An Analytic Description of Steady-State Imaging with Dual RF Pulses and Gradient Spoiling

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Introduction: Small-tip fast recovery (STFR) imaging is a recently proposed steady-state sequence that has similar T2/T1 contrast as bSSFP but has the potential to simultaneously remove banding artifacts and transient fluctuations [1,2]. At the end of each TR, the STFR method tips the magnetization back to the longitudinal axis using a tailored RF pulse that is designed according to the local off-resonance as determined by a measured B0 map of the imaging slice (Fig. 1). After the tip-up pulse, a gradient spoiler is applied. We are developing two versions of the STFR sequence, one that uses RF spoiling (applies linear phase increment to the RF pulse), which we call RF-STFR, and another that uses gradient spoiling only, called G-STFR. Our Bloch simulations demonstrate that the G-STFR sequence is less sensitive to tip-up phase mismatch than RF-STFR, however the analysis of G-STFR is complicated by the need to account for TR-to-TR transverse signal pathways (ideal spoiling cannot be assumed) [3]. Using symbolic computation, we have derived an analytic expression for the G-STFR signal and we have verified the equation with simulations and phantom data. This new signal equation is useful for analyzing the properties of G-STFR, and potentially for model-based image reconstruction and quantitative imaging applications (e.g. T1 and T2 mapping).

Theory & Methods: The analytical signal model for G-STFR is derived by first developing the expression for the steady state transverse magnetization $M_1(\phi)$ for a single isochromat (defined by the local phase ϕ induced by the gradient crusher after the tip-up pulse). We then integrate $M_1(\phi)$ over 2π, to obtain the total voxel signal. The simplified spin path for a single isochromat is illustrated in Fig.1. M_1 to M_2 is the free procession period, M_2 to M_3 is caused by the tip-up pulse, M_3 to M_4 is a rotation around z-axis caused by the gradient crusher, and M_4 to M_1 is caused by the tip down pulse. By modeling each part of the spin path with the Bloch equation and then combining them together, we can get an expression for M_1. In the ideal case, the difference in phase between the tailored tip-up pulse and tip-down pulse should match with the accumulated phase $\theta(x,y)$ during the free procession period. But in reality, there may be a phase mismatch θ_m between them. In our derivation, we will consider a spin that precesses θ_m, but the phase of the actual tip up pulse and tip down pulse are both 0. Define β and - β to be the flip angle of tip-down pulse and tip-up pulse respectively, T_{free} as the time between the tip-down pulse and the tip-up pulse, $E_1 = e^{-\frac{T_R}{T_1}}$, and $E_2 = e^{-\frac{T_{free}}{T_2}}$. The steady-state signal for a single spin is:

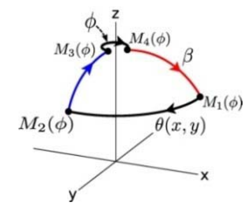

Fig 1: Spin path for G-STFR

$$\mathrm{M}_1(\phi) = \frac{a*\cos\phi + b*\sin\phi + c}{d*\cos\phi + e*\sin\phi + f} \qquad [1]$$

$$a = -2i(1-E_1)\sin 2\beta\left(1 + E_2 e^{i\theta_m}\right)$$
$$b = 4i(1-E_1)\sin\beta\left(E_2 e^{i\theta_m} - 1\right)$$
$$c = 2i(1-E_1)\sin 2\beta\left(1 + E_2 e^{i\theta_m}\right)$$
$$d = -4\sin^2\beta(E_1 - E_2^2) + (E_1 - 1)E_2\cos\theta_m(6 + 2\cos 2\beta)$$
$$e = -8(E_1 - 1)E_2\cos\beta\sin\theta_m$$
$$f = (E_1 - 1)E_2\cos(2 - 2\cos 2\beta) - 4(E_1 - E_2^2)\cos^2\beta - 4E_1E_2^2 + 4$$

