ONLINE DYNAMIC MRI RECONSTRUCTION VIA ROBUST SUBSPACE TRACKING

Greg Ongie, Saket Dewangan, Jeffrey A. Fessler, Laura Balzano

University of Michigan, EECS Department, Ann Arbor, MI, USA

ABSTRACT

We propose an efficient online reconstruction algorithm for the problem of highly undersampled dynamic magnetic resonance imaging (DMRI). Our approach reconstructs the dynamic time series by processing only a small batch of frames at a time. We adapt an online subspace tracking algorithm based on manifold optimization to the DMRI reconstruction setting and propose a novel extension of the algorithm to enable robust subspace tracking based on a local low-rank plus transform sparse model. Our experiments on real and synthetic data show that proposed approach gives comparable results to methods that reconstruct the entire image series at once while requiring only a fraction of the memory and computational demand. The dramatic memory savings allows robust subspace-based methods to be applied to much larger datasets than previously allowed.

***Index Terms*—** Online algorithms, dynamic MRI reconstruction, Grassmannian optimization, robust PCA, low-rank plus sparse.

1. INTRODUCTION

Low-rank models have been used extensively for reconstructing undersampled dynamic magnetic resonance imaging (DMRI) datasets [7, 9]. In these models, each vectorized temporal image frame is treated as a column of an approximately low-rank space-time matrix, where the low-rank property arises due to spatio-temporal correlations. These models have been extended by a variety of works to include additional priors on the low-rank factors, *e.g.*, the sparse and low-rank model of [10]. Recently, Otazo *et al.* [16] applied a low-rank plus sparse (L+S) model to DMRI reconstruction using a robust principal component analysis (RPCA) approach [2]. Different from traditional RPCA [2], in [16] the sparse component is modeled as *transform sparse*.

However, fitting a single low-rank or low-rank plus sparse model to a global DMRI dataset has several limitations. First, for large datasets, iterative reconstruction of the entire time series can be time consuming or even impossible due to excessive memory demand. Furthermore, a global low-rank model cannot efficiently accommodate changes that may occur in the low-rank components over time, *e.g.*, because of patient motion, physiological noise, or field inhomogeneities.

An online reconstruction method, *i.e.*, one reconstructs the data in a sequential or streaming fashion, has the potential to overcome these limitations. Rather than reconstructing the entire dataset at once, an online method reconstructs the data frame-by-frame, or in sequential batches of frames, so that only a fraction of the dataset needs to be stored in working memory at any given time. This can yield substantial reductions in computational and memory demand. An online approach also has benefits over independently reconstructing batches of the data, since implicitly an online approach maintains a model for the data (*e.g.*, a low-dimensional subspace), which allows it to share information from previous frames.

This paper adapts the Grassmannian rank one update subspace estimation (GROUSE) algorithm [1], originally proposed for online subspace tracking, to the problem of DMRI reconstruction. Our main contribution is a novel extension of GROUSE that incorporates batch processing of several frames under a low-rank plus transform sparse model. In particular, using tools from [6] we generalize the GROUSE subspace update step to batch sizes greater than one. The resulting framework gives a flexible, online, memory-efficient method for reconstructing compressively sampled DMRI datasets. Our experiments on real and simulated DMRI datasets demonstrate that the proposed online approach gives comparable results to state-of-the-art algorithms that fit a global low-rank plus sparse model.

A previous work [8] proposed a modification of GROUSE for robust subspace estimation with applications to foreground-background separation in compressed surveillance video streams. However, directly adapting the algorithm [8] to a DMRI compressed sensing setting is computationally prohibitive, since it requires solving a sequence of large-scale sparsity regularized inverse problems at each iteration. Furthermore, the algorithm in [8], which processes the data one frame at a time, is unable to model transform sparsity along the temporal dimension.

Several works have also investigated online reconstruction approaches for DMRI reconstruction. This includes approaches based on Kalman filtering combined with compressed sensing [12, 18], auto-regessive modeling combined with compressed sensing [11], sparisty under spatial and temporal finite differences [4, 5, 13], dictionary learning [15], and

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deep learning [14]. The present work differs from these in that it is based on a robust subspace tracking approach.

