Non-Parametric Tracking of Shift and Shape Functions in Medical Images

Jeffrey A. Fessler and Albert Macovski
Stanford University

Several important estimation problems, in particular the quantitation of blood vessel position and radius from projections, involve tracking of dynamics shift band shape parameters. Most of the published algorithms for blood vessel quantitation consist of two steps: 1) obtaining preliminary estimates of the position and radius parameters, and 2) smoothing those estimates using and *ad hoc* method such as local averaging. Bresler [1] has recently presented a minimum mean-square error algorithm for nonlinear estimation of these parameters, but his formulation assumes a known (parametric) Gauss- Markov stochastic state-space model for the dynamic evolution. The parameters of his Gauss-Markov model are unknown and unmotivated for biological phenomena such as the variations in radius along a stenotic (narrowed) section of a blood vessel. In this paper, we present an alternative algorithm for tracking shift and shape parameters that is based on non-parametric cubic-spline smoothing [2]. Rather than requiring a known Gauss-Markov model, our algorithm assumes only that the shift and shape functions be smoothly varying in a sense defined below.

The paper discusses the physical motivation for our (global) optimality criterion, derives an efficient algorithm for computing the optimal estimates, and demonstrates the performance on angiographic data. The performance of the algorithm is demonstrated on simulated angiogram data below.

References

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Jeffrey A. Fessler and Albert Macovski*
Information Systems Laboratory
Department of Electrical Engineering
Stanford University · Stanford, CA 94305
(415)-723-1904
ARPA: fessler@isl.stanford.edu

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1 Introduction

Several important estimation problems, in particular the quantization of blood vessel position and radius from projections, involve tracking of dynamic shift and shape parameters. Bresler [1] has recently presented an algorithm for nonlinear estimation of these parameters from a fixed set of measurements. His formulation assumes a known (parametric) discrete-time Gauss-Markov stochastic state-space model for the dynamic evolution of the shift and shape parameters, and he derives a minimum mean-square error estimator. This parametric approach is certainly appropriate for dynamic systems governed by differential equations. However, many biological phenomena, such as the variations in radius along a stenotic (narrowed) section of a blood vessel, are not well modeled by Gauss-Markov processes. In this paper, we present an alternative algorithm for tracking shift and shape parameters that is based on non-parametric cubic-spline smoothing [2, 3]. Rather than requiring a known Gauss-Markov model, our algorithm requires only that the shift and shape functions be smoothly varying in a sense defined below. The performance of the algorithm is demonstrated on simulated angiogram data.

2 The Problem

Consider Fig. 1; each row of this $(N \times M)$ image is a sampled semi-ellipse function of unknown radius and of unknown shift (position). Our goal is to estimate the shift and shape functions from such an image.

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Let $X(z) = [x_1(z)\ x_2(z)]'$, where $x_1(z)$ and $x_2(z)$ denote the shift and shape parameters for row z, where $z = 1, \dots, N$.

We use the following model for images of this type:

$$y(t; z) = s(t - x_1(z); x_2(z)) + v(t; z), \tag{1}$$

where $t = t_1, \dots, t_M$, and $s(t; x_2)$ is a known (nonlinear) function of t and x_2, and $v(t; z)$ is additive white Gaussian noise (AWGN). This measurement model can be rewritten

$$Y(z) = S(X(z)) + V(z),$$

where $S(X) = [s(t_1 - x_1; x_2), \dots, s(t_M - x_1; x_2)]'$, $Y(z) = [y(t_1; z), \dots, y(t_M; z)]'$, and $V(z)$ is similarly defined.

