Dynamic MRI Reconstruction using Low-Rank plus Sparse model with Optimal Rank Regularized Eigen-Shrinkage

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Purpose: Recent advances in dynamic contrast-enhanced MRI (DCE-MRI) algorithms have employed low-rank plus sparse matrix decomposition for joint reconstruction of multicoil data. Such methods model DCE-MRI data as the superposition of low-rank and sparse components because the high spatio-temporal correlations of the static background image are inherently low-rank while the remaining dynamic contrast component is often sparse with respect to an appropriate temporal transformation (*e.g.,* wavelet/Fourier transform). A predominant approach for performing low-rank plus sparse decomposition has been to adopt a low-rank regularization term involving the nuclear norm $\|.\|_*$ and a sparse regularization term involving the ℓ_1 norm $\|.\|_1$. In this work, we replace the popular nuclear norm with a recently developed[1] optimal rank-regularizer derived in the context of random matrix theory. We compare the performance of the resulting algorithm to recent work[2] based on nuclear norm rank regularization in the context of DCE-MRI for cardiac perfusion.

Theory: Low-rank plus sparse reconstruction for DCE-MRI was recently investigated[2] through solving the convex optimization problem

$$\arg\min_{L,S} \frac{1}{2}\|E(L+S)-d\|_F^2 + \lambda_L\|L\|_* + \lambda_S\|TS\|_1$$

where E is the multicoil encoding operator, d is k-space data, T is a sparsifying transformation, and λ_L and λ_S are regularization parameters that control the relative cost of the low-rankness of L and sparsity of S, respectively. The algorithm output is $M = L + S$, a matrix whose columns contain the reconstructed dynamic images. Efficient algorithms for solving this problem employ low-rank updates involving the proximity operator of the nuclear norm - singular value thresholding[3] – applied to a low-rank plus noise matrix. The use of the nuclear norm as a low-rank regularizer is often justified by noting that it is the tightest convex relaxation of the hard rank constraint. However, this does not guarantee that the nuclear norm is optimal. Indeed, it was recently shown[1] that the MMSE low-rank proximity operator is OptShrink, a singular value shrinkage function uniquely determined by the noise spectrum. We leverage this result in DCE-MRI by proposing a modified version of Otazo's algorithm[2] for low-rank plus sparse decomposition where we replace singular value thresholding with the data-driven OptShrink estimator from Algorithm 1 of [1]. In the resulting algorithm, the regularization parameter λ_L is replaced by a parameter r that directly specifies the desired rank of L (often small in practice due to the high spatio-temporal correlation of the background in DCE-MRI). The proposed algorithm can be seen as a first principles alternative to kt-SLR[4], a method that uses the Schatten p-norm $\|.\|_p$ with $p < 1$ as a replacement rank regularizer for $\|.\|_*$.

Methods: We compare the performance of Otazo's algorithm with our proposed algorithm on a cardiac perfusion data set.[2] The data d contains k-space data corresponding to 40 frames, each with resolution 128 x 128, acquired via DCE-MRI with 12 coils and Cartesian sampling. The data was retroactively downsampled by a factor of 8 with random subsampling patterns in k-space for each time point. The corresponding encoding operator E incorporates coil sensitivities and performs FFT operations, and the sparsifying transform T was a temporal FFT. For technical data acquisition specifications, see [2]. Parameters were chosen for each algorithm to yield qualitatively superior images. In particular, we set $r = 2$.

