Statistical Image Reconstruction with Sample-Based Beam-Hardening compensation for X-ray CT

C. Martinez, J. A. Fessler, M. Desco, and M. Abella

Abstract— **CT images are often affected by beam-hardening artifacts due to the polychromatic nature of the X-ray spectra. These artifacts appear in the image as cupping in homogeneous areas and streaks between dense parts in heterogeneous samples.**

This paper proposes a new statistical reconstruction method for X-ray CT based on Poisson statistics, taking into account the non-linearities caused by beam hardening. To avoid needing knowledge of the X-ray spectrum, the method obtains the 2D beam-hardening function using information provided by the acquired data itself.

Evaluation using simulations showed beam hardening artifact reductions similar to those achieved with conventional post-processing techniques while avoiding noise and artifacts in low-dose studies.

Index Terms—**Beam-hardening, CT, artifacts, penalized-likelihood, streaks, polychromatic.**

INTRODUCTION

The beam hardening effect in computed tomography derives from the polychromatic nature of the radiation produced by X-ray tubes. Due to the energy dependence of mass attenuation coefficients, low energy photons are preferably absorbed, causing a shift of the mean energy of the X-ray beam to higher values. This effect leads to two main artifacts in uncorrected reconstructed images: cupping in homogeneous regions and streaks between dense areas in heterogeneous regions [1].

Several strategies can be found in the literature to compensate for this effect. Physical filters are generally used to pre-harden the beam before reaching the sample, but this is not enough to remove the artifacts. Another method implemented in most commercial scanners is the water-linearization, based on a prior calibration with a water-equivalent phantom. This method models the object as homogeneous and corrects only cupping artifact [2]. To correct also streaks, Nalcioglu et al. [3] proposed a method that requires knowledge of the spectrum, the linear attenuation coefficients and the thickness of soft tissue and bone traversed estimated by means of a preliminary reconstruction. Joseph et al. [4] proposed a similar idea modeling the corrected data with a second-order polynomial dependent on the bone traversed thickness. However, the optimum parameters for this model could be exactly obtained only with a complete characterization of the spectrum. This need of knowledge of the spectra was avoided in [5, 6], also based on a linear combination of basis images to correct streaks. The coefficients of this linear combination are obtained iteratively maximizing the flatness of the soft tissue areas, which could reduce the soft-tissue contrast. Cupping correction is achieved using the water-linearization method, which needs a calibration step. We recently proposed two new methods extending the water-linearization to a 2D linearization [7, 8]. However, they require a good bone and soft tissue segmentation which may hinder their use in low-dose studies.

To deal with low-dose studies, Elbrakri et al. presented a statistical method that requires knowledge of the spectrum [9, 10]. This is avoided in [11, 12] with a simplified statistical algorithm that parametrizes the beam-hardening function following the model proposed by Joseph and Spital [4].

This paper presents a variation of [12] that replaces the approximation functions with the real measured line integrals of bone and soft tissue of the sample as proposed in [8].

MATERIALS AND METHODS

A. Forward model

We model the measurements as independently distributed Poisson random variables [13] contaminated by extra background counts, primarily scatter:

$$Y_i \sim Poisson\left\{\overline{Y_i}\right\}, i = 1, \dots, N \tag{1}$$

with

$$\overline{Y_i} = \int I_i(\varepsilon)e^{-\int_{L_i} \mu(\varepsilon)dl}d\varepsilon + r_i \tag{2}$$

where $\mu(\varepsilon)$ is the attenuation coefficient at each energy ε, the integral in the exponent is taken over the line L_i

followed by the ray, and $I_i(\varepsilon)$ is the incident intensity and the term r_i accounts for mean scatter and mean other background signals for the i-th ray.

Following [4, 14] we model the attenuation coefficient in Eq. 2 at each pixel j as:

$$\mu_j(\varepsilon) = \sum_{k=1}^{K} mac_k(\varepsilon) f_k^j \rho_j \qquad (3)$$

where mac_k is the mass attenuation coefficient of the material k, ρ the density and f_k^j is a unitless fraction that describes the contribution of the material k to attenuation in the pixel j. We assume $K=2$, i.e., the object contains only soft tissue (ST) and bone (B). The contribution of each tissue type to the line integral along the i-th ray is:

$$t_{ST}(\rho) = \sum_{j=1}^{p} a_{ij} f_{ST}^j(\rho_j) \rho_j \qquad (4)$$

$$t_B(\rho) = \sum_{j=1}^{p} a_{ij} f_B^j(\rho_j) \rho_j \qquad (5)$$

where a_{ij} are the elements of the system matrix. Here we allow the unitless fraction (f_k^j) to only be 1 or 0, i.e., the pixels do not contain mixtures of tissues. Eq. 2 for the expected value of the measured data along the path i results in:

$$\overline{Y}_i(\rho) = \int I_i(\varepsilon) e^{-\int_{L_i} \mu(\varepsilon) dl} d\varepsilon + r_i =$$
$$= I_i e^{-F\left(t_{ST}^i(\rho), t_B^i(\rho)\right)} + r_i \qquad (6)$$

where

$$I_i \equiv \int I_i(\varepsilon) d\varepsilon \qquad (7)$$

and the beam-hardening function, F, is:

$$F(t_{ST}, t_B) = -\log \int \frac{I(\varepsilon)}{I} e^{-mac_{ST}(\varepsilon) t_{ST} - mac_B(\varepsilon) t_B} \, d\varepsilon \qquad (8)$$

dropping the dependence on ray i for simplicity.

