Detection Performance Prediction for CdZnTe Array

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***Abstract*—The complex system response of 3D position–sensitive gamma–ray detectors complicates the model for the recorded measurements and makes exact expressions for detection performance intractable. This makes source detection performance difficult and expensive to compute. Asymptotic analysis has the potential to simplify detection performance prediction with complex systems and has previously been applied to detection performance prediction with simulated gamma–ray detectors. In this work, we use asymptotic performance prediction methods to predict points on the receiver operating characteristic (ROC) curve for the illustrative task of detecting a Cs-137 source in background with an 18-detector CdZnTe array. We assume that the source position, background spectrum, and background spatial distribution are known. Our results show that the asymptotic performance prediction method accurately predicts the empirically observed performance even with real data recorded with a real system. Our results also characterize the performance of the detector array for the task of source detection. The accuracy and computational efficiency of the asymptotic detection performance prediction method make it a viable alternative to empirical performance evaluation.**

***Index Terms*—Compton imaging, Detection, Performance Prediction**

Gamma–ray source detection problems arise in security screening, nuclear nonproliferation, and medical diagnostics. Simple systems for radioactive source detection look for an increase in the rate of received photons due to a radiation source. More complex measurement systems use spatial and spectral information to achieve better performance, but these systems often have a complicated system response, making it difficult to compute detection performance analytically.

In this work, we quantify detection performance in terms of the receiver operating characteristic (ROC) curve, which is the probability of detection as a function of the probability of false alarm [1]. Previous work that characterized the detection performance of gamma–ray detectors relied on empirical ROC calculation, e.g., [2], [3]. Empirical ROC calculation is computationally expensive and provides only limited intuition about how detector or environment parameters affect detection performance.

Asymptotic ROC prediction is a computationally efficient alternative to empirical ROC computation for likelihood–based tests, or tests that are functions of estimates obtained by maximizing a modeled likelihood. We developed asymptotic approximations for the distributions of likelihood–based estimates in [4], and used these approximations to predict detection performance in the presence of model mismatch. It was shown in [4] that the asymptotic performance prediction method yields more accurate predictions in terms of mean–square error than empirical methods, especially when few measurements are available.

The simulation results of [4] do not demonstrate that the proposed method can accurately predict the performance of real detectors. In this work, we show that the performance prediction method that accounts for model mismatch developed in [4] can accurately predict source detection performance with a real Compton imaging system. To our knowledge, this work is the first to apply an asymptotic performance prediction method to characterize the performance of the source detection task with a real gamma–ray imaging system.

In addition to demonstrating the practical utility of the asymptotic performance predictions of [4], this work serves as an example application of the asymptotic performance prediction method to practical problems. We use the asymptotic method to predict the probability of detection as a function of scan time with a fixed false–alarm rate for various source–to–background ratios. These examples demonstrate how the asymptotic performance prediction method can be applied to evaluate the performance of real detectors in the field.

Demonstrating the accuracy of the asymptotic performance prediction method with real data is significant because there is more model mismatch than in the simulated case. For example, Doppler broadening [5] is not simulated in [4]. Room–temperature pixellated semiconductor detectors, including the detectors used in this

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work [6], have an area near the anode where interacting photons are not detected. To simplify computation, the model used in this work, based on [7], does not account for this non–ideal detector behavior. Furthermore, crystal defects can cause errors in the measured interaction positions that are not accounted for by the model. Our results show that the asymptotic prediction method is reasonably accurate in the scenarios considered despite the system response approximation and failure to account for all non–ideal detector behavior.

The contributions of this work are: (i) to show that the asymptotic performance prediction method developed in [4] gives reasonable predictions with a real system, (ii) to illustrate practical uses of this method, and (iii) to provide representative detection performance figures for a real CdZnTe gamma–ray imaging system. This paper is organized as follows: §I describes the experimental setup, §II shows predicted and empirical performance of the detector for various tasks, and §III gives our conclusions and plans for future work.

