Discussion of “The EM algorithm - An old folk song sung to a fast new tune”
by Xiao-Li Meng and David van Dyk in JRSSB 59(3) 1997

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May 3, 2000

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In the medical imaging field, an EM algorithm has finally reached the “pop charts.” Companies that make instruments for single photon emission computed tomography (SPECT) now sell EM algorithm software for image reconstruction. (Interestingly, it was the nonuniform attenuation problem in SPECT that drove this evolution, not the considerations of noise or spatial resolution addressed in many EM related papers for PET.) Sadly, it is the “golden oldies” version of that EM algorithm [1] that is commercially available—without regularization or acceleration. (In fact, the imaging community parochially calls it “the” EM algorithm!) Our efforts to “jazz up” EM algorithms with penalty functions and faster convergence [2, 3] have yet to make the commercial hit parade.

Within part of the medical imaging community, the continuing prevalence of classical EM over contemporary faster renditions is due to a lack of acceptance of methods for regularization. Without regularization, methods for accelerating EM algorithms for tomography are of limited use, because they only provide faster convergence to lousy images (since the ML estimate has unacceptably high variance). One problem with regularization has been that conventional penalty functions result in nonuniform spatial resolution, with the poorest resolution in regions of highest intensity. This property is counter-intuitive and undesirable, and we have partially addressed it in [4]. Further work in understanding and improving the penalty functions will be needed before regularized image reconstruction methods and the accompanying fast algorithms will see widespread use in medical imaging.

In classical missing-data problems in statistics, there is often exactly one natural choice for the augmented data, so the EM algorithm is appealing. In contrast, in the image reconstruction problem described in [3] and summarized in section 3.5, the augmented data has no physical interpretation. As noted in [3], one can derive the SAGE algorithm for image reconstruction from a non-statistical perspective using only the concavity of the log-likelihood and the convexity inequality. In some respects this derivation is considerably simpler than the EM-based approach of section 3.5, since it avoids the artificial augmented data. We refer the reader to [5] for details about this alternate derivation, which encompasses a large class of problems.

Finally, the reader should be aware that the convergence results in section 3.3 do not apply to the image reconstruction problem in section 3.5 due to the nonnegativity constraint. A general proof in [3] establishes convergence of the SAGE algorithm for image reconstruction.

References

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