Water-Fat Decomposition With MR Data Based Regularized Estimation In MRI

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INTRODUCTION: The ability of MRI to separate chemical components, such as IDEAL and Region Growing [1,2], is important in clinical use. However, conventional approaches can be sensitive to field inhomogeneity. Typically, these methods ignore any temporal changes during the readout in their model. We describe a new regularized iterative algorithm, termed k-space method, which reconstructs water and fat and field map. The k-space method has been implemented with MR data to consider different data acquisition time. In the method, the cost function includes a regularized term due to prior knowledge that off-resonance is usually smooth. We use iterative approach with field map estimation, and water and fat estimation. By providing readout information to the cost function, the method yields improved estimates of the chemical components.

METHODS: The input data consists of L MR data with different echo times τ_l, denoted by $\underline{y} = (y^1, y^2, \dots, y^L)$, where typically $L=3$. Let Δm denote chemical shift of mth chemical component relative to water (Hz) and ω_n denote the off-resonance of the nth voxel. We model the L MR data by $y_s^l = \Phi(\vec{k}(t_s))\Sigma_n \left((\Sigma_m X_{nm} e^{-i\,2\Delta_m\, t_l}) e^{-i\omega_n(t_s+\tau_l)} e^{-j2\pi(\vec{k}(t_s)\cdot\vec{r}_n)} \right) + \epsilon_s^l$ for $l = 1,2,..\ L$, where $\Phi(\vec{k}(t_s))$ denotes the Fourier transform of the voxel basis function, t_l denotes the echo time difference of the lth scan relative to the original scan, X_{nm} denotes the mth chemical components in the nth voxel, and ε_s^l denotes the complex noise. We assume that additive complex noise, ε_s^l, are independent white Gaussian. The goal here is to estimate the chemical components $\underline{\boldsymbol{X}} = (X_{11}, X_{12}, \dots, X_{NM})$ and the field map $\boldsymbol{\omega} = (\omega_1, \omega_2, \dots, \omega_N)$ from the MR data $\underline{y}$. To separate chemical components, we repeatedly carry out two major steps; chemical component estimation step and field map estimation step.
First, we estimate chemical components based on the estimated field map in the previous iteration by minimizing the following penalized least-squares (PL) cost function:

$$\hat{\underline{X}}^{(i)} = \underset{\mathbf{x}\in\mathbb{C}^{N\cdot M}}{\arg\min}\, \Psi_{\mathrm{PL}}^1(\underline{X}; \hat{\omega}^{(i-1)}), \Psi_{\mathrm{PL}}^1(\underline{X}; \hat{\omega}^{(i-1)}) = \sum_{l=1}^{L} \left\| \underline{y}^l - A^l(\hat{\omega}^{(i-1)}) B^l \underline{X} \right\|^2 + \beta_1 R(\underline{X}), \quad (1)$$

where $[A^l(\omega)]_{s,n} = \Phi(\vec{k}(t_s)) e^{-j2\Pi(\vec{k}(t_s)\cdot r_n)} e^{-j\omega_n(t_s+\tau_l)}$, $[B^l] = diag(D_1, D_2, \dots, D_N)$, $D_k = e^{-j\underline{\Delta}\tau_l}$ for all k, and $\underline{\Delta} = [\Delta_1, \Delta_2, \dots, \Delta_M]$. Second we can form a new PL cost function based on the estimated chemical components in the previous iteration for estimating field map as follows:

$$\hat{\omega}^{(i)} = \underset{\boldsymbol{\omega}\in\mathbb{R}^N}{\arg\min}\, \Psi_{\mathrm{PL}}^2(\boldsymbol{\omega}; \hat{\underline{X}}^{(i-1)}), \Psi_{\mathrm{PL}}^2(\boldsymbol{\omega}; \hat{\underline{X}}^{(i-1)}) = \sum_{l=1}^{L} \left\| \underline{y}^l - A^l(\boldsymbol{\omega}) B^l \hat{\underline{X}}^{(i-1)} \right\|^2 + \beta_2 R(\boldsymbol{\omega}). \quad (2)$$

