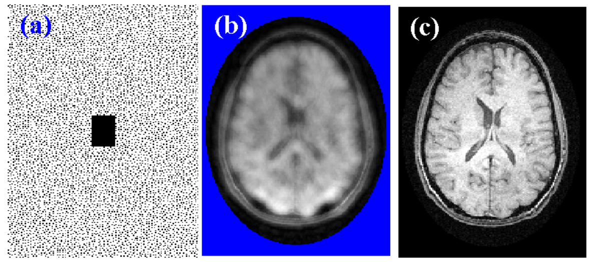
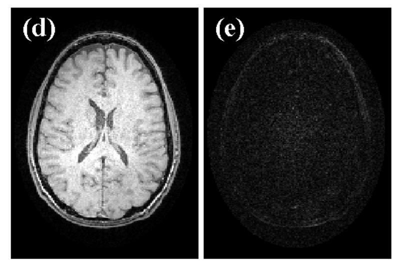
An Efficient Variable Splitting Based Algorithm for Regularized SENSE Reconstruction with Support Constraint

Mai T. Le[1], Sathish Ramani[1], and Jeffrey A. Fessler[1]
[1]*EECS, University of Michigan, Ann Arbor, MI, United States*

Introduction: SENSitivity Encoding (SENSE) based reconstruction for parallel MRI requires regularization for improved image quality at high acceleration (undersampling) factors [1,2]. Sparsity promoting regularizers are attractive from a compressed sensing perspective [3], but they in turn demand computation intensive, non-linear optimization algorithms [2]. Previous algorithms such as the split-Bregman (SB) [4] and those based on the *augmented Lagrangian* (AL) framework [2] reconstructed the entire rectangular image ignoring prior information that patients are not rectangular. In this work, we focus on regularized SENSE reconstruction that explicitly includes a *support constraint* in terms of a *spatial mask* in the problem formulation. We propose a specific *variable splitting* (VS) strategy that when combined with the AL framework and *alternating minimization* (AM), yields an algorithm with simple and *non-iterative* update steps that can be implemented efficiently. We present numerical results with real in-vivo data to demonstrate the efficacy of the proposed method.



Problem Formation: We formulate regularized SENSE reconstruction with support constraint as the following optimization problem: $\mathbf{x}^{(*)} = \mathit{argmin}_{\mathbf{x}}\|\mathbf{y} - \mathbf{ASMx}\|^2 + \lambda\|\mathbf{RMx}\|_1$, where $\mathbf{y}$ is the undersampled k-space data from all coils, $\mathbf{A}$ is a block diagonal matrix consisting of undersampled DFT matrices, $\mathbf{S}$ is a stack of diagonal matrices containing the sensitivity maps, $\mathbf{M}$ is an undersampled identity matrix denoting the user-selected image (support) mask, $\mathbf{R}$ is a regularization operator (e.g., finite differences) such that $\mathbf{R'R}$ is circulant, λ is the regularization parameter, and $\mathbf{x}^{(*)}$ is the reconstructed image. The proposed algorithm can also tackle other types of convex regularizers such as total variation.



Method: We reformulate the above problem as an equivalent constrained optimization task via VS as $\mathit{min}\ \|\mathbf{y} - \mathbf{Au}\|^2 + \|\mathbf{z}\|_1$ s.t. $\mathbf{u} = \mathbf{SMx}$, $\mathbf{v} = \mathbf{Mx}$, $\mathbf{z} = \mathbf{Rv}$. We now employ the AL framework discussed in Sec. IV-B of [2] with AM. This leads to an iterative algorithm like the one in Sec. IV-B of [2] but involving inversion of the following matrices at each iteration: $\mathbf{H} = \mathbf{A'A} + \alpha\mathbf{I}$, and $\mathbf{G} = \mathbf{R'R} + \beta\mathbf{I}$ and $\mathbf{K} = \mathbf{M'S'SM} + \gamma\mathbf{I}$, where $\alpha, \beta, \gamma > 0$ are penalty parameters like those in Sec. IV-B of [2]. Due to the design of the above splitting, these matrices can be inverted non-iteratively, e.g., $\mathbf{H}$ and $\mathbf{G}$ can be inverted using FFTs, while $\mathbf{K}$ is diagonal. In comparison, the algorithm **AL-P2** in [2] for the above reconstruction problem with support constraint involves (among other simple update steps) the inversion of $\mathbf{G_1} = \mathbf{M'R'RM} + \beta\mathbf{I}$ which is not circulant; the large size of $\mathbf{G_1}$ for typical reconstruction setups precludes direct inversion and the corresponding variable update in **AL-P2** requires an iterative method. We chose to execute that variable update by running the preconditioned conjugate gradient (PCG) method with the inverse of the circulant matrix $\mathbf{R'R} + \beta\mathbf{I}$ as the preconditioner.

