Homework #3, EECS 755, W13. Due **Wed. Oct. 23** by 5:00 PM

Hand in solutions to at least 3 of these.

Optimization

1. [10] Consider the differentiable cost function $\Psi(x) = \frac{1}{1+x^2}$. Determine the (smallest possible) Lipschitz constant $\mathcal{L}$ for its derivative $\dot{\Psi}$. What happens if we apply GD with $\alpha = 1/\mathcal{L}$?

2. [10] The PGD method is applied to the cost function $\Psi(\boldsymbol{x}) = \|\boldsymbol{y} - \boldsymbol{A}\boldsymbol{x}\|^2 + \beta \|\boldsymbol{C}\boldsymbol{x}\|^2$ where $\mathsf{F} = \boldsymbol{A}'\boldsymbol{A}$ and $\boldsymbol{R} = \boldsymbol{C}'\boldsymbol{C}$ are both circulant. The preconditioner is simply $\boldsymbol{P} = \boldsymbol{I}$ and suppose the optimal step size $\alpha_\star$ is used from §11.3.3. Analyze (11.3.9) in the frequency domain to determine which spatial frequency components converge quickly and slowly. As an example, consider the case of Tikhonov regularization where $\boldsymbol{C} = \boldsymbol{I}$ and assume that F has a low-pass nature.

3. [10] Determine a Lipschitz constant $\mathcal{L}_{\dot{f}}$ for the derivative of the 1D function $f(\alpha) = \Psi(\boldsymbol{x} + \alpha \boldsymbol{d})$ when Ψ is the following regularized LS cost function: $\Psi(\boldsymbol{x}) = \frac{1}{2}\|\boldsymbol{y} - \boldsymbol{A}\boldsymbol{x}\|_2^2 + \beta \, \mathsf{R}(\boldsymbol{x}), \quad \mathsf{R}(\boldsymbol{x}) = \sum_k \psi([\boldsymbol{C}\boldsymbol{x}]_k)$. Assume that $\dot{\psi}$ is Lipschitz continuous with constant $\mathcal{L}_{\dot{\psi}}$. As discussed in §11.6, the Lipschitz constant $\mathcal{L}_{\dot{f}}$ enables a simple descent method for the line search.

4. [10] Use §11.3.8 to derive the preconditioned BB algorithm of §11.7.1.

5. [10] The formula for γ_n^{HZ} given in [1, p. 2] was for the case $\boldsymbol{P} = \boldsymbol{I}$. Use the coordinate transformation ideas of §11.3.8 to derive the preconditioned version (11.8.10).

[1] W. W. Hager and H. Zhang. A survey of nonlinear conjugate gradient methods. *Pacific J Optimization*, 2(1):35–58, January 2006.