2. PROBLEM FORMULATION

2.1. Signal model

We model the DMRI time-series to be reconstructed $\boldsymbol{X} = [\boldsymbol{x}_1 \; \cdots \; \boldsymbol{x}_T]$ as being the sum of a component belonging to a r-dimensional subspace $\mathcal{S}$ and a (transform) sparse residual component:

$$\boldsymbol{x}_t = \boldsymbol{U}\boldsymbol{w}_t + \boldsymbol{s}_t, \;\text{ for all }\; t = 1, ..., T. \tag{1}$$

Here $\boldsymbol{U} \in \mathbb{C}^{n\times r}$ is a matrix representative of the subspace $\mathcal{S}$, *i.e.*, a matrix having orthonormal columns such that span$(\boldsymbol{U}) = \mathcal{S}$, $\boldsymbol{w}_t \in \mathbb{C}^r$ is a vector of weights at frame t, and $\boldsymbol{s}_t \in \mathbb{C}^n$ represents a sparse residual at frame t.

We also assume at each frame we receive $m_t < n$ compressive measurements $\boldsymbol{b}_t \in \mathbb{C}^{m_t}$ of the form

$$\boldsymbol{b}_t = \boldsymbol{A}_t\boldsymbol{x}_t + \boldsymbol{n}_t, \;\text{ for all }\; t = 1, ..., T, \tag{2}$$

where $\boldsymbol{A}_t : \mathbb{C}^n \to \mathbb{C}^{m_t}$, is a time-dependent linear measurement operator and $\boldsymbol{n}_t$ is a vector of additive noise, which we assume to be i.i.d. complex mean-zero white Gaussian of uniform variance across all time frames. In this work, we focus on the SENSE parallel MRI setting [17] where $\boldsymbol{A}_t = [\boldsymbol{A}_{t,1}^T, ..., \boldsymbol{A}_{t,C}^T]^T$ is the block column matrix whose ith block $\boldsymbol{A}_{t,i}$ is given by $\boldsymbol{A}_{t,i} = \boldsymbol{P}_{\Omega_t}\boldsymbol{F}\boldsymbol{D}_i$, for all $i = 1, ..., C$, where $\boldsymbol{D}_i \in \mathbb{C}^{n\times n}$ is a diagonal matrix representing multiplication by the known spatial sensitivity profile of the ith receive coil, $\boldsymbol{F}$ is a 2-D DFT matrix, and $\boldsymbol{P}_{\Omega_t}$ is projection onto the frame-dependent index set Ω_t and C is the number of coils.

2.2. Batch model for online reconstruction

One possible online reconstruction approach would be to recover the model (1) from the compressive measurements (2) by processing one frame at a time, as in [8]. However, in a DMRI context, the sparse component $\boldsymbol{s}_t$ is better modeled as transform sparse [16], where the transform occurs in the temporal dimension (*e.g.*, temporal finite differences).

To incorporate temporal transform sparsity into a robust online subspace tracking approach, we instead process a *batch* of measurements $\boldsymbol{B}_j = [\boldsymbol{b}_t]_{t\in\mathcal{I}_j}$ coming from temporally adjacent image frames $\boldsymbol{X}_j = [\boldsymbol{x}_t]_{t\in\mathcal{I}_j}$, where $\mathcal{I}_j$ is the index set corresponding to the jth batch, with $|\mathcal{I}_j| = T_j$. In particular, we model the jth batch of frames $\boldsymbol{X}_j$ as

$$\boldsymbol{X}_j = \boldsymbol{U}\boldsymbol{W}_j + \boldsymbol{S}_j \in \mathbb{C}^{n\times T_j}, \;\text{ for all }\; j = 1, ..., B, \tag{3}$$

