An Augmented Lagrangian Method for MR Coil Sensitivity Estimation

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INTRODUCTION

Accurate coil sensitivity estimates are necessary to avoid artifacts from parallel imaging techniques such as SENSitivity Encoded reconstruction (SENSE) [1]. The simplest sensitivity estimate is the quotient of an image obtained using the surface coil (**z**) and an image obtained using a body coil (**b**) with near uniform sensitivity over the field-of-view (FOV), **z**/**b**. However, such an estimate will be highly corrupted in areas of low signal and will have discontinuities at object edges, contrary to the smooth nature of true coil sensitivities. Several improved sensitivity estimation methods have been developed (see [2] for a summary), the most accurate of these being regularized methods such as the variational approach [2] and inpainting [3]. These regularized methods require significant computation in cases of high noise or when the FOV has large regions of low signal. We therefore propose a method that uses the Augmented Lagrangian (AL) formalism to efficiently compute regularized estimates. We also present an orthonormal polynomial fitting procedure that provides cost effective initializations for iterative estimation methods.

METHODS

We pose the sensitivity estimation problem as the minimization of a regularized cost function, **C0**: $\hat{\mathbf{s}} = \operatorname{argmin}_{\mathbf{s}}\ 1/2\,\|\mathbf{z} - \mathbf{DMs}\|^2 + \lambda\|\mathbf{Rs}\|^2$, where **s** is the coil sensitivity, $\mathbf{D} = \operatorname{diag}\{\mathbf{b}\}$ is a diagonal matrix, λ is a regularization parameter, **R** corresponds to a finite differencing matrix, and $\mathbf{M} = \operatorname{diag}\{\boldsymbol{\phi}\}$ is a binary mask with $\phi_i = 1$ for object pixels. The mask helps ensure that the estimate is based only on pixels that provide meaningful sensitivity information. As in [2], we use quadratic regularization as it provides accurate estimates while enabling efficient implementations. The non-iterative solution to **C0** requires the inversion of a large matrix, thus we propose an iterative AL solution method.

We rewrite **C0** as the equivalent constrained optimization problem **C1**: $\hat{\mathbf{s}} = \operatorname{argmin}_{\mathbf{s},\mathbf{u_0},\mathbf{u_1}}\ 1/2\,\|\mathbf{z} - \mathbf{DMs}\|^2 + \lambda\|\mathbf{u_0}\|^2$ subject to $\mathbf{u_0} = \mathbf{Ru_1}$ and $\mathbf{u_1} = \mathbf{s}$. Following [4], we then construct an AL function for **C1** as $\mathrm{L}(\mathbf{u}, \boldsymbol{\eta}, \mu) = 1/2\,\|\mathbf{z} - \mathbf{DMs}\|^2 + \lambda\|\mathbf{u_0}\|^2 + \mu\nu_0/2\,\|\mathbf{u_0} - \mathbf{Ru_1} - \boldsymbol{\eta}_\mathbf{0}\|^2 + \mu\nu_1/2\,\|\mathbf{u_1} - \mathbf{s} - \boldsymbol{\eta}_\mathbf{1}\|^2$, where $\boldsymbol{\eta}=[\boldsymbol{\eta}_\mathbf{0}, \boldsymbol{\eta}_\mathbf{1}]$ are Lagrange multipliers, $\mathbf{u}=[\mathbf{s}, \mathbf{u_0}, \mathbf{u_1}]$, and $\mu, \nu_0, \nu_1 > 0$ are scalar parameters that control the convergence properties. We use an alternating minimization scheme to solve the AL function resulting in the final estimation algorithm (see below). In step 2 of the algorithm, we avoid inverting a large matrix by approximating the update using a few iterations of the conjugate gradient (CG) method with a circulant pre-conditioner. Step 3 can be efficiently computed as the matrix requiring inversion is diagonal. We choose the AL parameters by first setting $\nu_1 = 0.05$ and then selecting ν_0 and μ so that the condition numbers of the matrices in step 2 and step 3 are 120 and 10, respectively. These parameter values have worked well for several distinct datasets.

To reduce the convergence time of the iterative methods, we initialize with an estimate based on polynomial fitting. We avoid computing the ratio **z**/**b** by posing the initialization problem as $\hat{\boldsymbol{\theta}} = \operatorname{argmin}_{\boldsymbol{\theta}}\ \|\mathbf{M}(\mathbf{z} - \mathbf{DP}\boldsymbol{\theta})\|^2$, where $\mathbf{P} = [\mathbf{p_1}, \mathbf{p_2}, \ldots, \mathbf{p_n}]$ is a matrix of 2D-Chebyshev polynomials of the first kind. The least squares solution to this problem is well conditioned due to the orthonormality of the Chebyshev polynomials.

AL Estimation Method

Set $\mathbf{s}^{(0)}$, $\mathbf{u}_1^{(0)}$ *to the initialization image. Set* $\boldsymbol{\eta}_0^{(0)}, \boldsymbol{\eta}_1^{(0)}$, j *to zeros.*
Repeat until stop criterion is achieved.