Results: We used a 3D in-vivo human data set acquired from a GE 3T scanner (T_R = 25 ms, T_E = 5.172 ms, and voxel size = 1x1.35x1mm^3) with an 8-channel coil. The slices were 256x144 in size with 128 samples in the read-out direction. We performed regularized SENSE reconstruction of a single 2D slice from undersampled data. We used a Poisson-disk-based (nearly random) undersampling pattern (reduction factor ≈ 5.65, Fig. 1a) in the phase-encode plane that included the central 32x18 phase-encodes. We used the central phase-encodes to generate low resolution images that were then used to estimate smooth sensitivity maps using the method in [5]. We compared the following algorithms: **SB-*n***, with ***n*** CG inner iterations, **AL-P2-*n*** with ***n*** PCG inner iterations, the Chambolle-Pock Primal Dual Algorithm (**CP-PDA**) [6] and the proposed method, **AL-Mask**. To measure the convergence rate of these algorithms, we computed the normalized root mean squared-distance (NRMSD) between a given iterate and the solution, $\mathbf{x}^{(*)}$, that was in turn obtained by running **SB-10** for 2000 iterations. We then performed a convergence rate comparison of **CP-PDA**, **SB-5**, **AL-P2-5**, **AL-Mask** (Fig. 1f). For fair comparison, the penalty parameters for all these algorithms were chosen such that the threshold levels were the same in all the algorithms for the l_1-regularizer. We used finite differences for $\mathbf{R}$ and adjusted λ manually. The mask $\mathbf{M}$ was chosen to be an ellipse that snugly fit the object, based on the sum-of-squares (SoS) of the zero-filled iFFT-reconstructed coil images (Fig. 1b); we also used this SoS estimate to initialize all algorithms. Aliasing effects and blur in the SoS estimate are suppressed in the regularized reconstruction (Fig. 1d), which is visually similar to the body coil image reconstructed from the fully sampled data (Fig.1c). Fig. 1f indicates that **AL-Mask** converges to $\mathbf{x}^{(*)}$ faster than the others. The increase in speed is due to the quick, non-iterative update that efficiently inverts the circulant matrix $\mathbf{G}$ using FFTs. We observed similar promising results for reconstruction of other slices in this volume.

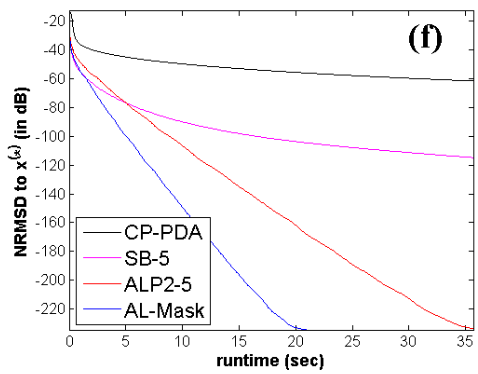


Fig.1 *Experiment with a 2D slice of an in-vivo human brain dataset. (a) Poisson disk undersampling pattern; (b) Mask superimposed on initial estimate; (c) Body-coil image (d) Regularized SENSE-reconstruction $x^{(*)}$; (e) Absolute difference between (c) and (d); (f) RMSD versus runtime for AL-Mask and competing algorithms*

Conclusion: We proposed a VS- and AL-based iterative algorithm that admits non-iterative update steps for Regularized SENSE reconstruction with support constraint. The proposed method is simple to implement, converges faster than some of the existing state-of-the-art VS-based methods for the same problem, and is attractive for 3D reconstruction.

References: [1] L. Ying *et al*, *MRM,* 60:414-21, 2008; [2] S. Ramani *et al*, *IEEE,* 30:694-706, 2011; [3] Lustig *et al*, *MRM*, 58:1182-1195, 2007; [4] T. Goldstein *et al*, *J. Img. Sci*., 2:323-343, 2009; [5] M.J. Allison *et al*, *ISBI*, 394:397, 2012; [6] A. Chambolle *et al*, *MIV*, 40:120-145, 2011. This work is supported by NIH/NCI P01 CA87634.