where $\boldsymbol{W}_j \in \mathbb{C}^{r\times T_j}$ is a matrix of subspace weights for each frame in the batch, and we assume the component $\boldsymbol{S}_j = [\boldsymbol{s}_t]_{t\in\mathcal{I}_j}$ is *transform sparse*, *i.e.*, $\mathcal{T}(\boldsymbol{S}_j)$ is sparse, where $\mathcal{T}$ is some linear operator. This model can be viewed as a local version of the low-rank plus sparse model investigated in [16]. In this work we focus on the case where $\mathcal{T}$ is a finite differencing in the temporal dimension over a batch of frames, *i.e.,* $\mathcal{T}(\boldsymbol{S}_j)$ is a concatenation of the vectors $\boldsymbol{s}_t - \boldsymbol{s}_{t+1}$ for all consecutive indices $t, t+1 \in \mathcal{I}_j$.

3. ALGORITHM

We adapt the online Grassmannian approach of [1, 20] to perform DMRI reconstruction with the batch low-rank plus transform sparse model (3). Our approach is motivated by the following global optimization problem:

$$\min_{\boldsymbol{U}} \sum_{j=1}^{B} \min_{\boldsymbol{S}_j, \boldsymbol{W}_j} \sum_{t\in\mathcal{I}_j} \|\boldsymbol{A}_t(\boldsymbol{U}\boldsymbol{w}_t + \boldsymbol{s}_t) - \boldsymbol{b}_t\|_2^2 + \lambda\|\mathcal{T}(\boldsymbol{S}_j)\|_1 \quad s.t. \;\text{span}(\boldsymbol{U}) \in \mathcal{G}(n, r). \tag{4}$$

Here $\mathcal{G}(n, r)$ denotes the *Grassmannian*, the space of all r-dimensional subspaces in $\mathbb{C}^n$. Rather than computing a global solution to (3), we attempt to find an approximate solution to (3) in an online fashion by processing only one batch of frames at a time. In particular, the algorithm alternates between solving for the optimal subspace weights and sparse component for a single batch assuming $\boldsymbol{U}$ is fixed, and updating an estimate of the low-rank subspace representative $\boldsymbol{U}$ via gradient descent on the Grassmannian. Algorithm 1 summarizes the proposed approach, which we call Robust Upwards of one Frame Fed GROUSE (RUFFed GROUSE).

3.1. Estimation of weights and sparse component

With $\boldsymbol{U}$ fixed, the cost function (3) is separable in each batch. For notational simplicity in this section we drop the batch index subscript j. Let $\mathcal{I}$ be the index set of the current batch, with $|\mathcal{I}| = T$. Restricting the optimization (3) to the current batch, yields the convex optimization problem

$$\min_{\boldsymbol{S}, \boldsymbol{W}} \sum_{t\in\mathcal{I}} \|\boldsymbol{A}_t(\boldsymbol{U}\boldsymbol{w}_t + s_t) - \boldsymbol{b}_t\|_2^2 + \lambda\|\mathcal{T}(\boldsymbol{S})\|_1. \tag{5}$$

The minimum of (5) over $\boldsymbol{W} = [\boldsymbol{w}_t]_{t\in\mathcal{I}}$ is separable in each $\boldsymbol{w}_t$, and has the exact solution

$$\boldsymbol{w}_t = \boldsymbol{Q}_t^+(\boldsymbol{b}_t - \boldsymbol{A}_t\boldsymbol{s}_t), \;\text{ for all }\; t \in \mathcal{I}, \tag{6}$$

where $\boldsymbol{Q}_t = \boldsymbol{A}_t\boldsymbol{U}$ and $(\cdot)^+$ denotes the Moore-Penrose pseudoinverse. Substituting (6) into (5) gives the simplified optimization problem

$$\min_{\boldsymbol{S}} \sum_{t\in\mathcal{I}} \|\boldsymbol{P}_t(\boldsymbol{A}_t\boldsymbol{s}_t - \boldsymbol{b}_t)\|_2^2 + \lambda\|\mathcal{T}(\boldsymbol{S})\|_1, \tag{7}$$