I. METHODS

We recorded gamma–ray interaction data with a Compton imaging system consisting of an 18 detector CdZnTe array similar to the system described in [8]. We obtained list–mode measurements of the natural background in a room with concrete walls, and measurements in the same position with a Cs-137 source located 1.83 meters from the front of the detector. We use the events obtained from these measurements to evaluate the source detection performance of the system.

A. Measurement Model

There are many aspects of the gamma–ray source detection problem that one can model. The system model and sensitivity are necessary for the likelihood–based detection methods used in this work. We also model the background spatial and energy distributions because this improves detection performance when the modeling is reasonably accurate. Background modeling is beneficial in applications where the detector and environment are stationary.

1) Model Parameters: There are several parameters that characterize the gamma–ray source detection problem. We characterize the source by its intensity α with units of counts *emitted* per unit time and position[1] $\boldsymbol{\phi} \in \Phi$. In the 3D far–field with a known source energy, the set Φ could be $[0, 2\pi] \times [0, \pi]$, representing all possible azimuth and polar angles on a sphere. We parameterize the background intensity by the background count rate λ_b with units of gamma–ray counts *recorded* per unit time. We assume that the background spectrum is known. Let $\boldsymbol{\theta}$ be the vector of all parameters, where $\boldsymbol{\theta}$ lies in the d-dimensional parameter space Θ. In what follows, we assume that $\boldsymbol{\theta}$ takes the form:

$$\boldsymbol{\theta} = (\alpha, \boldsymbol{\phi}, \lambda_b), \tag{1}$$

for which $d = 3$. Throughout this work, we assume that the source position in space and energy $\boldsymbol{\phi}$ and the background intensity λ_b are known. Let the modeled sensitivity $\tilde{s}(\boldsymbol{\phi})$ approximate the probability that a photon emitted from a source positioned at $\boldsymbol{\phi}$ is recorded. We model the total rate of recorded photons by the sum of the rates of recorded source and background photons

$$\tilde{\lambda}(\boldsymbol{\theta}) \triangleq \lambda_b + \alpha \tilde{s}(\boldsymbol{\phi}). \tag{2}$$

2) System Model: We use the model given in in [4] to describe the system used in this work. Let $\boldsymbol{r}$ be a vector of recorded attributes associated with a single photon interaction. In a position–sensitive Compton detector, the attribute vector $\boldsymbol{r}$ contains the interaction positions and deposited energies for a single interacting photon. In fixed–time mode, the number of recorded photons J is reasonably modeled as a random variable, where $J \sim \text{Poisson}(\tilde{J}(\boldsymbol{\theta}))$. The mean number of recorded photons $\tilde{J}(\boldsymbol{\theta})$ is given by

$$\tilde{J}(\boldsymbol{\theta}) \triangleq \tilde{\lambda}(\boldsymbol{\theta})\tau,$$

where τ is the scan time. Let $\tilde{\boldsymbol{r}} = [\boldsymbol{r}_1, \boldsymbol{r}_2, \ldots, \boldsymbol{r}_J]$ be a list of the recorded attributes for all interacting photons during a fixed–time scan. By the statistics of list–mode data [9], a reasonable model for the list of recorded attributes $\tilde{\boldsymbol{r}}$ is

$$\tilde{\mathsf{p}}\left(\tilde{\boldsymbol{r}}; \boldsymbol{\theta}\right) \triangleq e^{-\tau\tilde{\lambda}(\boldsymbol{\theta})}[\tau\tilde{\lambda}(\boldsymbol{\theta})]^J/J! \prod_{j=1}^{J} \tilde{\mathsf{p}}(\boldsymbol{r}_j; \boldsymbol{\theta}). \tag{3}$$

We model the probability density of the individual recorded attributes $\tilde{\mathsf{p}}\left(\boldsymbol{r}; \boldsymbol{\theta}\right)$ using the approximate model in [7], which makes approximations to achieve computational efficiency. We use this approximate model because it results in reasonable detection performance and is much faster than computing the true density exactly.