Equation [1] reflects the relation between the transverse magnetization of a single spin and the accumulated phase induced by gradient spoiling, which is plotted and validated by Bloch simulation in Fig. 2 for θ_m= 60º. To get the signal strength M_t for a voxel, we integrate $M_1(\phi)$ over ϕ from 0 to 2π, which means integrating the curve in Fig. 2. This was done by Matlab symbolic integration. The final result is:

$$\begin{aligned} M_t &= \frac{1}{2\pi}\int_0^{2\pi} \mathrm{M}_1(\phi)d \\ &= \frac{1}{2\pi}\int_0^{2\pi} \frac{a*\cos\phi + b*\sin\phi + c}{d*\cos\phi + e*\sin\phi + f}\,d\phi \\ &= \frac{1}{2\pi}\int_0^{2\pi} \frac{a*\cos\phi + b*\sin\phi + c}{\sqrt{d^2+e^2}\cos(\phi - \tan^{-1}(\frac{e}{d})) + f}\,d\phi \\ &= \frac{c}{\sqrt{f^2-d^2-e^2}} - \frac{ad+be}{d^2+e^2}\left(\frac{f-\sqrt{f^2-d^2-e^2}}{\sqrt{f^2-d^2-e^2}}\right) \qquad [2] \end{aligned}$$

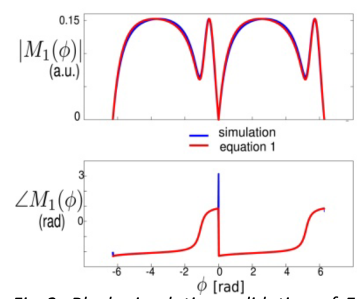

Fig 2: Bloch simulation validation of Eq. [1]. Transverse magnetization $M_1(\phi)$ for a spin isochromat, as a function of the gradient-induced precession ϕ.

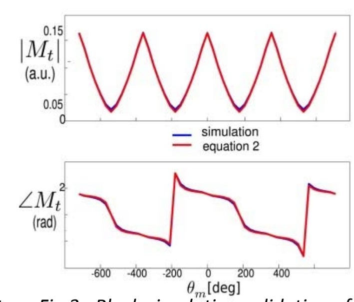

Fig 3: Bloch simulation validation of Eq. [2]. Total voxel signal as a function of phase mismatch θ_m.

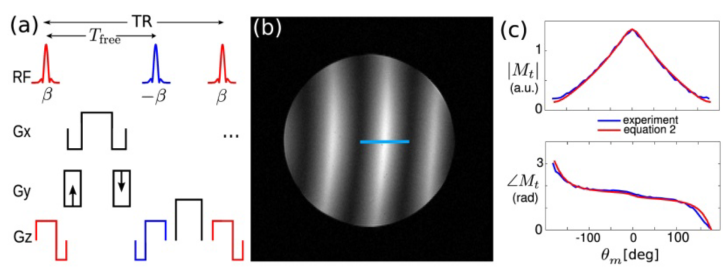

Fig 4: Experimental validation of Eq. [2], the total voxel signal. (a) Pulse sequence (b) G-STFR phantom image (c) Signal profile along the blue line in (b). (T1/T2 = 500/50ms, T_{free}/TR =7/10ms, β = 16º)

Results: Equation [2] is verified by Bloch simulation, which is shown in Fig.3. As we can see, the analytic plot agrees simulation. A phantom experiment was also conducted by applying a linear gradient shim to a gel phantom and then imaging the phantom using the G-STFR sequence (Fig.4a). Experimental and analytic profiles are compared in Fig. 4(c), and we observe good agreement in both magnitude and phase.

Discussion and Conclusion: An analytic expression for the G-STFR is derived and validated by simulations and phantom data. This expression may also be used to analyze the extended chimera SSFP sequence [4], which uses the triangular signal profile as a source of contrast.

References: [1] Heilman et al, ISMRM 2009; [2] Nielsen et al, ISMRM 2010; [3] Scheffler et al, NMR Biomed (2001); [4] Bieri et al, ISMRM 2009