B. Beam-Hardening function

The beam-hardening function $F(t_{ST}, t_B)$ could be analytically calculated from a known spectrum, but often this information is not available. To avoid assuming spectrum knowledge, the proposed method determines $F(t_{ST}, t_B)$ experimentally using the acquired data following the process outlined in Fig. 1.
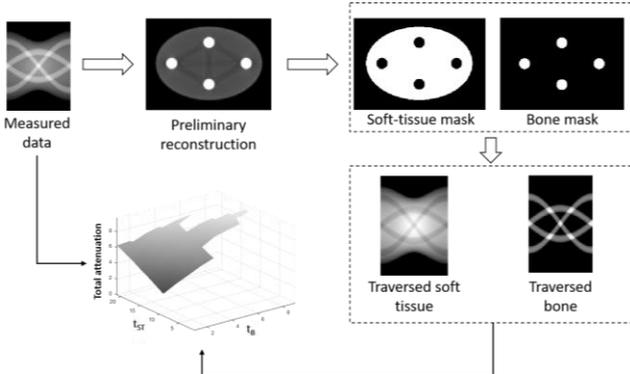

Fig. 1. Workflow for the generation of the beam-hardening function.

First bone and soft-tissue masks are obtained by thresholding a preliminary reconstructed image. These two masks are then multiplied by the density of each tissue and projected, which will be the x and y axis, where x corresponds to t_{ST} and y corresponds to t_B. The value in the original projection will be the z axis, corresponding to $F(t_{ST}, t_B)$.

The generated $F(t_{ST}, t_B)$ will not cover the whole space, since a specific acquisition will not have all possible combinations of soft tissue and bone (Fig. 2, left). To completely characterize the beam-hardening function, we "extrapolate" the incomplete function F using a quadratic function (Fig. 2, right).
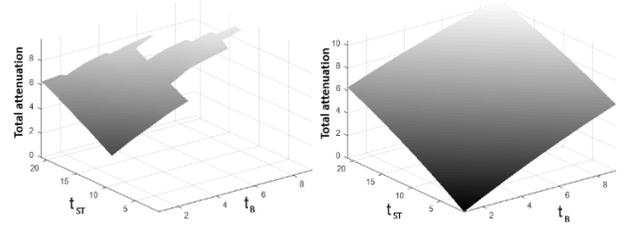

Fig. 2. Measured (left) and extrapolated (right) beam-hardening function.

C. Cost function

The negative log-likelihood for independent Poisson measurement is:

$$L(\rho) = -\sum_{i=1}^{N} h_i\left(F\left(t_{ST}(\rho), t_B(\rho)\right)\right) \qquad (9)$$

where

$$h_i(d) = -Y_i \log(I_i e^{-d} + r_i) + I_i e^{-d} + r_i \qquad (10)$$

Since minimizing $L(\rho)$ is generally an ill-posed problem, regularization is included by adding a penalty term to control how much the object ρ departs from our assumptions about image properties. In this work, we use a 3D roughness penalty function with the convex edge-preserving Huber potential. The resulting penalized cost function is:

$$\Phi(\rho) = L(\rho) + \beta R(\rho) \qquad (11)$$

where β is a scalar that controls the tradeoff between the data-fit and penalty terms.

D. Algorithm

We derive an iterative algorithm based on separable quadratic surrogates using the principles of optimization transfer [14], resulting in the following update:

$$\rho^{n+1} = \rho^n - D^{-1} \nabla \Phi(\rho^n) \qquad (12)$$

where D is a diagonal matrix that influences the rate of convergence. We originally designed D to ensure that the algorithm monotonically decreases the cost function. As in [9], in practice we choose the elements of D approximately by using the precomputed curvature:

$$d_j = \left(mac_{ST}^2(\varepsilon_{eff}) + mac_B^2(\varepsilon_{eff})\right) \sum_{i=1}^{N} a_{ij} \sum_j a_{ij} Y_i \qquad (13)$$

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where the effective $mac(\varepsilon_{eff})$ values for each tissue are approximated using the derivative of beam-hardening function at (0,0).

Artifacts in the preliminary reconstruction may hinder the segmentation to obtain the bone and soft-tissue masks, resulting in an erroneous beam-hardening function. To tackle this problem, new bone and soft-tissue masks are obtained from the solution at each iteration and are used to recalculate the beam-hardening function.

