Separate Magnitude and Phase Regularization via Compressed Sensing

F. Zhao[1], J. A. Fessler[2], J-F. Nielsen[1], and D. C. Noll[1]
[1]Biomedical Engineering, University of Michigan, Ann Arbor, Michigan, United States, [2]Electrical Engineering, University of Michigan

Introduction: Compressed sensing (CS) [3] is a general framework for speeding up MRI acquisition, but its use in applications with rapid spatial phase variations is challenging, e.g. B0 map estimation, PRF-shift thermometry [2], and velocity mapping [4]. Previously, an iterative MRI reconstruction with separate magnitude and phase regularization [1] was proposed for applications when magnitude and phase maps are both of interest, but it requires fully sampled data and unwrapped phase maps. In this work, we extend [1] by using CS to speed up the imaging acquisition, and using new phase regularization terms and optimization algorithms to improve its performance in imaging applications with wrapped phase. The method is demonstrated by simulated thermometry data and real velocity mapping data.

Theory: The cost function of conventional CS is $\Psi(\boldsymbol{f}) = \|\boldsymbol{y} - A\boldsymbol{f}\|^2 + \beta\|U\boldsymbol{f}\|_1$, where $\boldsymbol{y}$ is randomly sampled k-space data, $\boldsymbol{f}$ is the complex image, A is Fourier transform matrix, U is the sparse transform matrix (e.g. wavelet transform), $\|\cdot\|$ and $\|\cdot\|_1$ denote L2 and L1 norm, and β is a scalar regularization parameter; in contrast, our cost function is $\Psi(\boldsymbol{x}, \boldsymbol{m}) = \left\|\boldsymbol{y} - A\boldsymbol{m}e^{i\boldsymbol{x}}\right\|^2 + \beta_1 R(\boldsymbol{x}) + \beta_2\|U\boldsymbol{m}\|_1$, where $\boldsymbol{m}$ and $\boldsymbol{x}$ are magnitude and phase of $\boldsymbol{f}$, β_1 and β_2 are scalar regularization parameters, and $R(\cdot)$ is the regularizer for phase. Conventional CS works by exploiting sparsity of complex images in the sparse transform domain. This sparsity needs to be intensified by phase correction when complex images contain high phase variation [3]. Thus, our method, which enforces sparsity directly for magnitude, potentially works better. Meanwhile, features of the phase are exploited by $R(\cdot)$, which is $\|C\boldsymbol{x}\|^2$ in [1] (C is finite differencing matrix which penalizes roughness) assuming phase map is smooth. However, it cannot handle big jumps in the wrapped phase map, due to non-convexity of the cost function for x. Alternatively, we propose $R(\boldsymbol{x}) = \left\|Ce^{ix}\right\|^2$, which can solve this problem and approximates $\|C\boldsymbol{x}\|^2$ very well when neighboring difference is below 1 radian (Fig.1). Moreover, this method is general enough to design different regularizers for specific types of phase maps. For example, we designed a regularizer for applications that have distinct areas on top of smooth background in the phase map, e.g. hot spots in temperature maps and arteries in velocity maps; the regularizer is designed to be edge-preserving, that is, $R(\boldsymbol{x}) = \sum_{k=1}^{K}\psi\left(\left|[Ce^{ix}]_k\right|\right)$ where $\psi(\cdot)$ is an edge-preserving potential function and k is the index of pairs of neighboring pixels. Fig.1 shows the edge-preserving property of this regularizer. We estimate $\boldsymbol{x}$ *and* $\boldsymbol{m}$ from data $\boldsymbol{y}$ by minimizing $\Psi(\boldsymbol{x}, \boldsymbol{m})$. In each iteration, we alternate updating $\boldsymbol{x}$ and $\boldsymbol{m}$ where magnitude is optimized by iterative soft threshholding algorithm [5] and phase by conjugate gradient with monotonic line search (CG-MLS) [6]. Here we use CG-MLS because it converges much faster than the separable optimization transfer algorithm used in [1] for such particular problems.

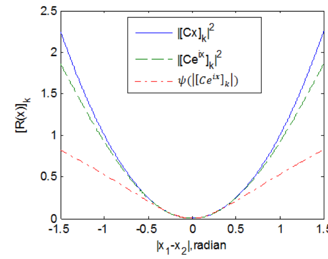

Fig. 1: comparison of 3 regularizers; $\psi(\cdot)$ is hyperbola function (δ=0.3)