where $\boldsymbol{P}_t = (\boldsymbol{I} - \boldsymbol{Q}_t\boldsymbol{Q}_t^+)$. Note (7) is a convex sparsity-regularized linear least-squares problem that one can solve efficiently with a variable splitting technique; we use the modified Arrow-Hurwicz primal-dual algorithm of [3]. Finally, after computing the sparse residuals $\boldsymbol{S} = [\boldsymbol{s}_t]_{t\in\mathcal{I}}$, the optimal weights $\boldsymbol{W} = [\boldsymbol{w}_t]_{t\in\mathcal{I}}$ are obtained directly from (6).

3.2. Descent on the Grassmannian

Similar to [1], we propose updating the subspace estimate by computing a gradient step along the Grassmannian $\mathcal{G}(n,r)$ of the following subspace cost:

$$F(\boldsymbol{U};j) = \sum_{t\in\mathcal{I}_j} \|\boldsymbol{A}_t\boldsymbol{U}\boldsymbol{w}_t - \boldsymbol{v}_t\|_2^2, \tag{8}$$

where $\boldsymbol{U}\in\mathbb{C}^{n\times r}$ satisfying $\boldsymbol{U}^H\boldsymbol{U}=\boldsymbol{I}$ is a representative of the subspace $\mathcal{S}\in\mathcal{G}(n,r)$, and where have set $\boldsymbol{v}_t=\boldsymbol{b}_t-\boldsymbol{A}_t\boldsymbol{s}_t$ for all $t\in\mathcal{I}_j$. Define $\boldsymbol{R}_j=[\boldsymbol{r}_t]_{t\in\mathcal{I}_j}=[\boldsymbol{A}_t^H(\boldsymbol{A}_t\boldsymbol{U}\boldsymbol{w}_t-\boldsymbol{v}_t)]_{t\in\mathcal{I}_j}$. Taking derivatives of (8) with respect to the components of $\overline{\boldsymbol{U}}$ (the complex conjugate of $\boldsymbol{U}$) we get

$$\frac{dF}{d\overline{\boldsymbol{U}}} = -\sum_{t\in\mathcal{I}_j}\boldsymbol{r}_t\boldsymbol{w}_t^H = -\boldsymbol{R}_j\boldsymbol{W}_j^H.$$

The gradient on the Grassmanian is given as[1] $\boldsymbol{G}$

$$\boldsymbol{G} = (\boldsymbol{I}-\boldsymbol{U}\boldsymbol{U}^H)\frac{dF}{d\overline{\boldsymbol{U}}} = -\boldsymbol{R}_j\boldsymbol{W}_j^H,$$

where the last equality follows since each residual $\boldsymbol{r}_t$ is in the orthogonal complement of $\boldsymbol{U}$. Set $\boldsymbol{H}=-\boldsymbol{G}$, the negative gradient, and let $\boldsymbol{H}=\tilde{\boldsymbol{U}}\boldsymbol{\Sigma}\boldsymbol{V}^H$ be the thin SVD of $\boldsymbol{H}$ where $\tilde{\boldsymbol{U}}\in\mathbb{C}^{n\times r}$, $\boldsymbol{\Sigma}\in\mathbb{C}^{r\times r}$, and $\boldsymbol{V}\in\mathbb{C}^{r\times r}$. If $\boldsymbol{U}_j$ denotes the current subspace estimate, a step along a geodesic of the Grassmannian manifold in the direction of negative gradient with step-size η is given by

$$\boldsymbol{U}_{j+1} = \boldsymbol{U}_j\boldsymbol{V}\cos(\eta\boldsymbol{\Sigma})\boldsymbol{V}^H + \tilde{\boldsymbol{U}}\sin(\eta\boldsymbol{\Sigma})\boldsymbol{V}^H \tag{9}$$

(see Eqn. 2.65 in [6]). Here $\cos(\cdot)$ and $\sin(\cdot)$ denote the matrix cosine and sine respectively. The subspace update (9) generalizes the update derived in [1]: for a batch size of one, (9) reduces to the rank one update of [1]. In general, the update (9) is low-rank, with rank at most the size of the batch.