Let $\tilde{\mathsf{p}}_{\mathrm{S}}(\boldsymbol{r}; \boldsymbol{\phi})$ denote the modeled density of a recorded attribute vector $\boldsymbol{r}$ given it originated from a source at position $\boldsymbol{\phi}$ and let $\tilde{\mathsf{p}}_{\mathrm{B}}(\boldsymbol{r})$ denote the modeled density of a recorded attribute vector $\boldsymbol{r}$ given that it originated from the background. Note that $\tilde{\mathsf{p}}_{\mathrm{S}}(\boldsymbol{r}; \boldsymbol{\phi})$ depends only on the source position, and $\tilde{\mathsf{p}}_{\mathrm{B}}(\boldsymbol{r})$ does not depend on any of the parameters in (1).

We model the overall distribution of recorded attributes as a mixture of $\tilde{\mathsf{p}}_{\mathrm{S}}(\boldsymbol{r}; \boldsymbol{\phi})$ and $\tilde{\mathsf{p}}_{\mathrm{B}}(\boldsymbol{r})$ given by

$$\tilde{\mathsf{p}}\left(\boldsymbol{r}; \boldsymbol{\theta}\right) = \frac{\alpha \tilde{s}(\boldsymbol{\phi}) \tilde{\mathsf{p}}_{\mathrm{S}}(\boldsymbol{r}; \boldsymbol{\phi}) + \lambda_b \tilde{\mathsf{p}}_{\mathrm{B}}(\boldsymbol{r})}{\alpha \tilde{s}(\boldsymbol{\phi}) + \lambda_b}. \tag{4}$$

[1] $\boldsymbol{\phi}$ could also denote a vector containing both spatial position and energy

3) Sensitivity Model: We computed the sensitivity model $\tilde{s}(\phi)$ by simulating the detector system in a uniform background using GEANT4 [10]. We used simple back projection [11] to reconstruct the sensitivity as a function of position and energy. We normalized the sensitivity so the sensitivity to the source at its true position and energy is one. This method of computing the sensitivity is approximate and it too may be a source of model mismatch.

4) Background distribution: The simulated results in [4] assumed a monoenergetic source and background. The natural radiation background is polyenergetic, so we discretize the energy spectrum into 80 uniformly–spaced bins from 0keV to 2000 keV. We assume that the source is monoenergetic and that its energy is known. We assume that the shape of the background spectrum is known, and perform detection with and without the assumption that its intensity is known. We examine the performance difference between a uniform spectral model and a spectral model based an independent measurement of the same environment.

We measured the natural background using 10,000 recorded background photons with two or more interactions. We used an expectation maximization (EM) algorithm [12] to reconstruct the emission density as a function of energy. Figure 1 shows the modeled probability density of recorded energy given that the photon originated from background $\tilde{\mathsf{p}}\,(\phi|B)$ computed using the measured background spectrum.

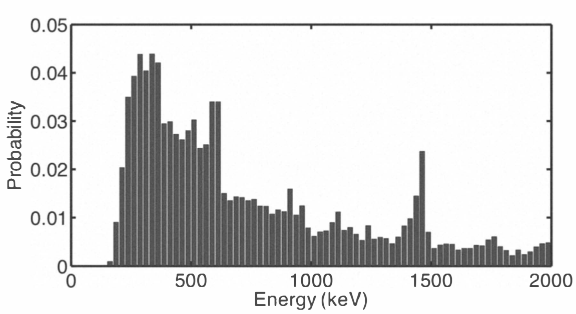


Fig. 1: Estimated probability density function for the incident energy of recorded background photons. This density was estimated using 10,000 recorded events from the natural background.

B. Detection Methods

We analyze the performance of the source intensity test (SIT) [13] applied to the gamma–ray source detection problem. The SIT is based on the quasi maximum–likelihood (QML) estimate for the source intensity. A QML estimator is equivalent to the ML estimator if the modeled distribution of the observations is equal to the true distribution. The QML estimate for the parameter vector $\boldsymbol{\theta}$ is defined as [14]

$$\tilde{\boldsymbol{\theta}}_\tau \triangleq \arg\max_{\boldsymbol{\theta}\in\Theta} \; \log \tilde{\mathsf{p}}(\tilde{\boldsymbol{r}};\boldsymbol{\theta}). \tag{5}$$

We use the SIT because experiments showed that its performance was superior to the generalized likelihood ratio test (GLRT) when applied to simple systems [13].