This can be solved using the approximation in [4]. To differentiate between the refinements, we denote i as the refinement index, $\hat{\underline{X}}^{(i)}$ and $\hat{\underline{\omega}}^{(i)}$ as the estimated chemical component and the field map in the ith iteration. When ω is an $N_1 \times N_2$ field map $\omega[n_1, n_2]$, the regularizing roughness penalty uses differences between horizontal and vertical neighboring voxel values as follows:

$$R(\boldsymbol{\omega}) = \sum_{n_1}\sum_{n_2} (\psi(R_{10}[n_1,n_2]) + \psi(R_{-1-1}[n_1,n_2]) + \psi(R_{01}[n_1,n_2]) + \psi(R_{-11}[n_1,n_2])), \quad (3)$$

where $R_{kl}[n_1,n_2] = 2\cdot\omega[n_1,n_2] - \omega[n_1-k, n_2-l] - \omega[n_1+k, n_2+l]$ and for simplicity $\psi(t) = t^2/2$. $R(\underline{X})$ can be defined similarly. To simplify the selection of the regularization parameter β_2 in (2), we normalize the magnitudes of the MR data, $\underline{y}$, by the median of the values so that the "typical" value is unity. Our algorithm repeatedly minimizes (1) and (2) until the difference between iterations estimated value of chemical components and field map are less than stopping criteria. To minimize (1) and (2), we use an optimization transfer iteration that decreases the cost function monotonically. Thus the estimated chemical components and field map converge to a local minimizer of the PL cost functions, (1) and (2). To encourage the iteration to converge to a desirable local minimum, we choose the estimated field map in [3] as our initial field map, $\hat{\omega}^{(0)}$, for the iteration.

RESULTS: We applied the k-space method to a simulated data set. The simulated data were generated with the IDEAL method (B_0 = 1.5T; phase shifts = $-\pi/6$, $\pi/2$, $7\pi/6$; time shifts = -0.4ms, 1.2ms, and 2.8ms). Fig. 1 shows the improvement in NRMSE using the k-space method over Region Growing method and LP method [3]. The NRMSE of the k-space method was lower than the other methods for all levels of SNR. Fig. 2 shows the simulated fat image and the errors of estimated fat images of the k-space method and other methods when SNR = 26.

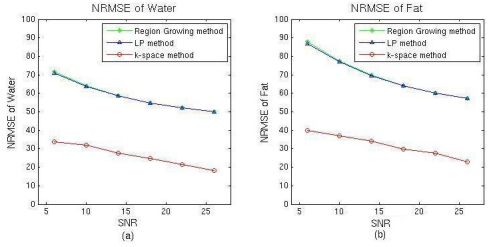

Fig.1.(a) NRMSE of water, (b) NRMSE of fat.

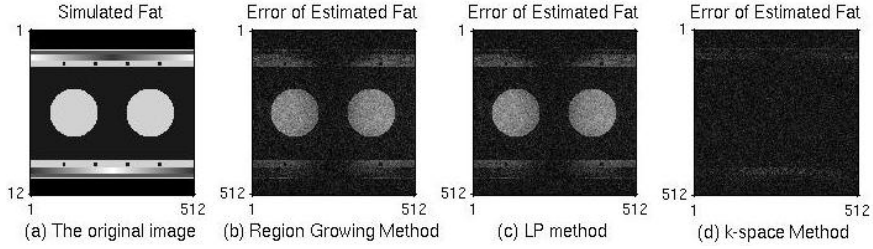

Fig.2. (a) The simulated fat, (b) Error of Region Growing method, (c) Error of LP method, (d) Error of k-space method.

CONCLUSION: We proposed a method for water-fat decomposition with MR data based method. The experimental results show that the k-space method can lead to improved chemical components estimates than Region Growing method and LP method as seen Fig. 1. The experimental results show that consideration of the data acquisition time can lead to enhanced chemical components estimates. Another potential advantage of the technique is that the RMSE of field map can be improved based on well estimated chemical components.

REFERENCES: [1]Reeder, *MRM*, 51(1): 35-45, 2004. [2] Yu, *MRM*, 54(4): 1032-9, 2005. [3] Hernando, *MRM*, 59(3):571-580, 2008. [4] Olafsson, *TMI*, 27(9):1177-1188, 2008.