3.3. Initialization strategies

The performance of RUFFed GROUSE will depend heavily on the quality of the initialization of the subspace estimate $\boldsymbol{U}_0$. When calibration data is present, *i.e.*, k-space data is collected at common low-pass locations in each frame, we can perform an SVD on an initial low-resolution reconstruction of the frames to obtain an estimate of $\boldsymbol{U}_0$. To further refine this initialization, we propose passing over the frames *in a randomized order* using the GROUSE algorithm, which returns an estimate of the subspace $\boldsymbol{U}_0$. Using a randomized order ensures the subspace estimate will not be biased towards certain temporal regions of the dataset. While this initialization procedure requires a full pass over data, the GROUSE algorithm is very efficient, and for the data sizes investigated in this work, the GROUSE initialization took only a few seconds.

[1] For derivation of this formula and other involving differential properties of the Grassmannian manifold, see [6].

Algorithm 1 RUFFed GROUSE

Input: An $n\times r$ matrix $\boldsymbol{U}_0$ representing an initial estimate of the low-rank subspace; sequence of compressive measurements $\boldsymbol{b}_t$, subspace update step-size η.

Output: Subspace representative $\boldsymbol{U}_j$, weights $\boldsymbol{W}_j$, and sparse residuals $\boldsymbol{S}_j$ at the j-th batch.

for batch $j=1,2,\dots,B$ **do**

Compute subspace projectors

$$\boldsymbol{Q}_t=\boldsymbol{A}_t\boldsymbol{U}_j,\qquad \boldsymbol{P}_t=\boldsymbol{I}-\boldsymbol{Q}_t^+\boldsymbol{Q}_t \text{ for all } t\in\mathcal{I}_j$$

Compute sparse residuals via iterative solver:

$$\boldsymbol{S}_j=\underset{\boldsymbol{S}=[\boldsymbol{s}_t]_{t\in\mathcal{I}_j}}{\arg\min}\sum_{t\in\mathcal{I}_j}\|\boldsymbol{P}_t(\boldsymbol{A}_t\boldsymbol{s}_t-\boldsymbol{b}_t)\|_2^2+\lambda\|\mathcal{T}(\boldsymbol{S})\|_1.$$

Compute subspace weights:

$$\boldsymbol{W}_j=[\boldsymbol{w}_t]_{t\in\mathcal{I}_j}=[\boldsymbol{Q}_t^+(\boldsymbol{b}_t-\boldsymbol{A}_t\boldsymbol{s}_t)]_{t\in\mathcal{I}_j}$$

Update subspace via gradient step on Grassmannian:

$$\boldsymbol{R}_j=[\boldsymbol{A}_t^H(\boldsymbol{A}_t\boldsymbol{s}_t+\boldsymbol{Q}_t\boldsymbol{w}_t-\boldsymbol{b}_t)]_{t\in\mathcal{I}_j}$$
$$(\tilde{\boldsymbol{U}},\boldsymbol{\Sigma},\boldsymbol{V})=\text{svd}(\boldsymbol{R}_j\boldsymbol{W}_j^H)\quad\text{(thin SVD)}$$
$$\boldsymbol{U}_{j+1}=\boldsymbol{U}_j\boldsymbol{V}\cos(\eta\boldsymbol{\Sigma})\boldsymbol{V}^H+\tilde{\boldsymbol{U}}\sin(\eta\boldsymbol{\Sigma})\boldsymbol{V}^H.$$

end for

3.4. Memory complexity

Supposing we obtain m measurements on average per frame having n voxels, then with uniform batch size T and subspace rank r the RUFFed GROUSE algorithm requires storing variable $\boldsymbol{U}\in\mathbb{C}^{n\times r}$, $\boldsymbol{S}_j\in\mathbb{C}^{n\times T}$, and T variables of size $\mathbb{C}^{m\times r}$ corresponding to the $\boldsymbol{Q}_t$. This yields an overall memory complexity of $O(mrT+n(r+T))$. In contrast, for an acquisition consisting of F frames, a global L+S method requires storing all $m\times F$ measurements and one variable having the size of the full dataset $\mathbb{C}^{n\times F}$, which has complexity at least $O((m+n)F)$. When $F\gg\max(r+T,rT)$ this shows the memory savings offered by RUFFed GROUSE algorithm are substantial over a global L+S method.