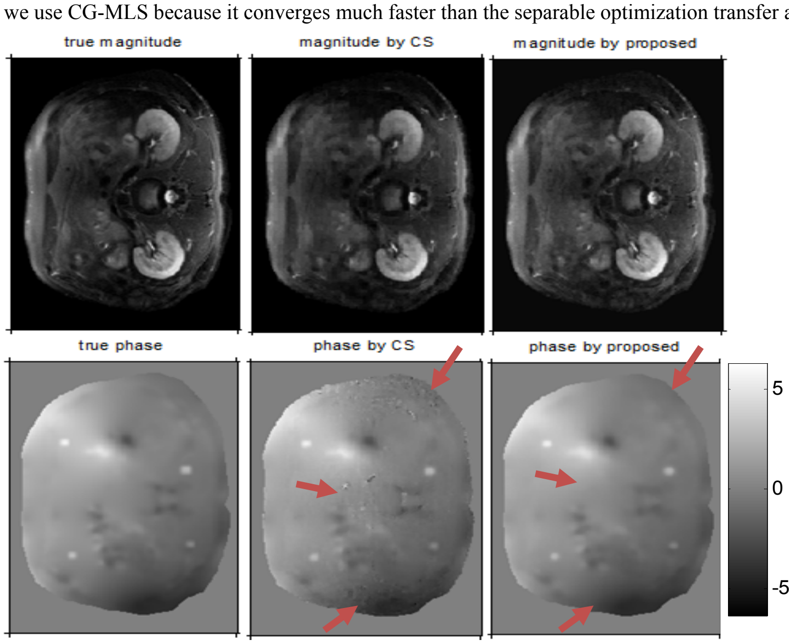

Fig.2: 0.4 sampling rate, background is masked out, the unit of phase is radian

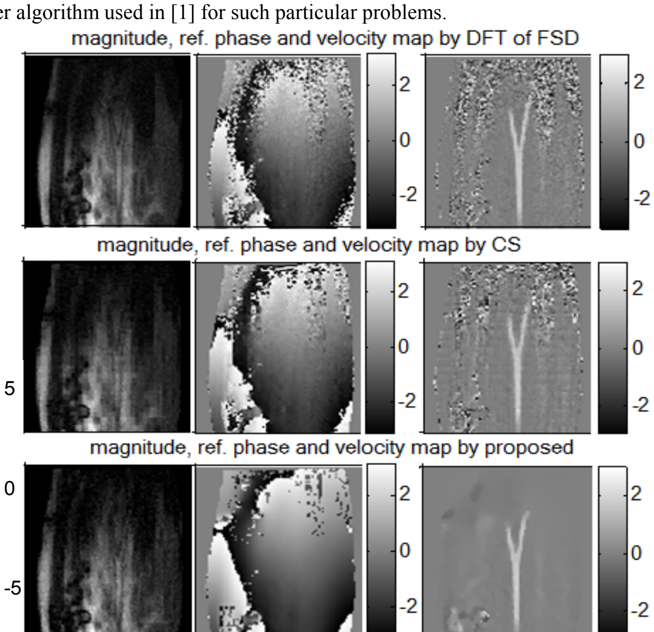

Fig. 4: the units of 2nd and 3rd columns are radian and cm/s respectively

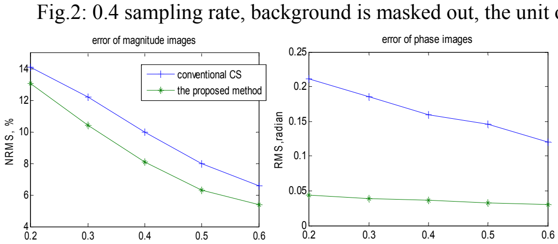

Fig.3: NRMS or RMS at different sampling rate

Methods and Results: First, we tested this method by a simulated thermometry data from an abdomen magnitude image and its smooth field map [7]. The field map is scaled to be $(-2\pi, 2\pi)$ to test performance of the proposed algorithm for wrapped phase maps; and then four "hot spots" are manually added onto this smooth phase map to simulate thermal ablation. Both magnitude and phase maps are cropped to be 320*208 matrix (40cm *26cm FOV). A single coil k-space data is simulated by randomly sampling in Cartesian grid with complex Gaussian noise added (SNR≈ $24dB$). $R(x) = \sum_{k=1}^{K}\psi\left(\left|[Ce^{ix}]_k\right|\right)$ where $\psi(t) = \delta^2\left(\sqrt{1 + |t/\delta|^2} - 1\right)$ with $\delta = 0.15$, and U is wavelet transform. The proposed method was compared with conventional phase corrected CS at different sampling rates. Images shown in Fig. 2 demonstrate that the proposed method produces much cleaner phase maps than convention CS and almost the same magnitude images. MSE plots in Fig. 3 show the results at different sampling rates in which magnitude images are also improved but not very visible in images. We also collected real velocity mapping data of the abdominal aorta (160*160 Cartesian grid with FOV of 16*16 cm) by phase-contrast bSSFP sequence in 3T GE scanner. We used the 1st frame (as reference frame) and the 6th frame of the data to test the algorithm; here $R(x) = \sum_{k=1}^{K}\psi\left(\left|[Ce^{ix}]_k\right|\right)$ with the same potential function was used for velocity mapping and $R(x) = \left\|Ce^{ix}\right\|^2$ was for reference frame, and U is wavelet transform in both. Fig. 4 shows magnitude maps, reference phase maps (wrapped) and the final velocity maps reconstructed by (a) DFT on fully sampled data (FSD), (b) conventional CS and (c) the proposed method on undersampled data (33% sampled). Good magnitude image and much better reference phase and velocity maps are reconstructed by the proposed method.

Conclusions: The proposed method can produce an improved phase map (thermometry or velocity map) while preserving a good magnitude map on undersampled data, compared to conventional phase corrected CS.

References: [1]Fessler et al. IEEE ISBI, 2004 [2]Poorter et al. MRM, 1995 [3]Lustig et al. MRM, 2007 [4] Nielsen et al. MRM, 2009 [5]Daubechies et al. Comm. Pure Appl. Math, 2004 [6]Fessler et al. IEEE Trans. Im. Proc.1999 [7] Data from "ISMRM Reconstruction Challenge 2010"