Homework #7, EECS 556, W21. Due **Thu. Mar. 18**, by 9:00AM

Skills and Concepts

- edge detection
- corner detection

Problems

1. [10] One way to approximate the Laplacian $\nabla^2 f(x,y)$ in discrete space for **edge detection** is to convolve $f[m,n]$ with the "discrete Laplacian" $h[m,n]$ shown here. Use frequency-domain analysis to show that this filter provides a reasonable approximation.

$$h[m,n] = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \underline{-8} & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

2. [30] In an attempt to develop an improved method of **edge detection**, an image processing engineer considers the following three-step 1D algorithm for computing "derivatives" of a discrete-space signal $f[n]$.
   - Up-sample the signal by a factor of two using sinc-based up-sampling:

$$f_2[n] \triangleq \begin{cases} f[n/2], & n \text{ even} \\ \sum_{k=-\infty}^{\infty} f[k] \operatorname{sinc}(n/2 - k), & n \text{ odd.} \end{cases}$$

   - Apply a central difference filter $h_d[n] = \begin{bmatrix} 1 & \underline{0} & -1 \end{bmatrix}$ to the up-sampled signal, to form a signal $g[n]$.
   - Downsample the filtered signal $g[n]$ by a factor of two by discarding the odd samples, to form a signal $d[n]$.

   To determine whether this approach is beneficial, we analyze it in the frequency domain.

   (a) [0] Rewrite $f_2[n]$ in a simpler form by examining the "n odd" expression above for the case where n is even. Hint: your simple expression should have no braces in it.

   (b) [5] Express the spectrum of $f_2[n]$ in terms of that of $f[n]$. Hint. Think about first up-sampling $f[n]$ (by zero insertion), and then convolving with something.

   (c) [5] Express the spectrum of $g[n]$ in terms of that of $f[n]$.

   (d) [5] Express the spectrum of $d[n]$ in terms of that of $f[n]$.

   (e) [5] Does this process correspond to an overall LSI system?

   (f) [5] Compare graphically (by plotting) the frequency-domain properties of this system with the frequency response of the "ideal" sinc-based derivative method discussed in the lecture notes, and with the frequency response of the conventional central-difference method (without the up/down sampling).

   (g) [5] What are the advantages and disadvantages of this "new" approach?

3. [10] Determine if the **Harris corner detection** method described in the course notes is a shift invariant DS operation. Explain.

4. [40] Download the m-file `edge_template.m` or JULIA code `edge_template.jl` from the web site and run it to generate a noisy image of rectangles.
498: Experiment with the **edge detection** methods in MATLAB's `edge` command on this noisy rectangles image.
556: Use kernels like `sobel` and `prewitt` (and perhaps `LoG`) in the `ImageFiltering.jl` package to implement simple edge detection methods and apply them to the original noisy image `yy`, to a median filtered version of it and to a denoised version of it (see below). See these examples.
   (a) [20] Show the results of the version that you think works the best, as well as one or two other methods for comparison.
   (b) [10] Apply 3×3 median-filter preprocessing to the image `yy` and investigate how edge-detection performance is affected. Show some representative results. For MATLAB, use `medfilt2`.
   For JULIA, use `ImageFiltering.MapWindow.mapwindow`. See this example.
   (c) [10] Download the MATLAB m-file `npls_sps.m` or JULIA code `npls_sps.jl` from the web site. This code provides a nonlinear image "denoising" method called nonquadratic penalized least squares (NPLS) that we will discuss later in the course. Apply this method to the noisy data by a command something like `newimage = npls_sps(oldimage)`. Display the denoised result for yourself. Investigate how edge-detection performance is affected by this preprocessing. Show some representative results.
   (d) [0] Could you make any *quantitative* evaluation of the performance of various methods and combination methods in this problem? Hint: consider `ss`.

Optional problem

5. [0] Consider the following continuous-space non-directional **edge enhancement** operation:

$$g(x, y) = |\nabla f(x, y)| = \sqrt{\left(\frac{\partial}{\partial x} f(x, y)\right)^2 + \left(\frac{\partial}{\partial y} f(x, y)\right)^2}.$$

Prove that this is a rotationally invariant operation. This property has practical importance because lack of rotation invariance would be considered a notable deficiency of a "non-directional" edge-detection method.