4. RESULTS

Fig. 1 shows the results of DMRI recovery experiments performed on two datasets: (1) a synthetic dataset generated using the MRXCAT phantom [19] that simulates a breath-held cardiac cine DMRI acquisition (409×409 pixels, 4 coils, and 24 temporal frames), and (2) a gated multicoil Cartesian-sampled cardiac perfusion DMRI dataset used in prior work [16] (128×128 pixels, 12 coils, and 40 temporal frames). We retrospectively undersample each dataset in k-t space at an acceleration factor of 8 using a variable-density random Cartesian lines undersampling pattern changing with each time frame. As an error metric, we use the signal to noise ratio (SNR) defined as $-20\log_{10}(\|\boldsymbol{x}-\boldsymbol{x}_{\text{ref}}\|_2/\|\boldsymbol{x}\|_2)$, where $\boldsymbol{x}_{\text{ref}}$ is a reference reconstruction computed from the fully

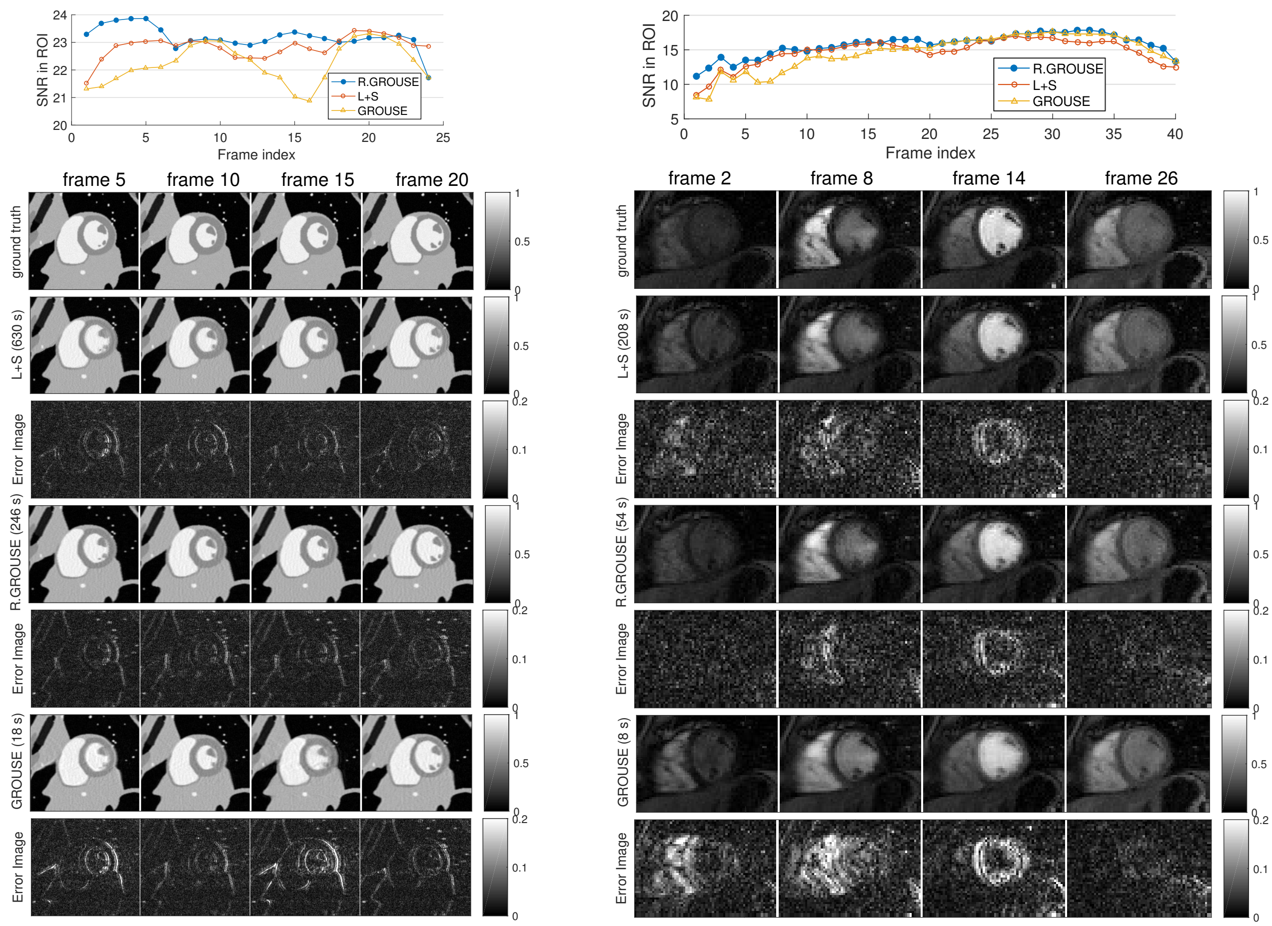

Fig. 1: Comparison of global L+S [16] reconstruction versus online algorithms RUFFed GROUSE (this work) and GROUSE [20] on a synthetic cardiac cine DMRI dataset (left) and a real cardiac perfusion DMRI dataset (right). Images are cropped to a region of interest (ROI).

sampled data. We compute the SNR of the reconstruction restricted to pixels in a region of interest (ROI) containing the myocardium, which we call the SNR in ROI.

For RUFFed GROUSE we partitioned the dataset into batches of b temporally adjacent frames and processed the batches in sequence using the following settings: MRXCAT dataset, $r = 1, b = 6, \eta = 0.0002, \lambda = 0.005$; perfusion dataset $r = 3, b = 5, \eta = 0.001, \lambda = 0.005$. We initialize the subspace estimate $\boldsymbol{U}_0$ using GROUSE [20]. We also compare with the GROUSE run frame-by-frame in sequence, using the same rank r and initialization of $\boldsymbol{U}_0$ (*i.e.