One trivial approach to estimating X is to use the Maximum Likelihood estimator:

$$\hat{X}_{\text{ML}}(z) = \arg\min_X \|Y(z) - S(X)\|^2.$$

However, this approach is suboptimal since it ignores the *a priori* knowledge that the shift and shape parameters, $x_1(z)$ and $x_2(z)$, are smooth functions of z. Shmueli et. al. [4] first attempted to quantify smoothness with a Gauss-Markov model: $X(z+1) = A_z X(z) + B_z v(z)$. We propose instead to use the following estimator:

$$\hat{X} = \arg\min_X \sum_{z=1}^{N} \|Y(z) - S(X(z))\|^2 + \alpha_1 \int_1^N \ddot{x_1}^2(z)\, dz + \alpha_2 \int_1^N \ddot{x_2}^2(z)\, dz. \tag{2}$$

This formulation is generalization of the criterion used for non-parametric smoothing with cubic splines. The solution is a compromise between fit-to-the-data and the roughness (as measured by the integrated squared second derivate) of the estimated parameters. The tradeoff is controlled by $\boldsymbol{\alpha} = [\alpha_1, \alpha_2]$; $\boldsymbol{\alpha}$ can be estimated from the data using cross validation [3], so that the relative smoothness of the shift and shape functions is not predetermined, in contrast to parametric approaches. This tradeoff is similar to the regularization methods being applied to machine vision [5].

Equation (2) is very similar to the formulation of spline smoothing [2], except that the signal S is a nonlinear function of X. In the next section we present an iterative algorithm for computing $\hat{X}$.

3 Computation

To compute the solution to (2), we make the same approximation used in the derivation of the extended Kalman filter (EKF) [6]:

$$S(X) \approx S(\bar{X}) + H(\bar{X}) \cdot (X - \bar{X}), \tag{3}$$

where $\bar{X}$ is an initial estimate for X and $H(X)$ is the $(M \times 2)$ Jacobian of $S(X)$. With this approximation, we can rewrite (2) as

$$\hat{X} = \arg\min_X \sum_{z=1}^{N} \|C(z) - H(\bar{X}(z))X(z)\|^2 + \alpha_1 \int_1^N \ddot{x_1}^2(z)\, dz + \alpha_2 \int_1^N \ddot{x_2}^2(z)\, dz. \tag{4}$$

where $C(z) = Y(z) - S(\bar{X}(z)) + H(\bar{X}(z))\bar{X}(z)$. The solution to (4) is now feasible since it is linear in X.

We can simplify the solution by noting (due to the approximation (3)) that: $C(z) \approx H(\bar{X}(z))X(z) + V(z)$. Let $D(z) = (H(\bar{X}(z))'H(\bar{X}(z)))^{-1}H(\bar{X}(z))'C(z)$, then $D(z) \approx X(z) + N(z)$, where $N(z)$ is AWGN with covariance $\Sigma(z) = (H(\bar{X}(z))'H(\bar{X}(z)))^{-1}$. This results in the final form for the estimator:

$$\hat{X} = \arg\min_{X} \sum_{z=1}^{N} (D(z) - X(z))'\Sigma(z)^{-1}(D(z) - X(z)) + \alpha_1 \int_1^N \ddot{x}_1{}^2(z)\, dz + \alpha_2 \int_1^N \ddot{x}_2{}^2(z)\, dz. \quad (5)$$

This minimization is performed using a vector measurement generalization [7] of the scalar spline smoothing algorithm [8].

Since (3) is most accurate for $\bar{X}$ very close to X, we evaluate (5) iteratively, using the estimate $\hat{X}$ of one iteration as the initial estimate $\hat{\bar{X}}$ of the next iteration.

4 Frequency space algorithm

As explained in detail by Bresler [1], the procedure described above is not robust to significant errors in the initial estimate $\bar{X}$, for the same reasons that an EKF can "lose track" and diverge. This is due to the fact that the support of the function s is typically a small fraction of the window $[t_1, t_M]$, as described in [9]. Bresler proposed using the discrete Fourier transform (DFT) of the measured data, and showed that the transformed data is more amenable to linearization [1, 9]. This approach can be directly applied to make our algorithm more robust to poor initial estimates.

Let

$$\{\tilde{y}_u(z)\}_{u=0}^{M-1} = DFT\{y(t_i; z)\}_{i=1}^{M}$$

and similarly define $\tilde{s}_u$ and $\tilde{y}_u$. Then

$$\tilde{y}_u(z) = \tilde{s}_u(X(z)) + \tilde{y}_u(z),$$

where

$$\tilde{s}_u(X) \approx \tilde{s}_a\left(\frac{2\pi}{N}u; x_2\right) \exp\left(-j\frac{2\pi}{N}ux_1\right),$$

and $\tilde{s}_a(\omega; x_2)$ is the continuous Fourier trasnform of $s(t; x_2)$.

