$\boxed{\text{Homework \#1, EECS 556, W21. Due } \textbf{Thu. Jan. 28}, \text{ by 9:00AM}}$

_____ **Notes** _____

- The lecture notes have several problems interspersed throughout. These are all important (and mostly easy). I recommend you solve them for yourself as practice.
- You must upload your answers to each HW twice! (Refer to Chapter 0 for details.)
  - First you upload your individual draft answers to https://gradescope.com by the deadline above.
  - After correcting your answers in class, you add your self reflection form to the front and upload to gradescope by 1PM Friday. Please indicate which parts you changed, e.g., by using a different color pen for your self-corrected answers.
- The problem points indicated below are directly relevant because HW is graded based on effort, not accuracy, but the points might give a rough sense of relative effort needed.

_____ **Skills and Concepts** _____

- 2D signals and systems

_____ **Problems** _____

1. [5]   Determine the **energy** of the 2D signal $f(x, y) = 3 \operatorname{rect}\left(\frac{r-7}{4}\right) e^{\imath 2\pi(3x-y)}$ defined on $\mathbb{R}^2$. Hint. The shape is not a disk!

2. [5]   Determine the **average power** of the 2D signal $g(x, y) = a \cos(2\pi(3x + 4y) + \pi/3)$.

3. [5]   Find a suitable 2D **sifting property** for $\frac{1}{r} \delta(r)$. In other words, simplify $\iint \frac{1}{r} \delta(r) \, g(x, y) \, dx \, dy$.
          Hint. There is a π in the answer.

4. [5]   Find a suitable 2D **sifting property** for $\delta(r - r_0) = \delta\left(\sqrt{x^2 + y^2} - r_0\right)$. Consider *all* possible values $r_0 \in \mathbb{R}$.
          Hint. For $r_0 = 0$ the answer differs from that of the previous problem.

5. [5]   Let $g(x, y)$ denote a (T_X, T_Y)-**periodic** signal that is also **separable**, *i.e.*, $g(x, y) = a(x)b(y)$.
          Express the **average power** of $g(x, y)$ in terms of simple properties of the component functions $a(x)$ and $b(x)$.

6. [20]   Use JULIA or MATLAB to display an image of the **separable** signal $g(x, y) = g_1(x) \, g_1(2y)$, where $g_1(t)$ is the following periodic signal consisting of semicircles of radius 1.
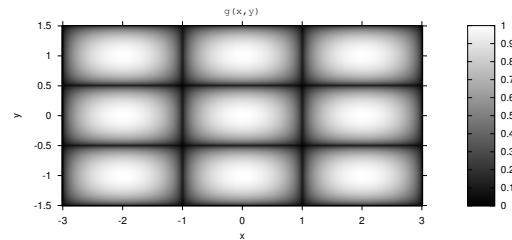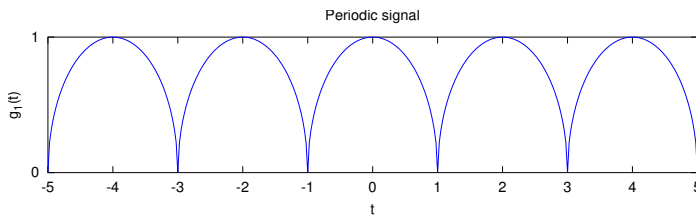
          Your answer should look like the grayscale image shown above, except that you must show at least 5 periods of the signal along each axis. Hint. Study example code on page 1.18.

          In all such coding problems, unless otherwise specified, JULIA is required for 556 students, and 498 students may choose JULIA or MATLAB.

In all such problems throughout the course, turn in your analytical work, a printout of your code and your plots.
Always label your image axes (*e.g.*, x and y or n and m), and include units when applicable.
Whenever you display images, always show the range of grayscale values using a colorbar.
In JULIA, a colorbar is shown by default with `MIRT.jim` and by default it uses `color=:grays`
In MATLAB, use the `colorbar` command and use `colormap(gray)` or `colormap(1-gray)` for grayscale images.

7. [40] The pdf file at url `https://web.eecs.umich.edu/~fessler/course/556/r/ieee-t-ip-2020.pdf`
has the table of contents of the entire 2020 volume of the IEEE Transactions on Image Processing, with embedded links
to the papers on IEEE Xplore.

Skim through the paper titles and then pick a couple of papers to skim that look like they might be interesting topics to
you for a 556 course project. (This is just a preliminary exploration.)

Ignore papers that solely use deep learning methods to solve a problem. If the paper just puts training data into a black
box network then it will not be acceptable as a 556 project. Also avoid papers that are clearly higher-level computer
vision topics like "object recognition" "image classification" "action prediction." (The topic "image segmentation" is in
the gray area between image processing and computer vision and is acceptable if not just deep learning based.) Video
processing is allowable, but keep in mind that processing videos (2D RGB + time) takes *much* more computation than
processing 2D images, so would be challenging as a course project.

Then pick one paper to skim more completely and write a paragraph or two summary of it. Your summary must include:
  - Your name
  - Full paper citation including the url (cut and paste from pdf is fine)
  - What problem are they trying to solve? (e.g., "removing noise from satellite images")
  - What are a few of the key image processing tools used? (*e.g.*, "wavelets", "FFT", optimization method XYZ)
  - What metrics did they use to quantify the effectiveness of their method? (*e.g.*, PSNR, compute time, DICE, etc.)
  - Did they compare their proposed method(s) to existing methods? (Probably so.) Which methods?
  - Did you find the comparison convincing?

If you are unsure where to begin, then focus on papers with topics mentioned in Ch. 0, like **denoising**, **deblur-
ring**/**restoration**, **registration**, **super-resolution**, etc.

Submit your short summary to gradescope, and also submit to this google form:
`https://forms.gle/Tv6mUk5kLT9oiyiB9`.

──────────────── **Optional problems** ────────────────

(Solutions will be provided for these, but it is unlikely there will be time to work on them in groups in class. I encourage you to
try them for yourself, but please do not submit solutions to these to gradescope.)

8. [0] An image rotation system has the input-output relationship $g(x, y) = f(x \cos \theta + y \sin \theta, -x \sin \theta + y \cos \theta)$.
  - Determine the **PSF** of this (linear) system.
  - Is it a **shift-invariant** system?
  - Is it a **rotation invariant** system?
  - Is it a **BIBO stable** system?
  - Is it an **invertible** system?
  - Is it a "**memoryless**" system?
  - Is it a **separable** system?

9. [0] Some, but not most, LSI systems have very many **eigenfunctions** in addition to complex exponentials. Give two distinct
examples of such LSI systems. One example is the "do nothing" system where $g(x, y) = f(x, y)$. For this system,
every 2D signal is an eigenfunction. (You provide two other different systems.)

10. [0] State and prove the condition on the impulse response $h(x, y)$ that ensures a 2D LSI system is **rotationally invariant**.

11. [0] Find a system that satisfies the **scaling property** but not the **additivity property**.