Homework #9, EECS 556, W21. Due **Thu. Apr. 15**, by 9:00AM

## Skills and Concepts

- image restoration

Problem #1 is optional for 498-556 students.

## Problems

1. [15] Consider the measurement model $g[m,n] = b[m,n] ** f[m,n] + v[m,n]$, where $b[m,n]$ denotes a known blur and $m,n \in \mathbb{Z}$. To perform **image restoration** to recover $f[m,n]$ from $g[m,n]$, for **AWGN**, a quadratic **regularized** cost function is:

$$\hat{f} = \arg\min_f \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} (g[m,n] - (b ** f)[m,n])^2$$
$$+ \beta \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \left(f[m,n] - \frac{1}{2} f[m-1,n] - \frac{1}{2} f[m+1,n]\right)^2$$
$$+ \beta \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \left(f[m,n] - \frac{1}{2} f[m,n-1] - \frac{1}{2} f[m,n+1]\right)^2.$$

(a) [10] Show that $\hat{f}[m,n]$ is a LSI function of $g[m,n]$, *i.e.*, $\hat{f}[m,n] = (h ** g)[m,n]$ and derive the frequency response $H(\Omega_1, \Omega_2)$ of the corresponding filter. Hint. Use **Parseval's theorem**.

(b) [5] Your final form should be closely related to the form of a **Wiener filter**. Assuming that $v[m,n]$ is zero-mean white noise with unit variance, what (possibly hypothetical) **power spectrum** model $P_f(\Omega_1, \Omega_2)$ would make the Wiener filter equivalent to this QPLS method?

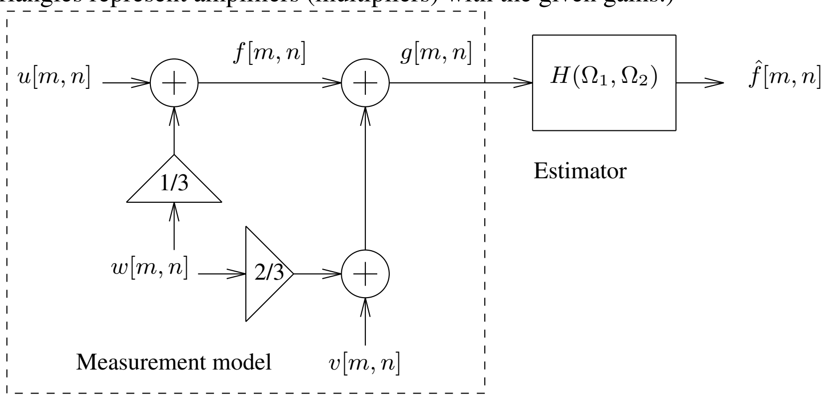
2. [20] Please complete the course evaluation online. Your comments are especially important to me this year!
Please write a note on your HW solutions confirming that you did it; the honor code applies.

3. [10] Each project group needs to have one member email me (in plain text) a project *title*, alphabetical list of *group members*, and short (1 paragraph) *abstract* by the due date of this HW, so that I can prepare a "program" for the video presentations. (These will be added to the web for future year students so write clearly.)
Review the presentation guidelines on the course web site for details about talk time limits etc.

## Optional problems

(Ignore the points listed below.)

4. [15] Consider the following statistical model for a measured image $g[m,n]$ related to a function of an unknown signal of interest $f[m,n]$. (The triangles represent amplifiers (multipliers) with the given gains.)




Assume that $u[m,n]$, $v[m,n]$, and $w[m,n]$ are all mutually uncorrelated, zero-mean WSS random processes with known autocorrelation functions.

(a) [10] Find the frequency response $H(\Omega_1,\Omega_2)$ of the **Wiener filter** for estimating the signal $f[m,n]$ from the measurement $g[m,n]$ under the above model.

(b) [5] Suppose $u[m,n]$, $v[m,n]$, $w[m,n]$ all have the same autocorrelation function. (This is rather hypothetical.)
Determine (exactly) the impulse response of the Wiener filter.
Explain whether you made any additional assumptions in your solution.

(c) [0] Optional. Postulate some other set of autocorrelation functions (not all the same) for $u[m,n]$, $v[m,n]$ and $w[m,n]$, for which the Wiener filter has an easily determined impulse response.

5. [15] A sequence of noisy images can be expressed: $g_k[m,n] = f[m,n] + v_k[m,n]$, where $k = 1,2,\ldots$ is the time index, $v_k[m,n]$ is zero-mean WSS white Gaussian noise with variance $\sigma_v^2$ that is independent from frame to frame.

(a) [5] One way to reduce noise is to average the $K > 1$ most recent frames:

$$\hat{f}_k[m,n] = \frac{1}{K} \sum_{l=k-K+1}^{k} g_l[m,n],$$

where $\hat{f}_k[m,n]$ is the estimate at time $k$. For this "frame averaging" method, determine, for $k > K$, the SNR improvement achieved in dB, defined by $\mathsf{SNR} = 10 \log \dfrac{\sigma_v^2}{\mathsf{Var}\left\{\hat{f}_k[m,n] - f[m,n]\right\}}$.

(b) [10] The preceding method requires storage of $K$ frames. An alternative approach that requires only one extra frame is the following recursion that involves a "forgetting factor" $\alpha \in [0,1]$:

$$\hat{f}_k[m,n] = \alpha\, \hat{f}_{k-1}[m,n] + (1-\alpha)\, g_k[m,n]\,.$$

Determine the value of $\alpha$ that yields the same SNR improvement as in part (a), in steady state, *i.e.*, in the limit as $k \to \infty$ where $\mathsf{Var}\left\{\hat{f}_k - f\right\} = \mathsf{Var}\left\{\hat{f}_{k-1} - f\right\}$.

(c) [0] Of the two preceding methods, which would be preferable in the presence of motion?

6. [40] Generate a couple realizations of a WSS random process with "fractal-like" power spectral density

$$P(\Omega_1,\Omega_2) = \frac{1}{0.001 + (\tilde{\Omega}_1/2)^2 + (6\tilde{\Omega}_2)^2}.$$

Display at least a $256 \times 256$ region of the process.
Hint: use FFT operations. Make sure your process is real-valued: any significant imaginary part is a bug.
Can you imagine any possible applications of this type of image?