Non-convex greedy compressed sensing for phase contrast MRI

D. Yoon[1], J. Fessler[1], J-F. Nielsen[2], A. Gilbert[3], and D. Noll[2]
[1]Electrical Engineering, University of Michigan, Ann Arbor, Michigan, United States, [2]Biomedical Engineering, University of Michigan, [3]Mathematics, University of Michigan

Introduction

We propose a novel compressed sensing algorithm for Phase Contrast MRI(PC-MRI) to estimate blood flow velocities. Blood flow velocities provide clinically useful information such as pressure gradients and PC-MRI has become an established technique to measure them. In conventional PC-MRI, velocity information is computed by comparing the phases of the velocity-encoded image and the reference image without velocity encoding. This procedure requires multiple scans of the imaged object, which is time-intensive. For example, it takes about 20 minutes to cover a 3D volume of 16x12x6cm^3 with a previously proposed PC-MRI sequence[1]. We have observed empirically that velocity encoding brings about phase changes only in blood vessel regions, which are sparse in the image domain[2]. Exploiting this sparse phase differences, we developed a non-convex greedy compressed sensing image reconstruction algorithm to accelerate the acquisition of velocity encoded images. Simulation results show that with random k-space sampling, our algorithm can perform well even with a high undersampling factor 15. We have also investigated an alternative convex optimization approach and compared its performance with our greedy algorithm. The simulation results show that our proposed greedy algorithm is more robust in high undersampling factors compared to the convex optimization method.

Theory

Let x_{ref} denote the reference image acquired without velocity encoding. The signal equation for the k-space data, y, of a velocity encoded object in the matrix-vector form can be modeled as $\mathbf{y} = \mathbf{F}\cdot[x_{ref}e^{i\boldsymbol{\theta}}] = \mathbf{A}[e^{i\boldsymbol{\theta}}]$, $\mathbf{A} = \mathbf{F}\cdot\mathbf{Diag}(x_{ref})$ where F is a undersampled Fourier encoding matrix, Diag(x_{ref}) is a diagonal matrix with entries populated with x_{ref}, $\boldsymbol{\theta}$ is the unknown velocity encoding phase, and $[e^{i\boldsymbol{\theta}}]$ is a column vector representation of $e^{i\boldsymbol{\theta}}$ (each entry corresponds to $e^{i\boldsymbol{\theta}}$ for each entry of $\boldsymbol{\theta}$).We assume $\boldsymbol{\theta}$ is sparse due to sparse blood vessel distributions but $[e^{i\boldsymbol{\theta}}]$ is not as most of its entries are 1. To generate a sparse parameter vector, we restate the problem as $\mathbf{y} - \mathbf{A[1]} = \mathbf{A}\cdot[e^{i\boldsymbol{\theta}} - 1]$ where [1] is a column vector of entries 1. Rewriting this with a change of variables yields $\hat{\mathbf{y}} = \mathbf{A}\hat{\mathbf{x}}$ where $\hat{\mathbf{y}} = \mathbf{y} - \mathbf{A[1]}$ and $\hat{\mathbf{x}} = [e^{i\boldsymbol{\theta}} - 1]$. Now $\hat{\mathbf{x}}$ is a sparse vector so we may apply compressed sensing, but we have to change the optimization procedure in order to constrain the entries of $\hat{\mathbf{x}}$ to be $[e^{i\boldsymbol{\theta}} - 1]$. We modified a greedy algorithm[3] for this purpose as described on the left. The algorithm iteratively selects columns of A until the chosen columns span most of $\hat{\mathbf{y}}$. In step (I), we estimate how much each column can span the residual of $\hat{\mathbf{y}}$ with a coefficient in the constrained form and select the one spanning the most in step (II). In step (III) we project $\hat{\mathbf{y}}$ onto the selected columns with constraints on their coefficient set and update the residual. Step (I) has a closed form solution $\theta_j = angle((\mathbf{r}+\mathbf{a_j},\mathbf{a_j}))$. In step (III), we iteratively update the intermediate solution by linearizing the equation with respect to a small update as in [4]. We have also noticed that the difference image, Δx, between the reference image and velocity encoded image is sparse, and implemented a convex optimization method expressed as $\min\|\Delta\mathbf{x}\|_1 \; s.t. \; \|\mathbf{y} - \mathbf{A}(\mathbf{x_{ref}} + \Delta\mathbf{x})\|_2 \le \varepsilon$, and solved it with the spgl1 package[5] We compared its performance with our non-convex greedy method.