1. $\mathbf{u_0}^{(j+1)} = \left(\mathbf{Ru}_1^{(j)} + \boldsymbol{\eta}_0^{(j)}\right)\mu\nu_0/(\mu\nu_0 + 2\lambda)$,
2. $\mathbf{u_1}^{(j+1)} = \operatorname{argmin}_{\mathbf{u_1}}\ \mu\nu_0 \left\|\mathbf{u_0}^{(j+1)} - \mathbf{Ru_1} - \boldsymbol{\eta}_0^{(j)}\right\|^2 + \mu\nu_1\left\|\mathbf{u_1} - \mathbf{s}^{(j)} - \boldsymbol{\eta}_1^{(j)}\right\|^2$,
3. $\mathbf{s}^{(j+1)} = \left((\mathbf{DM})^\mathbf{H}\mathbf{DM} + \mu\nu_1\mathbf{I}_\mathrm{N}\right)^{-1}\left[(\mathbf{DM})^\mathbf{H}\mathbf{z} + \mu\nu_1\left(\mathbf{u}_1^{(j+1)} - \boldsymbol{\eta}_1^{(j)}\right)\right]$,
4. $\boldsymbol{\eta}_0^{(j+1)} = \boldsymbol{\eta}_0^{(j)} - \left(\mathbf{u_0}^{(j+1)} - \mathbf{Ru}_1^{(j+1)}\right)$,
5. $\boldsymbol{\eta}_1^{(j+1)} = \boldsymbol{\eta}_1^{(j)} - \left(\mathbf{u_1}^{(j+1)} - \mathbf{s}^{(j+1)}\right)$,
6. $\mathrm{j} = \mathrm{j} + 1$.

RESULTS

We evaluated our algorithm with a set of contrast-enhanced breast phantom images acquired using four surface coils and one body coil and reconstructed with an iFFT (Philips 3T, TR = 4.6 ms, matrix = 94 × 384). Figs. 1b and 1c present the body coil image and a representative surface coil image, respectively. This dataset is particularly challenging for sensitivity estimation due to the large regions with both low and zero signal. Based on extensive simulation, we chose **R** to be a second order finite differencing matrix, $\lambda = 2^6$, and selected **M** by thresholding the body coil image. Estimates of the coil sensitivities were found using both the AL method and by applying CG to **C0**. For step 2 of the AL method, one iteration of CG provided the fastest overall convergence time. Polynomials of up to degree three provided the best initialization for both the AL and CG algorithms (Fig. 1d), saving 40 seconds of computation time. Fig. 1e presents the resulting AL estimate for the surface coil in Fig. 1c. The four AL estimates were used in a SENSE reconstruction with acceleration factor of two. The resulting reconstructed image contained no noticeable artifacts, indicating sufficient estimation accuracy. Since the true coil sensitivities are unknown, we ran both algorithms to convergence and used an average of their final estimates as the "truth", $\mathbf{s}^*$. Fig. 1a compares the convergence properties of the AL and CG algorithms by plotting the normalized L_2-distance between the estimate and the converged image ($\|\mathbf{s}^{(j)} - \mathbf{s}^*\|_2/\|\mathbf{s}^*\|_2$) against the elapsed time including initialization. It is apparent that the AL algorithm converged in approximately half the time of the CG method. Similar results (not shown) were obtained for the other breast phantom surface coils, as well as for simulated brain data with known sensitivity maps.

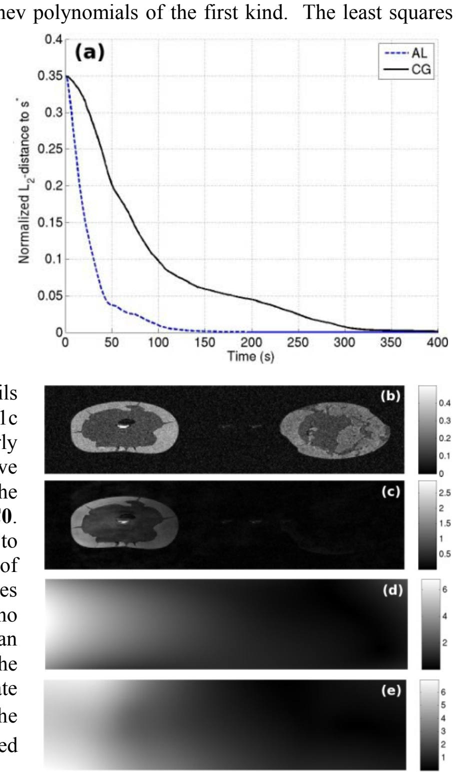

Fig. 1: *Results of experiment with breast phantom dataset. (a) Plot comparing the convergence rates of the AL and CG algorithms. (b) Body coil image. (c) One surface coil image. (d) Polynomial fit initialization image. (e) AL sensitivity estimate.*

CONCLUSIONS

Our Augmented Lagrangian estimation method provides accurate coil sensitivity estimates even in challenging cases where the images have large regions of low signal. Furthermore, the AL method exhibits significantly improved convergence times compared to CG. We have also developed an efficient initialization method that can notably reduce the time to convergence.

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REFERENCES

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