EVALUATION

Preliminary evaluation used simulations of a 2D phantom with two cortical bone inserts having density 1.9 gr/cm^3, one trabecular bone insert with density 1.5 gr/cm^3 and one adipose-tissue insert with density 0.9 gr/cm^3, inside of a soft-tissue ellipse with density 1.06 gr/cm^3 (Fig. 3).
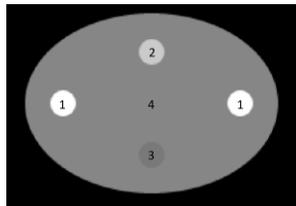

Fig. 3. Test phantom with two cortical bone inserts (1), one trabecular bone insert (2), and one adipose-tissue insert (3) inside of a soft-tissue ellipse (4).

Four polyenergetic X-ray datasets were generated using MIRT*(http://www.eecs.umich.edu/~fessler/code/index.html)* with a 50 kVp spectrum and 0.1 mm aluminum filtration, typically used in preclinical studies. The number of counts per ray were 10^5 and 10^6 to simulate low-SNR and high-SNR scenarios respectively. For each scenario, we obtained 45 and 180 projections in a span of 180 degrees with a matrix size of 256×256 pixels and 0.1×0.1 mm pixel size.

The data were reconstructed with FBP, with FBP corrected by the free calibration method (fCM) proposed in [8] and by the proposed statistical algorithm.

RESULTS

Fig. 4 shows the segmented soft-tissue mask along different iterations for the low-SNR scenario with 45 projections. The mask in the first iteration has holes due to the streaks from beam-hardening effect and low sampling, which are removed in subsequent iterations.

Fig. 4. Soft tissue mask in iteration 1 (left), 5 (center) and 15 (right).

Fig. 5 shows the results for 180 projections. The result of FBP corrected with fCM shows a good compensation of beam-hardening artifacts but it fails to correct streaks with high noise (low SNR scenario), while the proposed method results in good quality reconstructions in high-SNR and low-SNR scenarios.

When the number of projection is reduced to 45 (Fig. 6) FBP+fCM eliminates the beam hardening artifacts in the high-SNR scenario, but the reconstruction is contaminated with streaks due to the lack of projections. For the low-SNR scenario, FBP+fCM also fails to compensate the streaks due to beam-hardening effect, because of the wrong segmentation used to create the beam-hardening function. The proposed method eliminates the beam hardening and the streaks in all cases.
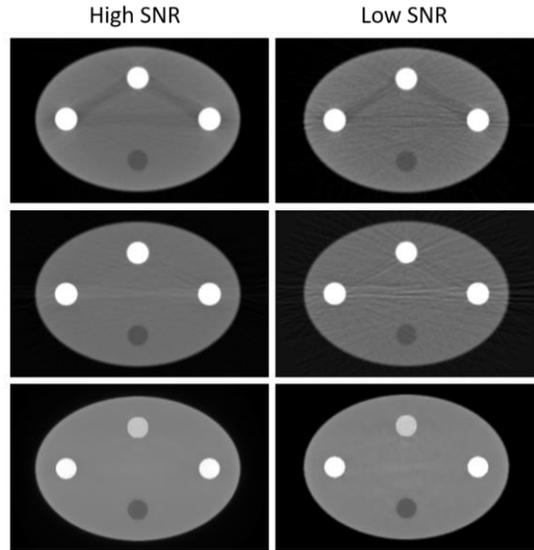

Fig. 5. Results for the 180 projections datasets using FBP (top), FBP + fCM (center) and the proposed algorithm (bottom).
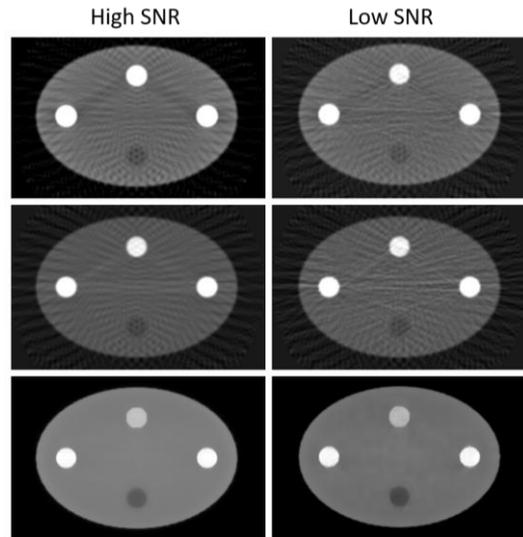

Fig. 6. Results for the 45 projections datasets using FBP (top), FBP +fCM (center) and the proposed algorithm (bottom).

CONCLUSIONS

We present a new statistical reconstruction algorithm that includes beam-hardening correction without needing any spectrum knowledge or correction parameters optimization.

The method models the polychromatic effect via a beam-hardening function determined from the acquired data and a segmentation of bone and soft-tissue masks, which are

iteratively improved. Results on simulated data show a reduction of streaks due to both beam-hardening effect and low number of projections.

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