In the absence of model mismatch, the parameter estimate vector $\tilde{\boldsymbol{\theta}}_\tau$ converges in probability to the true parameter values as the scan time goes to infinity. However, when model mismatch is present, the estimates may converge to some value that is not the true parameter. For example, when there is no source present, the true source intensity is zero, but model mismatch may cause the source intensity estimate to converge to some nonzero value. To precisely define the value to which the parameter estimates converge, we first define the expected log–likelihood by

$$\tilde{g}\,(\boldsymbol{\theta}) \triangleq \mathsf{E}\left[\log \tilde{\mathsf{p}}\,(\tilde{\boldsymbol{r}};\boldsymbol{\theta})\right], \tag{6}$$

where the expectation is with respect to the true distribution and $\mathcal{R}$ is the set of all lists of recorded attributes. The parameter estimates converge to the asymptotic mean, which is given by

$$\tilde{\boldsymbol{\mu}} \triangleq \arg\max_{\boldsymbol{\theta}\in\Theta} \tilde{g}\,(\boldsymbol{\theta})\,. \tag{7}$$

The asymptotic mean is an important component of the performance prediction method.

The source intensity test (SIT) [13] for detecting the presence of a radiation source of unknown intensity α is given by

$$\tilde{\alpha}_\tau \underset{H_0}{\overset{H_1}{\gtrless}} \gamma, \tag{8}$$

where $\tilde{\alpha}_\tau$ is the QMLE for α, which is the first element of $\tilde{\boldsymbol{\theta}}_\tau$, and γ is a threshold chosen by the user to obtain the desired false alarm rate. The user postulates that a source is present, or H_1 is true, when the source intensity estimate $\tilde{\alpha}_\tau$ is greater than the threshold γ. The distribution of $\tilde{\alpha}_\tau$ determines the threshold value that satisfies the desired false alarm rate, but the distribution of $\tilde{\alpha}_\tau$ is intractable in the gamma–ray imaging problem.

C. Performance Measure

We state our results in terms of the probability of detection as a function of scan time in contrast to previous works that state performance prediction [2], [4], [13] in terms of ROC and the area under the ROC curve (AUC). We choose to fix a false alarm probability and examine how the probability of detection varies as a function of scan time because the probability of detection is arguably more important to the practitioner than the AUC.

D. Performance Prediction

We predict the ROC by approximating the distribution of $\tilde{\alpha}_\tau$, justified by Theorems 1 and 2 of [4], by

$$\tilde{\alpha}_\tau \overset{\text{approx}}{\sim} \mathcal{N}\left(\tilde{\mu}_{[1]}, \frac{1}{\tau}\Sigma(\tilde{\boldsymbol{\mu}})_{[1,1]}\right), \tag{9}$$

where $\tilde{\mu}_{[1]}$ is the first element of $\tilde{\boldsymbol{\mu}}$, which corresponds to the asymptotic mean of the source intensity. The covariance is

$$\Sigma(\boldsymbol{\theta}) \triangleq \tilde{\mathsf{H}}^{-1}(\boldsymbol{\theta})\,\tilde{\mathsf{G}}(\boldsymbol{\theta})\,\tilde{\mathsf{H}}^{-1}(\boldsymbol{\theta}), \tag{10}$$

where

$$\tilde{\mathsf{G}}(\boldsymbol{\theta}) \triangleq \lambda_{\mathsf{s}}\mathsf{E}\Big[\left(\nabla_{\boldsymbol{\theta}}\log\tilde{\mathsf{p}}(\boldsymbol{r};\boldsymbol{\theta}) + \nabla_{\boldsymbol{\theta}}\log\tilde{\lambda}(\boldsymbol{\theta})\right) \left(\nabla_{\boldsymbol{\theta}}\log\tilde{\mathsf{p}}(\boldsymbol{r};\boldsymbol{\theta}) + \nabla_{\boldsymbol{\theta}}\log\tilde{\lambda}(\boldsymbol{\theta})\right)^T\Big], \tag{11}$$