Set $\Lambda = \{\}$ $\quad \mathbf{r} = \hat{\mathbf{y}}$

While $\|\mathbf{r}\| < error\_threshold$

I) For each $j \notin \Lambda$, find $\theta_j = \arg\min_{\omega} \|\mathbf{r} - \mathbf{a_j}(e^{i\omega} - 1)\|$

where $\mathbf{a_j}$ is the j-th column of $\mathbf{A}$

II) Find $\hat{j} = \arg\min_{j\notin\Lambda} \|\mathbf{r} - \mathbf{a_j}(e^{i\theta_j} - 1)\|$ and add $\hat{j}$ to Λ

III) do $\min_{\theta_j \; \forall j\in\Lambda} \left\|\hat{\mathbf{y}} - \sum_{j\in\Lambda}\mathbf{a_j}(e^{i\theta_j} - 1)\right\|$ and update $\mathbf{r}$ as

the minimization residual.

End

Simulation Results and Discussion

We reconstructed the velocity encoded image, $x_{ref}e^{i\boldsymbol{\theta}}$, with the greedy method and the convex method with 2D random k-space sampling having undersampling factors ranging from 1 to 18. For each undersampling factor, we generated ten k-space sampling instances to quantify average performance of each method. We computed the root mean squared error(RMSE) of the intra-vascular phase and the normalized root mean squared error(NRMSE) of the estimated image. Fig.1 shows that the average RMSE plot of our greedy method works well with a high undersampling factor 15 while the convex method starts to break down around an undersampling factor between 6-8. The NRMSE plot is not presented, but it showed a very similar pattern with the RMSE plot.

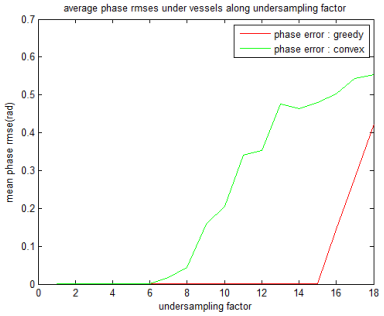

Figure 1. RMSEs v.s. undersampling factor

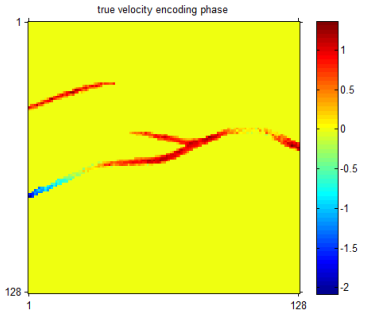

Figure 2. Truth set for $\boldsymbol{\theta}$

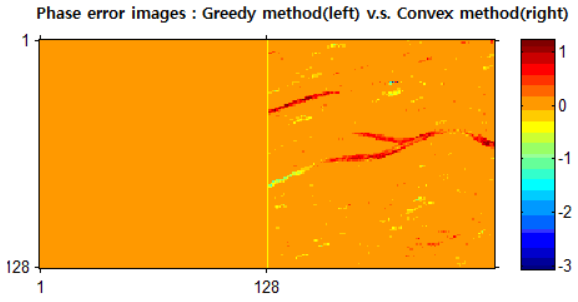

Figure 3. differences in radian between the true $\boldsymbol{\theta}$ and the estimated $\boldsymbol{\theta}$ from the greedy method(left) and the convex method(right) with undersampling factor 15

Conclusion

We have demonstrated in simulation that our proposed compressed sensing algorithm for velocity mapping in PC-MRI allows high acceleration rates when vessels are sparsely distributed. We expect our algorithm to be more optimal in 3D imaging because random k-space sampling can be used for determining phase encoding locations of the 3D k-space trajectory. Future work includes testing our method with in-vivo data and developing feasible random k-space trajectories for 2D PC-MRI.

References & Acknowledgements: [1] Nielsen, *Mag.Res.Med*, 61(5):1096-1102(2009) [2] King, ISMRM 2009, 2817 [3] Pati, *Proceedings of 27th Asilomar Conference on Signals, Systems and Computers*, 1993 [4] Olafsson, *IEEE Trans.Med.Imag*.,27(9):1177-88 2008 [5] http://www.cs.ubc.ca/labs/scl/spgl1/
The author thanks Dr. Krishna Nayak for suggesting this problem. This work is supported by NIH grant R01NS058576.