If we let $Y_f(z) = [\tilde{y}_1(z), \ldots, \tilde{y}_f(z)]'$, then we can apply the algorithm of the previous section to $Y_f(z)$, for increasing values f (more and more frequency components), to get estimates of increasing accuracy. In practice, only the first few frequency components are needed.

5 Experimental Results

Fig. 1 is a simulated noisy X-ray projection of a cylindrical blood vessel, with $N = M = 64$. For this example,

$$s(t; r) = 2r\sqrt{1 - (t/r)^2}\, 1_{\{|t|<r\}}.$$

Pseudo random AWGN with standard deviation 4.0 was added to the measurements; the resulting signal to noise ratio (height of projection / noise standard deviation) varied from 2.7 to 4.

Fig. 2 compares the real and estimated shift functions, using only 4 of the 64 frequency components. Despite the severe background noise, the errors are all less than the dimension of a pixel.

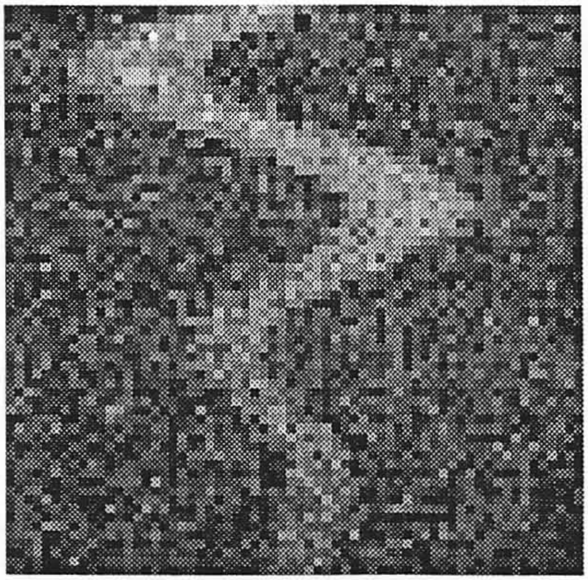

Figure 1: Noisy measurement image.

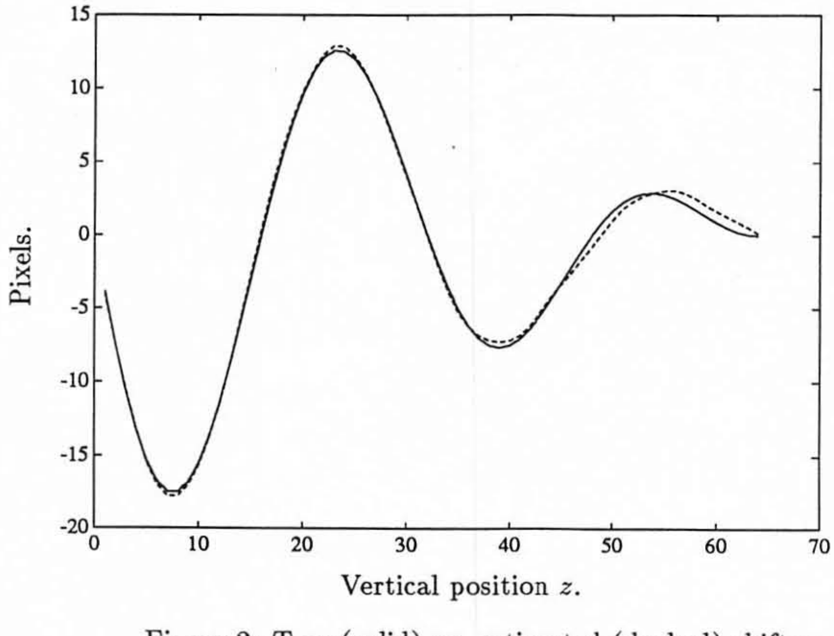

Figure 2: True (solid) vs. estimated (dashed) shift parameter.

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