*, two full passes of GROUSE). Finally, we compare with the low-rank plus sparse (L+S) method of [16] with temporal finite differences as the sparsifying transform, using publicly available code[2]. All experiments were run in MATLAB on a MacBook Pro laptop (3.1 GHz Intel Core i7 CPU, 16 GB RAM).

The reconstructions obtained using RUFFed GROUSE are comparable to the L+S method, both visually and in terms of SNR in ROI. The GROUSE reconstruction, which uses only a low-rank model, gives lower quality reconstructions in frames containing significant dynamics (*e.g.*, frame 15 for the MRXCAT phantom dataset, and frame 8 for the perfusion dataset). This demonstrates the benefit of including the (transform) sparse component in RUFFed GROUSE. However, the improvement in reconstruction quality comes at the expense of greater computation time: RUFFed GROUSE is several times slower than GROUSE (*e.g.*, 246 s vs. 18 s on the synthetic dataset), yet still 2-4 times faster than the L+S reconstruction on these datasets.

[2] `http://cai2r.net/resources/software/ls-reconstruction-matlab-code`

5. CONCLUSION

We extend the GROUSE subspace tracking algorithm to incorporate batch processing under a low-rank plus transform sparse model with applications to DMRI reconstruction. We show that the proposed approach gives comparable reconstruction quality to the approach of [16] based on a global low-rank plus sparse model that performs iterative reconstruction of the entire time series. The proposed approach requires storing and processing only small batches of frames sequentially, which allows robust subspace methods to be applied to much larger DMRI datasets than previously possible.

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