and

$$\tilde{\mathsf{H}}(\boldsymbol{\theta}) \triangleq -\lambda_{\mathsf{s}}\nabla^2_{\boldsymbol{\theta}}\log\tilde{\lambda}(\boldsymbol{\theta}) + \nabla^2_{\boldsymbol{\theta}}\tilde{\lambda}(\boldsymbol{\theta}) - \lambda_{\mathsf{s}}\mathsf{E}\left[\nabla^2_{\boldsymbol{\theta}}\log\tilde{\mathsf{p}}(\boldsymbol{r};\boldsymbol{\theta})\right], \tag{12}$$

$\nabla_{\boldsymbol{\theta}}$ is the column gradient with respect to $\boldsymbol{\theta}$, $\nabla^2_{\boldsymbol{\theta}}$ is the Hessian with respect to $\boldsymbol{\theta}$, λ_{s} is the true recorded count rate, and expectations are with respect to the true distribution. If the source position $\boldsymbol{\phi}$ and background intensity λ_b are known, then $\Sigma(\boldsymbol{\theta})$, $\tilde{\mathsf{G}}(\boldsymbol{\theta})$, and $\tilde{\mathsf{H}}(\boldsymbol{\theta})$ are scalar, otherwise they are matrices.

We used the following procedure to apply the asymptotic approximation in (9) for detection performance prediction.

1) Obtain 10,000 recorded events with a source placed 1.83 meters from the front of the detector.
2) Evaluate the asymptotic mean $\tilde{\mu}_{[1]}$ by solving (5) using the recorded events.
3) Evaluate the asymptotic covariance $\Sigma(\tilde{\boldsymbol{\mu}})$ using (10), (11), and (12). Use Monte Carlo integration with the recorded data to evaluate the expectations.
4) Compute the true count rate λ_{s} by averaging the count rate over two hours.
5) Obtain 10,000 recorded events with the detector in the same position as step 1 but without a source present.
6) Repeat steps 2-4 in the absence of a source.
7) Use the computed asymptotic means and covariances to compute the approximate distribution of $\tilde{\alpha}_\tau$ with and without a source present.
8) Use the approximate distributions to compute the probabilities of false alarm and detection.

E. Source Intensity Variation

We recorded data with and without a source present to predict detection performance. We achieved the desired source–to–background ratio by combining events from the measurements with and without a source. For example, the measurements of a Cs-137 source placed 1.83 meters from the detector contain approximately 48% source events and 52% background events. We observed the recorded background count rate λ_b to be approximately 9114 counts per minute using a measurement of 90.7 hours. The observed count rate of the measurement containing both source and background events is 18984 counts per minute, which we obtained from a measurement of 121 minutes. We combined events from the lists obtained with and without a source explore a range of source–to–background ratios.

F. Empirical Calculations

We compared the predicted performance to the empirical performance in terms of probability of detection, or equivalently, ROC. We computed the empirical performance by simulating 100 scans with a source present and 100 scans without a source present. This method is similar to the method used in [2], [4], and [13]. We used the following procedure to simulate a scan:

1) Draw the number of recorded source counts from a Poisson distribution with mean $\tau\alpha^{\mathsf{t}}s^{\mathsf{t}}$, where s^{t} is the true sensitivity, and $\tau\alpha^{\mathsf{t}}s^{\mathsf{t}}$ is the mean number of received counts from the source.
2) Draw the number of recorded background counts from a Poisson distribution with mean equal to the background count rate $\tau\lambda_b$, which we measured over a period of two hours.
3) Generate a list of events that contains $\tau\alpha^{\mathsf{t}}s^{\mathsf{t}}$ source counts and $\tau\lambda_b$ background counts using the recorded events in the presence and absence of a source. Combine the events from the measurements with and without a source to achieve the proper mean number of source counts.
4) Solve (5) using the list generated in the previous step.