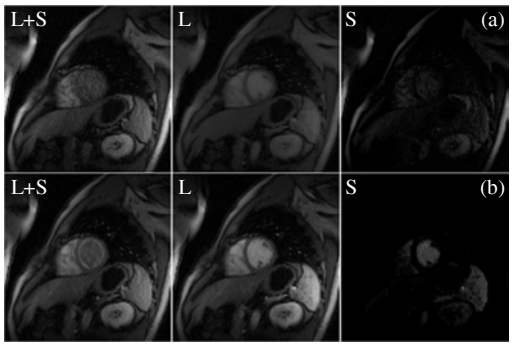
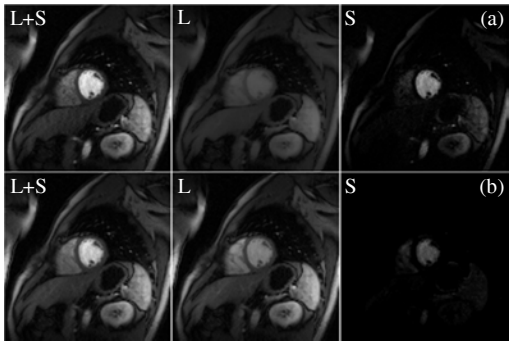


Figure 1: *Cardiac perfusion reconstruction via (a) Otazo's algorithm[2] and (b) the proposed method. In each row, (L+S): reconstructed image; (L): low-rank component; (S): sparse component.* ***Above****: Frame 2/40.* ***Below****: Frame 14/40.*

Results: Figure 1 shows two representative reconstructed frames for each algorithm. The proposed algorithm improves clarity of the myocardial wall. A drawback of singular value thresholding is that all singular values are shrunk uniformly to zero, resulting in unnecessary degradation of high SNR image components. OptShrink[1] avoids this phenomenon by shrinking singular values in an SNR-dependent fashion that maximizes the accuracy of the resulting low-rank image. OptShrink yields sharper low-rank images without sacrificing compressibility (*i.e.,* without increasing the rank of L). Comparing the sparse components of the two approaches suggests that OptShrink more fully exploits the spatio-temporal correlation of the frame backgrounds. Indeed, more frame-independent body regions were absorbed in the low-rank component of the proposed approach, while the resulting sparse component contains a focused visualization of the dynamic contrast enhancement. To quantitatively compare the performance of the algorithms, we retroactively added Gaussian noise over a range of variances to the k-space data and measured the resulting NRMSE of the algorithm outputs, using the output of Otazo's algorithm on the original data as ground truth. Table 1 shows that the proposed approach produces lower NRMSE over the range of noise levels.



Conclusion: We proposed a new optimal rank penalty for low-rank plus sparse decomposition in DCE-MRI. The resulting algorithm was shown to outperform existing techniques in both qualitative image quality and quantitative robustness to noise. Our low-rank penalty preserves the quality of high SNR image features without sacrificing compressibility and produces sparse components with fewer temporally static elements.

Normalized Noise Variance		5	7	9	11	13	15	17	19	21	23	25
NRMSE (%)	**Otazo[2]**	11.2	12.6	14.5	16.8	19.3	21.6	23.0	23.7	24.1	24.9	25.7
	Proposed	10.4	12.1	14.1	16.5	18.1	18.9	20.0	20.9	21.7	22.5	23.7

Table 1: *Comparison of reconstruction errors for cardiac MRI data with additional synthetic noise*

References: [1] R. R. Nadakuditi, "OptShrink: An algorithm for improved low-rank signal matrix denoising by optimal, data-driven singular value shrinkage," preprint, http://arxiv.org/abs/1306.6042, 2013; [2] R. Otazo, E. J. Candès, and D. K. Sodickson, "Low-rank and sparse matrix decomposition for accelerated dynamic MRI with separation of background and dynamic components," preprint, http://www.cai2r.net/research/ls-reconstruction, 2013; [3] E. J. Candès, X. Li, Y. Ma, and J. Wright, "Robust principal component analysis?," J. ACM, vol. 58,no. 3, pp. 11:1–11:37, 2011; [4] S. G. Lingala, Y. Hu, E. DiBella, and M. Jacob, "Accelerated dynamic MRI exploiting sparsity and low-rank structure: kt-SLR," IEEE Trans. Med. Imag., 30(5):1042–1054, 2011.