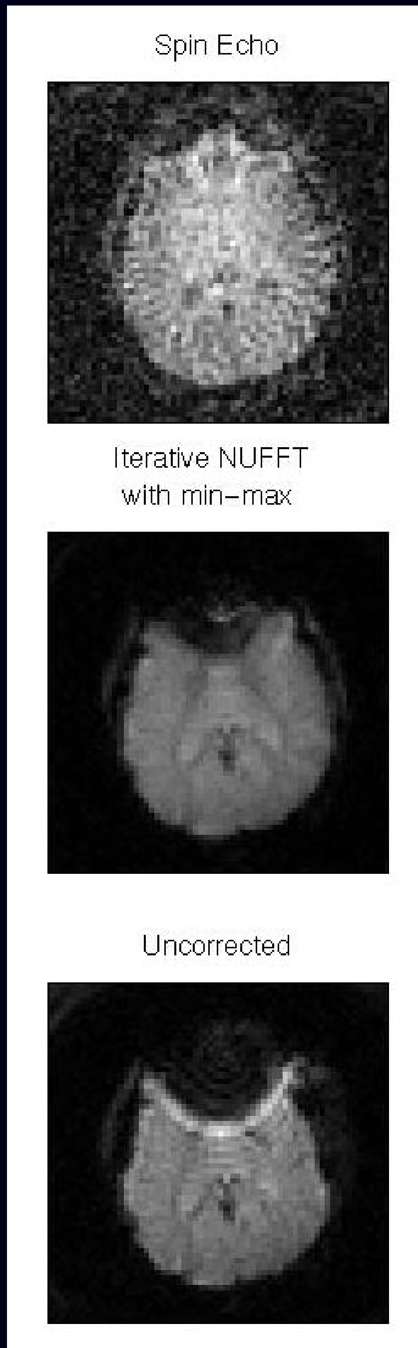
(Brad Sutton)

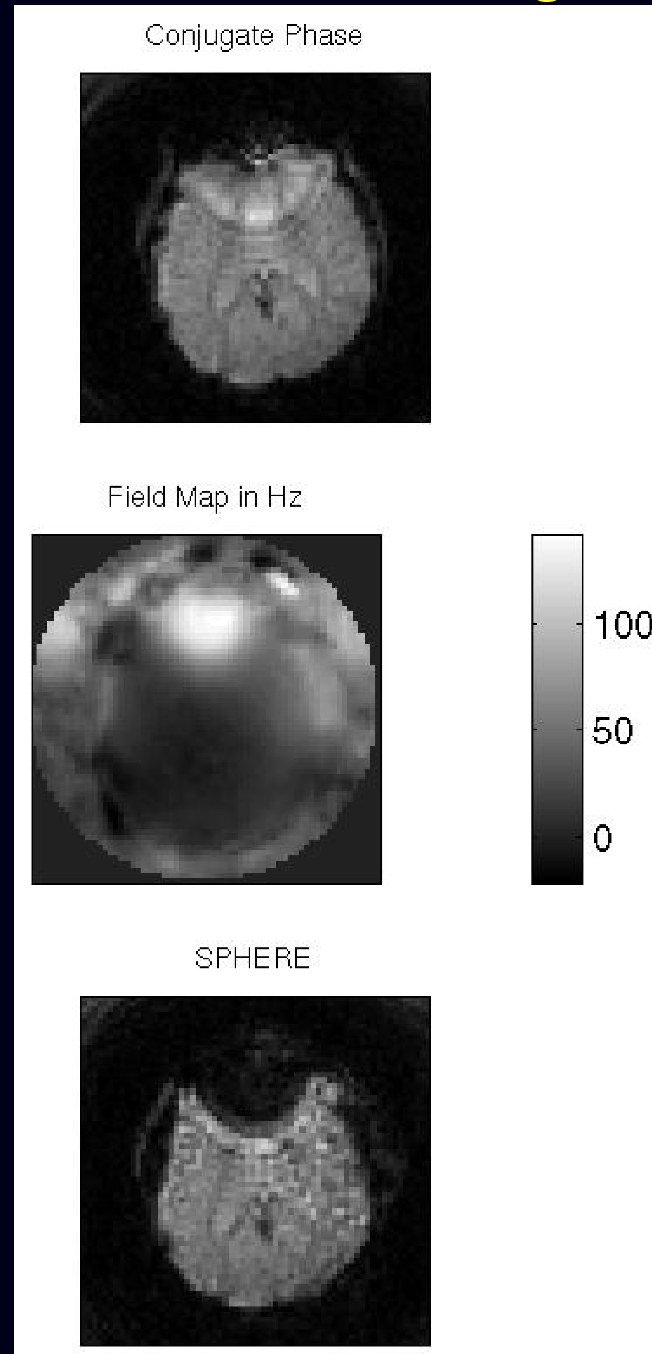
MR signal equation:

$$s(t) = \int f(\vec{x}) \exp(-\imath \omega(\vec{x}) t) \exp(-\imath 2\pi k(\vec{x}) \cdot \vec{x}) \, d\vec{x}$$

- Due to field inhomogeneity, signal is *not* Fourier transform of object.
- Measure off-resonance field-map $\omega(\vec{x})$ using two displaced echos
- Penalized WLS cost function minimized by conjugate gradient
- System matrix $\boldsymbol{A}$ includes off-resonance effects
- Fast algorithm using NUFFT and time-segmentation

Iterative MRI Reconstruction with Field Inhomogeneity





Other Ongoing Topics

- Confocal microscopy 3D image restoration
- Volume-to-projection registration for radiotherapy treatment positioning
- 3D SPECT reconstruction for I-131 imaging with high-energy collimators
- Signal detection methods for direct brain-computer interfaces
- Dual-energy X-ray CT image reconstruction
- PET-CT image reconstruction
- Design of regularization methods to optimize detectability
- ...