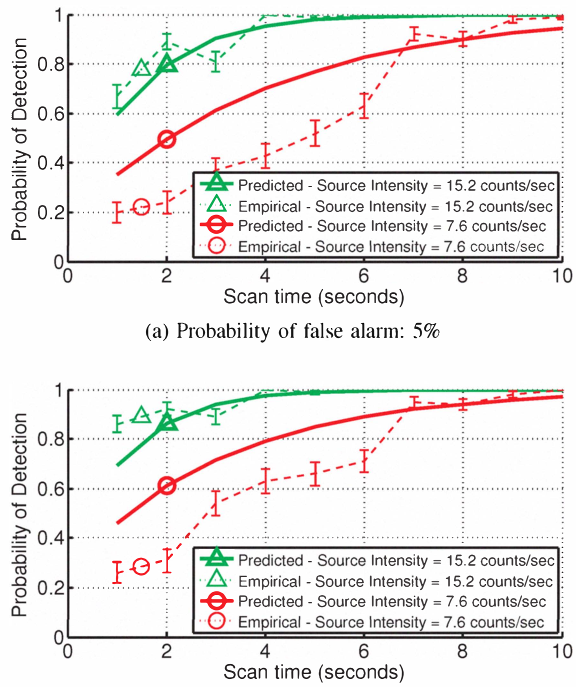
After simulating 100 scans with a source present and 100 scans without a source present, we used the empirical source intensity estimates obtained in step 4 to compute the empirical probabilities of detection and false alarm.

This empirical calculation method requires simulation of 200 scans for each point on the graph of probability of detection versus scan time. In contrast, the asymptotic prediction method based on (9) requires one computation with approximately 20,000 recorded events for all scan times because the asymptotic mean is invariant to scan time and the asymptotic covariance in (10) scales as the inverse of the scan time. Thus, computing detection performance as a function of scan time with the asymptotic prediction method requires much less computation than computing the performance empirically.

II. Results

We computed the probability of detection as a function of scan time when the source position, source energy, and background spectrum are assumed known. We examined the case of a known background intensity. We found that the performance predicted using (9) agreed well with the empirical performance in the known background case. Additional results with unknown background and background spectrum model mismatch are given in [15].

We examined the problem of detecting a Cs-137 source when the background spectrum is modeled by the measured background spectrum in Figure 1, and the background intensity is assumed known. Figure 2 shows the probability of detection as a function of scan time for probability of false alarms of 5% and 10% with source intensities of 7.6 counts per second and 15.2 counts per second. The background intensity is 152 counts per second. The agreement between the empirical and predicted probability of detection is better with the higher source intensity for both false alarm rates. This is likely due to the fact that the Gaussian approximation for the distribution of the source intensity estimate (9) improves as the number of recorded counts increases [4].



(a) Probability of false alarm: 5%

(b) Probability of false alarm: 10%

Fig. 2: Probability of detection versus scan time for detecting a Cs-137 source in a natural background with intensity 152 counts per second using an 18 detector CdZnTe array with various false alarm rates and source intensities. The background shape and intensity are assumed known and the background shape is modeled using a prior spectral measurement.

III. Conclusion and Future Work

We applied the asymptotic detection performance prediction method developed in [4] to performance prediction of a real system with real recorded data. Our results showed that the asymptotic prediction method accurately predicts detection performance for modest scan times when the background intensityis known.

This work serves as an example application of the theory of [4] to performance characterization of real systems. Our results show that the asymptotic performance prediction method gives reasonably accurate performance predictions that one can use to determine sensor placement, configuration, or viability.

We considered the case where the source position and energy are known. Future work would investigate the accuracy of the detection performance prediction method of [4] when the source position and energy are unknown. The unknown energy case presents additional challenges because in a typical application, only a set of possible sources are of interest, and each source may emit gamma–rays with multiple energies. The prediction method of [4] only applies to continuous parameters, but an unknown isotope is a discrete parameter. Future work would extend the performance prediction method to the case of a discrete parameter.

We also assumed that the background spectrum, spatial distribution, and intensity are known. We used this assumption because background shape and spectrum estimation is a high–dimensional problem. Future work would investigate methods of reducing the dimensionality of the background estimation problem so reasonable estimates could be obtained with a reasonable amount of data and computation.

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